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# Zero-field steps in long Josephson junctions

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IV-characteristics of a long Josephson junction with normal edges were calculated. Boundary conditions were modeled as shunts which consist of resistances, inductances and capacitances. IV-curves revealed self-induced resonant steps (zero-field steps) near voltages corresponding to frequencies of even-order modes of geometrical resonances (even Fiske steps). The dependence of the difference between resonant frequencies on the reciprocal length of the junction is in agreement with the theory. Conditions of the application of the model for the description of intrinsic Josephson junctions in high-temperature superconductors are discussed.
**Keywords:** Josephson junctions, high-temperature superconductors, transmission line, resonant modes.

Розраховано вольт-амперні характеристики довгого джозефсонівського контакту з нормальними краями. Граничні умови моделювалися як шунти, що мають електричний опір, індуктивність та ємність. На вольт-амперних характеристиках виявлено самоіндуковані резонансні сходинки (сходинки нульового поля) поблизу напруг, які відповідають частотам парних мод геометричних резонансів (парні сходинки Фіске). Залежність різниці між частотами резонансів від зворотної довжини контактів узгоджується з теорією. Обговорюються умови використання моделі для опису внутрішніх джозефсонівських контактів у високотемпературних надпровідниках.
**Ключові слова:** джозефсонівські контакти, високотемпературні надпровідники, довга лінія, резонансні моди.

Рассчитаны вольт-амперные характеристики длинного джозефсоновского контакта с нормальными краями. Граничные условия моделировались как шунты, которые имеют электрическое сопротивление, индуктивность и ёмкость. На вольт-амперных характеристиках найдены самоиндуцированные резонансные ступеньки (ступеньки нулевого поля) вблизи напряжений, которые отвечают частотам чётных мод геометрических резонансов (чётные ступеньки Фиске). Зависимость разницы между частотами резонансов от обратной длины контактов согласуется с теорией. Обсуждаются условия использования модели для описания внутренних джозефсоновских переходов в высокотемпературных сверхпроводниках.
**Ключевые слова:** джозефсоновские переходы, высокотемпературные сверхпроводники, длинная линия, резонансные моды.

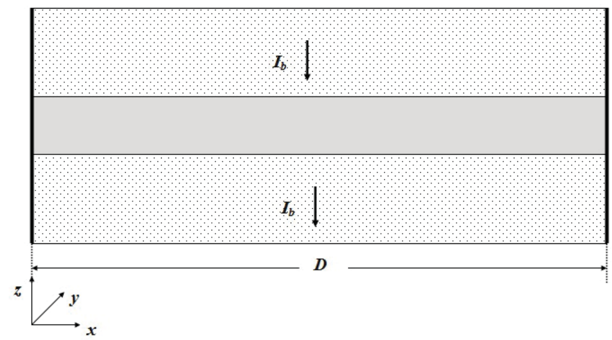
## Introduction

It was recently found [1-3] that intrinsic Josephson junctions in mesas of high-temperature superconductors (HTSC) revealed coherent emission at voltages corresponding to frequencies of geometrical resonances of microwaves. Self-resonant steps in IV-characteristics were observed at these resonances [1]. In experiments [1-3] mesas were biased only by the direct current. External magnetic field was not applied. The theory of resonant steps in IV-curves of junctions placed in external magnetic field (so-called Fiske steps) was developed in Refs. 4, 5. However, the appearance of self-resonant steps in the absence of external applied magnetic field (so-called zero-field steps) was also observed experimentally [6,7]. The proposed mechanisms of the vortex motion inside the junction [8] allowed explaining the appearance of zero-field steps at even resonant frequencies. The quantitative explanation of the appearance of zero-field steps referring to the general theory [4, 5] was made in Ref. 9.

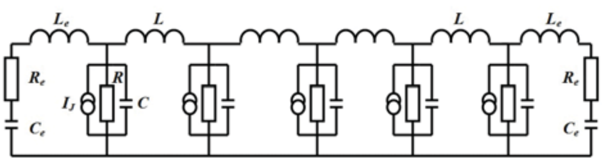
In theoretical investigations the resistive shunt over the whole stack of intrinsic Josephson layers was used to model their properties [10]. Earlier we showed theoretically that coherent radiation appeared at self-resonant steps of the IV-curve of the chain of Josephson junctions placed in the transmission line [11-12]. The existence of self-resonant steps was proved experimentally [13, 14]. In the present paper we show that in the long Josephson junction zero-field steps can appear if plausible boundary conditions are fulfilled. The described below conditions can be valid in high-temperature superconductors. Superconducting properties of these systems are strongly dependent on the content of the oxygen in superconducting layers. Diffusion of the oxygen out of superconducting layers leads to the change of the critical temperature at ends [1]. Moreover, one can expect that some small parts of superconducting layers at ends of the superconductor can be in the normal state. One can model boundary conditions of an intrinsic Josephson junction in such a superconductor as shunts consisting of inductances, resistances and capacitances (so-called RLC-shunts). We used the 'lumped' model of a long Josephson junction [15] in which the junction was represented as a set of radio-technical elements. The RLC-shunt is a model of the state when the insulating barrier is placed between non-superconducting parts of layers at the ends of the tunnel junction. We calculated IV-characteristics of the junction with the mentioned boundary conditions and showed that zero-field steps appeared at voltages corresponding to even modes of geometrical resonances in the junction. We also discussed the dependence of the distance between zero-field steps on the reciprocal dimension of the system.

**The model.**

The geometry of the long Josephson junction is presented in Fig. 1a, and the high-frequency scheme of the junction is shown in Fig. 1b. The junction with the critical current $I_{ctot}$ is divided into $n$ 'elementary junctions' with critical currents $I_{ctot}/n$. 'Elementary junctions' in Fig. 1b are presented in the range of the resistively and capacitively shunted model [15] with the resistance $R_k$, capacitance $C_k$ and the source of the Josephson current $I_{ck}\sin\varphi_k$ for the $k$-th junction ($k$ = 1…$n$). In the following consideration we assume that for all $k$ the condition $C_k = C$, $R_k = R$ and $I_{ck}=I_c$ is valid. 'Elementary junctions' are divided by the distance $\zeta = \overline{c}\sqrt{CL}$, where $\overline{c}$ is the velocity of light in the




*a*




*b*

*Fig. 1.* (a)- the long Josephson junction. Insulator is shaded. Black solid lines at ends of the junction symbolize the $R_eL_eC_e$-shunting discussed in the paper. (b)- the electrical scheme of the junction. The shown circuit consists of 4 cells and 5 junction. The additional boundary circuit contours are also shown. In calculations the number of cells was 200-500.

junction and $L$ is the inductance of the 'elementary cell' between junctions. Then the current conservation conditions for junctions together with equations for circulating currents in loops between junctions are as follows:

$$\frac{\Phi_0 C}{2\pi}\frac{d^2\varphi_k}{dt^2} + \frac{\Phi_0}{2\pi R}\frac{d\varphi_k}{dt} + I_c \sin\varphi_k = = I_b - I_k^R + I_{k+1}^R , \quad (1a)$$

$$I_k^R = -\frac{\Phi_0}{2\pi L}\left[\varphi_{k-1} - \varphi_k\right] , \quad (1b)$$

where $k = 2…n$-1, $\varphi_k$ is the phase difference across the $k$-th junction and $\Phi_0$ is the quantum of magnetic flux.

It can be shown that equations (1a) and (1b) represent the written in finite differences main equation of the electrodynamics of the long Josephson junction [15]. Let us introduce parameters $\overline{c}^2 = 1/\left(\mu_0 C^S d\right)$, $\lambda_J = \left[\hbar/\left(2\mu_0 e J_c d\right)\right]^{1/2}$ and $\beta = 1/\left(RC\right)$ with $d = l + \lambda_{L1} + \lambda_{L2}$ is the sum of London depths of penetration of electromagnetic field into the first and the second superconductors $\lambda_{L1}$ and $\lambda_{L2}$, correspondingly, $l$ is the thickness of the insulator barrier, $\mu_0$ is magnetic permeability of vacuum, $J_c$ is the density of the critical current, $C^S$ is the capacitance per unit area and $\lambda_J$ is the Josephson depth of penetration of magnetic field in the junction. With the use of these parameters for $\zeta\to 0$ one obtains the equation

$$\frac{\partial^2\phi(x,t)}{\partial x^2} - \frac{1}{\overline{c}^2}\left[\frac{\partial^2\phi(x,t)}{\partial t^2} + \beta\frac{\partial\phi(x,t)}{\partial t}\right] = = \frac{1}{\lambda_J^2}\left[\sin\phi(x,t) - \frac{I_b}{I_c}\right] . \quad (2)$$

Note that the well-known sine-Gordon equation [16] is obtained from Eq. (2) if terms containing β and $I_b/I_c$ are omitted.

There are not analytical solutions of Eq. (2). Therefore, we solved the corresponding system of Eqs. (1a) and (1b) numerically.

To form boundary conditions, additional contours with capacitances $C_{ek}$, inductances $R_{ek}$ and inductances $L_{ek}$ were added to both ends of the line. The Kirchhoff's circuit laws for these contours are following:

$$L_{ek}\frac{d^2q_k}{dt^2} + R_{ek}\frac{dq_k}{dt} + \frac{q_k}{C_{ek}} = = \mp\frac{\Phi_0}{2\pi}\frac{d\varphi_k}{dt} , \quad k = 1, n, \quad (3)$$

where the upper signs in the right side relate to $k$ =*1* (the left end of the junction), the lower signs relate to $k = n$ (the right side of the junction) and $q_k$ is the charge flowing through the inductance $L_{ek}$. In the beginning of calculations we set the length D and the width W of the long junction, the density of the critical current $J_c$, the critical voltage $V_c$, the inductance of the junction per unit of length $L_{ul}$ and the velocity of light in the junction $\bar{c}$ . Note that our model is quasi-one-dimensional, so the introduction of the width $W$ is made merely to calculate $J_c$. Then the junction was divided into $n$ cells with the length $\zeta = D/n$ . Parameters of 'elementary junctions' and cells were equal to $I_c = J_c \cdot W \cdot \zeta$, $R = V_c/I_c$, $L = L_{ul} \cdot \zeta$ and $C = \zeta^2 / \left(\bar{c}^2 L\right)$. The last relation follows from the constant velocity of light in the long junction that should provide the transmission line (see Fig. 1b). Note that the value of the McCumber parameter $\beta_C = (2\pi I_c R^2_n C)/\Phi_0$ does not depend on $\zeta$. Then Eqs. (1a) and (1b) with boundary conditions (3) were solved by the method of Runge-Kutta for different values of the bias current. IV-characteristics were obtained in calculations. The voltage over the junction was calculated as $V = \frac{\Phi_0}{2\pi n} \left\langle \sum_{k=1}^{n} \frac{d\varphi_k}{dt} \right\rangle$ , where the sign $\langle ... \rangle$ means full averaging over the large interval of time $T_J >> (1/\nu_J)$ with $\nu_J$ is the characteristic frequency of Josephson generation.

**Results and discussion.**

Values of parameters for calculations were chosen as follows. The parameter $D$ was changed from $50 \cdot 10^{-6}$ m to $650 \cdot 10^{-6}$m. Other parameters were $W = 300 \cdot 10^{-6}$ m, $J_{ctot} = 10^5$ A/m$^2$, $L_{ul} = 1.25 \cdot 10^{-8}$ H/m, $V_c$ = 4.74 mV, $\bar{c} = 6.708 \cdot 10^7$ m/sec [1], $\beta_C = 40.34$, $n = 200 \div 500$. Calculated parameters $\lambda_J$ and $d$ were $2.97 \cdot 10^{-5}$ m and $2.98 \cdot 10^{-6}$ m, correspondingly. The wavelength of the first harmonic of Josephson generation $\lambda = \bar{c}\Phi_0 / V_c$ at the characteristic voltage $V_c$ was equal to $\lambda = 29.3 \cdot 10^{-6}$ m.

Now we discuss the choice of parameters. We chose the described values because they are close to corresponding parameters of intrinsic Josephson junctions in high-temperature superconductors [17]. At the same time, it is known that the model of the resistively and capacitively shunted Josephson junction is not quite adequately describe intrinsic junctions [17], so some parameters like $\lambda_J$ and $\beta_C$ are different from those in HTSC. We note also that the thickness of the superconducting layer in HTSC is only two or three tenths of nanometer, whereas in the model we use the value $d$ of order of micrometers. Therefore, we found IV-characteristics of the Josephson junction which has some characteristic parameters of intrinsic Josephson junctions. For the adequate description of intrinsic junctions the present model should be changed to take into account the narrow layers of the superconductor and the periodic layered structure of HTSC.

For boundary conditions we chose values of $R_{ei}$= 100 Ohm, $L_{ei}$ = 0.2 pH, $C_{ei}$ = 3pF for both boundaries with $i$=1 and $i$=$n$ which correspond to approximately 2-5 micrometers of the non-superconducting part at edges. The resonant frequency of the $L_{ei}C_{ei}$-resonance contour is much larger than the characteristic frequency of Josephson generation. We checked that the choice of values of the parameter $n$ did not influence results of calculations.

The example of the IV-characteristic of the junction with the length $1.80 \cdot 10^{-4}$ m is shown in Fig. 2. It is seen that there are zero-field steps in the IV-characteristic. In Fig. 2 these steps are numbered from 1 to 4. Note that Eqs. (3) describe additional ac current contours through which some electromagnetic excitations enter the long junction. The frequency of these excitations coincides with the frequency of Josephson generation, and they propagate along the long junction and reflect from its ends. Resonance voltages in this model are described by such an expression [8,9]:

$$V_m = \frac{\Phi_0 \bar{c} m}{D}, \tag{4}$$

where $m$=1,2,3… is an integer. These voltages correspond to even Fiske steps $V_p = (\Phi_0 \bar{c} p)( 2D)$, with $p$ is an integer, so $p = 2m$. We noted positions of $V_m$ by short arrows in Fig. 2. It is seen that positions of $V_m$ do not coincide with steps obtained from solutions of Eqs. (1a)-(1b). More

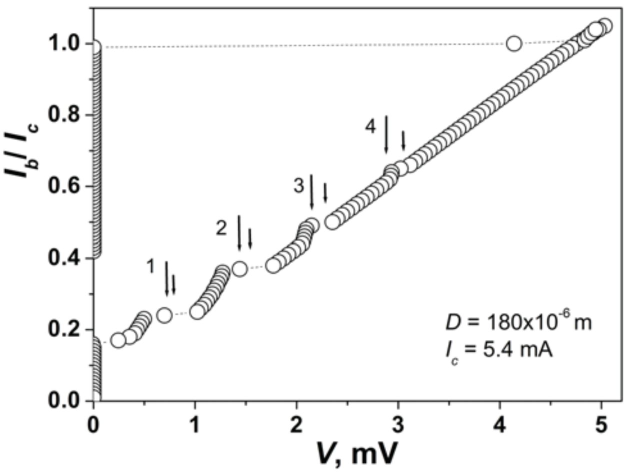



*Fig. 2.* The IV-characteristic (circles) of the junction with the length 180 micrometers. The jump of voltages at $I_b = I_c$ is marked by dashed line. Steps are numbered from $m$ =1 to $m$ = 4. Short arrows show positions of voltages $V_m = \Phi_0 \bar{c} m / D$, long arrows show equidistant steps found empirically.

adequate equidistant steps were found from data of Fig. 2 empirically (they are shown by long arrows). We explain this deviation by the influence of higher harmonics of Josephson generation in the hysteretic region.

To show that the distance between steps in IV-characteristics is proportional to the reciprocal length of the junction (see Eq. (4)), we calculated averaged values of $\Delta V = V_m - V_{m-1}$ for the junction with the given $D$ and plotted the difference of frequencies $\Delta f = \Delta V/\Phi_0$ on the value of $1/D$ (Fig. 3). For the comparison the dependence $\Delta f' = \frac{\Delta V}{\Phi_0} = \frac{\bar{c}}{D}$ obtained from Eq. (4) is also plotted in

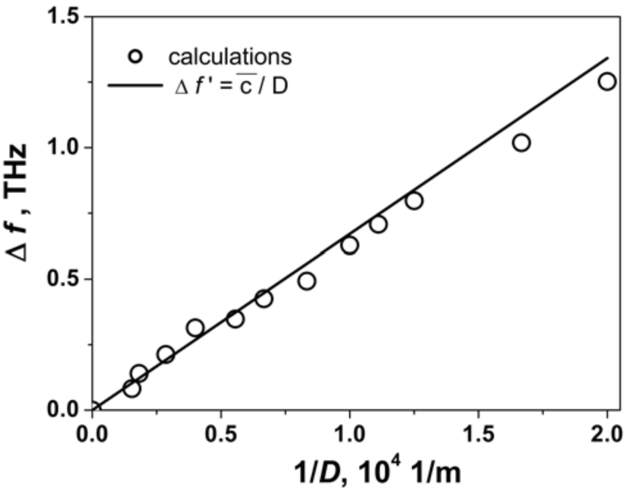



*Fig. 3.* The dependence of $\Delta f$ on the reciprocal width of the junctions $1/D$ (circles). Solid line is the dependence $\Delta f' = \bar{c}/D$.

the same figure. It is seen from Fig. 3 that data obtained from the numerical model is in accordance with the theoretical plot.

We found that the investigated effect of zero-field steps is observed in the very wide range of parameters $R_{ek}$, $L_{ek}$ and $C_{ek}$ for boundary conditions Eq. (3) providing and $R_{ek} \neq \infty$. Mentioned parameters can change by many orders of magnitude, but the change of both positions of steps and heights of steps is small. This result proves the supposition that the additional boundary contours plays the role of generators of excitations and makes more probable our supposition that described boundary conditions are responsible for the appearance of zero-field steps in HTSC. We checked that zero-field steps appeared also if ends of the long junction were superconducting but with slightly different critical currents. Thus, self-resonant steps can appear if electromagnetic excitations enter the junction from ends.

## Conclusions

In the present paper we calculated IV-characteristics of the long Josephson junction with boundary conditions which correspond to shunts which consist of resistances, inductances and capacitances. These boundary conditions model the single intrinsic Josephson junction with the insufficient content of the oxygen at edges. IV-curves reveal zero-field resonant steps. Frequencies at which these steps are observed correspond to the even geometrical resonances in the structure (even Fiske steps). We analyzed the dependence of the difference between frequencies of steps on the reciprocal length of the junction. This dependence is in agreement with predictions of the theory.


1. L. Ozyuzer, A. E. Koshelev, C. Kurter, N. Gopalsami, Q. Li, M. Tachiki, K. Kadowaki, T. Yamamoto, H. Minami, H. Yamaguchi, T. Tachiki, K. E. Gray, W.-K. Kwok, U. Welp. Science, 318, 1291 (2007).
2. B. Gross, S. Guénon, J. Yuan, M. Y. Li, J. Li, A. Ishii, R. G. Mints, T. Hatano, P. H. Wu, D. Koelle, H. B. Wang, and R. Kleiner. Phys. Rev. B 86, 094524 (2012).
3. Mengyue Li, Jie Yuan, Nickolay Kinev, Jun Li, Boris Gross, Stefan Guénon, Akira Ishii, Kazuto Hirata, Takeshi Hatano, Dieter Koelle, Reinhold Kleiner, Valery P. Koshelets, Huabing Wang, and Peiheng Wu. Phys. Rev. B 86, 060505(R) (2012).
4. I. O. Kulik. Zh. Eksp. Teor. Fiz. Pis. Red., 2, 134 (1965) [JETP Lett., 2, 84 (1965)].
5. I. O. Kulik. Zh. Tekh. Fiz., 37, 157 (1967) [Sov. Phys. Tech. Phys., 12, 111 (1967)].
6. I. M. Dmitrenko, S. I. Bondarenko and T. P. Narbut. Zh. Eksp. Teor. Phys. 57, 1513 (1969). [Sov. Phys. JETP, 30, 817 (1970)].
7. J. T. Chen, T. F. Finnegan and D. N. Langenberg. Physica (Utr.) 55, 413 (1971).
8. T. A. Fulton and R. C. Dynes. Solid State Commun., 12, 57 (1973).
9. K. Takanaka. Solid State Commun., 29,443 (1979).
10. B. Gross, J. Yuan, D. Y. An, M. Y. Li, N. Kinev, X. J. Zhou, M. Ji, Y. Huang, T. Hatano, R. G. Mints, V. P. Koshelets, P. H. Wu, H. B. Wang, D. Koelle, and R. Kleiner. Phys. Rev. B 88, 014524 (2013).
11. Alexander Grib and Paul Seidel. Low Temp. Phys. 38, 321 (2012).
12. Alexander Grib and Paul Seidel., J. Phys.: Conf. Ser., 507, 042038 (2014).
13. A. Grib, M. Mans, M. Büenfeld, J. Scherbel, F. Schmidl, H. Schneidewind, and P. Seidel. IEEE Trans. Appl. Supercond., 24, 1800205 (2014).
14. Alexander Grib, Michael Mans, Jens Scherbel, Matthias Büenfeld, Frank Schmidl and Paul Seidel. Supercond. Sci. Technol., 19, S200 (2006).
15. K. K. Likharev. Dynamics of Josephson junctions and circuits, Gordon and Breach, Philadelphia. (1991), 750 p**.**
16. Antonio Barone and Gianfranco Paternò. Physics and applications of the Josephson effect, A Wiley-Interscience Publication, New York (1982), 529 p.
17. R. Kleiner and P. Müller, Phys. Rev. B, 49, 1327 (1994).