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Towards to the theory of magnetoplasma waves on the semiconductor nanotube surface

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The effective mass approximation is used to consider plasma and magnetoplasma waves in electron system on the surface of the cylindrical semiconductor nanotube. The electron-electron interaction is taken into account within the framework of the random phase approximation. In the case of degenerate electron gas the spectral windows on the wavevector-frequency plane and the spectra of the waves are obtained. Their frequencies undergo quantum oscillations of the de Haas-van Alfven type which are attributed to the Fermi level traversing the sub-zone boundaries in the electron energy spectrum. The spectrum and the damping of waves in the non-degenerate electron gas were found. In a magnetic field parallel to the cylinder axis the frequencies of the magnetoplasma waves undergoes the Aharonov-Bohm type oscillations appearing with changing magnetic field strength.

Keywords: magnetoplasma waves, semiconductor nanotube, random phase approximation, spectral windows, spectrum and the damping of waves, de Haas-van Alfven type oscillation, Aharonov-Bohm type oscillation.

У наближенні ефективної маси розглядаються плазмові і магнітоплазмові хвилі у системі електронів на поверхні циліндричної напівпровідникової нанотрубки. Електрон-електронна взаємодію враховується у наближенні хаотичних фаз. У випадку виродженого електронного газу знайдені положення вікон прозорості для хвиль на площині хвильовий вектор-частота та спектри цих хвиль. Їх частоти випробовують квантові осциляції типу де Гааза-ван Альфена, обумовлені перетином границь підзон у спектрі енергії електронів рівнем Фермі. Знайдено спектр і згасання хвиль у невиродженому електронному газі. У магнітному полі, паралельному осі циліндра, частоти магнітоплазмових хвиль випробовують осциляції типу Ааронова-Бома зі зміною напруженості магнітного поля.

Ключові слова: магнітоплазмові хвилі, напівпровідникова нанотрубка, наближення хаотичних фаз, вікна прозорості, спектр і згасання хвиль, осциляції типу де Гааза-ван Альфена, осциляції типу Ааронова-Бома.

В приближении эффективной массы рассматриваются плазменные и магнитоплазменные волны в системе электронов на поверхности цилиндрической полупроводниковой нанотрубки. Электрон-электронное взаимодействие учитывается в приближении хаотических фаз. В случае вырожденного электронного газа найдены положения окон прозрачности для волн на плоскости волновой вектор-частота и спектры волн. Их частоты испытывают квантовые осцилляции типа де Гааза-ван Альфена, обусловленные пересечением границ подзон в спектре энергии электронов уровнем Ферми. Найдены спектр и затухание волн в невырожденном электронном газе. В магнитном поле, параллельном оси цилиндра, частоты магнитоплазменных волн испытывают осцилляции типа Ааронова-Бома с изменением напряженности магнитного поля.

Ключевые слова: магнитоплазменные волны, полупроводниковая нанотрубка, приближение хаотических фаз, окна прозрачности, спектр и затухание волн, осцилляции типа де Гааза-ван Альфена, осцилляции типа Ааронова-Бома.

Introduction

A keen interest of researchers in electron nanosystems on curving surfaces [1,2] is attributed to a number of reasons. These systems are functional elements of many devices and engineering gadgets. The perfection of experimental setup allows for production of these systems in a laboratory framework. For theorists these systems are convenient objects for field-testing of novel methods of computations of physical values. The curvature of the structure and external magnetic field enrich the picture of phenomena occurring in nanostructures. The control methods of their properties become more diverse.

Various effects have been found on curving surfaces in such electron systems, which cannot be reproduced in those of the plane geometry. Among them are the effects of hybridization of the spatial and magnetic quantization of electron motion, the modification of the hamiltonian of the electron system [2,3], the unusual performance of conductance [4] and magnetic response of the system [5], the peculiarities of the screening of electron-electron interaction [6], the specific resonances in electron scattering produced in carbon nanotubes [7] and quantum wires [8] by impurity atoms, etc.

The typical examples of the curving surface nanosystems are carbon and semiconductor nanotubes [1-7]. The electron energy spectrum in these systems is zoned.

It allows due to a small number of electrons near the zone bottom for use of the effective mass approximation. This approximation enables to describe qualitatively, and often quantitatively, the properties of this kind of systems.

Plasma oscillations of the electron gas density on the curving surface were investigated in Ref. [9]. However, the dispersion relationships of the spectrum and damping of plasmons are not given therein [9]. The present paper considers plasma and magnetoplasma waves on the surface of a cylindrical nanotube. We employ the effective mass approximation with the electron-electron interaction being considered in the random phase approximation. We consider the spectrum of density oscillations of degenerate electron gas in Section I and of the non-degenerate gas in Section II. The magnetoplasma waves on cylindrical surface in the longitudinal magnetic field are considered in Section III.

Degenerate electron gas

In the effective mass approximation the wave function of stationary electron state on the surface of cylindrical nanotube has the following form:

$$\psi_{mk}(\varphi, z) = \frac{1}{\sqrt{2\pi}} \mathrm{e}^{im\varphi} \frac{1}{\sqrt{L}} \mathrm{e}^{ikz}, \tag{1}$$

where $m = 0, \pm 1, \ldots$ is the azimuthal quantum number, k is the projection of electron wave vector to the cylinder axis z, φ is the polar angle, L is the length of the tube. The electron energy in the state (1) is:

$$\varepsilon_{mk} = \varepsilon_0 m^2 + \frac{k^2}{2m_*}. \tag{2}$$

Here m_* is the effective electron mass, $\varepsilon_0 = 1/2m_* a^2$ is the rotational quantum, a is the tube radius. Herein the quantum constant is assumed to be equal to unity.

The spectrum (2) consists of many one-dimensional subzones. The electron state density

$$\nu(\varepsilon) = \frac{\sqrt{2m_*}L}{\pi} \sum_m \frac{1}{\sqrt{\varepsilon - \varepsilon_0 m^2}} \tag{3}$$

has the square root singularities at the subzone boundaries $\varepsilon_m = \varepsilon_0 m^2$. In this formula the summation is done over those values of m for which the expression under the root is positive. By calculating the sum according to the Poisson formula, we obtain:

$$\nu(\varepsilon) = 2m_* aL \left[1 + 2\sum_{l=1}^{\infty} \mathrm{J}_0 \left(2\pi l \sqrt{\frac{\varepsilon}{\varepsilon_0}} \right) \right],$$

where J_0 is the Bessel function. In the limit $\varepsilon >> \varepsilon_0$, we have

$$\nu(\varepsilon) = 2m_* aL \left[1 + \frac{2}{\pi} \left(\frac{\varepsilon_0}{\varepsilon} \right)^{1/4} \sum_{l=1}^{\infty} \frac{1}{\sqrt{l}} \cos \left(2\pi l \sqrt{\frac{\varepsilon}{\varepsilon_0}} - \frac{\pi}{4} \right) \right].$$

The density of states oscillates with varying $\sqrt{\varepsilon}$ with the period $\left(\sqrt{2m_*}a\right)^{-1}$. The monotonous term $2m_* aL$ is equal to the density of states of the two-dimensional electron gas on the area $S = 2\pi aL$.

In the random phase approximation, the dispersion equation for the spectrum of plasma waves on the surface of the nanotube has the form [6]:

$$1 - \frac{\upsilon_m(q)}{2\pi L} P_m(q, \omega) = 0, \tag{4}$$

where

$$\upsilon_m(q) = 4\pi \bar{e}^2 \mathrm{I}_m(a|q|) \mathrm{K}_m(a|q|) \tag{5}$$

is the cylindrical harmonic of the electron Coulomb interaction energy, $\bar{e}$ is the electron charge divided by the background dielectric constant, I_m and K_m are the modified Bessel functions [10], P_m is the delayed polarization operator which depends on the projection q of the wave vector to the axis z and frequency ω. With the aim to consider further the magnetoplasma waves, we shall derive the value of P_m for the electron gas in the magnetic field B, which is parallel to the cylinder axis:

$$P_m(q,\omega)=\sum_{m'k\sigma}\frac{f\left(\varepsilon_{(m'+m)(k+q)\sigma}\right)-f\left(\varepsilon_{m'k\sigma}\right)}{\varepsilon_{(m'+m)(k+q)\sigma}-\varepsilon_{m'k\sigma}-\omega-\mathrm{i}0}, \quad (6)$$

where

$$\varepsilon_{mk\sigma}=\varepsilon_0\left(m+\frac{\Phi}{\Phi_0}\right)^2+\frac{k^2}{2m_*}+\sigma\mu_B B \quad (7)$$

is the electron energy in magnetic field [5], $\Phi=\pi a^2 B$ is the magnetic flux through the tube cross-section, $\Phi=2\pi c/e$ is the flux quantum (e is the electron charge, c is the speed of light), μ_B is the electron spin magnetic moment, $\sigma=\pm 1$ is the spin quantum number, f is the Fermi function.

The function (6) depends on $|q|$ and satisfies the following symmetry properties:

$$\begin{aligned}\operatorname{Re}P_m(q,-\omega,-\Phi)&=\operatorname{Re}P_m(q,\omega,\Phi),\\ \operatorname{Im}P_m(q,-\omega,-\Phi)&=-\operatorname{Im}P_m(q,\omega,\Phi),\\ P_{-m}(q,\omega,-\Phi)&=P_m(q,\omega,\Phi).\end{aligned} \quad (8)$$

Using $P_m=\sum_{m'\sigma}P_{mm'}^{\sigma}$, we obtain at zero temperature:

$$\begin{aligned}&\operatorname{Re}P_{mm'}^{\sigma}(q,\omega)=-\frac{m_*L}{2\pi q}\left(\ln\left|\frac{-qv_{m'+m}^{\sigma}-\omega_q+\Omega_{mm'}-\omega}{-qv_{m'}^{\sigma}+\omega_q+\Omega_{mm'}-\omega}\right|+\ln\left|\frac{qv_{m'}^{\sigma}+\omega_q+\Omega_{mm'}-\omega}{qv_{m'+m}^{\sigma}-\omega_q+\Omega_{mm'}-\omega}\right|\right),\\ &\qquad 0<q<k_{m'}^{\sigma}-k_{m'+m}^{\sigma};\\ &\operatorname{Re}P_{mm'}^{\sigma}(q,\omega)=-\frac{m_*L}{2\pi q}\ln\left|\frac{qv_{m'}^{\sigma}+\omega_q+\Omega_{mm'}-\omega}{qv_{m'+m}^{\sigma}-\omega_q+\Omega_{mm'}-\omega}\right|,\\ &\qquad k_{m'}^{\sigma}-k_{m'+m}^{\sigma}<q<k_{m'}^{\sigma}+k_{m'+m}^{\sigma};\\ &\operatorname{Re}P_{mm'}^{\sigma}(q,\omega)=-\frac{m_*L}{2\pi q}\ln\left|\frac{qv_{m'}^{\sigma}+\omega_q+\Omega_{mm'}-\omega}{-qv_{m'}^{\sigma}+\omega_q+\Omega_{mm'}-\omega}\right|,\\ &\qquad q>k_{m'}^{\sigma}+k_{m'+m}^{\sigma};\end{aligned} \quad (9)$$

$$\begin{aligned}&\operatorname{Im}P_{mm'}^{\sigma}(q,\omega)=-\frac{m_*L}{2|q|}\Big\{\Theta\Big[\omega-\left(-qv_{m'}^{\sigma}+\omega_q+\Omega_{mm'}\right)\Big]\Theta\Big[-qv_{m'+m}^{\sigma}-\omega_q+\Omega_{mm'}-\omega\Big]+\\ &\qquad+\Theta\Big[\omega-\left(qv_{m'+m}^{\sigma}-\omega_q+\Omega_{mm'}\right)\Big]\Theta\Big[qv_{m'}^{\sigma}+\omega_q+\Omega_{mm'}-\omega\Big]\Big\},\\ &\qquad 0<q<k_{m'}^{\sigma}-k_{m'+m}^{\sigma};\\ &\operatorname{Im}P_{mm'}^{\sigma}(q,\omega)=-\frac{m_*L}{2|q|}\Theta\Big[\omega-\left(qv_{m'+m}^{\sigma}-\omega_q+\Omega_{mm'}\right)\Big]\Theta\Big[qv_{m'}^{\sigma}+\omega_q+\Omega_{mm'}-\omega\Big],\\ &\qquad k_{m'}^{\sigma}-k_{m'+m}^{\sigma}<q<k_{m'}^{\sigma}+k_{m'+m}^{\sigma};\\ &\operatorname{Im}P_{mm'}^{\sigma}(q,\omega)=-\frac{m_*L}{2|q|}\Theta\Big[\omega-\left(-qv_{m'}^{\sigma}+\omega_q+\Omega_{mm'}\right)\Big]\Theta\Big[qv_{m'}^{\sigma}+\omega_q+\Omega_{mm'}-\omega\Big],\\ &\qquad q>k_{m'}^{\sigma}+k_{m'+m}^{\sigma}.\end{aligned} \quad (10)$$

Here

$$\upsilon_m^\sigma = \frac{k_m^\sigma}{m_*} = \sqrt{\frac{2}{m_*}}\sqrt{\mu_0 - \varepsilon_{m\sigma}} \qquad (11)$$

is the electron limiting velocity in the subzone with the number (m,σ),

$$\varepsilon_{m\sigma} = \varepsilon_0\left(m + \frac{\Phi}{\Phi_0}\right)^2 + \sigma\mu_B B$$

is the subzone boundary, $\omega_q = q^2 / 2m_*$, μ_0 is the chemical potential at zero temperature,

$$\Omega_{mm'} = \varepsilon_{(m'+m)\sigma} - \varepsilon_{m'\sigma} \qquad (12)$$

are the frequencies of electron vertical transitions, Θ is the Heaviside function.

From the Eqs. (9) and (10) one can see that the polarization operator real part as function of the frequency ω has logarithmic singularities at the region boundaries on the plane (q,ω), in which the collisionless damping of plasma waves is absent. The boundaries of these regions are derivable from the formula (10). The polarization operator imaginary part is zero in the spectral windows for such plasma waves that are limited by parabolas on the plane (q,ω).

It follows from the equation (10) that each of the subzones in the electron energy spectrum is connected with a branch of plasma waves that propagate along the tube axis. In particular, for the branch with the number $m = 0$ the frequencies of the transitions (12) are zero and the dispersion equation (4) is reduced. Let us present the solution of this equation in the absence of magnetic field in the ultra-quantum limits, where only the lowermost subzone $m' = 0$ is filled up. In this case, there is a parabolic spectral window which is limited by the parabola $\omega = q\upsilon_0 - \omega_q$ and by the axis q on the plane (q,ω), the region is above the parabola $\omega = q\upsilon_0 + \omega_q$ and to the right of the parabola $\omega = -q\upsilon_0 + \omega_q$, in which the collisionless damping of the waves is absent. The analysis of the equation (4) indicates that in this case there are no solutions for the equation (4) in the parabolic spectral window. They do exist above the parabola $\omega = q\upsilon_0 + \omega_q$ and in the region of the collisionless damping. Above the parabola $\omega = q\upsilon_0 + \omega_q$, we have the following solution:

$$\omega_0(q) = q\upsilon_0 + \omega_q \operatorname{cth}\frac{2\pi^2 q}{m_*\upsilon_0(q)}. \qquad (13)$$

In the long-wavelength approximation $qa << 1$, we shall use the expansions [10]:

$$\begin{gathered} I_0(x)K_0(x) \approx \ln\frac{2}{x\mathrm{e}^\gamma}, \\ I_1(x)K_1(x) \approx \frac{1}{2}\left[1 + \frac{x^2}{2}\ln\frac{x}{2}\right], \\ I_m(x)K_m(x) \approx \frac{1}{2m}\left[1 - \frac{\left(\frac{x}{2}\right)^2}{m-1}\right] \quad (m = 2,3,...), \end{gathered} \qquad (14)$$

where $x << 1$, $\gamma = 0,577...$ is the Euler number. In this approximation using the formula (13) we obtain the plasmon spectrum:

$$\omega_0(q) = \frac{\overline{e}^2}{\pi} q \ln\frac{2}{aq\mathrm{e}^{\gamma}}. \tag{15}$$

The dispersion of this wave is normal. In the region $qa \gg 1$ the dispersion curve (13) comes close to the parabola $\omega = q\upsilon_0 + \omega_q$ from above.

The plasma wave collisionless damping region is situated on the plane (q,ω) between the parabolas $\omega = q\upsilon_0 + \omega_q$, $\omega = q\upsilon_0 - \omega_q$ and $\omega = -q\upsilon_0 + \omega_q$. In this region the solution of the dispersion equation (4) in the case under consideration has the form:

$$\omega_0(q) = q\upsilon_0 + \omega_q \mathrm{th}\frac{2\pi^2 q}{m_* \upsilon_0(q)}. \tag{16}$$

In the long-wavelength limit we obtain hence:

$$\omega_0(q) = q\upsilon_0 + \frac{\pi q^3}{4m_*^2 \overline{e}^2 \ln\frac{2}{aq\mathrm{e}^{\gamma}}}.$$

In the limit $qa \gg 1$ the dispersion curve (16) approaches the parabola $\omega = q\upsilon_0 + \omega_q$ from below. It corresponds to the damping plasma wave.

In the absence of magnetic field in the case of long wavelengths $q\upsilon \ll \omega$ (υ – the Fermi electron velocity) we obtain the following equation from the formula (9):

$$\mathrm{Re}\, P_0(q,\omega) = \frac{qL}{\pi\omega} \sum_{m'} \Theta(\mu_0 - \varepsilon_{m'}), \tag{17}$$

where $\varepsilon_{m'} = \varepsilon_0 m'^2$ is the subzone boundary with the number m'. In case of a large number of filled subzones, we employ the Poisson formula for calculation of the sum:

$$\sum_{m=-\infty}^{\infty} \varphi(m) = \sum_{l=-\infty}^{\infty} \int_{-\infty}^{\infty} dx\, \varphi(x) \mathrm{e}^{2\pi \mathrm{i} lx}. \tag{18}$$

Then for the mode $m = 0$ at $qa \ll 1$ we obtain:

$$\omega_0(q) = \frac{4\overline{e}^2}{\pi} q \ln\frac{2}{qa\mathrm{e}^{\gamma}} \sqrt{\frac{\mu_0}{\varepsilon_0}} \left(1 + \frac{1}{\pi}\sqrt{\frac{\varepsilon_0}{\mu_0}} \sum_{l=1}^{\infty} \frac{1}{l} \sin 2\pi l \sqrt{\frac{\mu_0}{\varepsilon_0}}\right). \tag{19}$$

The wave frequency undergoes quantum oscillations of the de Haas-van Alfven type with the Fermi energy variations being μ_0, as accounted for by crossing of the subzone boundaries by the Fermi level. The Fermi energy is associated with the linear density $n = N/L$ of electrons via the relationship:

$$n = \frac{2}{\pi}\sqrt{2m_*\varepsilon_0} \sum_m \left(\frac{\mu_0}{\varepsilon_0} - m^2\right)^{1/2}.$$

The period of oscillations with varying $\sqrt{\mu_0}$ is equal to $1/\sqrt{2m_* a}$. It is determined by the electron effective mass and by the tube radius. The relative oscillation amplitude $\sim\sqrt{\dfrac{\varepsilon_0}{\mu_0}}$ is small at $\mu_0 \gg \varepsilon_0$.

Let us consider the dispersion of modes with the numbers $m>0$. In the long-wavelength limit, we obtain from the formula (9) that

$$\operatorname{Re} P_m = \frac{2m_* L}{\pi}\sum_{m'}\frac{\upsilon_{m'}-\upsilon_{m'+m}}{\omega-\Omega_{mm'}}, \tag{20}$$

where the summation is performed over those values of m', at which the sub-root expressions in (11) are positive. The sum in Eq. (20) is calculated according to the Poisson formula (18). In the dispersion equation (34) we still consider that $qa \ll 1$. In addition, we shall restrict ourselves to the case of high frequencies that satisfy the inequality:

$$\omega >> \varepsilon_0\left[\left(m+\sqrt{\frac{\mu_0}{\varepsilon_0}}\right)^2-\frac{\mu_0}{\varepsilon_0}\right].$$

Here $\left[\sqrt{\dfrac{\mu_0}{\varepsilon_0}}\right]$ is the number of filled subzones. Then from the formula (20) we obtain:

$$\operatorname{Re} P_m = \frac{2\sqrt{2m_*\varepsilon_0}\,m^2\mu_0 L}{\left(\omega^2-\varepsilon_m^2\right)}\left[1+\frac{2}{\pi}\sqrt{\frac{\varepsilon_0}{\mu_0}}\sum_{l=1}^{\infty}\frac{1}{l}\mathrm{J}_1\left(2\pi l\sqrt{\frac{\mu_0}{\varepsilon_0}}\right)\right], \tag{21}$$

where J_1 is the Bessel function. In this case, the solutions of the equation (4) have the form:

$$\omega_1^2(q) = \varepsilon_1^2 + 2\overline{e}^2\sqrt{2m_*\varepsilon_0}\,\mu_0\left[1+\frac{1}{2}(aq)^2\ln\frac{aq}{2}\right]\left[1+\frac{2}{\pi}\sqrt{\frac{\varepsilon_0}{\mu_0}}\sum_{l=1}^{\infty}\frac{1}{l}\mathrm{J}_1\left(2\pi l\sqrt{\frac{\mu_0}{\varepsilon_0}}\right)\right],$$

$$\omega_m^2(q) = \varepsilon_m^2 + 2\overline{e}^2\sqrt{2m_*\varepsilon_0}\,m\mu_0\left[1-\frac{(aq)^2}{4(m-1)}\right]\left[1+\frac{2}{\pi}\sqrt{\frac{\varepsilon_0}{\mu_0}}\sum_{l=1}^{\infty}\frac{1}{l}\mathrm{J}_1\left(2\pi l\sqrt{\frac{\mu_0}{\varepsilon_0}}\right)\right] \tag{22}$$

$$(m = 2,3,\ldots).$$

The cut-off wave frequencies with the spectrum (22) are equal to:

$$\omega_m^2(0) = \varepsilon_m^2 + 2\overline{e}^2\sqrt{2m_*\varepsilon_0}\,m\mu_0\left[1+\frac{2}{\pi}\sqrt{\frac{\varepsilon_0}{\mu_0}}\sum_{l=1}^{\infty}\frac{1}{l}\mathrm{J}_1\left(2\pi l\sqrt{\frac{\mu_0}{\varepsilon_0}}\right)\right] \tag{23}$$

$$(m = 1,2,\ldots).$$

The dispersion of these waves is anomalous. Their frequencies undergo the above-considered oscillations associated with the traversal of the subzone boundaries by the Fermi level.

Non-degenerate electron gas

In this Section we employ the Boltzmann distribution function to compute the polarization operator (9), (10) in the absence of magnetic field. The real and imaginary parts of the polarization operator of the Boltzmann electron gas are as follows:

$$\operatorname{Re} P_m(q,\omega) = \frac{N}{q}\sqrt{\frac{m_*\beta}{2}}\left\langle \mathrm{F}\left(x_{mm'}^-\right)-\mathrm{F}\left(x_{mm'}^+\right)\right\rangle, \tag{24}$$

$$\operatorname{Im} P_m(q,\omega) = \frac{N}{|q|}\sqrt{\frac{\pi m_*\beta}{2}}\left\langle \exp\left(-\beta\frac{k_{mm'}^{+2}}{2m_*}\right)-\exp\left(-\beta\frac{k_{mm'}^{-2}}{2m_*}\right)\right\rangle, \tag{25}$$

where

$$\langle ... \rangle = \frac{\sum_{m'} e^{-\beta\varepsilon_0 m'^2} ...}{\sum_{m'} e^{-\beta\varepsilon_0 m'^2}},$$

$$F(x) = \frac{1}{\sqrt{\pi}} P. \int_{-\infty}^{\infty} dy \frac{e^{-y^2}}{x-y},$$

$$x_{mm'}^{\pm} = \frac{1}{q}\sqrt{\frac{m_*\beta}{2}}\left(\omega_{\pm} \pm \Omega_{mm'}^{\pm}\right),\ \Omega_{mm'}^{\pm} = 2m\varepsilon_0\left(\frac{m}{2} \mp m'\right),\ k_{mm'}^{\pm} = \sqrt{\frac{2m_*}{\beta}} x_{mm'}^{\pm},\ \omega_{\pm} = \omega \pm \omega_q,$$

β is the inverse temperature. The number of electrons N, as included in (24) and (25), is associated with the chemical potential μ via the relation:

$$N = \sqrt{\frac{2m_*}{\pi\beta}} L e^{\beta\mu} \sum_{m=-\infty}^{\infty} e^{-\beta\varepsilon_0 m^2} = \sqrt{\frac{2m_*}{\varepsilon_0}} \frac{L}{\beta} e^{\beta\mu} \sum_{l=-\infty}^{\infty} \exp\left(-\frac{\pi^2 l^2}{\beta\varepsilon_0}\right).$$

Here we use the formula [11]:

$$\sum_{m=-\infty}^{\infty} \exp\left[-(m+b)^2 x\right] = \sqrt{\frac{\pi}{x}} \sum_{l=-\infty}^{\infty} \exp\left(-\frac{\pi^2 l^2}{x}\right) \cos 2\pi l b. \tag{26}$$

To calculate the spectrum and the damping of the mode with the number $m = 0$ we use the following expressions:

$$\begin{aligned} \operatorname{Re} P_0(q,\omega) &= \frac{N}{q}\sqrt{\frac{m_*\beta}{2}}\left[F\left(\sqrt{\frac{m_*\beta}{2}}\frac{\omega_-}{q}\right) - F\left(\sqrt{\frac{m_*\beta}{2}}\frac{\omega_+}{q}\right)\right], \\ \operatorname{Im} P_0(q,\omega) &= -\frac{N}{|q|}\sqrt{2\pi m_*\beta}\, \operatorname{sh}\frac{\beta\omega}{2} \exp\left(-\frac{\beta m_*\omega^2}{2q^2} - \frac{\beta q^2}{8m_*}\right). \end{aligned} \tag{27}$$

In the long-wavelength limit $qr_D \ll 1$ (r_D is the Debye screening radius), we obtain the following from the formula (27):

$$\begin{aligned} \operatorname{Re} P_0(q,\omega) &= \frac{Nq^2}{m_*\omega^2}, \\ \operatorname{Im} P_0(q,\omega) &= -\frac{N}{|q|}\sqrt{\frac{\pi m_*\beta}{2}}\beta\omega \exp\left(-\frac{\beta m_*\omega^2}{2q^2}\right). \end{aligned} \tag{28}$$

Then from the dispersion equation in the case of $qa \ll 1$, we obtain the plasma wave spectrum

$$\omega_0^2(q) = \frac{2\bar{e}^2 n}{m_*} q^2 \ln\frac{2}{aqe^{\gamma}}. \tag{29}$$

The decrement of the damping of this wave

$$\gamma(q) = \frac{\operatorname{Im} P(q,\omega(q))}{\frac{\partial}{\partial\omega(q)} \operatorname{Re} P(q,\omega(q))} \tag{30}$$

is equal to

$$\gamma_0(q)=\sqrt{\frac{\pi}{8}}\left(\frac{m_*\beta}{q^2}\right)^{3/2}\omega_0^4(q)\exp\left[-\frac{\beta m_*}{2}\left(\frac{\omega_0(q)}{q}\right)^2\right]. \quad (31)$$

The dispersion of the wave with the spectrum (29) is normal, the damping decrement (31) diminishes exponentially at $q\to 0$.

The real part of the polarization operator (24) at $|m|>0$ within the long-wavelength limit is:

$$\operatorname{Re}P_m=\frac{N}{2\varepsilon_0}\left\langle\left\{\left[m'-\frac{\omega}{2m\varepsilon_0}\right]^2-\frac{m^2}{4}\right\}^{-1}\right\rangle. \quad (32)$$

The included sums $\sum\limits_{m'}$ in the case of $\beta\varepsilon_0\ll 1$ can be replaced by integrals. Therefore

$$\operatorname{Re}P_m=-\frac{N}{2m}\sqrt{\frac{\beta}{\varepsilon_0}}\left\{\mathrm{F}\left(\sqrt{\beta\varepsilon_0}\left[\frac{\omega}{2m\varepsilon_0}+\frac{m}{2}\right]\right)-\mathrm{F}\left(\sqrt{\beta\varepsilon_0}\left[\frac{\omega}{2m\varepsilon_0}-\frac{m}{2}\right]\right)\right\}. \quad (33)$$

In the case of the high frequencies that satisfy the inequality

$$\omega>>2m\varepsilon_0\left(\frac{m}{2}+\frac{1}{\sqrt{\beta\varepsilon_0}}\right),$$

one can use the asymptote of the function $\mathrm{F}(x)\approx x^{-1}$ at $x\gg 1$. Then the expression (33) is approximated by

$$\operatorname{Re}P_m=\frac{2N\varepsilon_0 m^2}{\omega^2-\varepsilon_m^2}. \quad (34)$$

By substituting this expression in the dispersion equation (4) at $qa\ll 1$, we obtain:

$$\omega_1^2(q)=\varepsilon_0^2+2\overline{e}^2\varepsilon_0 n\left[1+\frac{(aq)^2}{2}\ln\frac{aq}{2}\right], \quad (35)$$

$$\omega_m^2(q)=\varepsilon_m^2+2\overline{e}^2\varepsilon_0 n|m|\left[1-\frac{a^2q^2}{4(|m|-1)}\right] \qquad (m=\pm 2,\pm 3,\dots). \quad (36)$$

The wave dispersion with the spectrum (35), (36) is anomalous. The cut-off frequencies in the spectrum of these waves are

$$\omega_m^2(0)=\varepsilon_m^2+2\overline{e}^2\varepsilon\, n|m|. \quad (37)$$

To compute the decrement of the damping of the modes with the numbers $|m|>0$, we shall make use of the formula (30), (34) and of the imaginary part of the polarization operator (25) that is equal in the long-wavelength limit to:

$$\operatorname{Im}P_m=-\frac{N}{|m|}\sqrt{\frac{\pi\beta}{\varepsilon_0}}\operatorname{sh}\frac{\beta\omega}{2}. \quad (38)$$

Then the decrement of the damping of the waves with the spectrum (35), (36) is as follows:

$$\gamma_1(q)=\frac{\overline{e}^4\varepsilon_0 n^2}{\omega_1(q)}\sqrt{\frac{\pi\beta}{\varepsilon_0}}sh\frac{\beta\omega_1(q)}{2}\left[1+\frac{(aq)^2}{2}\ln\frac{aq}{2}\right]^2, \quad (39)$$

$$\gamma_m(q)=\frac{\bar{e}^4\varepsilon_0 n^2}{|m|\omega_m(q)}\sqrt{\frac{\pi\beta}{\varepsilon_0}}\,\mathrm{sh}\frac{\beta\omega_m(q)}{2}\left[1-\frac{a^2q^2}{4(|m|-1)}\right]^2 \qquad (m=\pm2,\pm3,\ldots). \tag{40}$$

The ratio ${\gamma_m}/{\omega_m}$ decreases with increasing mode number in proportion to $|m|^{-3}$.

Magnetoplasma waves

In the magnetic field, which is parallel to the cylinder axis, the energy of electron is given by Eq. (7), while the polarization operator of degenerate electron gas is derived in Eq.(9), (10).

Let us obtain the solution of the dispersion equation (4) in the ultra-quantum limit, when only two subzones are filled with the numbers $(m,\sigma)=(0,\pm)=0^{\pm}$. In this case, there are lobe and triangular spectral windows in the wave vector-frequency plane besides the parabolic spectral window which is found between the parabola $\omega=q\upsilon_0^{+}-\omega_q$ and the axis q, and also besides the collisionless wave damping region above the parabola $\omega=q\upsilon_0^{-}+\omega_q$ and to the right of the parabola $\omega=-q\upsilon_0^{-}+\omega_q$. The lobe spectral window is confined within the parabolas $\omega=q\upsilon_0^{+}+\omega_q$ and $\omega=q\upsilon_0^{-}-\omega_q$, and the triangular spectral window is limited by the parabolas $\omega=-q\upsilon_0^{+}+\omega_q$, $\omega=q\upsilon_0^{-}-\omega_q$ and the axis q. The coordinates of the lobe window uppermost boundary in the plane (q,ω) are $\left(m_*\left(\upsilon_0^{-}-\upsilon_0^{+}\right),\frac{m_*}{2}\left(\upsilon_0^{-2}-\upsilon_0^{+2}\right)\right)$. If $q<m_*\left(\upsilon_0^{-}-\upsilon_0^{+}\right)$, solutions for the equation (4) exist in the lobe spectral window above the parabola $\omega=q\upsilon_0^{-}+\omega_q$ and in the regions of collisionless damping. The dispersion law of magnetoplasma wave in the lobe spectral window is

$$\omega_0(q)=q\frac{\upsilon_0^{+}+\upsilon_0^{-}}{2}+\omega_q\mathrm{cth}\frac{2\pi^2q}{m_*\upsilon_0(q)}-\left[\frac{q^2}{4}\left(\upsilon_0^{+}-\upsilon_0^{-}\right)^2+\frac{\omega_q^2}{\mathrm{sh}^2\frac{2\pi^2q}{m_*\upsilon_0(q)}}\right]^{1/2}. \tag{41}$$

At small values of q the spectrum of this wave is linear. With increasing q, the dispersion curve (41) comes closer to the lobe boundary $\omega=q\upsilon_0^{+}+\omega_q$. The appearance of the quantum number of the magnetic flux ${\Phi}/{\Phi_0}$ in the formula (7) has its effect on the wave spectrum brought for consideration in Section I. In particular, the formula (19), in the presence of magnetic field and taking into account (26), takes the following form:

$$\omega_0(q)=\frac{4\bar{e}^2}{\pi}q\ln\frac{2}{qa\mathrm{e}^{\gamma}}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left(1+\frac{1}{\pi}\sqrt{\frac{\varepsilon_0}{\mu_0}}\sum_{l=1}^{\infty}\frac{1}{l}\sin 2\pi l\sqrt{\frac{\mu_0}{\varepsilon_0}}\cos 2\pi l\frac{\Phi}{\Phi_0}\right). \tag{42}$$

This formula does not account for the spin splitting of the levels. Besides the oscillations of the de Haas-van Alfven type, which were considered in Section I, appearing with changing μ_0, the wave frequency undergoes oscillations of the Aharonov-Bohm type appearing with changing magnetic field. The period of these oscillations is equal to one quantum of the magnetic flux Φ_0. In the magnetic field, the linear electron density n is connected with the chemical potential μ_0 via the relation:

$$n = \frac{2}{\pi}\sqrt{2m_*\varepsilon_0}\sum_m\left[\frac{\mu_0}{\varepsilon_0} - \left(m + \frac{\Phi}{\Phi_0}\right)^2\right]^{1/2}.$$

The frequencies of modes with the numbers $|m| > 0$ also undergo the Aharonov-Bohm type oscillations. In the formulae (22), (23), under the sign of the sum $\sum_l$, it appears the factor $\cos 2\pi l \frac{\Phi}{\Phi_0}$.

Conclusions

The energy spectrum (2) of electron on the nanotube surface is a set of one-dimensional subzones, the positions of the boundaries of which are not equidistant. As a result, the density of states (3) oscillates with changing $\sqrt{\varepsilon}$ with the period $\left(\sqrt{2m_*}a\right)^{-1}$. This accounts for the oscillations of plasma wave frequencies in the degenerate electron gas with changing value of $\sqrt{\mu_0}$. These oscillations resemble the de Haas-van Alfven oscillations of electron gas magnetization that emerge with changing magnetic field strength. This distinction is attributed to existence of the non-equidistant boundaries of the subzones in the spectrum (2). The cause of the oscillations is a jumping density variation of the density of states, when the Fermi level traverses the subzone boundaries. The measurement of the period of the oscillations allows to obtain $\sqrt{m_*}a$. To be able to observe these oscillations, it is necessary to have the capability to change the Fermi level of electrons on the tube like it is done in the two-dimensional electron gas [12]. While measuring the plasmon frequencies on tubes with different values of m_*, a, μ_0, a spread of the frequency values should be expected, caused by the oscillations.

In a magnetic field, which is parallel to the cylinder axis, the Aharonov-Bohm oscillations appearing with changing magnetic field will be superposed on the magnetoplasma wave frequency oscillations of the de Haas-Van Alfven type. The reason for the former oscillations differs. Their period does not depend on μ_0 and is equal to one quantum of the magnetic flux Φ_0. It related to the area πa^2, which is occupied by the projection of the orbit of electron to the plane $z = 0$, and it does not depend on the energy of the electron.

The oscillations described here can be observed in experiments on the measurement of the cross-section of scattering of the light and electrons by plasma waves on the nanotube surface.

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References

1. M.S. Dresselhaus, G. Dresselhaus, and P.C. Eklund, Science of Fullerenes and Carbon Nanotubes (Academic, New York, 1996).
2. L.I. Magarill, A.V. Chaplik, M.V. Entin, Uspehi fizicheskikh nauk **175**, 995 (2005).
3. R.C.T. da Kosta, Phys. Rev. **A23**, 1982 (1981).
4. L.I. Magarill, D.A. Romanov, A.V. Chaplik, JETP **113**, 1411 (1998).
5. V.A. Geyler, V.A. Margulis, A.B. Shorohov, JETP **115**, 1450 (1999).
6. R.Z. Vitlina, L.I. Magarill, A.V. Chaplik, JETP letters **86**, 132 (2007).
7. G.D. Mahan, Phys. Rev. **B69,** 125407 (2004).
8. A.M. Ermolaev and G.I. Rashba, J. Phys.: Condens. Matter **20**, 175212 (2008).
9. V.V. Rotkin, R.A. Suris, Fizika tverdogo tela **36**, 3569 (1994).
10. H. Bateman, A. Erdelyi, Higher Transcendental Functions (Mc Graw-Hill Book Company, New York, 1953), Vol. 1,2.
11. A.P. Prudnikov, U.A. Brichkov, O.I. Marichev, Integrali i ryadi (Nauka, Moskva, 1981).
12. T. Ando, A. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).