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EVOLUTIONARY APPROACH FOR THE PROBLEM OF ELECTROMAGNETIC FIELD PROPAGATION THROUGH NONLINEAR MEDIUM

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The problem of transient electromagnetic wave propagation in unbounded nonlinear medium is solved by means of Evolutionary Approach. It consists in the conversion of the initial three-dimensional nonstationary problem to one-dimensional one for the set of evolutionary equations with initial and boundary conditions by the modal expansion of the initial problem. The construction of the analytical solution of a nonlinear problem in time domain is proposed by the step-by-step method in approximation of weak nonlinearity and losses taking into account new sources of current inside the medium.
KEYWORDS*:* transient electromagnetic field, time domain, evolutionary approach, nonlinear medium.

Задача поширення нестаціонарної електромагнітної хвилі у необмеженому просторі, що заповнений нелінійним середовищем, розв'язується за допомогою еволюційного підходу. Його сутність полягає у перетворенні вихідної тривимірної нестаціонарної задачі в одновимірну нестаціонарну задачу для системи еволюційних рівнянь, доповнених початковими та граничними умовами, за допомогою модового розкладу вихідної тривимірної задачі. Побудову аналітичного розв'язку нелінійної задачі пропонується здійснити методом послідовних наближень у припущенні слабкої нелінійності середовища із втратами, враховуючи нові джерела струму всередині цього середовища.
КЛЮЧОВІ СЛОВА: нестаціонарне електромагнітне поле, часовий простір, еволюційний підхід, нелінійне середовище

Задача распространения нестационарной электромагнитной волны в неограниченном пространстве, заполненном нелинейной средой, решается при помощи эволюционного подхода. Его суть состоит в преобразовании исходной трехмерной нестационарной задачи в одномерную нестационарную задачу для системы эволюционных уравнений, дополненных начальными и граничными условиями, посредством модового разложения исходной трехмерной задачи. Построение аналитического решения нелинейной задачи во временной области предлагается осуществить методом последовательных приближений в допущении слабой нелинейности среды с потерями, учитывая новые источники тока внутри этой среды.
КЛЮЧЕВЫЕ СЛОВА: нестационарное электромагнитное поле, временная область, эволюционный подход, нелинейная среда

INTRODUCTION

To solve the radiation problems in unbounded layered transient inhomogeneous nonlinear medium in time domain the method of evolutionary equations is applied. Initially it was applied for resonator and waveguide problems with the same medium. The essence of the method consists in constructing the basis in transversal plane to decrease the dimension of initial problems holding the explicit time dependence of all variables. For the free space problems it was used in [1] by constructing the modal basis in cylindrical coordinate system and proving of its completeness by Weyl's theorem about orthogonal splitting in Hilbert space. The method was applied to solve the transient problems for given sources [2]-[4] and obtain the analytical solution of transient wave diffraction problem [5] by mode matching technique in time domain. In [6] the evolutionary equation set was obtained in spherical coordinate system, and some analytical solutions of the transient radiation problems were received by Laplace Transform [7]. Transient wave propagation problem in inhomogeneous nonstationary medium was solved analytically and numerically with noticeable advantages concerning calculation time and RAM consuming in comparison with direct three-dimensional numerical method [8].

The present work is devoted to construction of the analytical solution of a nonlinear problem [9]-[10] by the step-by-step method in approximation of weak nonlinearity.

THE STATEMENT OF THE PROBLEM

The initial problem is stated for the set of three-dimensional nonstationary Maxwell's equations

$$\mathrm{rot}\vec{\mathbf{H}} = \frac{\partial}{\partial t}\vec{\mathbf{D}} + \vec{\mathbf{J}}^{\sigma} + \vec{\mathbf{J}}^{e}\,;\ \text{-}\mathrm{rot}\vec{\mathbf{E}} = \frac{\partial}{\partial t}\vec{\mathbf{B}} + \vec{\mathbf{J}}^{h}\,;$$
$$\mathrm{div}\vec{\mathbf{D}} = \rho^{\sigma} + \rho^{e}\,;\ \mathrm{div}\vec{\mathbf{B}} = \rho^{h}\,, \tag{1}$$

supplemented by constitutive relations and equations of continuity

$$\vec{\mathbf{D}} = \varepsilon_0\vec{\mathbf{E}} + \vec{\mathbf{P}}\left(\vec{\mathbf{E}}\right);\ \vec{\mathbf{B}} = \mu_0\left(\vec{\mathbf{H}} + \vec{\mathbf{M}}\left(\vec{\mathbf{H}}\right)\right);\ \frac{\partial}{\partial t}\rho^{e,h} = -\mathrm{div}\,\vec{\mathbf{J}}^{e,h}\,, \tag{2}$$

where $\vec{\mathbf{E}}$ and $\vec{\mathbf{H}}$ are electrical and magnetic field strength vectors, $\vec{\mathbf{D}}$ and $\vec{\mathbf{B}}$ are electric displacement field and magnetic field, $\vec{\mathbf{P}}$ and $\vec{\mathbf{M}}$ are polarization and magnetization, ε_0, μ_0 are electric and magnetic free-space constants, $\vec{\mathbf{J}}^{e,h}$ is density of electric or magnetic current, $\vec{\mathbf{J}}^{\sigma}$ is conductivity current density, $\rho^{e,h}$ is density of electric or magnetic charges. All these functions are depended on position vector $\vec{R} = \vec{r} + z\vec{z}_0$ and time t. The problem for correct statement is completed by initial and boundary conditions. Their solution will be found in the class of quadratically integrable vector functions that satisfy the condition

$$\int_{t_1}^{t_2} dt \int_{z_1}^{z_2} dz \int_{0}^{2\pi} d\varphi \int_{0}^{\infty} \rho d\rho \left(\varepsilon_0\vec{\mathbf{E}}\cdot\vec{\mathbf{E}}^{*} + \mu_0\vec{\mathbf{H}}\cdot\vec{\mathbf{H}}^{*}\right) < \infty\,. \tag{3}$$

EVOLUTIONARY EQUATION SET

We can separate the linear and nonlinear part in polarization and magnetization vectors as follows

$$\vec{\mathbf{P}}\left(\vec{\mathbf{E}}\right) = \varepsilon_0\alpha(z,t)\vec{\mathbf{E}} + \vec{\mathbf{P}}'\left(\vec{\mathbf{E}}\right);\ \vec{\mathbf{M}}\left(\vec{\mathbf{H}}\right) = \chi(z,t)\vec{\mathbf{H}} + \vec{\mathbf{M}}'\left(\vec{\mathbf{H}}\right), \tag{4}$$

where $\alpha(z,t)$ and $\chi(z,t)$ are electric and magnetic susceptibilities. It gives possibility to rewrite the constitutive equations in the following form:

$$\vec{\mathbf{D}}\left(\vec{\mathbf{E}}\right) = \varepsilon_0\varepsilon(z,t)\vec{\mathbf{E}} + \vec{\mathbf{P}}'\left(\vec{\mathbf{E}}\right);\ \vec{\mathbf{B}} = \mu_0\mu(z,t)\vec{\mathbf{H}} + \mu_0\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}\right), \tag{5}$$

where $\varepsilon(z,t) = 1 + \alpha(z,t)$ is relative permittivity, $\mu(z,t) = 1 + \chi(z,t)$ is relative permeability. Using these notations the Maxwell's equations can be rewritten to the form of

$$\mathrm{rot}\vec{\mathbf{H}} = \varepsilon_0\frac{\partial}{\partial t}\left(\varepsilon(z,t)\vec{\mathbf{E}}\right) + \left\{\frac{\partial}{\partial t}\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}\right) + \vec{\mathbf{J}}^{\sigma}\left(\vec{\mathbf{E}},\vec{\mathbf{H}}\right) + \vec{\mathbf{J}}^{e}\right\};\ \text{-}\mathrm{rot}\vec{\mathbf{E}} = \mu_0\frac{\partial}{\partial t}\left(\mu(z,t)\vec{\mathbf{H}}\right) + \left\{\frac{\partial}{\partial t}\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}\right) + \vec{\mathbf{J}}^{h}\right\};$$
$$\varepsilon_0\mathrm{div}\left(\varepsilon(z,t)\vec{\mathbf{E}}\right) = -\mathrm{div}\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}\right) + \rho^{\sigma} + \rho^{e};\ \mu_0\mathrm{div}\left(\mu(z,t)\vec{\mathbf{H}}\right) = -\mathrm{div}\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}\right) + \rho^{h}\,. \tag{6}$$

So, we can introduce the equivalent densities of electric and magnetic currents and charges in right-hand sides of the equation by the following way:

$$\vec{\mathbf{J}} = \frac{\partial}{\partial t}\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}\right) + \vec{\mathbf{J}}^{\sigma}\left(\vec{\mathbf{E}},\vec{\mathbf{H}}\right) + \vec{\mathbf{J}}^{e};\ \vec{\mathbf{I}} = \frac{\partial}{\partial t}\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}\right) + \vec{\mathbf{J}}^{h};\ \varrho = -\mathrm{div}\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}\right) + \rho^{\sigma} + \rho^{e};\ g = -\mathrm{div}\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}\right) + \rho^{h}\,. \tag{7}$$

So, the set of Maxwell's equations acquires the form

$$\mathrm{rot}\vec{\mathbf{H}} = \varepsilon_0\frac{\partial}{\partial t}\left(\varepsilon\vec{\mathbf{E}}\right) + \vec{\mathbf{J}}\,;\ \text{-}\mathrm{rot}\vec{\mathbf{E}} = \mu_0\frac{\partial}{\partial t}\left(\mu\vec{\mathbf{H}}\right) + \vec{\mathbf{I}}\,;\ \varepsilon_0\mathrm{div}\left(\varepsilon\vec{\mathbf{E}}\right) = \varrho\,;\ \mu_0\mathrm{div}\left(\mu\vec{\mathbf{H}}\right) = g\,. \tag{8}$$

To project the equations onto transversal plane and longitudinal axis one should present all vectors in the form of sum

$$\vec{\mathbf{A}}\left(\vec{R},t\right) \equiv \vec{\mathbf{A}}\left(\vec{\mathrm{r}},\mathrm{z},t\right) = \vec{A}\left(\vec{\mathrm{r}},\mathrm{z},t\right) + \vec{z}_0 A_z\left(\vec{\mathrm{r}},\mathrm{z},t\right), \tag{9}$$

and nabla operator in form of sum $\nabla = \nabla_{\perp} + \vec{z}_0\dfrac{\partial}{\partial z}$.

The last notation gives us possibility to transform the Maxwell's equations:

$$\left[\nabla_{\perp}\times\vec{z}_0\right]H_z = \varepsilon_0\frac{\partial}{\partial t}\left(\varepsilon\vec{E}\right) + \frac{\partial}{\partial z}\left[\vec{H}\times\vec{z}_0\right] + \vec{J}\,; \tag{10}$$

$$\mu_0\frac{\partial}{\partial z}\left\{\mu H_z\right\} = -\mu_0\mu\nabla_{\perp}\cdot\vec{H} + g\,; \tag{11}$$

$$\mu_0\frac{\partial}{\partial t}\left(\mu H_z\right) = \nabla_{\perp}\cdot\left[\vec{z}_0\times\vec{E}\right] - I_z\,; \tag{12}$$

$$\left[\vec{z}_0 \times \nabla_\perp\right] E_z = \mu_0 \frac{\partial}{\partial t}\left(\mu \vec{H}\right) + \frac{\partial}{\partial z}\left[\vec{z}_0 \times \vec{E}\right] + \vec{I}\,; \tag{13}$$

$$\varepsilon_0 \frac{\partial}{\partial t}\left(\varepsilon E_z\right) = \nabla_\perp \cdot \left[\vec{H} \times \vec{z}_0\right] - J_z\,; \tag{14}$$

$$\varepsilon_0 \frac{\partial}{\partial z}\left\{\varepsilon E_z\right\} = -\varepsilon_0 \varepsilon \nabla_\perp \cdot \vec{E} + \varrho\,. \tag{15}$$

The same equations can be written in matrix form

$$\begin{pmatrix} \varepsilon_0^{-1}\left[\vec{z}_0 \times \nabla_\perp\right]\nabla_\perp \cdot \vec{H} \\ \mu_0^{-1}\nabla_\perp\left[\vec{z}_0 \times \nabla_\perp\right] \cdot \vec{E} \end{pmatrix} = \begin{pmatrix} \mu^{-1}\partial_z\left\{\mu\varepsilon_0^{-1}\vec{F}_H\right\} + \left(\varepsilon_0\mu_0\mu\right)^{-1}\left[\vec{z}_0 \times \nabla_\perp g\right] \\ -\partial_t\left\{\mu\left[\vec{z}_0 \times \vec{F}_H\right]\right\} - \mu_0^{-1}\nabla_\perp I_z \end{pmatrix}; \tag{16}$$

$$\begin{pmatrix} \varepsilon_0^{-1}\nabla_\perp\left[\nabla_\perp \times \vec{z}_0\right] \cdot \vec{H} \\ \mu_0^{-1}\left[\nabla_\perp \times \vec{z}_0\right]\nabla_\perp \cdot \vec{E} \end{pmatrix} = \begin{pmatrix} -\partial_t\left\{\varepsilon\left[\vec{F}_E \times \vec{z}_0\right]\right\} - \varepsilon_0^{-1}\nabla_\perp J_z \\ \varepsilon^{-1}\partial_z\left\{\varepsilon\mu_0^{-1}\vec{F}_E\right\} + \left(\varepsilon_0\mu_0\varepsilon\right)^{-1}\left[\nabla_\perp \varrho \times \vec{z}_0\right] \end{pmatrix}, \tag{17}$$

where $\vec{F}_H = \varepsilon_0 \frac{\partial}{\partial t}\left(\varepsilon \vec{E}\right) + \frac{\partial}{\partial z}\left[\vec{H} \times \vec{z}_0\right] + \vec{J}$, $\vec{F}_E = \mu_0 \frac{\partial}{\partial t}\left(\mu \vec{H}\right) + \frac{\partial}{\partial z}\left[\vec{z}_0 \times \vec{E}\right] + \vec{I}$.

Introducing four dimensional vectors of electromagnetic field

$$\mathbf{X} = \begin{pmatrix} \vec{E} \\ \vec{H} \end{pmatrix} \tag{18}$$

with the scalar product definition $\langle \mathbf{X}_1, \mathbf{X}_2 \rangle = \frac{1}{4\pi}\int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \left(\varepsilon_0 \vec{E}_1 \cdot \vec{E}_2^* + \mu_0 \vec{H}_1 \cdot \vec{H}_2^*\right)$ originated from (3) and the matrix operations for the vectors

$$W_H \mathbf{X} = \begin{pmatrix} \mathbf{0} & \varepsilon_0^{-1}\left[\vec{z}_0 \times \nabla_\perp\right]\nabla_\perp \cdot \\ \mu_0^{-1}\nabla_\perp\left[\vec{z}_0 \times \nabla_\perp\right] \cdot & \mathbf{0} \end{pmatrix}\mathbf{X};\ W_E \mathbf{X} = \begin{pmatrix} \mathbf{0} & \varepsilon_0^{-1}\nabla_\perp\left[\nabla_\perp \times \vec{z}_0\right] \cdot \\ \mu_0^{-1}\left[\nabla_\perp \times \vec{z}_0\right]\nabla_\perp \cdot & \mathbf{0} \end{pmatrix}\mathbf{X}, \tag{19}$$

Using these matrix operations we can rewrite the Maxwell’s equation set to the form of two matrix equations with four-dimensional operators containing partial differentiation by transversal coordinates and with right sides consisted of partial differentiation by longitudinal coordinate and time:

$$W_H \mathbf{X} = \begin{pmatrix} \mu^{-1}\partial_z \mu\left\{\partial_t\left(\varepsilon \vec{E}\right) + \varepsilon_0^{-1}\partial_z\left[\vec{H} \times \vec{z}_0\right]\right\} + \left\{\left(\varepsilon_0\mu\right)^{-1}\partial_z\left(\mu \vec{J}\right) + \left(\varepsilon_0\mu_0\mu\right)^{-1}\left[\vec{z}_0 \times \nabla_\perp g\right]\right\} \\ -\partial_t \mu\left\{\varepsilon_0\partial_t\varepsilon\left[\vec{z}_0 \times \vec{E}\right] + \partial_z \vec{H}\right\} - \left\{\partial_t\mu\left[\vec{z}_0 \times \vec{J}\right] + \mu_0^{-1}\nabla_\perp I_z\right\} \end{pmatrix};$$

$$W_E \mathbf{X} = \begin{pmatrix} -\partial_t\varepsilon\left\{\partial_z \vec{E} + \mu_0\partial_t\mu\left[\vec{H} \times \vec{z}_0\right]\right\} - \left\{\partial_t\varepsilon\left[\vec{I} \times \vec{z}_0\right] + \varepsilon_0^{-1}\nabla_\perp J_z\right\} \\ \varepsilon^{-1}\partial_z\varepsilon\left\{\mu_0^{-1}\partial_z\left[\vec{z}_0 \times \vec{E}\right] + \partial_t\mu \vec{H}\right\} + \left\{\left(\mu_0\varepsilon\right)^{-1}\partial_z\varepsilon \vec{I} + \left(\varepsilon_0\mu_0\varepsilon\right)^{-1}\left[\nabla_\perp \varrho \times \vec{z}_0\right]\right\} \end{pmatrix}. \tag{20}$$

It is easy to prove using Fourier-Bessel transform and the one of definitions of Dirac delta function

$$\delta\left(\chi_1 - \chi_2\right) = \sqrt{\chi_1\chi_2}\int_0^{\infty} \rho d\rho J_m\left(\chi_1\rho\right) J_m\left(\chi_2\rho\right) \tag{21}$$

that the operators are self-adjoint ones with eigen functions

$$\mathbf{Y}_{\pm m} = \begin{pmatrix} \sqrt[-2]{\varepsilon_0}\left[\nabla_\perp \Psi_m \times \vec{z}_0\right] \\ \pm\sqrt[-2]{\mu_0}\nabla_\perp \Psi_m \end{pmatrix} \text{ and } \mathbf{Z}_{\pm n} = \begin{pmatrix} \sqrt[-2]{\varepsilon_0}\nabla_\perp \Phi_n \\ \pm\sqrt[-2]{\mu_0}\left[\vec{z}_0 \times \nabla_\perp \Phi_n\right] \end{pmatrix} \tag{22}$$

correspondingly, where $J_m(x)$ is Bessel function, Ψ_m and Φ_n are scalar function that satisfy the equations in transversal plane

$$\left(\Delta_\perp + \sqrt{\varepsilon_0\mu_0}\,p\right)\Psi_m = 0 \text{ and } \left(\Delta_\perp + \sqrt{\varepsilon_0\mu_0}\,q\right)\Phi_n = 0, \tag{23}$$

p and q are the eigen numbers.

The properties of the expansion permit to apply Weyl Theorem for the Hilbert Space L_2^4 to prove the completeness of the expansion

$$\mathbf{X}\left(\vec{r}, z, t\right) = \sum_{m=-\infty}^{\infty}\int_0^{\infty} d\chi A_m(z,t,\chi)\mathbf{Y}_m(\vec{r},\chi) + \sum_{n=-\infty}^{\infty}\int_0^{\infty} d\nu B_n(z,t,\nu)\mathbf{Z}_n(\vec{r},\nu) \tag{24}$$

or

$$\begin{pmatrix}\vec{E}\\ \vec{H}\end{pmatrix}=\sum_{m=1}^{\infty}\int_0^{\infty}d\chi\left\{A_m\begin{pmatrix}\nabla_\perp\Psi_m\times\vec{Z}\\ \nabla_\perp\Psi_m\end{pmatrix}+A_{-m}\begin{pmatrix}\nabla_\perp\Psi_m\times\vec{Z}\\ -\nabla_\perp\Psi_m\end{pmatrix}\right\}+\sum_{n=1}^{\infty}\int_0^{\infty}d\nu\left\{B_n\begin{pmatrix}\nabla_\perp\Phi_n\\ \nabla_\perp\Phi_n\times\vec{Z}\end{pmatrix}+B_{-n}\begin{pmatrix}\nabla_\perp\Phi_n\\ -\nabla_\perp\Phi_n\times\vec{Z}\end{pmatrix}\right\} \quad (25)$$

that can be rewritten for different field components

$$\vec{E}=\sqrt[-2]{\varepsilon_0}\left\{\sum_{m=1}^{\infty}\int_0^{\infty}d\chi V_m^h\left[\nabla_\perp\Psi_m\times\vec{z}_0\right]+\sum_{m=1}^{\infty}\int_0^{\infty}d\nu V_n^e\nabla_\perp\Phi_n\right\};$$

$$\vec{H}=\sqrt[-2]{\mu_0}\left\{\sum_{m=1}^{\infty}\int_0^{\infty}d\nu I_m^h\nabla_\perp\Psi_m+\sum_{m=1}^{\infty}\int_0^{\infty}d\chi I_n^e\left[\vec{z}_0\times\nabla_\perp\Phi_n\right]\right\};$$

$$E_z(\rho,\phi,z,t)=\sqrt[-2]{\varepsilon_0}\sum_{n=1}^{\infty}\int_0^{\infty}\chi^2d\chi e_n(z,t;\chi)\Phi_n(\rho,\phi;\chi);$$

$$H_z(\rho,\phi,z,t)=\sqrt[-2]{\mu_0}\sum_{m=1}^{\infty}\int_0^{\infty}\nu^2d\nu h_m(z,t;\nu)\Psi_m(\rho,\phi;\nu) \quad (26)$$

by resigning the coefficients

$$\begin{aligned}A_m+A_{-m}&=V_m^h; \quad B_n+B_{-n}=V_n^e;\\ A_m-A_{-m}&=I_m^h; \quad B_n-B_{-n}=I_n^e.\end{aligned} \quad (27)$$

To obtain the evolutionary equation set that is the one dimensional nonlinear equations of the first order, we should substitute the fields in (11-16) by the received above expansions. The application of orthogonal properties of the scalar functions

$$\frac{\chi\chi'}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Psi_m(\chi)\Psi_{m'}^*(\chi')=\delta_{mm'}\delta(\chi-\chi');$$

$$\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\nabla_\perp\Psi_m(\chi)\cdot\nabla_\perp\Psi_{m'}^*(\chi')=\delta_{mm'}\delta(\chi-\chi');$$

$$\frac{1}{2\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\left[\nabla_\perp\Psi_m(\chi)\times\vec{z}_0\right]\cdot\left[\nabla_\perp\Psi_{m'}^*(\chi')\times\vec{z}_0\right]=\delta_{mm'}\delta(\chi-\chi');$$

$$\frac{1}{4\pi}\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\nabla_\perp\Phi_n(\nu)\cdot\left[\nabla_\perp\Psi_{m'}^*(\chi')\times\vec{z}_0\right]+\left[\vec{z}_0\times\nabla_\perp\Phi_n(\nu)\right]\cdot\nabla_\perp\Psi_{m'}^*(\chi')=0 \quad (28)$$

helps to derive the system of evolutionary equations [9]:

$$\begin{cases}\partial_z\left\{\mu h_m\right\}=\mu I_m^h+\sqrt[-2]{\mu_0}\,\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Psi_m^*(\nu)g\\ \partial_{ct}\left\{\mu h_m\right\}=-V_m^h-\sqrt{\varepsilon_0}\,\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Psi_m^*(\nu)I_z\\ -\partial_{ct}\left\{\varepsilon V_m^h\right\}-\partial_z I_m^h+\nu^2h_m=\sqrt{\mu_0}\,\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\left[\vec{z}_0\times\vec{J}\right]\cdot\nabla_\perp\Psi_m^*(\nu)\\ \partial_{ct}\left\{\varepsilon V_n^e\right\}+\partial_z I_n^e=-\sqrt{\mu_0}\,\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\vec{J}\cdot\nabla_\perp\Phi_n^*(\chi)\\ \partial_{ct}\left\{\varepsilon e_n\right\}=-I_n^e-\sqrt{\mu_0}\,\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Phi_n^*(\chi)J_z\\ \partial_z\left\{\varepsilon e_n\right\}=\varepsilon V_n^e+\sqrt[-2]{\varepsilon_0}\,\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\Phi_n^*(\chi)\varrho\\ -\partial_{ct}\left\{\mu I_n^e\right\}-\partial_z V_n^e+\nu^2e_n=\sqrt{\varepsilon_0}\,\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\left[\vec{I}\times\vec{z}_0\right]\cdot\nabla_\perp\Phi_n^*(\chi)\\ \partial_{ct}\left\{\mu I_m^h\right\}+\partial_z V_m^h=-\sqrt{\varepsilon_0}\,\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi}d\varphi\int_0^{\infty}\rho d\rho\vec{I}\cdot\nabla_\perp\Psi_m^*(\nu)\end{cases} \quad (29)$$

where

$$\Phi_n = \frac{J_n(\chi\rho)}{\sqrt{\chi}} e^{in\varphi}; \quad \Psi_m = \frac{J_m(\nu\rho)}{\sqrt{\nu}} e^{in\varphi}. \tag{30}$$

ITERATIVE SOLUTION SCHEME

The solution of nonlinear problem is proposed to find in form of sum of solution for linear medium and l-th solution for equivalent sources of nonlinearity [9] $\vec{\mathbf{E}} = \vec{\mathbf{E}}^0 + \vec{\mathbf{E}}^l$; $\vec{\mathbf{H}} = \vec{\mathbf{H}}^0 + \vec{\mathbf{H}}^l$. The last solution can be obtained for given sources

$$\vec{\mathbf{J}}^l = \frac{\partial}{\partial t}\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}^0 + \vec{\mathbf{E}}^{l-1}\right);\ \mathbf{I}^l = \frac{\partial}{\partial t}\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}^0 + \vec{\mathbf{H}}^{l-1}\right);\ \varrho^l = -\mathrm{div}\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}^0 + \vec{\mathbf{E}}^{l-1}\right);\ \mathrm{g}^l = -\mathrm{div}\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}^0 + \vec{\mathbf{H}}^{l-1}\right), \tag{31}$$

where for l=1 one should take into account the influence of the linear part of field only

$$\vec{\mathbf{J}}^1 = \frac{\partial}{\partial t}\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}^0\right);\ \mathbf{I}^1 = \frac{\partial}{\partial t}\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}^0\right);\ \varrho^1 = -\mathrm{div}\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}^0\right);\ \mathrm{g}^1 = -\mathrm{div}\vec{\mathbf{M}}'\left(\vec{\mathbf{H}}^0\right). \tag{32}$$

Using the analytical solution of the linear problem obtained by Riemann function [1] one can account the equivalent nonlinear sources.

PLANE SOURCE RADIATION PROBLEM

Let consider the problem of transient radiation of plane source of current into the medium with weak nonlinearity and constant μ. As a source one can consider plane disk with homogeneous current distribution [1]:

$$\vec{j}_0 = \vec{x}_0 H(t)\delta(z)\big(H(\rho) - H(\rho - R)\big), \tag{33}$$

where $H(*)$ is the Heaviside's step functions.

The source generates TE waves only. The right-hand side of evolutionary equation is

$$j_m(z,t;\nu) = \frac{\sqrt{\mu_0}}{2\pi}\int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho\, \overrightarrow{j_0}\left[\nabla_\perp \Psi_m{}^* \times \overrightarrow{z_0}\right]. \tag{34}$$

The linear solution of Klein-Gordon equation

$$\frac{\varepsilon\mu}{c^2}\frac{\partial^2 h_m}{\partial t^2} - \frac{\partial^2 h_m}{\partial z^2} + \nu^2 h_m = j_m \tag{35}$$

is known and received by Riemann function method

$$h_m = \frac{c}{2}\int_0^{\infty} dz' \int_0^{\infty} dt' J_0\left(\nu\sqrt{c^2(t-t')^2 - (z-z')^2}\right) j_m. \tag{36}$$

The expression can be used to take into account additional field caused by conductivity and nonlinearity of unbounded medium placing in the integrand the expression $\vec{\mathbf{J}}^l = \frac{\partial}{\partial t}\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}^0 + \vec{\mathbf{E}}^{l-1}\right) + \vec{\mathbf{J}}^\sigma\left(\vec{\mathbf{E}}^0 + \vec{\mathbf{E}}^{l-1}\right)$. But the initial field excited by plane source can be obtained by the following expression

$$h_m = \sqrt{\mu_0} J_1(\nu R)\frac{ic\delta_{m,1}}{4\sqrt{\nu}} \int_0^{t-\frac{z}{c}} dt' J_0\left(\nu\sqrt{c^2(t-t')^2 - z^2}\right). \tag{37}$$

We rewrite it in terms of Lommel's function of two variables

$$h_m = -\sqrt{\mu_0} J_1(\nu R)\frac{\delta_{m,1}}{2\sqrt{\nu^3}} U_1\left[-i\nu(ct - z), \nu\sqrt{c^2t^2 - z^2}\right] \tag{38}$$

for the purpose of obtaining the analytical linear solution for amplitude of electrical field to find new equivalent sources of current. It is easy to demonstrate that the sources do not generate TM waves, so

$$e_n(z,t;\chi) = I_n^e = V_n^e = 0, \tag{39}$$

I_m^h, V_m^h can be found as

$$\begin{aligned} I_m^h &= \mu^{-1}\partial_z\{\mu h_m\}\big|_{\mu=const} = \frac{\partial h_m}{\partial z}; \\ V_m^h &= -\partial_{ct}\{\mu h_m\}\big|_{\mu=const} = -\frac{\mu}{c}\frac{\partial h_m}{\partial t}. \end{aligned} \tag{40}$$

It permits to receive the linear solution for the field

$$\vec{E}^0 = \sqrt[-2]{\varepsilon_0}\sum_{m=1}^{\infty}\int_0^{\infty} d\nu V_m^h \left[\nabla_\perp \Psi_m \times \vec{z}_0\right] \tag{41}$$

to obtain the source $\vec{\mathbf{J}}^1 = \frac{\partial}{\partial t}\vec{\mathbf{P}}'\left(\vec{\mathbf{E}}^0\right) + \vec{\mathbf{J}}^\sigma\left(\vec{\mathbf{E}}^0\right)$ for the first nonlinear approximation of solution. It can be obtained by the expression

$$h^1_{\ m} = \frac{c}{2}\int_0^{\infty} dz' \int_0^{\infty} dt' J_0\left(\nu\sqrt{c^2\left(t-t'\right)^2 - \left(z-z'\right)^2}\right) j^1_{\ m}, \tag{42}$$

where $j^1_{\ m}(z,t;\nu) = \frac{\sqrt{\mu_0}}{2\pi}\int_0^{2\pi} d\varphi \int_0^{\infty} \rho d\rho \vec{j}^1 \left[\nabla_\perp \Psi_m^{\ *} \times \vec{z}_0\right]$.

One can repeat the procedure to receive the second and further approximations of the solution.

CONCLUSIONS

The evolutionary approach to electromagnetics was used to receive the system of evolutionary equations that describes the propagation of transient electromagnetic field in nonlinear nonstationary inhomogeneous medium. The equations contain the sought coefficients of electromagnetic field expansion in the basis on transversal plane. Right-hand sides consist of the evolutionary coefficients and the expansion of given sources of electric and magnetic current and charge, current of conductivity and nonlinear reaction of medium. The total problem should contain the initial and boundary conditions for the evolutionary coefficients that can be derived as the expansion of given fields in the basis. The equation set can be transformed to the system of linear equations of second order as was done in [1]. The algorithm of the receiving of the general iterative analytical solution and its first approximation for weak nonlinearity of medium was presented.

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