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"FRACTALIZATION" OF THE SOLID-STATE PHYSICS

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It has been demonstrated that in modern solid-state physics, in accordance with the nonlinear and systems paradigms formulated by L. F. Chernogor in the late 1980s, many processes in open, nonlinear, dynamical systems are occurred to be very complex, nonlinear, short-time, ultra-wideband, or fractal.

Moreover, from the point of view of the fractal paradigm put forward in the early 2000s by V. V. Yanovsky, fractality is generally considered as one of the fundamental properties of the surrounding world. Therefore, the study of fractal characteristics, in particular, of natural physical processes and objects in the field of solid-state physics, is appeared to be relevant, interesting, useful and promising.

The main stages of the development of the fractal approach in general are briefly discussed. It is pointed that it was occurred to be too hard to formulate a strict and clear mathematical definition of a fractal. The definition of a fractal been formulated by. K. Falconer and been agreed by the most speciallists as the best for practical usage is considered in detail. Main numerical characteristics of a fractal as well as the modern classification of fractals are considered. The general principal differences existing between the mathematical fractals and physical (or natural) ones as well as between the mono-fractals and the multi-fractals are clearly explained. A review of the main methods for estimating the Hurst fractal dimension is provided.

The main existing directions of "fractalization" of modern solid-state physics are highlighted. Relevant examples are given.

It is noted that images with certain fractal properties play an important role in the "fractalization" of solid-state physics. The use of the two-dimensional Weierstrass function is proposed for modeling images with fractal properties. As an example, the modelling of the unidirectional twin structure observed in the $YBa_2Cu_3O_{7-\delta}$ crystal is considered. A comparison between the model image based on one-dimensional Weierstrass function with defined value of the Hurst fractal dimension and real experimental one is demonstrated.

Keywords: *nonlinear paradigm, fractal paradigm, fractal approach, Hurst fractal dimension, fractal image, two-dimensional Weierstrass function.*

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INTRODUCTION

It is well known that being a valuable and important part of modern physics, the solid-state physics studies the open, dynamical, non-lianear, complex, quantum-mechanical systems, which consist a number of the open, non-lianear, complex, quantum-mechanical subsystems (see, for example, [1, 2]). These subsystems interact strongly with each other and, therefore, can be successfully described in bounds of so called the non-linear and the system paradigms been formulated in 1980s by Prof. L. F. Chernogor (Kharkiv, Ukraine). The non-linear and the system paradigms claim that many processes as natural as artificial origin which have been generated in the open, complex, non-linear, dynamical system on influence, in particular, of the powerful, non-stationary source of energy release are appeared to be short-time, ultra-wideband, non-linear and fractal ones [3].

Namely the last possibility is appeared to be very important in bounds of present paper. Moreover, as it has been claimed in 2003 in so called fractal paradigm proposed by Prof. V. V. Yanovsky (Kharkiv, Ukraine), the fractality at all is appeared to be one of the most important initial properties of the world around us [4, 5].

The purposes of the paper are to demonstrate a present state of the "fractalization" process occurring in the solid-state physics and to attract the attention of the specialists to new occurring possibilities for successful investigations.

FRACTAL APPROACH EVOLUTION

Now it is well known that the term "fractal" was introdused by world famous American mathematican and physicist B. Mandelbrot (1924–2010) in 1975 in a rather philosophical essay [6] published initial in French. In 1977, this work was translated into English [7], but did not gain a wide popularity. For this reason, the true "fractal revolution" is considered to have begun in 1982 with the publication of a book [8] that gained worldwide fame.

It is very important to point that comparing with traditional Euclidian geometry, which was a basis of all human civilization up to present time, the fractal concept is principally a new approach to describing of the surrounding world [8]. In comparison with familiar triangles, rectangles, circles, pyramids, prisms, spheres etc, the fractals being very complex fractured and disordered structures are appeared to be an unique tool, which has much more high level of complexity, and, thus, allows describing the world around us much more accurately and adequately [8]. Application of the fractal approach called frequently as "fractalization" requires to provide a real revolution in the chosen investigation direction, where it is assumed to be provided.

As it should be waited, the process of "fractalization" in different brunches of modern science and engineering was neither simple, nor smooth and unambiguous (see, for example, [9]). Historically, the "fractalization" process can be divided on the next four stages [9].

During first of them called as "The era of monsters" (mid-19th century – early 1960s) in mathematics, the set of first fractals had been created. For example, in 1873, the first continuous nowhere differentiable function was proposed by world famous German mathematician K. Weierstrass [10]. Moreover, there is an information that famous Italian mathematician B. Bolzano had constructed a function with similar properties, probably before 1830, although this fact appeared to be known only in 1930 [11]. Nevertheless, despite of appearance of the works related to fractal ideas and created by such worldwide known specialists as K. Weierstrass (1815–1897), G. Cantor (1845–1918), G. Peano (1858–1932), A. Lebesgue (1875–1941), F. Hausdorff (1868–1942), A. Besicovitch (1891–1970), B. Bolzano (1781–1848), E. Cesaro (1859–1906), H. von Koch (1870–1924), W. F. Ostgood (1864–1943), W. F. Sierpinski (1882–1969), K. Menger (1903–1985), P. S. Urysohn (1898–1924), A. Poincaré (1854–1912), G. Riemann (1826–1866), J. Darboux (1842–1917), L. Bachelier (1870–1946), F. d'Alb (1868–1933), P. Levy (1886–1971), L. Richardson (1881–1953), J. Zipf (1902–1950), G. Hurst (1880–1978), G. Julia (1893–1978), P. Fatu (1878–1929) etc, the point of view of the world scientific society about fractals at all had remained extremally negative and disparaging. In particular, in 1893, a real example of such non-constructive approach had been shown by another worldwide known mathematician C. Hermite (1822–1901), which called the fractals even as "monsters" [12]

Thus, that time the fractals had been considered as a pathology of interest only to researchers who abused mathematical quirks, and not to true scientists at all [6–8, 13]. By the way, even today some respected specialists repeat sometimes these ideas (see, for example, [14]). A wish of authors to prevent the quite possible appearing of this unacceptable situation among the solid-state physics specialists is another (hidden) aim of this paper. On our opinion, most likely, the main cause of negative relation of somebody to the fractal concept can be simply explained rather by insufficient amount of available information and time needed to understand it than by existence of real difficulties, troubles and contradictions.

The second stage called as "The preparatory stage" (early 1960s – 1975) deals with the information accumulation. Namely that times, B. Mandelbrot, in his own words [8, 15], collected and analyzed any information related the fractals in any way, tried to generalize it and to do somewhat acceptable conclusions.

The third stage named as "The stage of formation and development" (1975 – early 2000s), more likely, was the most difficult and problematic as well as at first time the fractal approach was appeared to be misunderstood and unaccepted by most scientific society.

However, thanks to Mandelbrot's enormous efforts, connected, first, with publication of a number of his new books and papers (see, for example, [16–21]) and, second, with many organized scientific conferences on fractal themes, the situation gradually began to change. Even his consistent opponents gradually turned into his no less ardent supporters. In that time, many different objects and structures have been discovered not only in physics, but also in astronomy, electronics, materials science, signal and image processing, computer networks, chemistry, physical chemistry, biology, physiology, psychiatry, biophysics, biochemistry, medicine, geology, geography, geophysics, geochemistry, climatology, meteorology, soil science, ecology, computer science, finance, archeology, architecture and design, and even in music and literary works, as well as in painting (for example, in the paintings of M. Escher and S. Dali, in the canvases of K. Hokusai, A. Durer and Leonardo da Vinci), urban culture, psychology, management and other humanities and social sciences. In fact, it was a real worldwide triumph of the fractal concept, which has been called by some authors (see, for example, [22]) as “a fractal fever”.

Today we are living on the fourth (the last, but not the least!) stage called as “The modern stage” (early 2000s – present day). As a result of the successful numerous applications of the methods of fractal geometry, fractal analysis, fractional calculus, in various fields of science and engineering, many separate “fractal” directions are formed and distinguished, for example, fractal electrodynamics, fractal radiophysics, fractal radar, fractal physics, etc. (see, for example, [4, 9, 23–26]). Many real effective technologies based on the use of the fractal approach have been created and implemented in practice. Thus, the fractals have been finally transformed from an abstract mathematical idea, understandable only to a narrow circle of specialists, into a significant and powerful force capable of changing the surrounding world.

More close review regarding a modern state of the fractal approach implementation in science and engineering and covering a number of useful references can be found, for example, in the papers [9, 23–25].

It is necessary to say a few words about the generally accepted terminology, which will be used here later.

It seems to be some surprising and strange, but it is very difficult to define a fractal strongly from mathematical point of view. Moreover, in 1999 after some attempts, B. Mandelbrot just refused to do this at all [17]. So, although on the present day, there are near ten such definitions [9], as a rule, the most generally accepted from them was recognized as the one given by K. Falconer in 1990 [9]. According that [27], a set R is called a fractal if, among its properties, there are the following: 1) the set R has a fine structure, i.e. it is detailed at the smallest scales; 2) the set R is a sufficiently irregular structure that it could be described by traditional geometric methods (Euclidean or Lobachevsky geometry, etc.) both at the local level and at the level of the entire structure; 3) the set R has a property of self-similarity both in an approximate form and in a statistical one; 4) usually the fractal dimension of a set R (defined in some way) turns out to be greater than its topological dimension; 5) in most cases the set R is defined in a very simple way, possibly recursively. In 1996, K. Falconer has added the sixth property: 6) often the set R is of natural origin [28].

On the origin, all the fractals can be divided on mathematical and physical (or natural) ones (see, for, example [7, 8, 13, 15, 16]). Mathematical fractal is a fractal, which exists in imagination of the mathematicians only. This is an ideal abstract object, which has the self-similarity (or self-affinity) properties on the scales from some maximal value to the minimal value, which towards fundamentally to zero. Physical fractal is a fractal really existing in surrounding world. It is important to point that it’s minimal scale, in which their fractal properties exist, is appeared to be greater than zero. A physical fractal can have as natural origin, as artificial one. All mathematical fractals can be considered as convenient models of the really existing physical fractals.

Both the mathematical and the physical fractals are occurred to be divided on deterministic (or regular) and stochastic (or unregular random) ones (see, for, example [7, 8, 13, 15, 16]). Regular fractal is a fractal, which has a fully deterministic algorithm for their creation. The regular fractals are occurred to be divided on algebraic and geometric ones. For geometric fractals, self-similarity (or self-affinity) is covered in their geometric structure, for algebraic fractal, this occurs for their numerical characteristics. At the same time, stochastic fractal is a fractal, in which there is at least one stochastic parameter. The stochastic fractals are self-similar (or self-affine) in stochastic sense. In the most cases, all natural and artificial physical processes, objects and phenomena are appeared to be physical fractals.

To describe the fractal properties of the objects investigated, a set of numerical characteristics are used (see, for, example [7, 8, 13, 15, 16]). The most known and usable characteristic is a fractal dimension. For mathematical fractals, the Hausdorff – Besicovitch dimension is usually applied. For physical fractals, a number of other fractal dimensions are used. There are many such dimensions, for example, the cluster dimension [29], the capacity dimension [13, 30], the box-counting dimension [31, 32], the Hurst dimension [6–8], the pointwise dimension [33], the informational dimension [33], the generalized Renyi dimensions [34], the mass dimension [35] and many others. On the number of fractal dimension values needed for describing the fractal properties of an object investigated, all fractals are appeared to be divided on mono-fractals and multi-fractals ones (see, for, example [7, 8, 13, 15, 16]). A mono-fractal needs only one such value, but a multi-fractal requires more than one. At the same time, all the fractal analysis methods are divided on mono-fractal and multi-fractal ones too. In general, the firsts of them are based on the fractal dimension estimation, the seconds – on application of more complex approach grounded, in particular, on so called 'traditional multi-fractal formalism' (see, for example, [36, 37]). In bounds of the traditional multi-fractal formalism (or so called 'P-model'), there are a set of different numerical characteristics including, in particular, the spectrum of generalized fractal dimensions, the scaling exponent, the multi-fractal spectrum function and others. It is important to point that as the mono-fractal analysis can be used to the multifractals, as the multi-fractal analysis can be applied to the mono-fractals. In such cases there are no principally problems and prohibitions. By the way, there are another multi-fractal approaches, for example, the multifractal formalism by Chabra and Jensen [29], the informational interpretation of multifractal formalism [29], the L-model of multifractal formalism [29], the Mandelbrot's approach (or so called 'Holder analysis') [16, 19, 38, 39].

THE HURST FRACTAL DIMENSION

In fractal analysis at all, one of the most known and frequently applied numerical characteristics is the Hurst fractal dimension D_H. In bounds of the so called generalized (or fractional) Brownian motion model (see, for example, [6–8]), it is related to another popular numerical characteristic, the Hurst exponent H, using two simple relations: $D = 2 - H$ for the signals and $D = 3 - H$ for the images.

As a numerical characteristics, the Hurst exponent H was introduced by H. E. Hurst in 1951 (in the "pre-fractal" era) in the paper [40] and was used to describe the statistical properties of random signals and processes (see, for example, [41, 42].

It is well known (see, for, example, [6–8]), that for fractals, the Hurst exponent H satisfies the inequality $0 < H < 1$. If the fractal object has the property of persistence, then $0.5 < H < 1$, if it has the property of anti-persistence, then $0 < H < 0.5$. When an object investigated is appeared to be a signal or a process, it is said to have a "memory", i.e., the preservation of the tendency of changes in its persistence and the non-preservation of the tendency in its anti-persistence. It is important that in both cases the process becomes predictable in a certain sense. If $H = 0.5$, then the process is occurred to be purely random and there is no tendency in its changes. If the Hurst exponent H for the process is appeared to be outside the interval $H \in (0.1)$ (this is also quite possible), then this means that the object is non-fractal in this sense. This approach was later extended to other cases, in particular, to image analysis.

Today, the Hurst exponent H is successfully used in a large number of modern methods of signal and image analysis (see, for example, [43–44]). The most known of them are the R/S analysis [41], the coarse graining spectral analysis [46, 47], rescaled-range R/S analysis [48], the auto-correlation analysis [49], the detrended moving average method [50], the second moment method [51], the Peltier – Levi-Vehel's method [52], the variance plot method [53], the variogram method [54], the adaptive fractal analysis [55], the generalized the variogram method [56], the aggregate variance method [57], the periodogram method [58], the wavelet transform based methods [29], the fractional Fourier transform based method [59], the empirical mode decomposition based method [60], the dispersion analysis method [61], the scaled windowed variance method [62] and the signal summation conversion method [63].

FRACTALS ANS SOLID-STATE PHYSICS

In the solid-state physics, as in other branches of science and technologies, the "fractalization" process moves in some different directions.

In condensed matter physics [30, 65–69], the fractal approach has been successfully used to describe the path of ramdom walk, the self-avoiding random walk, the percolation clusters and networks, the random aggregates, the porous materials, the rough surfaces and the fractures. The fractal properties are present, for example, in the small-angle light or neutron scattering, in the growth of fractal aggregates, in the kinetic theory of dendritic growth, in the fractal concepts of the structure of crystalline bodies (in particular, of nano-particles), in the fractal approach to Debye's theory of heat capacity, in the diffusion on fractal networks, in the elastic properties of fractal networks, in the magnetic ordering on fractal networks, in the particle-cluster and cluster-cluster aggregation models, in the growth of fractal structures, in the nucleation-induced crystallization, in the atomic vibrations of percolating networks (a fracton appearence; fracton is a phonon in a fractal medium), in the Anderson transition, in the spin waves in diluted Heisenberg antiferromagnets, in the transport on fractal structures, in the diffusion-limited aggregation, in the Josephson junction, in the viscous fingering, in the dielectric breakdown, in the surfaces of solids, in the phase transition of percolation, in the molecular spectra, in the crystallography of quasicrystals etc. Yet another modern object for fractal properties investigations is the crystalline metamaterials [70].

Some fractal properties were discovered in the phase transitions [71]. For example, fractal structures can be observed in dendrites formed during crystallization of a metal or a binary mixture. They have strong similarities to colloidal aggregates. The analysis of digitalized images of dendritic structures formed in thin films of Zn and $NbGe_2$ gives fractal exponents of 1.66 and 1.69, respectively. Another suitable example is connected with phase transition between magnetic and non-magnetic states of the substance [72].

The fractality is occurred to be in so called interphase too. The interphase or the “interface phase” is understood to be the region between the “perfect” solid phase and the “perfect” liquid phase. The structure of the interphase has been studied using the theory of fractals with the result that different growth models led to the formation of clusters in the interphase with different fractal dimensions. These characterize the shape of clusters or the roughness of the interphase [64].

It is important to point, that the “fractalization” of the solid-state physics is not over yet and is continued just now. So, two years ago, an interesting concept of a generalization of the crystalline order, the ground-state fractal crystal, were proposed. The authors claim that they created a simple continuous-space-discrete-field model whose ground state is a crystal where each unit cell is a fractal [73].

IMAGE MODELLING WITH THE WEIERSTRASS FUNCTION

As it was noted above, the images with certain fractal properties play an important role in the solid-state physics. To create the simple and convenient mono-fractal and multi-fractal models of such images, the usage of the one- and two-dimensional Weierstrass function is proposed.

In the simple case (see, for example, [74]) for a 8-bit gray-scale image, the intensity $z = f(x,y)$ takes on integer values in the range from 0 to 255. Then, in a rectangular Cartesian coordinate system in $(x, y, f(x,y))$ space, the image as an object is occurred to be a surface.

The one-dimensional Weierstrass function is given by the relation (see, for example, [27]):

$$W(x) = \sum_{n=1}^{\infty} b^{(D-2)n} \sin\left(b^n x\right),$$

where $b > 1$, $1 < D < 2$, D is the capacity fractal dimension of this function. The two-dimensional Weierstrass function $W(x,y)$ can be obtained as multiplication of two Weierstrass functions of two different arguments (x and y correspondently):

$$W(x,y) = W_1(x)W_2(y),$$

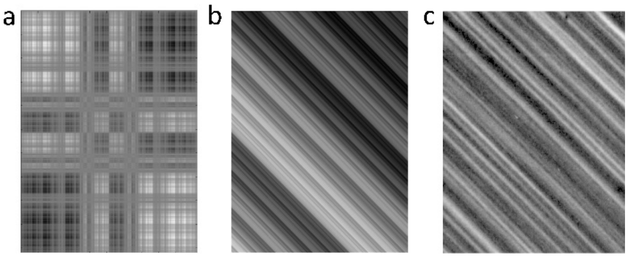

Fig. 1. Comparison of the mono-fractal two-dimensional Weierstrass function model with $D_1 = D_2 = 1.6$ (a), the model twin structure based on the one-dimensional Weierstrass function with $D = 1.6$ (b) and real experimental unidirectional twin structure in the $YBa_2Cu_3O_{7-\delta}$ crystal (c).

where

$$W_1(x) = \sum_{n=1}^{\infty} b_1^{(D_1-2)n} \sin\left(b_1^{\,n} x\right),$$

$$W_2(y) = \sum_{n=1}^{\infty} b_2^{(D_2-2)n} \sin\left(b_2^{\,n} y\right).$$

If $D_1 = D_2$, the model proposed is appeared to be mono-fractal, in opposite case ($D_1 \neq D_2$), it is occurred to be multi-fractal. An example of such mono-fractal two-dimensional Weierstrass function with $D_1 = D_2 = 1.6$ is shown at the Fig. 1a.

As an example of the useful practical usage of the model proposed, we consider the following.

It is well known, that in some crystals, the so called “twin structure” can appear. For, example, the twin planes in the $YBa_2Cu_3O_{7-\delta}$ crystals are occurred to be always oriented at an angle of 45° to the crystallographic *a* and *b* axes and oriented parallel to the *c* axis [75]. Such real experimental unidirectional twin structure in the $YBa_2Cu_3O_{7-\delta}$ crystal is shown at the Fig. 1c. This structure can be successfully modelled with usage of the model given by the relation $F(x,y) = W(x) \cdot 1$ and then rotated anti-clockwise at the angle of 45°, where $W(x)$ is the one-dimensional Weierstrass function. At the Fig. 1, b, an example of such model with $D = 1.6$ is shown. Both images, model (Fig. 1b) and real (Fig. 1c), are seen to be very similar. No doubts, the fractal dimension of the model image can be varied to look for the most optimal value, when the model and the real image will be most similar.

Moreover, there are no prohibitions to use another continuous no-where differentiable functions instead of the Weierstrass function. As such functions, for example, the Riman function, the Riman – Weierstrass functuion, the Weierstrass – Mandelbrot function, the Cellerier function, the Darboux function, the Takagi function, the Van der Waerden function, the first and the second Faber functions, the Schoenberg function, the McCarthy function, the Liu Wen function can be successfully used.

Thus, the “fractalization” of the solid-state physics is shown to be interesting, useful and perspective.

CONCLUSIONS

1. The fractality is occurred to be one of the most important initial and fundamental properties of the world around us.

2. In the process of “fractalization” of modern science and technologies, today we are living on the fourth stage called as “the modern stage” (early 2000s – present day).

3. There are a lot of the fractal analysis methods, as mono-fractal, as multi-fractal, which are successfully applied for investigations of the fractal properties (mono-fractal and multi-fractal) of different objects, systems and processes in the world around us. The number of such methods is continuously increasing.

4. In the solid-state physics, the fractal approach has been successfully applied in the past years, and the new ideas related to it continue to be used even more widely in many directions at the present day too.

5. As an example of such new ideas, the proposition to create the simple and convenient mono-fractal and multi-fractal models images based on usage of the one- and two-dimensional Weierstrass function are considered.

6. For the specialists in the solid-state physics, expanding the use of the fractal approach is insistently proposed, as well as this promises to obtain a lot of new and useful information about the objects and the systems investigated.

7. To avoid accusations of the authors exaggerating the role of fractals for modern human civilization, it is worth quoting B. Mandelbrot's own words on this subject: “Fractals are not a panacea. I do not recommend fractal methods to everyone and certainly have never tried to impose them on anyone” [39].

CONFLICT OF INTEREST

The authors declare that they have no conflict of interests.

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«ФРАКТАЛІЗАЦІЯ» ФІЗИКИ ТВЕРДОГО ТІЛА

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Продемонстровано, що в сучасній фізиці твердого тіла, відповідно до нелінійної та системної парадигм, сформульованих Л. Ф. Черногором наприкінці 1980-х років, багато процесів у відкритих, нелінійних, динамічних системах виявляються дуже складними, нелінійними, короткочасними, надширокосмуговими або фрактальними.

Більш того, з точки зору фрактальної парадигми, висунутої на початку 2000-х років В. В. Яновським, фрактальність взагалі розглядається як одна з фундаментальних властивостей навколишнього світу. Тому вивчення фрактальних характеристик, зокрема, природних фізичних процесів та об'єктів у галузі фізики твердого тіла виявляється актуальним, цікавим, корисним і перспективним.

Коротко розглянуто основні етапи розвитку фрактального підходу загалом. Зазначається, що виявилося занадто складним сформулювати суворе та чітке математичне визначення фрактала. Детально розглянуто визначення фрактала, сформульоване К. Фалконером і визнане більшістю спеціалістів найкращим для практичного використання. Розглянуто основні числові характеристики фрактала, а також сучасну класифікацію фракталів. Чітко пояснено загальні принципові відмінності між математичними та фізичними (або природними) фракталами, а також між моно- та мультифракталами. Наведено огляд основних методів оцінки фрактальної розмірності Херста.

Висвітлено основні існуючі напрямки «фракталізації» сучасної фізики твердого тіла. Наведено відповідні приклади.

Відзначено, що важливу роль у «фракталізації» фізики твердого тіла відіграють зображення, що мають певні фрактальні властивості. Для моделювання зображень із фрактальними властивостями запропоновано використання двовимірної функції Вейєрштрасса. Як приклад, розглянуто моделювання однонаправленої двійникової структури, що спостерігається в кристалі $YBa_2Cu_3O_{7-\delta}$. Продемонстровано порівняння модельного зображення, заснованого на одновимірній функції Вейєрштрасса із заданим значенням фрактальної розмірності Херста, з реальним експериментальним зображенням.

Ключові слова: *нелінійна парадигма, фрактальна парадигма, фрактальний підхід, фрактальна розмірність Херста, фрактальне зображення, двовимірна функція Вейєрштрасса.*