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# APPLYING SPLINE INTERPOLATION TO INCREASE ACCURACY OF CORRELATION-EMERGENCY NAVIGATION SYSTEMS

**Relevance.** Spline interpolation is used to improve the accuracy of correlation-extreme navigation systems. A two-stage algorithm for combining images in correlation-extreme navigation systems is proposed. At the first stage, the surface of the decision function of the algorithm is constructed in the vicinity of its extremum using a quadratic interpolator by six points and its Gaussian curvature and extremum coordinates are estimated. These parameters are used to determine the optimal value of the parameter of the cubic spline interpolator used in the second stage in order to refine the rough estimate of the coordinates and improve the positioning accuracy of the navigation system.
**Purpose of the work:** The purpose of the work is to develop an algorithm for aligning images in correlation-extreme navigation systems, which makes it possible to realize a cubic spline parameter close to the optimal value for each of the possible shifts of the current image relative to the reference image and, as a result, to increase the accuracy of determining the coordinates.
**Materials and methods.** In correlation-extreme navigation systems, the coordinates of the aircraft are determined by calculating the mutual shift of the current image obtained using the sensor of the Earth's physical field and the reference image, which is known in advance. At the same time, the alignment accuracy of discrete current and reference images, which are usually used in practice, does not exceed half a pixel. Therefore, the problem of improving the accuracy of navigation systems is of great importance. One of the possible ways to solve this problem is to use methods for approximating the decision function of the image alignment algorithm in the vicinity of its global maximum.
**Results:** To illustrate the gain in the accuracy of the positioning of navigation systems, statistical tests of the algorithm with a 6-point interpolator and the above-described two-stage procedure for minimizing the decision function containing spline interpolation at the second stage were carried out. A typical image was used as a reference image. The coordinates of the center of the current and reference images $(x_0, y_0)$ were played randomly in accordance with the two-dimensional normal distribution law, the average value of which coincided with the center of the reference image; the standard deviation is also found. Then the current image was formed. The constructed current image was noisy with additive white Gaussian noise with zero mean value and the same standard deviation for each element $\sigma$. Image alignment was assumed to be correct if the following conditions were met: $|x - x_0| \le 2$, $|y - y_0| \le 2$, where $(x, y)$ – is the shift estimate generated by the algorithm. Then, the algorithms were repeatedly run with different realizations of the noise component of the current image, and the dependences of the root-mean-square error $\sigma_x = \left[\frac{1}{N}\sum_{i=1}^{N}\left(x_i - x_{0i}\right)^2\right]$, $\sigma_y = \left[\frac{1}{N}\sum_{i=1}^{N}\left(y_i - y_{0i}\right)^2\right]$ in each direction on the mean-square value were plotted $\sigma$.

The figures in the article show the dependencies $\sigma_x(\lg\sigma)$ for the algorithm with a 6-point interpolator (upper curve) and for a two-stage algorithm (lower curve). Analysis of the graphs allows us to conclude that the second algorithm wins in the accuracy of determining the coordinates of the shift by about 5 times. The dependencies $\sigma_y(\lg\sigma)$ for both algorithms practically coincide with those shown in the figure. It should be noted the weak dependence of the positioning accuracy on the change in the parameter $\sigma$ in the area $0 < \sigma < 10$.

**Conclusions:** It is shown that the optimal value of the parameter of the cubic spline interpolator depends to a lesser extent on the magnitude of the local shift of the images and, to a greater extent, on the correlation interval of the reference image in the vicinity of the image alignment point, which is proposed to be estimated using the Gaussian curvature parameter.

KEY WORDS: **correlation-extreme navigation systems (CENS), current image (CI), reference image (RI), decision function (DF), high-resolution interpolation cubic spline (HRICS).**

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In [1], the methods of interpolation of functions, including cubic splines, used for the local refinement of the image shift parameter, were analyzed. Methods for approximating the decision function of the algorithm in the vicinity of its global extremum based on the least squares method were studied in [2]. It is shown that among the interpolators the best characteristics are possessed by the spline, called the high-resolution interpolation cubic spline (HRICS), described in [3].

The spline is specified up to a parameter that is recommended to be selected in each specific case. Preliminary studies have shown that for a given EI the optimal value of this parameter depends on the local image shift, the image correlation interval at a given shift, and other parameters.

## FORMULATION OF THE PROBLEM

Let be $\nu_{ij}$, $i, j \in \overline{-n, n}$ – an array of samples of the two-dimensional function to be reconstructed. In the general case, the reconstructed function has the form:

$$\nu(x', y') = \sum_{i,j=-n}^{n} \nu_{ij} g'(x' - ih, y' - jh) . \quad (1)$$

Where $g'(t, s)$ – interpolation function (also called interpolation kernel), $h$ – sampling step of the original function. Usually, separable interpolation functions are used, represented as a product of two one-dimensional functions, i.e. $g'(s, t) = g(s)g(t)$. After passing to dimensionless variables $x = x'/h$, $y = y'/h$ expression (1) takes the form:

$$\nu(x, y) = \sum_{i,j=-n}^{n} \nu_{ij} g(x - i) g(y - j) . \quad (2)$$

The kernel of a high-resolution interpolation cubic spline is described by the relation [1]:

$$g(s,a) = \begin{cases} (a+2)|s|^3 - (a+3)s^2 + 1, & |s| \le 1; \\ a(|s|^3 - 5s^2 + 8|s| - 4, & 1 < |s| \le 2; \\ 0, & |s| > 2 \end{cases} \quad (3)$$

specified up to a parameter $a$. Since the kernel (3) is equal to zero outside the interval $[-2, 2]$, then in this case $n = 2$. The parameter value is suggested to be selected from the interval $[-1; -0,5]$ in each case. The spline derivative has the form:

$$\frac{dg(s,a)}{ds} = \begin{cases} 3\,\mathrm{sgn}(s)(a+2)s^2 - 2(a+3)s, & |s| \le 1; \\ a\,\mathrm{sgn}(s)(3s^2 - 10|s| + 8), & 1 < |s| \le 2; \\ 0, & |s| > 2, \end{cases} \quad (4)$$

Fourier transform of an even function $g(s)$ is real and is defined by the expression:

$$r(v,a) = -\frac{8\sin(v/2)}{v^4}\left[4av\cos^3(v/2) + (3v + 2a)\cos(v/2) - 6\sin(v/2) - 6a\sin(v)\right]. \quad (5)$$

As an alternative, consider a two-dimensional non-separable interpolator with respect to 6 points [2], numbered sample $(0,0)$, $(-1,0)$, $(0,-1)$, $(1,0)$, $(0,1)$, $(1,1)$, moreover:

$$g'(x,y) = 1 + xy - x^2 - y^2;\; g'(x+1,y) = y(y-1)/2;\; g'(x,y+1) = x(x-1)/2;$$
$$g'(x,y-1) = x(x-2y+1);\; g'(x-1,y) = y(y-2x+1);\; g'(x-1,y-1) = xy,\; n = 1.$$

From a geometric point of view, the specified interpolator constructs a surface of the second order:

$$\pi(x,y) = b_1 + b_2 x + b_3 x^2 + b_4 y + b_5 y^2 + b_6 xy ,$$

where

$$b_1 = \nu_{00},\ b_2 = \left(\nu_{-1,0} - \nu_{1,0}\right)/2,\ b_3 = \nu_{0,-1} + \nu_{1,0} - 2\nu_{00},\ b_4 = \left(\nu_{0,1} - \nu_{0,-1}\right)/2,$$
$$b_5 = \nu_{1,0} + \nu_{-1,0} - 2\nu_{00},\ b_6 = \nu_{00} + \nu_{-1,1} - \nu_{0,1} - \nu_{-1,0}.$$

In the case when the determinant of the Hessian matrix satisfies the condition:

$$\Delta = 4b_3b_5 - b_6^2 > 0, \qquad (6)$$

this surface is an elliptical paraboloid, otherwise it is a hyperbolic paraboloid. In the first case, the coordinates of the extremum of the function described by the indicated surface are determined by the expressions:

$$y = -\frac{b_4b_6 - 2b_2b_5}{4b_3b_5 - b_6^2},\ x = \frac{b_2b_6 - 2b_3b_4}{4b_3b_5 - b_6^2}. \qquad (7)$$

If condition (6) is not met, then the point with coordinates (7) is a saddle point.

Let us carry out a comparative analysis of the considered interpolators when using them to refine the integer shift $(k,l)$, generated by the image alignment algorithm in correlation-extreme navigation systems (CENS). Move the origin to a point $(k,l)$.

## 1. ERROR IN DETERMINING THE MAXIMUM OF A KNOWN FUNCTION.

We will conduct a study of the quality of restoration of function:

$$f(x,y) = \left[\frac{\operatorname{sinc}\left(p(x-x_0)\right)}{p(x-x_0)}\frac{\operatorname{sinc}\left(q(y-y_0)\right)}{q(y-y_0)}\right]^2 \qquad (8)$$

in area $[-1,1]\times[-1,1]$, using her counts $\nu_{ij} = f(i,j),\ i,j \in \overline{-2,2}$. If the width of the main lobe of function (8) in each of the planes does not exceed 2, then such a function approximately correctly describes the surface of the decision function in the vicinity of its extremum in practical applications for aligning images in the CENS. Using the parameter p, you can change the width of the main lobe of the function (8), and by changing the parameters $x_0,\ y_0$, which we will carry out within the region $[-0,5;0,5]\times[-0,5;0,5]$, you can move the coordinates of the point of its maximum.

In the case under consideration, the quality of restoration will be characterized by the value of the displacement $(\Delta x = \hat{x} - x_0, \Delta y = \hat{y} - y_0)$ maximum $(\hat{x},\hat{y})$ function $\nu(x,y)$ relative to the maximum point $(x_0,y_0)$ the original function (8), which is assumed to be specified in the region $[-2,2]\times[-2,2]$.

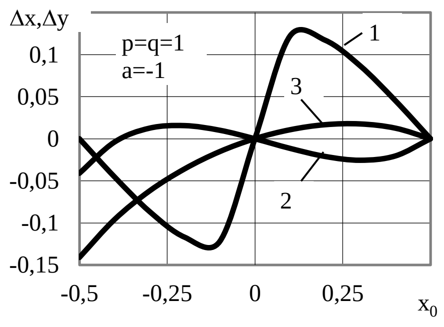



Fig. 1. Dependencies $\Delta x(x_0), \Delta y(x_0)$

In Fig. 1 shows the dependences of the displacement $\Delta x$ from the shift $x_0$ at a fixed $y_0 = 0$ and parameter values $p = q = 1$, and curve 1 corresponds to the HRICS with the parameter $a = -1$, curve 2 – 6-point interpolator. In the case under consideration, the interpolator of the first type is significantly inferior to the second in terms of displacement. However, due to the inseparability of the core of the latter, the maximum displacement is observed at $x_0 = -0,5$, and there is also a significant displacement along the axis (curve 3 in Fig. 1), which is absent in the HRICS.

Odds $\nu_{ij}$ in formula (2) depend on the form of the function $f(x,y)$, and if it is given in the form (8), then they depend on the parameters $p,\ q,\ x_0,\ y_0$. If we fix them, then we can solve the problem:

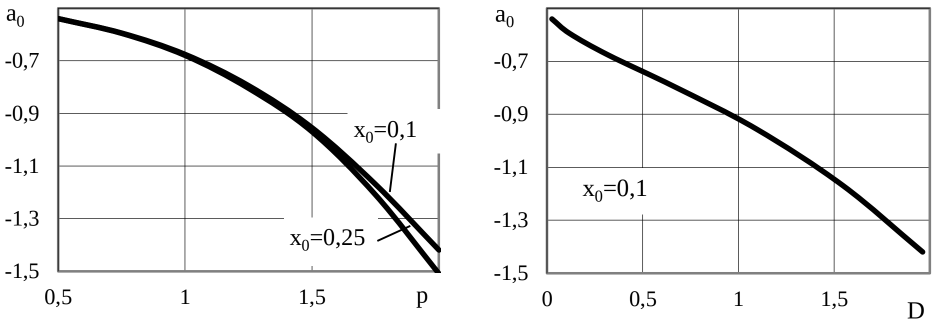
$$\left(\hat{x}(a),\hat{y}(a)\right) = \arg\max_{x,y\in[-1,1]} \nu(x,y,a). \qquad (9)$$

A necessary condition for the fulfillment of equality (9) is that $\left(\hat{x}(a),\hat{y}(a)\right)$ should be a solution to the system of equations:

$$\begin{cases} \dfrac{\partial v(x,y,a)}{\partial x} = \sum\limits_{i,j=-2}^{2} v_{ij} \dfrac{dg}{dx}(x-i,a)g(y-j,a) = 0; \\ \dfrac{\partial v(x,y,a)}{\partial y} = \sum\limits_{i,j=-2}^{2} v_{ij} g(x-i,a)\dfrac{dg}{dy}(y-j,a) = 0; \end{cases} \quad x,y \in [-1,1]. \tag{10}$$

If we now fix $y = y_0$, then you can find the optimal value $a_0 = a_0(x_0)$ as a solution to the equation:

$$x(a) - x_0 = 0, \ a \in [-1{,}5; -0{,}5]. \tag{11}$$




a) b)

Fig. 2. Dependency graphs $a_0(p)$ (a) and $a_0(D)$ (b)

Family of function graphs $a_0(p)$ at $q = p$; $y_0 = 0$ and for different values $x_0 = 0{,}1$ and $x_0 = 0{,}25$ are shown in Fig. 2a. It should be noted that the graphs differ slightly in the area $0{,}5 \le p < 1{,}5$ when changing the shift of the interpolated function. If we solve the problem of reconstructing an unknown function from a set of its samples, then instead of the parameters $p, q$, characterizing the curvature of the function in each direction, it is necessary to use other parameters. For example, can be used to describe the curvature of a surface $z = f(x, y)$ at the point $(x_0, y_0)$, wherein $\nabla f = 0$, determinant of the Hesse matrix of second derivatives:

$$D = \det \begin{bmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{bmatrix}_{x=x_0, y=y_0} = (f_{xx}f_{yy} - f_{xy}^2)_{x=x_0, y=y_0}, \tag{12}$$

called the Gaussian curvature [4]. In addition, the concepts of curvature are used $\lambda_1$, $\lambda_2$ (eigenvalues of the Hessian), as well as the mean curvature:

$$\rho = (f_{xx} + f_{yy})_{x=x_0, y=y_0} = \lambda_1 + \lambda_2 . \tag{13}$$

If you build a dependency $D(p)$ for function (8), then it can be used to obtain the graph of the function $a_0(D)$, shown in Fig. 2b for fixed shift $x_0 = 0{,}1$, which in the first approximation can be approximated by a straight line by the least squares method. If necessary, you can use the cubic parabola approximation.

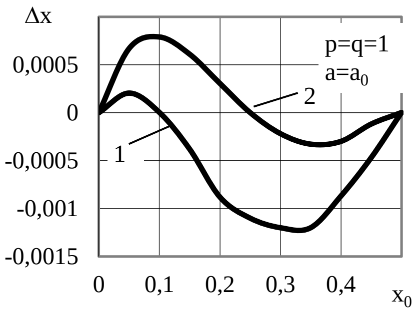



Fig. 3. Addiction $\Delta x(x_0)$

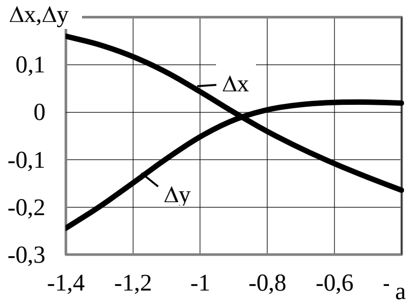
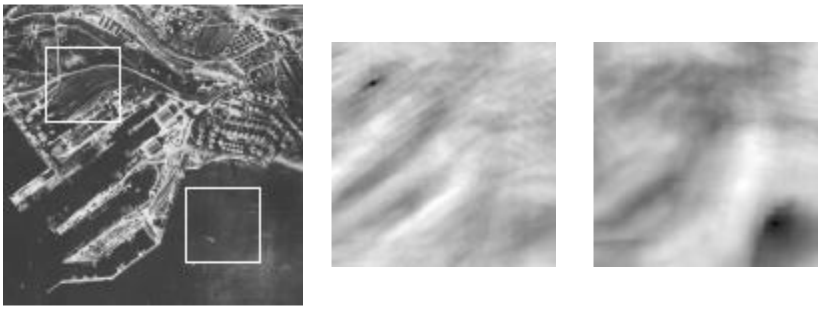



Fig. 4. Dependencies $\Delta x(x_0), \Delta y(x_0)$

Fig. 3 illustrates a graph of a function $\Delta x(x_0)$ for $a = a_0(0{,}1)$ (curve 1) and $a = a_0(0{,}25)$ (curve 2). Thus, the exact restoration of the original function is possible only with its displacements $x_0 = 0$; $x_0 = \pm 0{,}5$; $x_0 = \pm x_0'$, where $x_0'$ - the value of the shift of function (8), for which the optimal value of the parameter a was determined. At other points, the error in recovering a known function of the form (8) is significantly less than for an interpolator by 6 points.

## 2. THE ERROR IN DETERMINING THE EXTREMUM OF THE DECISION FUNCTION OF THE CORRELATION ALGORITHM.

Let us illustrate the possibility of using HRICS to refine the position of the extremum of the decision function (DF) corresponding to the correlation algorithm for aligning images in CENS. For example, let's take the port image as an RI $M_1 = 150 \times M_2 = 150$ elements shown in Fig. 4a.



a) b) c)

Fig. 4. RI (a) and the decision function (DF) corresponding to the shift $(\Delta x = 20,2;\ \Delta y = 20)$ (b) and $(\Delta x = 90,2;\ \Delta y = 90)$ (c)

Let us take the DF of the correlation algorithm in the form:

$$b_{kl} = \sum_{i=1}^{N_1}\sum_{j=1}^{N_2}(t_{ij} - e_{ij}^{kl})^2,\ (k,l) \in \overline{1, M_1 - N_1 + 1} \times \overline{1, M_2 - N_2 + 1}, \tag{14}$$

where $t_{ij}$ – elements of the centered and normalized CI [2], measuring $N_1 \times N_2$, $N_1 < M_1$, $N_2 < M_2$;

$\left[e_{ij}^{kl}\right]$, $(i, j) \in \overline{1, N_1} \times \overline{1, N_2}$ – RI sub matrix corresponding to its centered and normalized fragment obtained by shifting by $(k,l)$ elements relative to the upper left corner of the RI. The quadratic difference algorithm (14) is equivalent to the correlation one due to the centrality and normalization of the CI and RI fragments, and calculations in accordance with the algorithm (14) by the computer are carried out much faster than when using the correlation algorithm with the operation of multiplication under the sum.

The process of forming a CI using RI is described in [2], but instead of an interpolator by 6 points, the above-described HRICS with the parameter $a = -0,6$. If necessary, the generated CI can be noisy to simulate the noise component that occurs when CI is obtained in real conditions.

The image of a man-made object shown in fig. 4a indicates its significant inhomogeneity. So, in fig. 4b and fig. 4c are shown in the form of images of the DF algorithm (14) obtained for the CI with $N_1 = N_2 = 39$ elements and offsets $(\Delta x = 20,2;\ \Delta y = 20)$ and $(\Delta x = 90,2;\ \Delta y = 90)$ respectively. The positions of the CI on the RI are shown in Fig. 4a. It can be seen that in the second case, corresponding to sighting of the water area, the correlation radius is significantly larger than in the first. Therefore, when using HRICS to reconstruct DF from its discrete readings in the vicinity of the minimum, it is necessary to refine the parameter depending on the correlation interval of the RI at the extremum point.

The following two-stage procedure is proposed for using the HRICS in order to correct the position of the DF minimum.

At the **first** stage, a rough estimate of the shift parameter is determined $(\hat{x}, \hat{y})$ by formula (7) using a 6-point interpolator. Determinant $\Delta$ of an elliptic paraboloid (formula (6)) constructed by the interpolator does not depend on the coordinates $(x, y) \in [-1,1] \times [-1,1]$ and coincides with the determinant of the Hessian matrix (expression (12)). In particular, for the cases shown in Fig. 4b and 4c, determinant values $D$ made up 1,406 and 0,648 respectively.

If we now construct a calibration curve similar to that shown in Fig. 2b, then at the **second** stage according to the known parameter $D$ it is possible to find the optimal value of the parameter $a$, construct the surface (2) with the help of HRICS using 25 DF readings and determine the position of its minimum, for example, by one of the gradient-type algorithms, and use a rough estimate (7) as the initial value for this algorithm.

In the implemented algorithm, to minimize function (2), the iterative Newton-Raphson algorithm was used [5]:

$$\mathbf{x}_{i+1} = \mathbf{x}_i - \mathbf{D}^{-1}(\mathbf{x}_i)\nabla\nu(\mathbf{x}_i), \tag{15}$$

where $\mathbf{x} = [x, y]^T$, $\nabla\nu$ - gradient function (2), $\mathbf{D}$ – Hesse matrix. Equation (15) in coordinates has the form:

$$x_{i+1} = x_i - \left[ \frac{1}{D} \left( f_x f_{yy} - f_y f_{xy} \right) \right]_{x=x_i, y=y_i};$$

$$y_{i+1} = y_i - \left[ \frac{1}{D} \left( -f_x f_{xy} + f_y f_{xx} \right) \right]_{x=x_i, y=y_i},$$

where $D(x,y) = (f_{xx} f_{yy} - f_{xy}^2)$. These equations coincide with those obtained in another way in [6]. The second derivative of the spline (3) is described by the expression:

$$\frac{d^2 g(s,a)}{ds^2} = \begin{cases} 6(a+2)|s| - 2(a+3), & |s| \le 1; \\ 2a(3|s| - 5), & 1 < |s| \le 2; \\ 0, & |s| > 2 \end{cases}$$

and has discontinuities of the first kind at points $s = \pm 1,\ s = \pm 2$.

## 3. RESULTS OF STATISTICAL TESTS OF ALGORITHMS.

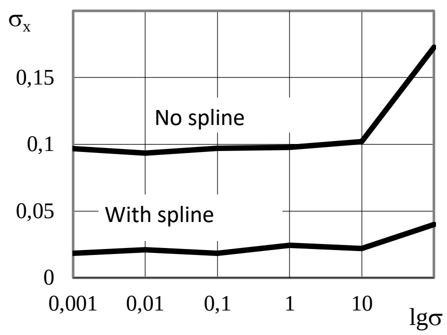



Fig. 5. Addiction $\sigma_x(\lg \sigma)$

To illustrate the gain in the accuracy of the CENS positioning, statistical tests of the algorithm (14) were carried out with an interpolator by 6 points and the above-described two-stage procedure for minimizing DF, containing spline interpolation at the second stage. As a reference image, we used the image shown in Fig. 4a.

Coordinates of CI center on RI $(x_0, y_0)$ were played randomly in accordance with the two-dimensional normal distribution law, the average value of which coincided with the center of the RI, and the standard deviation $\sigma_0$ was chosen from the condition $\sigma_0 \approx M_1 / 8$. Then the CI was formed in accordance with the methodology described in [2]. The constructed CI was noisy with additive white Gaussian noise with zero mean value and the same standard deviation for each element, expressed in units of image brightness, which can take values from the interval $[0, 255]$.

Image alignment was assumed to be correct if the conditions were met (see [16]):

$$\left| x - x_0 \right| \le 2,\ \left| y - y_0 \right| \le 2. \qquad (16)$$

Where $(x, y)$ – an estimate of the shift generated by the algorithm. Next, the algorithms were repeatedly run with different implementations of the noise component of the CI and the dependences of the root mean square error were plotted in each direction $\sigma_x = \left[ \frac{1}{N} \sum_{i=1}^{N} \left( x_i - x_{0i} \right)^2 \right]$, $\sigma_y = \left[ \frac{1}{N} \sum_{i=1}^{N} \left( y_i - y_{0i} \right)^2 \right]$ from rms $\sigma$.

In Fig. 5 shows the dependencies $\sigma_x(\lg \sigma)$ for the 6-point interpolator algorithm (upper curve) and for the two-stage algorithm (lower curve). Analysis of the graphs allows us to conclude that the second algorithm wins in the accuracy of determining the shift coordinates by about 5 times. Dependencies $\sigma_y(\lg \sigma)$ for both algorithms practically coincide with those presented in Fig. 5. It should be noted the weak dependence of the positioning accuracy on the change in the parameter $\sigma$ in area $0 < \sigma < 10$.

## CONCLUSIONS

It has been shown that the optimal value of the parameter of the cubic spline interpolator is less dependent on the magnitude of the local shift of the images and, to a greater extent, on the correlation interval of the RI in the vicinity of the image alignment point, which is proposed to be estimated using the Gaussian curvature parameter. A two-stage procedure has been developed for determining the coordinates in the CENS, in which, at the first stage, the decisive function is reconstructed in the vicinity of its global extremum using a two-dimensional quadratic interpolator for 6 counts and the coordinates of its extremum and the Gaussian curvature are estimated. These data are used at the second stage to determine the optimal value of the cubic spline, constructed from 25 DF readings, and to refine the coordinates of its extremum. It is shown that the positioning accuracy of the CENS can be increased by up to 5 times.

## СПИСОК ВИКОРИСТАНОЇ ЛІТЕРАТУРИ

1. Грузман ИС, Киричук ВС, Косих ВП, и др. Цифровая обработка изображений в информационных системах. Учебник. Новосибирск: Изд-во НГТУ; 2000. 168 с.
2. Антюфеев ВИ, Бакулин ИЕ, Быков ВН, Гричанюк АМ, Мирошник-Быкова ТВ. Повышение точности определения местонахождения радиометрических корреляционно-экстремальных систем навигации путем применения методов приближения решающей функции. (Сообщение 1). Радиотехника: Всеукр. Межвед. научно-техн. сб. 2002;124:84-89.
3. Keys RG. Cubic convolution interpolation for Digital Image Processing. IEEE Trans. Acoust., Speech, Signal Processing. 1981;29(6):1153-1160.
4. Дубровин БА, Новиков СП, Фоменко АТ. Современная геометрия. Москва: Наука; 1979. 760 с.
5. Поляк Е. Численные методы оптимизации. Единый подход. Москва: Мир; 1974. 376 с.
6. Jahne B. Digital Image Processing: Concepts, Algorithms, and Scientific Applications. Berlin: Springer-Verlag; 1993. (In Russian).
7. Антюфеев ВИ, Быков ВН, Чмиль ВВ. Вероятность правильного совмещения изображений в корреляционно-экстремальных системах навигации. Радиотехника: Всеукр. Межвед. научно-техн. сб. 2005;140:70-75.

## REFERENCES

1. Gruzman IS, Kirichuk VS, Kosykh VP, and other. Digital image processing in information systems. Tutorial. Novosibirsk: Publishing house of NSTU; 2000. 168 p. (In Russian).
2. Antyufeev VI, Bakulin IE, Bykov VN, Grichanyuk AM, Miroshnik-Bykova TV. Improving the positioning accuracy of radiometric correlation-extreme navigation systems by using methods of approximation of the decisive function. (Message 1). Radio engineering: Vseukr. mezhved. scientific and technical Sat. 2002;124:84-89. (In Russian).
3. Keys RG. Cubic convolution interpolation for Digital Image Processing. IEEE Trans. Acoust., Speech, Signal Processing. 1981;29(6):1153-1160.
4. Dubrovin BA, Novikov SP, Fomenko AT. Modern geometry. Moscow: Nauka; 1979. 760 p. (In Russian).
5. Poliak E. Numerical optimization methods. One approach. Moscow: Mir; 1974. 376 p. (In Russian).
6. Jahne B. Digital Image Processing: Concepts, Algorithms, and Scientific Applications. Berlin: Springer-Verlag; 1993.
7. Antyufeev VI, Bykov VN, Chmil VV. Probability of correct alignment of images in correlation-extreme navigation systems. Radio engineering: Vseukr. mezhved. scientific and technical Sat. 2005;140:70-75.

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## ЗАСТОСУВАННЯ СПЛАЙН – ІНТЕРПОЛЯЦІЇ ДО ПІДВИЩЕННЯ ТОЧНОСТІ КОРЕЛЯЦІЙНО-ЕКСТРЕМАЛЬНИХ СИСТЕМ НАВІГАЦІЇ

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**Актуальність:** Сплайн–інтерполяція застосовується для підвищення точності кореляційно-екстремальних систем навігації. Запропоновано двухетапний алгоритм суміщення зображень в кореляційно-екстремальних системах навігації. На першому етапі будується поверхня вирішальної функції алгоритму в околиці її екстремуму за допомогою квадратичного інтерполятора по шести точках і оцінюються її гаусова кривизна і координати екстремуму. Ці параметри використовуються для визначення оптимального значення параметра кубічного сплайн–інтерполятора, який використовується на другому етапі з метою уточнення грубої оцінки координат і підвищення точності визначення місцезнаходження системи навігації.

**Метою роботи є**: Розробка алгоритму суміщення зображень в кореляційно-екстремальних системах навігації, що дозволяє реалізувати близьке до оптимального значення параметру кубічного сплайну для кожного з можливих зрушень поточного зображення щодо еталонного зображення і в результаті підвищити точність визначення координат.

**Матеріали та методи:** У кореляційно-екстремальних системах навігації визначення координат літального апарату здійснюється шляхом обчислення взаємного зсуву поточного зображення, отриманого за допомогою датчика фізичного поля Землі, і еталонного зображення, відомого заздалегідь. При цьому точність суміщення дискретних поточного і еталонного зображення, які зазвичай використовуються на практиці, не перевищує половини пікселя. Одним з можливих шляхів вирішення проблеми підвищення точності систем навігації є застосування методів наближення вирішальної функції алгоритму суміщення зображень в околиці її глобального екстремуму.

**Результати:** Для ілюстрації виграшу в точності визначення місцезнаходження системи навігації були проведені статистичні випробування алгоритму з інтерполятором по шести точках і вищеописаної двоетапної процедури мінімізації вирішальної функції, що містить сплайн–інтерполяцію на другому етапі. В якості еталонного зображення використовувалося типове зображення. Координати центру поточного і еталонного зображень $(x_0, y_0)$ розігруються випадковим чином відповідно до двовимірного нормального закону розподілу, середнє значення якого збігається з центром еталонного зображення, знаходиться також середньоквадратичне відхилення. Потім формується поточне зображення по деяким значенням еталонного зображення. Побудоване таки чином поточне зображення зашумляють адитивним білим гаусівським шумом з нульовим середнім значенням і однаковим для кожного елемента середньоквадратичним відхиленням, що виражається в одиницях яскравості зображення. Поєднання зображень є правильним, якщо виконуються умови: $\left|x - x_0\right| \le 2, \left|y - y_0\right| \le 2$ , де $(x, y)$ – оцінка зсуву, яка формується алгоритмом. Далі здійснювався багаторазовий запуск алгоритмів з різними реалізаціями шумової компоненти поточного зображення і будуються залежності середньоквадратичної помилки по кожному напрямку від середньоквадратичного значення $\sigma$ , тобто

$$\sigma_x = \left[\frac{1}{N}\sum_{i=1}^{N}\left(x_i - x_{0i}\right)^2\right], \ \sigma_y = \left[\frac{1}{N}\sum_{i=1}^{N}\left(y_i - y_{0i}\right)^2\right].$$

На малюнках, приведених в статті, наведено залежності $\sigma_x(\lg\sigma)$ для алгоритму з інтерполятором по 6 точках (верхня крива) і для двоетапного алгоритму (нижня крива). Аналіз графіків дозволяє зробити висновок про те, що другий алгоритм виграє в точності визначення координат зсуву приблизно в 5 разів. Залежності для обох алгоритмів практично збігаються з представленими на малюнку. Слід зазначити слабку залежність точності визначення місцезнаходження від зміни параметра $\sigma$ в області $0 < \sigma < 10$ .

**Висновки:** Показано, що оптимальне значення параметру кубічного сплайн-інтерполятора в меншій мірі залежить від величини локального зсуву зображень і в найбільшій мірі від інтервалу кореляції еталонного зображення довкілля точки суміщення зображень, який запропоновано оцінювати за допомогою параметра гаусової кривизни.

Ключові слова: **кореляційно-екстремальні системи навігації, поточне зображення, еталонне зображення, вирішальна функція, високороздільний інтерполяційний кубічний сплайн.**

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