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CASIMIR HONEYCOMB DRIVE: ON THE FORCE ON PERFECTLY CONDUCTING HONEYCOMB ON A PLATE

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In this paper, the two-dimensional one-body Casimir effect is analyzed on the example of square-shaped nanocells. In the classical one-dimensional two-body Casimir effect a Casimir force appears between two plates as a difference of electromagnetic pressures of zero- point quantum-vacuum oscillation on different sides of each of the plates. The plates are pushed forwards each other by external quantum-vacuum oscillation fields, which in classical configuration exceed internal quantum-vacuum oscillation fields. It is possible to try to create a difference of electromagnetic pressures of quantum-vacuum oscillation on different sides of a single plate due to the difference of the geometry of vacuum resonators on different sides of the plate. For this purpose, it is necessary to grow nanocells on one of surfaces of a smooth metallic plate. As a result, it has been found that the formula for the force per unit area is very similar to the formula of the classical Casimir effect, except for the value of the proportionality coefficient.

The force applied to perfectly conducting honeycombs on a plate as a result of the difference in specific energy density on its different sides can be interpreted as the pressure of the zero-point electromagnetic oscillations. According to the formula presented in this work, for the gold nano-honeycomb with a size of about 2 microns the force should be equal to 8.55 dynes per square meter of the panel, which is quite an acceptable value for the practical use of the expected effect for satellite orbits correction.

Although the effect is small, an experimental confirmation could serve as a critical proof for the existence of Casimir's virtual quantum photons.

Keywords: *Two dimensional one body Casimir effect, nanocells, nanohoneycomb, Casimir thrust.*

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INTRODUCTION

The introduction of half-quanta in the context of black-body radiation by Planck in 1911 was fundamental for discovering the Casimir effect presented by Casimir in his seminal paper [1]. This is one of the most direct manifestations of a quantum and relativistic phenomenon caused by the zero-point oscillation of quantized fields.

The Casimir effect in its simplest form is the force between a pair of neutral, parallel conducting plates resulting from the modification of the electromagnetic vacuum properties caused by the change in boundary conditions.

The calculation of the Casimir force is a particularly complicated theoretical problem. Remarkably, for closed configurations, i.e., when the Casimir effect manifests for one body instead of two, the Casimir force can be not only attractive but repulsive as well. As it has been shown by Boyer [2], the latter is true for an ideal metal spherical shell.

Antipin A. V. [3] expects the appearance of the driving force/thrust (due to the Casimir effect on one body), as a result of the difference in the impact of virtual particles (photons) on external and internal reflecting surfaces of pyramidal, conical or V-shaped objects.

This work is dedicated to one-body Casimir effect for open configuration of perfectly conducting nanohoneycomb.

TWO DIMENSIONAL CASIMIR'S APPROACH

Let us consider a cubic cavity of volume L^3 bounded by perfectly conducting walls where perfectly conducting square plate with side L is placed in this cavity parallel to the xy face, and let the distance between the plate and the xy face be sufficiently large, $L/2$, for example. One side of this perfectly conducting square plate is a smooth plane and another is covered with perfectly conducting square-shaped honeycombs with a square side a.

On both sides of the plate the expressions $\sum \hbar\omega/2$ where the summation extends over all possible resonance frequencies of the cavity $L/2 \times L \times L$ (a large cavity: between smooth plane and xy face) and the cavity $L/2 \times a \times a$ (a small cavity, one honeycomb cell: between the bottom of the honeycomb and the opposite xy face) are divergent and devoid of physical meaning but the difference between these sums on the opposite sides of the plate, $(\sum \hbar\omega)_I/(2V_I) - (\sum \hbar\omega)_{II}/(2V_{II})$, will be shown to have a well-defined value and this value will be interpreted as the interaction between the plate and the both remote xy faces.

The possible oscillations inside cavities defined by $0 \le x \le L$, $0 \le y \le L$, $0 \le z \le L/2$ (a large cavity between smooth plane and xy face) and $0 \le x \le a$, $0 \le y \le a$, $0 \le z \le L/2$ (a small cavity, one honeycomb cell) have wave vectors $k_x = \frac{\pi}{L} n_x$, $k_y = \frac{\pi}{L} n_y$, $k_z = \frac{\pi}{L/2} n_z$ (a large cavity between smooth plane and xy face), and $k_x = \frac{\pi}{a} n_x$, $k_y = \frac{\pi}{a} n_y$, $k_z = \frac{\pi}{L/2} n_z$ (a small cavity, one honeycomb cell), where n_x, n_y, n_z are positive integers; $k = \sqrt{k_x^2 + k_y^2 + k_z^2} = \sqrt{\kappa^2 + k_z^2}$.

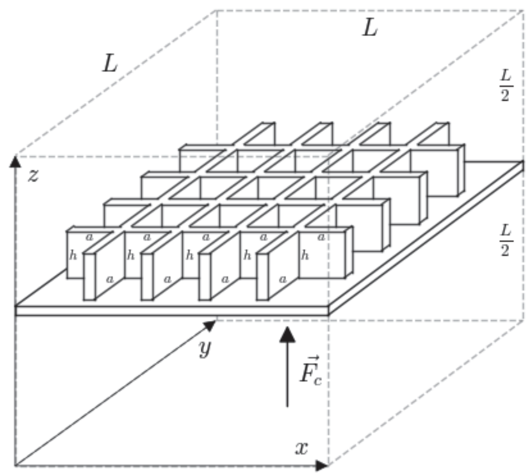

Fig.1. Cubic cavity with a plate covered by honeycomb.

Let us write the expression for the sum of zero-point energy in general form

$$E = \frac{1}{2}\sum \hbar\omega = \hbar c \frac{1}{2} \sum_{n_x}\sum_{n_y}\sum_{n_z} k. \qquad (1)$$

Two standing waves correspond to every k_x, k_y, k_z but in case when one of the n_i is zero, there is only one wave.

That is of no importance in case of one honeycomb cell cavity for k_z, since for very large $L/2$ we may regard k_z as continuous variable, replacing summation over n_z with integration. Thus, for a small cavity consisting of one honeycomb, we find

$$E = \frac{\hbar c}{2} \int_0^\infty \left[\sqrt{k_z^2} + 2 \sum_{n_x=1}^\infty \sum_{n_y=1}^\infty \sqrt{\frac{n_x^2\pi^2}{a^2} + \frac{n_y^2\pi^2}{a^2} + k_z^2} \right] dn_z.$$

Considering $dn_z = \frac{L/2}{\pi} dk_z$ we can find the specific energy density E/V, where $V = V_{small} = a^2 L/2$:

$$\frac{E}{V} = \frac{\hbar c}{a^2 \pi} \int_0^\infty \left[\frac{\sqrt{k_z^2}}{2} + \sum_{n_x=1}^\infty \sum_{n_y=1}^\infty \sqrt{\frac{n_x^2\pi^2}{a^2} + \frac{n_y^2\pi^2}{a^2} + k_z^2} \right] dk_z,$$

$$\frac{E}{V} = \frac{\hbar c}{a^2 \pi} \int_0^\infty \left[\sum_{n_x=(0)1}^\infty \sum_{n_y=(0)1}^\infty \sqrt{n_x^2 \frac{\pi^2}{a^2} + n_y^2 \frac{\pi^2}{a^2} + k_z^2} \right] dk_z,$$

where the notation $(0)\,1$ is meant to indicate that the term with $n_x = 0$ and $n_y = 0$ has to be multiplied by $1/2$. Thus, for a small cavity consisting of one honeycomb, we have

$$\frac{E}{V} = \frac{\hbar c}{a^2 \pi} \sum_{n_x=(0)1}^\infty \sum_{n_y=(0)1}^\infty \left[\int_0^\infty \sqrt{n_x^2 \frac{\pi^2}{a^2} + n_y^2 \frac{\pi^2}{a^2} + k_z^2}\, dk_z \right].$$

That is of no importance in case of a large cavity for k_x, k_y since for very large L we may regard k_x, k_y as continuous variables. Thus, for large cavity between smooth plane and xy face we find

$$\sum \frac{\hbar\omega}{2} = \frac{\hbar c}{2}\int_0^\infty\int_0^\infty \left[\sqrt{k_x^2+k_y^2} + 2\sum_{n_z=1}^{\infty}\sqrt{\frac{n_z^2\pi^2}{(L/2)^2}+k_x^2+k_y^2}\right] dn_x dn_y.$$

For very large $L/2$ the last summation may also be replaced by an integral and, therefore, it can be seen that energy of a large cavity is given by

$$\frac{1}{2}\sum\hbar\omega = \hbar c\int_0^\infty\int_0^\infty\int_0^\infty \sqrt{k_z^2+k_x^2+k_y^2}\, dn_x\, dn_y\, dn_z,$$

$$dn_x = \frac{L}{\pi}\, dk_x,\ dn_y = \frac{L}{\pi}\, dk_y,\ dn_z = \frac{L/2}{\pi}\, dk_z,$$

Now for the specific energy density E/V for a large cavity, where $V = V_{large} = L^3/2$ we can write the following sequence of transformations:

$$\frac{E}{V} = \frac{\hbar c}{L^3/2}\int_0^\infty\int_0^\infty\int_0^\infty \sqrt{k_z^2+k_x^2+k_y^2}\, dn_x\, dn_y\, \frac{L/2}{\pi}\, dk_z,$$

$$\frac{E}{V} = \frac{\hbar c}{L^2\pi}\int_0^\infty\int_0^\infty \left[\int_0^\infty \sqrt{k_x^2+k_y^2+k_z^2}\, dk_z\right]\left(\frac{L}{\pi}dk_x\right)\left(\frac{L}{\pi}dk_y\right),$$

$$\frac{E}{V} = \frac{\hbar c}{a^2\pi}\int_0^\infty\int_0^\infty \left[\int_0^\infty \sqrt{k_x^2+k_y^2+k_z^2}\, dk_z\right]\left(\frac{a}{\pi}dk_x\right)\left(\frac{a}{\pi}dk_y\right).$$

And finally for a large cavity between smooth plane and xy face we can formulate the energy density as following

$$\frac{\sum\hbar\omega}{2V} = \frac{\hbar c}{a^2\pi}\int_0^\infty\int_0^\infty \left[\int_0^\infty \sqrt{k_x^2+k_y^2+k_z^2}\, dk_z\right] dn_x\, dn_y,$$

but for a small cavity, one honeycomb cell we have $\frac{\sum\hbar\omega}{2V} = \frac{\hbar c}{a^2\pi}\sum_{n_x=(0)1}^{\infty}\sum_{n_y=(0)1}^{\infty}\left[\int_0^\infty \sqrt{n_x^2\frac{\pi^2}{a^2}+n_y^2\frac{\pi^2}{a^2}+k_z^2}\, dk_z\right]$.

Therefore, it is obvious that the interaction energy is determined by the following energy density difference:

$$\delta\frac{E}{V} = \frac{\hbar c}{a^2\pi}\left\{\sum_{n_x=(0)1}^{\infty}\sum_{n_y=(0)1}^{\infty}\left[\int_0^\infty\sqrt{\frac{n_x^2\pi^2}{a^2}+\frac{n_y^2\pi^2}{a^2}+k_z^2}dk_z\right] - \int_0^\infty\int_0^\infty\left[\int_0^\infty\sqrt{k_x^2+k_y^2+k_z^2}\, dk_z\right] dn_x dn_y\right\}. \tag{2}$$

This expression is clearly infinite, and to proceed with the calculation, it is convenient to introduce a regulator.

In order to receive a finite result, it is necessary to multiply the integrands by a regularization function $f(k/k_m)$ which is unity for $k << k_m$ but tends to zero sufficiently rapidly for $(k/k_m) \to \infty$. Where k_m may be defined by $f(1) = 1/2$. The physical meaning is obvious: our plate is hardly an obstacle for very short waves (X-rays e.g.) and, therefore, the zero-point energy of these waves will not be influenced by the position of this plate.

The purpose of regulator is to make the expression finite, and influence of its specific type will be removed by a limit transition in the end.

Introducing the variable $u^2 = a^2 k_z^2/\pi^2$, $du = a/\pi\, dk_z$, we have:

$$\delta\frac{E}{V} = \frac{\hbar c\pi}{a^4}\left\{\sum_{n_x=(0)1}^{\infty}\sum_{n_y=(0)1}^{\infty}\int_0^\infty\sqrt{n_x^2+n_y^2+u^2}\quad f\left(\frac{\pi\sqrt{n_x^2+n_y^2+u^2}}{a k_m}\right) du - \int_0^\infty\int_0^\infty\int_0^\infty\sqrt{n_x^2+n_y^2+u^2}\, f\left(\frac{\pi\sqrt{n_x^2+n_y^2+u^2}}{a k_m}\right) du\, dn_x\, dn_y\right\}. \tag{3}$$

If $\omega_{n_x,n_y} = c\sqrt{n_x^2\frac{\pi^2}{a^2}+n_y^2\frac{\pi^2}{a^2}+k_z^2}$ and $k_z^2 = u^2\frac{\pi^2}{a^2}$ we have $\omega_{n_x,n_y} = c\,\frac{\pi}{a}\sqrt{n_x^2+n_y^2+u^2}$ so $f\left(\frac{\pi\sqrt{n_x^2+n_y^2+u^2}}{a k_m}\right) = f\left(\frac{\omega_{n_x,n_y}}{c k_m}\right)$, where the cutting frequency is $\omega_m = c\, k_m$. Introducing function

$$F = \sqrt{n_x^2+n_y^2+u^2}\, f\left(\frac{\pi\sqrt{n_x^2+n_y^2+u^2}}{a k_m}\right) \tag{4}$$

we can write

$$\delta\frac{E}{V} = \frac{\hbar c\pi}{a^4}\left\{\sum_{n_x=(0)1}^{\infty}\sum_{n_y=(0)1}^{\infty}\left(\int_0^\infty F(u,n_x,n_y)\, du\right) - \int_0^\infty\int_0^\infty\left(\int_0^\infty F(u,n_x,n_y)\, du\right) dn_x\, dn_y\right\}.$$

And at least, introducing

$$G(n_x,n_y) = \int_0^\infty F(u,n_x,n_y)\, du \tag{5}$$

we have

$$\delta\frac{E}{V} = \frac{\hbar c\pi}{a^4}\left\{\sum_{n_x=(0)1}^{\infty}\sum_{n_y=(0)1}^{\infty} G(n_x,n_y) - \int_0^\infty\int_0^\infty G(n_x,n_y)\, dn_x\, dn_y\right\}. \tag{6}$$

Integration of $G(n_x,n_y)$ with example of regulator function is presented in Appendix A.

To receive a way of calculating $\delta(E/V)$, the Euler-Maclaurin 2D formula could be considered.

EULER-MACLAURIN 2D FORMULA

According to A.Bikyalis [4] we apply Euler-Maclaurin formula twice on n_x and on n_y. Starting from the following form of this formula

$$\sum_{i=a}^{b} f(i) = \int_a^b f(x)\,dx + \frac{f(a)+f(b)}{2} + \\ + \sum_{k=1}^{\lfloor p/2 \rfloor} \frac{B_{2k}}{(2k)!}\left(f^{(2k-1)}(b) - f^{(2k-1)}(a)\right) + \\ + R_p, \tag{7}$$

$$P_k(x) = B_k\left(x - \lfloor x \rfloor\right), \tag{8}$$

$$R_p = (-1)^{p+1} \int_a^b f^{(p)}(x) \frac{P_p(x)}{p!}\,dx, \tag{9}$$

since we are dealing with a very complex mathematical problem of integrating the function, which often oscillates and suffers discontinuities at the points of each integer value of the argument due to the presence of the multiplier $(x - \lfloor x \rfloor)$, hereafter, we will use the fact that the remainder can also be expressed in the form

$$R_p = (-1)^{p+1} \sum_{j=a}^{b-1} \int_0^1 f^{(p)}(v+j) \frac{B_p(v)}{p!}\,dv. \tag{10}$$

We can see that it consists of 4 parts:
the integral $\int\limits_{x} = \int_{x_a}^{x_b} f(x)\,dx$,
the half sum $\underset{x}{H_\Sigma} = \left(f(x_a) + f(x_b)\right)/2$,
the sum of Bernoulli polynomials
$\sum\limits_{x}^{B} = \sum_{k=1}^{\lfloor p/2 \rfloor} \frac{B_{2k}}{(2k)!}\left(f^{(2k-1)}(x_b) - f^{(2k-1)}(x_a)\right)$,
and the remainder
$\underset{x}{R_p} = (-1)^{p+1} \sum_{j=x_a}^{x_b-1} \int_0^1 f^{(p)}(v_x + j_x) \frac{B_p(v_x)}{p!}\,dv_x$.

When applying it to G twice on n_x and on n_y we should have the following summands which can be represented as the table:

$$\begin{array}{cccc}
\int\limits_{n_y}\int\limits_{n_x} G & \int\limits_{n_y}\underset{n_x}{H_\Sigma} G & \int\limits_{n_y}\sum\limits_{n_x}^{B} G & \int\limits_{n_y}\underset{n_x}{R_p} G \\
\underset{n_y}{H_\Sigma}\int\limits_{n_x} G & \underset{n_y}{H_\Sigma}\underset{n_x}{H_\Sigma} G & \underset{n_y}{H_\Sigma}\sum\limits_{n_x}^{B} G & \underset{n_y}{H_\Sigma}\underset{n_x}{R_p} G \\
\sum\limits_{n_y}^{B}\int\limits_{n_x} G & \sum\limits_{n_y}^{B}\underset{n_x}{H_\Sigma} G & \sum\limits_{n_y}^{B}\sum\limits_{n_x}^{B} G & \sum\limits_{n_y}^{B}\underset{n_x}{R_p} G \\
\underset{n_y}{R_p}\int\limits_{n_x} G & \underset{n_y}{R_p}\underset{n_x}{H_\Sigma} G & \underset{n_y}{R_p}\sum\limits_{n_x}^{B} G & \underset{n_y}{R_p}\underset{n_x}{R_p} G.
\end{array} \tag{11}$$

Taking into account that the function G is symmetric on its n_x and n_y arguments, so the two-dimentional Euler-Maclaurin marix presented above is symmetric too.

SUMMARY OF EULER-MACLAURIN 2D

Taking value of parameter $p = 1$ we have:

$$\int\limits_{n_y}\int\limits_{n_x} G = \int\limits_{a_y}^{b_y}\int\limits_{a_x}^{b_x} G(n_x, n_y)\,dn_x\,dn_y$$

$$\int\limits_{n_y}\underset{n_x}{H_\Sigma} G = \frac{1}{2}\int\limits_{a_y}^{b_y} G(a_x, n_y)\,dn_y$$

$$\int\limits_{n_y}\sum\limits_{n_x}^{B} G = 0$$

$$\int\limits_{n_y}\underset{n_x}{R_p} G = \sum_{j_x=a_x}^{b_x-1}\int\limits_{a_y}^{b_y}\int\limits_{0}^{1} \frac{1}{2}\left(2v_{n_x} - 1\right) G'\left(\widehat{j_x + v_{n_x}}, n_y\right) dv_{n_x}\,dn_y$$

$$\underset{n_y}{H_\Sigma}\int\limits_{n_x} G = \frac{1}{2}\int\limits_{a_x}^{b_x} G(n_x, a_y)\,dn_x$$

$$\underset{n_y}{H_\Sigma}\underset{n_x}{H_\Sigma} G = \frac{1}{4} G(a_x, a_y)$$

$$\underset{n_y}{H_\Sigma}\sum\limits_{n_x}^{B} G = 0$$

$$\underset{n_y}{H_\Sigma}\underset{n_x}{R_p} G = \sum_{j_{n_x}=a_x}^{b_x-1} \frac{1}{4}\int\limits_{0}^{1}\left(2v_{n_x} - 1\right) G'\left(\widehat{j_{n_x} + v_{n_x}}, a_y\right) dv_{n_x}$$

$$\sum\limits_{n_y}^{B}\int\limits_{n_x} G = 0$$

$$\sum\limits_{n_y}^{B}\underset{n_x}{H_\Sigma} G = 0$$

$$\sum\limits_{n_y}^{B}\sum\limits_{n_x}^{B} G = 0$$

$$\sum\limits_{n_y}^{B}\underset{n_x}{R_p} G = 0$$

$$\underset{n_y}{R_p}\int\limits_{n_x} G = \sum_{j_x=a_x}^{b_x-1}\int\limits_{a_y}^{b_y}\int\limits_{0}^{1} \frac{1}{2}\left(2v_{n_x} - 1\right) G'\left(\widehat{j_x + v_{n_x}}, n_y\right) dv_{n_x}\,dn_y$$

$$\underset{n_y}{R_p}\underset{n_x}{H_\Sigma} G = \sum_{j_{n_y}=a_y}^{b_y-1}\int\limits_{0}^{1} \frac{1}{4}\left(2v_{n_y} - 1\right) G'\left(a_x, \widehat{j_{n_y} + v_{n_y}}\right) dv_{n_y}$$

$$\underset{n_y}{R_p}\sum\limits_{n_x}^{B} G = 0$$

$$\underset{n_y}{R_p}\underset{n_x}{R_p} G = \sum_{j_y=a_y}^{b_y-1}\sum_{j_x=a_x}^{b_x-1}\int\limits_{0}^{1}\int\limits_{0}^{1} \frac{1}{4}\left(2v_y - 1\right)\left(2v_x - 1\right)\cdot \\ G''\left(\widehat{j_x + v_x}, \widehat{j_y + v_y}\right) dv_x\,dv_y$$

A WAYOF CALCULATING $\delta(E/V)$

Let us consider the expression

$$\sum_{n_x=(0)\,1}^{\infty}\sum_{n_y=(0)\,1}^{\infty} G - \int\limits_{0}^{\infty}\int\limits_{0}^{\infty} G\,dn_x\,dn_y \tag{12}$$

Firstly, we can see, that

$$\sum_{n_x=(0)\,1}^{\infty}\sum_{n_y=(0)\,1}^{\infty} G(n_x, n_y) = \\ \frac{1}{4}G(0,0) - \frac{1}{2}\sum_{n_y=0}^{\infty} G(0, n_y) - \\ \frac{1}{2}\sum_{n_x=0}^{\infty} G(n_x, 0) + \sum_{n_x=0}^{\infty}\sum_{n_y=0}^{\infty} G(n_x, n_y). \tag{13}$$

Therefore, we have

$$\sum_{n_x=(0)\,1}^{\infty}\sum_{n_y=(0)\,1}^{\infty} G(n_x,n_y) - \int_0^{\infty}\int_0^{\infty} G(n_x,n_y)\, dn_x\, dn_y =$$

$$\frac{1}{4}G(0,0) - \frac{1}{2}\sum_{n_y=0}^{\infty} G(0,n_y) - \frac{1}{2}\sum_{n_x=0}^{\infty} G(n_x,0) + \sum_{n_x=0}^{\infty}\sum_{n_y=0}^{\infty} G(n_x,n_y) - \int_0^{\infty}\int_0^{\infty} G(n_x,n_y)\, dn_x\, dn_y.$$

On the other hand, we have found that

$$\sum_{n_x=0}^{\infty}\sum_{n_y=0}^{\infty} G(n_x,n_y) - \int_0^{\infty}\int_0^{\infty} G(n_x,n_y)\, dn_x\, dn_y =$$

$$\begin{array}{llll}
\underset{n_y}{\int} \underset{n_x}{H_\Sigma} G & +\underset{n_y}{\int} \underset{n_x}{\Sigma^B} G & +\underset{n_y}{\int} \underset{n_x}{R_p} G \\
+\underset{n_y}{H_\Sigma} \underset{n_x}{\int} G & +\underset{n_y}{H_\Sigma} \underset{n_x}{H_\Sigma} G & +\underset{n_y}{H_\Sigma} \underset{n_x}{\Sigma^B} G & +\underset{n_y}{H_\Sigma} \underset{n_x}{R_p} G \\
+\underset{n_y}{\Sigma^B} \underset{n_x}{\int} G & +\underset{n_y}{\Sigma^B} \underset{n_x}{H_\Sigma} G & +\underset{n_y}{\Sigma^B} \underset{n_x}{\Sigma^B} G & +\underset{n_y}{\Sigma^B} \underset{n_x}{R_p} G \\
+\underset{n_y}{R_p} \underset{n_x}{\int} G & +\underset{n_y}{R_p} \underset{n_x}{H_\Sigma} G & +\underset{n_y}{R_p} \underset{n_x}{\Sigma^B} G & +\underset{n_y}{R_p} \underset{n_x}{R_p} G,
\end{array}$$

where $\underset{n_y}{H_\Sigma} \underset{n_x}{H_\Sigma} G$, $\underset{n_y}{\int} \underset{n_x}{H_\Sigma} G$ and $\underset{n_y}{H_\Sigma} \underset{n_x}{\int} G$ are:

$\frac{1}{4} G(a_x,a_y)$, $\frac{1}{2}\int_{a_y}^{b_y} G(a_x,n_y)\, dn_y$, $\frac{1}{2}\int_{a_x}^{b_x} G(n_x,a_y)\, dn_x$.

Now using 1D Euler-Maclaurin formula in the form (7) we can see that

$$\frac{1}{4}G(0,0) - \frac{1}{2}\sum_{n_y=0}^{\infty} G(0,n_y) - \frac{1}{2}\sum_{n_x=0}^{\infty} G(n_x,0)$$

will be equal to

$$\begin{array}{llll}
\underset{n_y}{H_\Sigma} \underset{n_x}{H_\Sigma} G \\
-\underset{n_y}{\int} \underset{n_x}{H_\Sigma} G & -\underset{n_y}{H_\Sigma} \underset{n_x}{H_\Sigma} G & -\frac{1}{2}\underset{n_y}{\Sigma^B} G & -\frac{1}{2}\underset{n_y}{R_p} G/2 \\
-\underset{n_y}{H_\Sigma} \underset{n_x}{\int} G & -\underset{n_y}{H_\Sigma} \underset{n_x}{H_\Sigma} G & -\frac{1}{2}\underset{n_x}{\Sigma^B} G & -\frac{1}{2}\underset{n_x}{R_p} G/2.
\end{array}$$

Now we can find expression (12) by using the following summation

$$\begin{array}{llll}
 & & +\underset{n_y}{\int} \underset{n_x}{\Sigma^B} G & +\underset{n_y}{\int} \underset{n_x}{R_p} G \\
 & & +\underset{n_y}{H_\Sigma} \underset{n_x}{\Sigma^B} G & +\underset{n_y}{H_\Sigma} \underset{n_x}{R_p} G \\
+\underset{n_y}{\Sigma^B} \underset{n_x}{\int} G & +\underset{n_y}{\Sigma^B} \underset{n_x}{H_\Sigma} G & +\underset{n_y}{\Sigma^B} \underset{n_x}{\Sigma^B} G & +\underset{n_y}{\Sigma^B} \underset{n_x}{R_p} G \\
+\underset{n_y}{R_p} \underset{n_x}{\int} G & +\underset{n_y}{R_p} \underset{n_x}{H_\Sigma} G & +\underset{n_y}{R_p} \underset{n_x}{\Sigma^B} G & +\underset{n_y}{R_p} \underset{n_x}{R_p} G \\
 & & -\frac{1}{2}\underset{n_y}{\Sigma^B} G & -\frac{1}{2}\underset{n_y}{R_p} G \\
 & & -\frac{1}{2}\underset{n_x}{\Sigma^B} G & -\frac{1}{2}\underset{n_x}{R_p} G.
\end{array}$$

It is easy to see that the sum of all summands without remainder

$$\begin{array}{lll}
 & & +\underset{n_y}{\int} \underset{n_x}{\Sigma^B} G \\
 & & +\underset{n_y}{H_\Sigma} \underset{n_x}{\Sigma^B} G \\
+\underset{n_y}{\Sigma^B} \underset{n_x}{\int} G & +\underset{n_y}{\Sigma^B} \underset{n_x}{H_\Sigma} G & +\underset{n_y}{\Sigma^B} \underset{n_x}{\Sigma^B} G \\
 & & -\frac{1}{2}\underset{n_y}{\Sigma^B} G \\
 & & -\frac{1}{2}\underset{n_x}{\Sigma^B} G
\end{array}$$

is 0. And, therefore, any possible non zero result of expression (12) should be attributed to the remainder

$$\sum_{n_x=(0)\,1}^{\infty}\sum_{n_y=(0)\,1}^{\infty} G(n_x,n_y) - \int_0^{\infty}\int_0^{\infty} G(n_x,n_y)\, dn_x\, dn_y =$$

$$\begin{array}{llll}
 & & & +\underset{n_y}{\int} \underset{n_x}{R_p} G \\
 & & & +\underset{n_y}{H_\Sigma} \underset{n_x}{R_p} G \\
 & & & +\underset{n_y}{\Sigma^B} \underset{n_x}{R_p} G \\
+\underset{n_y}{R_p} \underset{n_x}{\int} G & +\underset{n_y}{R_p} \underset{n_x}{H_\Sigma} G & +\underset{n_y}{R_p} \underset{n_x}{\Sigma^B} G & +\underset{n_y}{R_p} \underset{n_x}{R_p} G \\
 & & & -\frac{1}{2}\underset{n_y}{R_p} G \\
 & & & -\frac{1}{2}\underset{n_x}{R_p} G.
\end{array}$$

Or using symmetric properties of G, we can rewrite the resulting formula

$$\sum_{n_x=(0)\,1}^{\infty}\sum_{n_y=(0)\,1}^{\infty} G(n_x,n_y) - \int_0^{\infty}\int_0^{\infty} G(n_x,n_y)\, dn_x dn_y =$$

$$+2\cdot \underset{n_y}{R_p} \underset{n_x}{\int} G + 2\cdot \underset{n_y}{R_p} \underset{n_x}{H_\Sigma} G + 2\cdot \underset{n_y}{R_p} \underset{n_x}{\Sigma^B} G + \\ + \underset{n_y}{R_p} \underset{n_x}{R_p} G - \underset{n_y}{R_p} G$$

with the following summands:

$$2\underset{n_y}{R_p} \underset{n_x}{\int} G = \sum_{j_x=a_x}^{b_x-1} \int_{a_y}^{b_y}\int_0^1 (2v_{n_x}-1) G'\left(\widehat{j_x+v_{n_x}}, n_y\right) dv_{n_x} dn_y;$$

$$2\underset{n_y}{R_p} \underset{n_x}{H_\Sigma} G = \sum_{j_{n_y}=a_y}^{b_y-1} \int_0^1 \left(v_{n_y}-\frac{1}{2}\right) G'\left(a_x, \widehat{j_{n_y}+v_{n_y}}\right) dv_{n_y};$$

$$2\underset{n_y}{R_p} \underset{n_x}{\Sigma^B} G = 2\sum_{j_{n_y}=a_y}^{b_y-1} \int_0^1 0\, dv_{n_y} = 0;$$

$$\underset{n_y}{R_p} \underset{n_x}{R_p} G = \sum_{j_y=a_y}^{b_y-1} \sum_{j_x=a_x}^{b_x-1} \int_0^1\int_0^1 \left(v_y-\frac{1}{2}\right)\left(v_x-\frac{1}{2}\right) \cdot G''\left(\widehat{j_x+v_x}, \widehat{j_y+v_y}\right) dv_x\, dv_y;$$

$$-\underset{n_y}{R_p} G = -\sum_{j_{n_y}=a_y}^{b_y-1} \int_0^1 \left(v_{n_y}-\frac{1}{2}\right) G'\left(n_x, \widehat{j_{n_y}+v_{n_y}}\right) dv_{n_y}.$$

Considering that $\underset{n_y}{R_p} \underset{n_x}{\Sigma^B} G$ should be 0 because derivative with respect to n_x, for $n_x = 0$ is 0, and $2\cdot \underset{n_y}{R_p} \underset{n_x}{H_\Sigma} G - \underset{n_y}{R_p} G$ gives 0 in summation, we can simplify the result in the form

$$\sum_{n_x=(0)\,1}^{\infty}\sum_{n_y=(0)\,1}^{\infty} G(n_x,n_y) - \int_0^{\infty}\int_0^{\infty} G(n_x,n_y)\,dn_x\,dn_y = 2\cdot \underset{n_y}{R_p}\underset{n_x}{\int} G + \underset{n_y}{R_p}\underset{n_x}{R_p} G \tag{14}$$

or in detailed form

$$\sum_{n_x=(0)\,1}^{\infty}\sum_{n_y=(0)\,1}^{\infty} G - \int_0^{\infty}\int_0^{\infty} G\;dn_x dn_y = \sum_{j_x=0}^{\infty}\int_{n_y=0}^{\infty}\int_{v_x=0}^{1}(2v_{n_x}-1)\,G'\left(\widehat{j_x+v_{n_x}},n_y\right)dv_{n_x}dn_y + \\ + \sum_{j_y=0}^{\infty}\sum_{j_x=0}^{\infty}\int_0^1\int_0^1\left(v_y-\tfrac{1}{2}\right)\left(v_x-\tfrac{1}{2}\right)G''\left(\widehat{j_x+v_x},\widehat{j_y+v_y}\right)dv_x dv_y. \tag{15}$$

HOW $\delta(E/V)$ DEPENDS ON ak_m ?

Casimir in his original work [1] has provided his formula in assumption that $ak_m >> 1$. But now we can investigate how the expression (12) depends on ak_m.

Thus, for the energy density difference per cm^2 we find

$$\delta\frac{E}{V} = \frac{\hbar c\pi}{a^4}\int_0^{\infty}\left\{\sum_{n_x=(0)\,1}^{\infty}\sum_{n_y=(0)\,1}^{\infty} F(n_x,n_y) - \int_0^{\infty}\int_0^{\infty} F(n_x,n_y)\,dn_x dn_y\right\}du. \tag{16}$$

Table 1.

The result of evaluating the expression (12).

$a\cdot k_m$	$\sum_{(0)1}^{\infty}\sum_{(0)1}^{\infty}-\int_0^{\infty}\int_0^{\infty}G$	ε	j_{max}
0.25	0.00108989	4.99669e-07	2
0.5	0.00322377	7.99836e-07	6
0.75	0.00440392	8.72173e-07	11
1.0	0.00304837	8.49556e-07	17
1.25	-0.00148115	1.77116e-06	18
1.5	-0.00870823	3.67271e-06	18
1.75	-0.0172279	6.80405e-06	18
2.0	-0.0251746	9.98944e-06	19
2.25	-0.0308341	9.37033e-06	23
2.5	-0.0332867	1.42819e-05	23
2.75	-0.0324363	2.09102e-05	23
3.0	-0.0289864	2.96151e-05	23
3.25	-0.02404795	4.07897e-05	23
3.5	-0.0183092	8.56118e-06	45
3.75	-0.0135918	2.185368e-05	36
4	-0.00973151	1.46105e-05	45
4.5	-0.00634270	1.93298e-05	48
5	-0.00732334	2.32423e-05	52
6	-0.0124360	2.37328e-05	66
7	-0.0142094	3.85332e-05	69
8	-0.0136610	3.29375e-05	87
9	-0.0137827	3.58598e-05	99

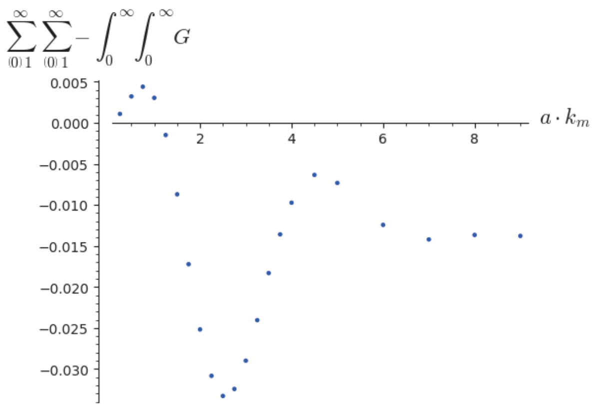

Fig.2. *How* $\sum_{(0)1}^{\infty}\sum_{(0)1}^{\infty}-\int_0^{\infty}\int_0^{\infty}G$ *depends on* $a\cdot k_m$.

According to our calculation we can see that

$$\int_0^{\infty}\left\{\sum_{n_x=(0)\,1}^{\infty}\sum_{n_y=(0)\,1}^{\infty} F(n_x,n_y) - \int_0^{\infty}\int_0^{\infty} F(n_x,n_y)\,dn_x dn_y\right\}du \approx R(ak_m), \tag{17}$$

where $R(ak_m)$ is a function depended on material properties with well defined limit at ak_m ˇ 1. So

$$\delta\frac{E}{V} = R(ak_m)\,\frac{\hbar c\pi}{a^4}. \tag{18}$$

For the energy density difference per cm^2 (in the limit at $ak_m \to 10$) we find that
$\delta\frac{E}{V} = \hbar c\pi\frac{R}{a^4} = 0.0136\,\frac{1}{a_\mu^4}\,dyne/cm^2$

where a_μ is a square side of honeycombs measured in microns.

Can this difference of specific energy density $\delta(E/V)$ be interpreted as the cause of the force F applied to perfectly conducting honeycomb on a plate? For example, my investigations of the configuration used by Casimir have shown that for the geometric configuration of two perfectly conducting plates $F/S = -3\cdot\delta(E/V)$. But what can be said about honeycomb configuration? Research of this question presented in appendixes B and C shows that $F/S \approx \delta(E/V)$.

CONCLUSION

Therefore, the following conclusions can be drawn: there is a force applied to perfectly conducting honeycombs on a plate as a result of the difference in specific energy density on its different sides. This force depends on the material of the plate. This force depends on the cutoff frequency ω_m of the honeycomb plate material at least. This force can be interpreted as the pressure of the zero-point electromagnetic oscillations.

Although the effect is small, an experimental confirmation seems not unfeasible and might be of a certain interest.

Tuo Qu, Fang Liu, Yuechai Lin, Yidong Huang [5] have reported the production of gold nano-honeycomb with a size of about 2 microns. According to the formula presented in this work that honeycomb should have Casimir energy density difference about $\delta(E/V) = 0.0136/(2^4) = 0.0008555\, dyne/cm^2 = 0.0008555 \cdot 10^4 = 8.55\, dyne/m^2$, that is 8.55 dynes per square meter of the panel, which is quite an acceptable value for the practical use of the expected effect for satellite orbits correction.

It is important to point out, that according to the proposed method, the $\delta(E/V)$ is calculated not for the total surface area of the honeycombs, but for a part of the panel occupied by cavities (minus the area of the honeycomb walls).

That does not mean, the honeycomb walls should be made as thin as possible, because with a decrease in the wall thickness, the value of k_m will also decrease.

For simplicity of calculations, square-shaped nanocells have been analyzed, but the obtained result can be used, with some correction unknown so far, to estimate the Casimir effect in honeycombs of a different shape (hexagonal, cylindrical, etc.). Such honeycombs are simpler to be manufactured, but require much more complex calculations to estimate the Casimir effect exactly. In addition, the bottom of the honeycomb is not necessarily flat, but spherically concave, for example, which does not fundamentally affect the magnitude of the thrust.

In addition, the appearance of Casimir thrust is expected not only in metal honeycombs, but in dielectric honeycombs as well, because the Casimir effect in dielectrics is well known.

It has been assumed that the height of the the walls of the honeycombs is $L/2$, while the real height of the walls h will be much smaller (say about a).

At the same time, by using the Antipin approach (see Appendix D) it can be shown that the effect should be observed at a smaller height of the ribs, although the dependence of the effect on the height may be the subject of further research.

To answer Hrvoje [6] who considers the Casimir effect as not a consequence of the existence of virtual quantum photons, but as manifestation of the London-Van der Waals dispersion forces, I would like to note that setting up an experiment to measure the thrust produced by nanocells grown on metal plate could serve as a critical experiment to find out which points of view on the nature of the Casimir force corresponds to reality.

APPENDIX A. INTEGRATION DETAILS

Let us use the regularization function in the form

$$f\left(\frac{k}{k_m}\right) = \frac{1}{\frac{k^4}{k_m^4}+1}. \tag{A.1}$$

Starting from (4) and by introducing a variable $n_{xy} = \sqrt{n_x^2+n_y^2}$ and $n = \sqrt{n_x^2+n_y^2+u^2} = \sqrt{n_{xy}^2+u^2}$, we have

$$F(u, n_{xy}, ak_m) = \frac{\sqrt{n_{xy}^2+u^2}}{\frac{\pi^4\left(n_{xy}^2+u^2\right)^2}{ak_m^4}+1}. \tag{A.2}$$

And using this variable, we can make the following substitution

$u = \sqrt{n^2-n_x^2-n_y^2} = \sqrt{n^2-n_{xy}^2}$, $\frac{du}{dn} = \frac{n}{\sqrt{n^2-n_{xy}{}^2}}$, $du = \frac{n\,dn}{\sqrt{n^2-n_{xy}{}^2}}$.

And now we can rewrite integral (5) in form

$$G = \int_0^\infty \sqrt{n_{xy}^2+u^2}\, f\left(\frac{\pi\sqrt{n_{xy}^2+u^2}}{a\,k_m}\right) du, \tag{A.3}$$

changing the integration variable from u to n

$G = \int_{n_{xy}}^{\infty} \sqrt{n_{xy}^2+u^2}\, f\left(\frac{\pi\sqrt{n_{xy}^2+u^2}}{a\,k_m}\right) dn\, \frac{n}{\sqrt{n^2-n_{xy}^2}}$

$$G(n_x,n_y) = \int_{n_{xy}}^{\infty} n\, f\left(\frac{\pi n}{a\,k_m}\right) dn\, \frac{n}{\sqrt{n^2-n_{xy}^2}}, \tag{A.4}$$

because in this form integral can be taken analytically. So, we have the following integrand

$$F(n, n_{xy}, ak_m) = \frac{n^2}{\left(\frac{\pi^4 n^4}{ak_m^4}+1\right)\sqrt{n^2-n_{xy}^2}} \tag{A.5}$$

and the following limits of integration by n: $n_a = n_{xy}$, $n_b = \infty$.

Let us use the Abel substitution:

$$t = \left(\sqrt{n^2 - n_{xy}^2}\right)', \quad t = \frac{n}{\sqrt{n^2 - n_{xy}^2}} \tag{A.6}$$

and the following limits of integration by t: $t_a = +\infty$, $t_b = +1$.

Let us denote dependency of n from t

$$n^2 = \frac{n_{xy}^2 t^2}{t^2 - 1}, \quad n = n_{xy}\sqrt{\frac{t^2}{t^2-1}} \tag{A.7}$$

and derivatives

$$\frac{dt}{dn} = \frac{d}{dn}\left(\frac{n}{\sqrt{n^2 - n_{xy}^2}}\right) = -\frac{n^2}{\left(n^2 - n_{xy}^2\right)^{\frac{3}{2}}} + \frac{1}{\sqrt{n^2 - n_{xy}^2}},$$

$$\frac{dn}{dt} = -\frac{n^4 - 2n^2 n_{xy}^2 + n_{xy}^4}{\sqrt{n^2 - n_{xy}^2} n_{xy}^2}. \tag{A.8}$$

Now we can rewrite the integrand, making it depending on t

$$F(t, n_{xy}, ak_m) = F(n, n_{xy}, ak_m) \cdot \left.\frac{dn}{dt}\right|_{n = n_{xy}\sqrt{\frac{t^2}{t^2-1}}}$$

$$F(n, n_{xy}, ak_m) \cdot \frac{dn}{dt} = -\frac{\left(n^4 - 2n^2 n_{xy}^2 + n_{xy}^4\right) n^2}{\left(\frac{\pi^4 n^4}{ak_m^4} + 1\right)\left(n^2 - n_{xy}^2\right) n_{xy}^2}$$

$$F(t, n_{xy}, ak_m) = -\frac{\left(\frac{n_{xy}^4 t^4}{\left(t^2-1\right)^2} - \frac{2 n_{xy}^4 t^2}{t^2-1} + n_{xy}^4\right) t^2}{\left(\frac{\pi^4 n_{xy}^4 t^4}{\left(t^2-1\right)^2 ak_m^4} + 1\right)\left(\frac{n_{xy}^2 t^2}{t^2-1} - n_{xy}^2\right)\left(t^2 - 1\right)}$$

$$F(t, n_{xy}, ak_m) = \frac{ak_m^4 n_{xy}^2 t^2}{2 ak_m^4 t^2 - \left(\pi^4 n_{xy}^4 + ak_m^4\right) t^4 - ak_m^4}$$

Let us extract coefficient near t^4 from the denominator.

Now let us move the above coefficient up to the numerator. So, the new numerator will be

$$-\frac{ak_m^4 n_{xy}^2 t^2}{\pi^4 n_{xy}^4 + ak_m^4}.$$

Accordingly, the new denominator will be

$$-\frac{2 ak_m^4 t^2}{\pi^4 n_{xy}^4 + ak_m^4} + t^4 + \frac{ak_m^4}{\pi^4 n_{xy}^4 + ak_m^4}.$$

Now we should convert this denominator to the following form

$$-\left(\alpha_1 t + t^2 + \beta_1\right)\left(\alpha_1 t - t^2 - \beta_1\right),$$

$$t^4 - \left(\alpha_1^2 - 2\beta_1\right) t^2 + \beta_1^2.$$

So, we have the following equation

$$-\frac{2 ak_m^4 t^2}{\pi^4 n_{xy}^4 + ak_m^4} + t^4 + \frac{ak_m^4}{\pi^4 n_{xy}^4 + ak_m^4} = t^4 - \left(\alpha_1^2 - 2\beta_1\right) t^2 + \beta_1^2$$

and its solution

$$\beta_1 = \frac{ak_m^2}{\sqrt{\pi^4 n_{xy}^4 + ak_m^4}}, \tag{A.9}$$

$$\alpha_1 = \sqrt{2} ak_m \sqrt{\frac{ak_m^2 + \sqrt{\pi^4 n_{xy}^4 + ak_m^4}}{\pi^4 n_{xy}^4 + ak_m^4}}. \tag{A.10}$$

After the conversion determined above, the integrand can be presented as

$$\frac{ak_m^4 n_{xy}^2 t^2}{\left(\pi^4 n_{xy}^4 + ak_m^4\right)\left(\alpha_1 t + t^2 + \beta_1\right)\left(\alpha_1 t - t^2 - \beta_1\right)}.$$

Let us check determinant $\alpha_1^2 - 4\beta_1$ by using the expression of α_1 and β_1 found above.

The determinant is negative and the integral can be easily calculated:

$$\int F(t, n_{xy}, ak_m)\, dt = -\frac{ak_m^4 n_{xy}^2}{4\left(\pi^4 n_{xy}^4 + ak_m^4\right)}\left(\frac{2\arctan\left(\frac{\alpha_1 + 2t}{\sqrt{-\alpha_1^2 + 4\beta_1}}\right)}{\sqrt{-\alpha_1^2 + 4\beta_1}} + \frac{2\arctan\left(-\frac{\alpha_1 - 2t}{\sqrt{-\alpha_1^2 + 4\beta_1}}\right)}{\sqrt{-\alpha_1^2 + 4\beta_1}} - \frac{\log\left(\alpha_1 t + t^2 + \beta_1\right)}{\alpha_1} + \frac{\log\left(-\alpha_1 t + t^2 + \beta_1\right)}{\alpha_1}\right).$$

APPENDIX B. ELECTROMAGNETIC PRESSURE CALCULATION

Let us consider a rectangular resonator with size $a \times b \times h$.

For the electric mode

$$\nabla \vec{E} + \frac{\omega^2}{c^2} \vec{E} = 0, \tag{B.1}$$

we have the following solution

$$E_x = A_x \cos\left(\frac{\pi n_x x}{a}\right) \sin\left(\frac{\pi n_y y}{b}\right) \sin(k_z z)$$

$$E_y = A_y \cos\left(\frac{\pi n_y y}{b}\right) \sin\left(\frac{\pi n_x x}{a}\right) \sin(k_z z)$$

$$E_z = A_z \cos(k_z z) \sin\left(\frac{\pi n_x x}{a}\right) \sin\left(\frac{\pi n_y y}{b}\right)$$

and

$$H_x = \frac{i(A_y b k_z - \pi A_z n_y) c \cos\left(\frac{\pi n_y y}{b}\right) \cos(k_z z) \sin\left(\frac{\pi n_x x}{a}\right)}{b \mu \omega}$$

$$H_y = -\frac{i\left(A_x k_z - \frac{\pi A_z n_x}{a}\right) c \cos\left(\frac{\pi n_x x}{a}\right) \cos(k_z z) \sin\left(\frac{\pi n_y y}{b}\right)}{\mu \omega}$$

$$H_z = -\frac{i\left(\frac{\pi A_y n_x}{a} - \frac{\pi A_x n_y}{b}\right) c \cos\left(\frac{\pi n_x x}{a}\right) \cos\left(\frac{\pi n_y y}{b}\right) \sin(k_z z)}{\mu \omega}$$

with

$$k_z^2 + \frac{\pi^2 n_x^2}{a^2} + \frac{\pi^2 n_y^2}{b^2} - \frac{\omega^2}{c^2} = 0, \tag{B.2}$$

by using $div \vec{E} = 0$, we have

$$A_z k_z + \frac{\pi A_x n_x}{a} + \frac{\pi A_y n_y}{b} = 0. \tag{B.3}$$

Field energy density $\left(\int \frac{E_x^2 + E_y^2 + E_z^2}{8\pi} dV\right)/V$ is

$$\frac{E}{V} = \frac{A_x^2 + A_y^2 + A_z^2}{64\pi}. \tag{B.4}$$

Full energy density $\left(\int \frac{|\vec{E}|^2}{8\pi} dV + \int \frac{|\vec{H}|^2}{8\pi} dV\right)/V$ is

$$\frac{E}{V} = \frac{A_x^2 + A_y^2 + A_z^2}{32\pi}. \tag{B.5}$$

Electromagnetic pressure $\left(\int \frac{H_x^2 + H_y^2}{8\pi} dS\right)/S$ on xy plate is

$$\frac{f_z}{S} = -\frac{2\pi A_x A_z a b^2 k_z n_x - \pi^2 A_z^2 b^2 n_x^2 + 2\pi A_y A_z a^2 b k_z n_y - \pi^2 A_z^2 a^2 n_y^2 - \left(A_x^2 + A_y^2\right) a^2 b^2 k_z^2}{32\left(\pi a^2 b^2 k_z^2 + \pi^3 b^2 n_x^2 + \pi^3 a^2 n_y^2\right)}. \tag{6}$$

Their relation $\frac{f_z/S}{E/V}$ is

$$\frac{f_z/S}{E/V} = \frac{A_x^2 a^2 b^2 k_z^2 + A_y^2 a^2 b^2 k_z^2 - 2\pi A_x A_z a b^2 k_z n_x + \pi^2 A_z^2 b^2 n_x^2 - 2\pi A_y A_z a^2 b k_z n_y + \pi^2 A_z^2 a^2 n_y^2}{\left(a^2 b^2 k_z^2 + \pi^2 b^2 n_x^2 + \pi^2 a^2 n_y^2\right)\left(A_x^2 + A_y^2 + A_z^2\right)}. \tag{7}$$

Considering solution with wave propagation in z-direction we have $H_z = 0$ which gives

$$\pi A_y b n_x - \pi A_x a n_y = 0$$

and

$$A_x = -\frac{A_z a b^2 k_z n_x}{\pi b^2 n_x^2 + \pi a^2 n_y^2}, A_y = -\frac{A_z a^2 b k_z n_y}{\pi b^2 n_x^2 + \pi a^2 n_y^2}.$$

In this case the relation of electromagnetic pressure per field energy density is equal to 1,

$$\frac{f_z/S}{E/V} = 1. \tag{8}$$

Considering solution with wave propagation in x-direction we have $H_x = 0$ which gives

$$A_y = -\frac{\pi^2 A_x b n_x n_y}{a b^2 k_z^2 + \pi^2 a n_y^2}, A_z = -\frac{\pi A_x b^2 k_z n_x}{a b^2 k_z^2 + \pi^2 a n_y^2}.$$

In this case the relation of electromagnetic pressure per field energy density is

$$\frac{f_z/S}{E/V} = \frac{b^2 k_z^2}{b^2 k_z^2 + \pi^2 n_y^2}. \tag{9}$$

Considering solution with wave propagation in y-direction, $H_y = 0$, as in x-direction

$$A_x = -\frac{\pi^2 A_y a n_x n_y}{a^2 b k_z^2 + \pi^2 b n_x^2}, A_z = -\frac{\pi A_y a^2 k_z n_y}{a^2 b k_z^2 + \pi^2 b n_x^2},$$

$$\frac{f_z/S}{E/V} = \frac{a^2 k_z^2}{a^2 k_z^2 + \pi^2 n_x^2}. \tag{10}$$

For the magnetic mode

$$\nabla \vec{H} + \frac{\omega^2}{c^2} \vec{H} = 0 \tag{11}$$

, we have the following solution

$$H_x = B_1 \cos\left(\frac{\pi n_y y}{b}\right)\cos(k_z z)\sin\left(\frac{\pi n_x x}{a}\right)$$
$$H_y = B_2 \cos\left(\frac{\pi n_x x}{a}\right)\cos(k_z z)\sin\left(\frac{\pi n_y y}{b}\right)$$
$$H_z = B_3 \cos\left(\frac{\pi n_x x}{a}\right)\cos\left(\frac{\pi n_y y}{b}\right)\sin(k_z z)$$

and

$$E_x = \frac{i\left(B_2 k_z - \frac{\pi B_3 n_y}{b}\right) c \cos\left(\frac{\pi n_x x}{a}\right)\sin\left(\frac{\pi n_y y}{b}\right)\sin(k_z z)}{\mu\omega}$$
$$E_y = -\frac{i\left(B_1 k_z - \frac{\pi B_3 n_x}{a}\right) c \cos\left(\frac{\pi n_y y}{b}\right)\sin\left(\frac{\pi n_x x}{a}\right)\sin(k_z z)}{\mu\omega}$$
$$E_z = -\frac{i\left(\frac{\pi B_2 n_x}{a} - \frac{\pi B_1 n_y}{b}\right) c \cos(k_z z)\sin\left(\frac{\pi n_x x}{a}\right)\sin\left(\frac{\pi n_y y}{b}\right)}{\mu\omega}$$

with

$$k_z^2 + \frac{\pi^2 n_x^2}{a^2} + \frac{\pi^2 n_y^2}{b^2} - \frac{\omega^2}{c^2} = 0, \qquad (12)$$

by using $div\vec{H} = 0$, we have

$$B_3 k_z + \frac{\pi B_1 n_x}{a} + \frac{\pi B_2 n_y}{b} = 0. \qquad (13)$$

Magnetic field energy density $\left(\int \frac{H_x^2+H_y^2+H_z^2}{8\pi} dV\right)/V$ is

$$\frac{E}{V} = \frac{B_1^2 + B_2^2 + B_3^2}{64\pi}. \qquad (14)$$

Full energy density $\left(\int \frac{|\vec{E}|^2}{8\pi} dV + \int \frac{|\vec{H}|^2}{8\pi} dV\right)/V$ is

$$\frac{E}{V} = \frac{B_1^2 + B_2^2 + B_3^2}{32\pi}. \qquad (15)$$

Electromagnetic pressure $\left(\int \frac{H_x^2+H_y^2}{8\pi} dS\right)/S$ on xy plate is

$$\frac{f_z}{S} = \frac{B_1^2 + B_2^2}{32\pi}. \qquad (16)$$

Their relation $\frac{f_z/S}{E/V}$ is

$$\frac{f_z/S}{E/V} = \frac{B_1^2 + B_2^2}{B_1^2 + B_2^2 + B_3^2}. \qquad (17)$$

Considering solution with wave propagation in z-direction, we have $E_z = 0$ which gives

$$\frac{\pi B_2 n_x}{a} - \frac{\pi B_1 n_y}{a} = 0$$

and

$$B_1 = -\frac{B_3 a k_z n_x}{\pi n_x^2 + \pi n_y^2}, B_2 = -\frac{B_3 a k_z n_y}{\pi n_x^2 + \pi n_y^2}.$$

In this case the relation of electromagnetic pressure per field energy density is

$$\frac{f_z/S}{E/V} = \frac{a^2 k_z^2}{a^2 k_z^2 + \pi^2 n_x^2 + \pi^2 n_y^2}. \qquad (18)$$

Considering solution with wave propagation in x-direction, we have $E_x = 0$ which gives

$$B_2 = -\frac{\pi^2 B_1 n_x n_y}{a^2 k_z^2 + \pi^2 n_y^2}, B_3 = -\frac{\pi B_1 a k_z n_x}{a^2 k_z^2 + \pi^2 n_y^2}.$$

In this case the relation of electromagnetic pressure per field energy density is

$$\frac{f_z/S}{E/V} = \frac{a^2 b^4 k_z^4 + 2\pi^2 a^2 b^2 k_z^2 n_y^2 + \pi^4 b^2 n_x^2 n_y^2 + \pi^4 a^2 n_y^4}{\left(a^2 b^2 k_z^2 + \pi^2 b^2 n_x^2 + \pi^2 a^2 n_y^2\right)\left(b^2 k_z^2 + \pi^2 n_y^2\right)}.$$

Considering solution with wave propagation in y-direction $E_y = 0$ as in x-direction

$$B_1 = -\frac{\pi^2 B_2 n_x n_y}{a^2 k_z^2 + \pi^2 n_x^2}, B_3 = -\frac{\pi B_2 a k_z n_y}{a^2 k_z^2 + \pi^2 n_x^2},$$

$$\frac{f_z/S}{E/V} = \frac{a^4 b^2 k_z^4 + 2\pi^2 a^2 b^2 k_z^2 n_x^2 + \pi^4 b^2 n_x^4 + \pi^4 a^2 n_x^2 n_y^2}{\left(a^2 b^2 k_z^2 + \pi^2 b^2 n_x^2 + \pi^2 a^2 n_y^2\right)\left(a^2 k_z^2 + \pi^2 n_x^2\right)}.$$

So, we can see that task of electromagnetic force calculation in the nanohoneycomb configuration is quit easy, because

$$\lim_{k_z \to \infty} \frac{f_z/S}{E/V} = 1. \qquad (19)$$

We can see that if we decrease a then $\frac{f_z/S}{E/V}$ also decreases and that leads to

$$\frac{F}{S} \geq \delta \frac{E}{V} = \hbar c \pi \frac{R}{a^4}. \qquad (20)$$

On the other hand, the same result can be shown by using Hamiltonian mechanics approach.

APPENDIX C. HAMILTONIAN MECHANICS APPROACH

Let us consider a cubic cavity of volume L^3 bounded by perfectly conducting walls where perfectly conducting square plate with side L is placed in this cavity parallel to the xy face, and let the distance between the plate and xy face be sufficiently large, say $l = L/2$, for example.

One side of this perfectly conducting square plate is a smooth plane and another is covered by perfectly conducting square-shaped honeycomb a square side a.

On both sided of the the plate the expressions $1/2\sum\hbar\omega$ where the summation extends over all possible resonance frequencies of the cavity $(L-l)\times L\times L$ (a large cavity between smooth plane and xy face) and the cavity $l\times a\times a$ (a small cavity, one honeycomb cell) are divergent and devoid of physical meaning, but it will be shown that for the both opposite sides the derivative $d\langle 0|\mathscr{H}|0\rangle/dl$ of the vacuums Hamiltonian of the whole system for these sums $\langle 0|\mathscr{H}|0\rangle = 1/2\left(\sum\hbar\omega\right)_I + 1/2\left(\sum\hbar\omega\right)_{II}$, has a well-defined value and this value will be interpreted as the interaction between the plate and the both xy faces.

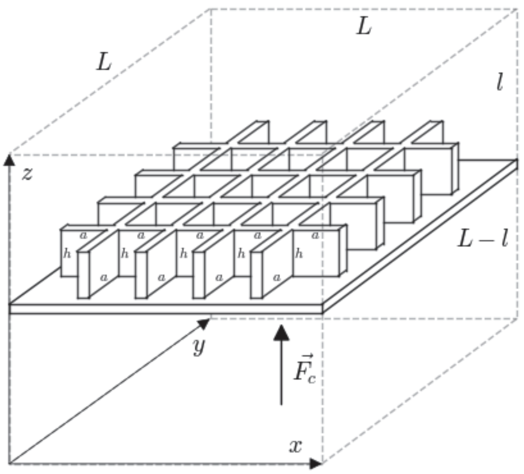

Fig.3. *Cubic cavity with a plate covered by honeycomb.*

The possible oscillations of the cavities defined by $0\le x\le L$, $0\le y\le L$, $0\ge z\ge -(L-l)$ (a large cavity between smooth plane and xy face) and $0\le x\le a$, $0\le y\le a$, $0\le z\le l$ (a small cavity, one honeycomb cell) have the wave vectors $k_x = \frac{\pi}{L}n_x$, $k_y = \frac{\pi}{L}n_y$, $k_z = \frac{\pi}{L-l}n_z$ (a large cavity between smooth plane and xy face), and $k_x = \frac{\pi}{a}n_x$, $k_y = \frac{\pi}{a}n_y$, $k_z = \frac{\pi}{l}n_z$ (a small cavity, one honeycomb cell), where n_x. n_y, n_z are positive integers; $k = \sqrt{k_x^2+k_y^2+k_z^2} = \sqrt{\kappa^2+k_z^2}$.

Let us write the expression for the sum of zero-point energy in general form

$$E = \frac{1}{2}\sum\hbar\omega = \hbar c\frac{1}{2}\sum_{n_x}\sum_{n_y}\sum_{n_z}k. \qquad \text{(C.1)}$$

Two standing waves correspond to every k_x, k_y, k_z, but in case when one of the n_i is zero, there is only one wave. That is of no importance in case of one honeycomb cell cavity for k_z, since for very large l we may regard k_z as continuous variable, replacing summation over n_z with integration. Thus, for a small cavity consisting of one honeycomb, we find

$$E = \frac{\hbar c}{2}\int_0^\infty\left[\sqrt{k_z^2} + 2\sum_{n_x=1}^\infty\sum_{n_y=1}^\infty\sqrt{\frac{n_x^2\pi^2}{a^2}+\frac{n_y^2\pi^2}{a^2}+k_z^2}\right]dn_z.$$

Considering $dn_z = \frac{l}{\pi}dk_z$ we can find the specific energy density E/S, where $S = S_{small} = a^2$:

$$\frac{E}{S} = \frac{\hbar c}{a^2}\int_0^\infty\left[\frac{\sqrt{k_z^2}}{2} + \sum_{n_x=1}^\infty\sum_{n_y=1}^\infty\sqrt{\frac{n_x^2\pi^2}{a^2}+\frac{n_y^2\pi^2}{a^2}+k_z^2}\right]\frac{l}{\pi}dk_z,$$

$$\frac{E}{S} = \frac{\hbar c l}{a^2\pi}\sum_{n_x=(0)1}^\infty\sum_{n_y=(0)1}^\infty\left[\int_0^\infty\sqrt{\frac{n_x^2\pi^2}{a^2}+\frac{n_y^2\pi^2}{a^2}+k_z^2}\,dk_z\right].$$

That is of no importance in case of a large cavity for k_x, k_y since for very large L we may regard k_x, k_y as continuous variables. Thus, for large cavity between smooth plane and xy face we find

$$\begin{aligned}\sum\frac{\hbar\omega}{2} = \frac{\hbar c}{2}\int_0^\infty\int_0^\infty\Bigg[&\sqrt{k_x^2+k_y^2}+\\ &+\ 2\sum_{n_z=1}^\infty\sqrt{\frac{n_z^2\pi^2}{(L-l)^2}+k_x^2+k_y^2}\Bigg]dn_x dn_y.\end{aligned} \qquad \text{(C.2)}$$

For very large $L-l$ the last summation may be replaced by an integral and, therefore, it can be seen that energy of a large cavity is given by

$$\sum\frac{\hbar\omega}{2} = \hbar c\int_0^\infty\int_0^\infty\int_0^\infty\sqrt{k_z^2+k_x^2+k_y^2}\,dn_x\,dn_y\,dn_z, \qquad \text{(C.3)}$$

where $dn_x = \frac{L}{\pi}dk_x$, $dn_y = \frac{L}{\pi}dk_y$, $dn_z = \frac{L-l}{\pi}dk_z$.

Now for the specific (per area) energy density E/S for a large cavity, where $S = S_{large} = L^2$ we can write the following sequence of transformations:

$$\frac{\sum\hbar\omega}{2S} = \frac{\hbar c}{L^2}\int_0^\infty\int_0^\infty\int_0^\infty\sqrt{k_z^2+k_x^2+k_y^2}\,dn_x\,dn_y\,\frac{L-l}{\pi}dk_z,$$

$$\frac{E}{S} = \frac{L-l}{L^2\pi}\hbar c\int_0^\infty\int_0^\infty\left[\int_0^\infty\sqrt{k_x^2+k_y^2+k_z^2}\,dk_z\right]\left(\frac{L}{\pi}dk_x\right)\left(\frac{L}{\pi}dk_y\right),$$

$$\frac{E}{S} = \frac{L-l}{a^2\pi}\hbar c\int_0^\infty\int_0^\infty\left[\int_0^\infty\sqrt{k_x^2+k_y^2+k_z^2}\,dk_z\right]\left(\frac{a}{\pi}dk_x\right)\left(\frac{a}{\pi}dk_y\right).$$

Therefore, it can be seen that specific (per area) vacuum Hamiltonian $\frac{\langle 0|\mathscr{H}|0\rangle}{S}$ of the whole system is given by

$$\frac{\langle 0|\hat{\mathscr{H}}|0\rangle}{S}=\frac{\hbar c}{a^2\pi}\cdot\left\{l\sum_{n_x=(0)1}^{\infty}\sum_{n_y=(0)1}^{\infty}\left[\int_0^{\infty}\sqrt{n_x^2\frac{\pi^2}{a^2}+n_y^2\frac{\pi^2}{a^2}+k_z^2}\,dk_z\right]+(L-l)\int_0^{\infty}\int_0^{\infty}\left[\int_0^{\infty}\sqrt{k_x^2+k_y^2+k_z^2}\,dk_z\right]\left(\frac{a}{\pi}dk_x\right)\left(\frac{a}{\pi}dk_y\right)\right\}$$

and interaction $\frac{F}{S}=-\frac{\partial}{\partial z}\frac{\langle 0|\hat{\mathscr{H}}|0\rangle}{S}=\frac{\partial}{\partial l}\frac{\langle 0|\hat{\mathscr{H}}|0\rangle}{S}$ is

$$\frac{F}{S}=\frac{\hbar c}{a^2\pi}\left\{\sum_{n_x=(0)1}^{\infty}\sum_{n_y=(0)1}^{\infty}\left[\int_0^{\infty}\sqrt{n_x^2\frac{\pi^2}{a^2}+n_y^2\frac{\pi^2}{a^2}+k_z^2}\,dk_z\right]-\int_0^{\infty}\int_0^{\infty}\left[\int_0^{\infty}\sqrt{k_x^2+k_y^2+k_z^2}\,dk_z\right]dn_x\,dn_y\right\} \quad (4)$$

Then, the formula for the force acting on a perfectly conducting honeycomb on a plate F/S obtained by using Hamiltonian mechanics approach is the same as formula (2) received for the difference of specific energy density on its different sides $\delta(E/V)$.

APPENDIX D. THE DERIVATION OF THE THRUST FORMULA FOR THE HONEYCOMB BASING ON THE ANTIPIN'S FORMULA FOR THE METAL ANGLE

Antipin [3] gives an estimated calculation of the thrust of the V-shaped angle by using the Casimir formula, "with the most general and natural approximations known as PFA (Proximity Force Approximation) or PAA (Pairwise Additive Approximation) calculation method [7], [8]".

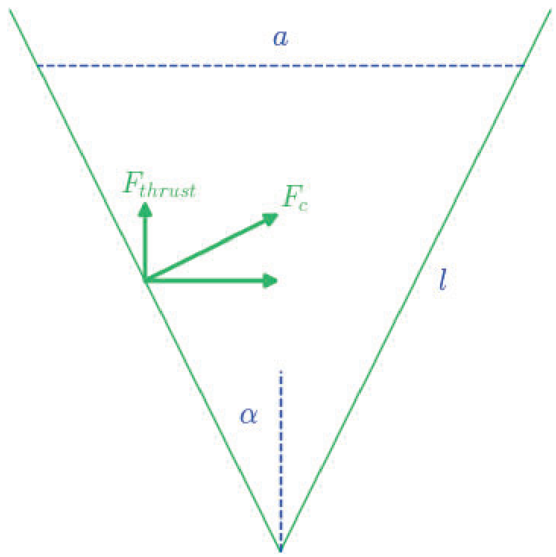

Fig.4. *Antipin's angle.*

Casimir's interaction energy is given by

$$\delta E/L^2=\hbar c\frac{\pi^2}{4a^3}\left\{\frac{-4}{24\times 30}\right\}. \quad (D.1)$$

Casimir's force is

$$F_c=\hbar c\frac{-3\pi^2}{4a^4}\left\{\frac{-4}{24\times 30}\right\}. \quad (D.2)$$

Thrust of the metal V-shaped angle is

$$F_{thrust}=2\int F_c\sin\alpha\,dS,\quad dS=b\,dz \quad (D.3)$$

$$F_{thrust}=2\frac{-3\pi^2\hbar cb}{4}\int_{z_{min}}^{z_{max}}\left\{\frac{-4}{24\times 30}\right\}\frac{\sin\alpha\,dz}{(a(z))^4}. \quad (D.4)$$

Let us make a substitution

$$a(z)=2z\,tg\,\alpha,$$

$$F_{thrust}=2\frac{-3\pi^2\hbar cb}{4}\int_{z_{min}}^{z_{max}}\left\{\frac{-4}{24\times 30}\right\}\frac{\sin\alpha\,dz}{(2z\,tg\alpha)^4},$$

$$F_{thrust}=2\frac{-3\pi^2\hbar cb}{4}\frac{\sin\alpha}{(2tg\,\alpha)^4}\int_{z_{min}}^{z_{max}}\left\{\frac{-4}{24\times 30}\right\}\frac{dz}{z^4},$$

$$F_{thrust}=-2\cdot 3\frac{\pi^2\hbar cb}{240}\frac{\sin\alpha}{(2tg\,\alpha)^4}\left(\frac{1}{z^3}\right)\Bigg|_{z_{min}}^{z_{max}}.$$

The following substitution can be made:

$$z=l\cos\alpha,$$

$$F_{thrust}=-2\cdot 3\frac{\pi^2\hbar cb}{240}\frac{\sin\alpha}{2^4(tg\,\alpha)^4(\cos\alpha)^3}\left(\frac{1}{l^3}\right)\Bigg|_{l_{min}}^{l_{max}},$$

$$F_{thrust}=-2\cdot 3\frac{\pi^2\hbar cb}{240}\frac{\cos\alpha}{2^4(\sin\alpha)^4}\left(\frac{1}{l^3}\right)\Bigg|_{l_{min}}^{l_{max}},$$

$$F_{thrust}=\frac{\pi^2\hbar cb}{640}\frac{\cos\alpha}{(\sin\alpha)^4}\left(\frac{1}{l_{min}^3}-\frac{1}{l_{max}^3}\right). \quad (D.5)$$

Thus, the formula for the thrust for the V-shaped angle is derived basing on the length of its wings.

Antipin indicates that the value of l_{min} is limited from below by the „cutoff" level, which is determined technologically:

- by the accuracy of plate manufacturing (their roughness, degree of flatness), as well as
- by the the minimum wavelength of photons that can effectively reflect the substance from which the V-shaped angle is made (by k_m value).

Investigating the dependence of the coefficient in the angle thrust formula, which depends on the half angle α, it can be seen that for a given length of the V-shape sides, it is more efficient to make the angle as small as possible. However, for the purposes of this work (studying the possibility of obtaining thrust by using nanocells), it is important to note that for a L-shaped angle with a right angle $\alpha = \pi/4$, the coefficient $(cos\,\alpha)/\left((sin\,\alpha)^4\right) = 2\sqrt{2}$. Thus, by composing a honeycomb structure from many rectangular L-shaped angles, it can be shown that the thrust of the panel consisting of rectangular honeycombs is not zero.

Indeed, a rectangular honeycomb with a cell size of $b \times b$ and with the same edge height equal to $b = l_{max}$ can be imagined as a combination of four L-shaped angles where a half-angle is equal to $\alpha = \pi/4$. The thrust of the every L-shaped angle

$$F_{thrust} = -\frac{\pi^2 \hbar c l_{max}}{640} 2\sqrt{2}\left(\frac{1}{l_{min}^3} - \frac{1}{l_{max}^3}\right) \qquad \text{(D.6)}$$

directed along the bisector of each angle must be multiplied by $sin(\pi/4) = \sqrt{2}/2$ and, when multiplied by 4, the thrust of such a honeycomb cell will be equal to

$$F_{thrust} = -\frac{\pi^2 \hbar c b}{80}\left(\frac{1}{l_{min}^3} - \frac{1}{b^3}\right). \qquad \text{(D.7)}$$

So, the formula for the specific thrust of cells obtained by using the PFA (Proximity Force Approximation) method or PAA (Pairwise Additive Approximation) method

$$\frac{F_{thrust}}{S} = -\frac{\pi^2 \hbar c}{80 b}\left(\frac{1}{l_{min}^3} - \frac{1}{b^3}\right) \qquad \text{(D.8)}$$

is to some extent similar to the formula for the magnitude of the two-dimensional Casimir effect on honeycombs presented in the first part of this work. At least the value of the exponent in the denominator is the same

$$\delta \frac{E}{V} \approx R(b \cdot k_m)\,\frac{\hbar c \pi}{b^4}. \qquad \text{(D.9)}$$

It should be noted, that this formula is received without taking into account the finiteness of the cell edge height (i.e., in the approximation of the infinite edge height), in contrast to the PFA version of the formula for which the edge height is assumed to be equal to the cell width.

The approximate agreement of these formulas indicates that the effect should also be observed at finite height of the ribs, although the dependence of the effect on this height may be the object of further research.

Theoretically it is possible to achieve a greater value of thrust with a panel produced from acute-angled V-shaped angles, but producing panels from honeycombs seems to be technologically simpler than from V-shaped angles.

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СТІЛЬНИКОВИЙ РУШІЙ КАЗИМИРА: ПРО СИЛУ, ЩО ДІЄ НА СОТИ НА ПЛАСТИНІ, ЯКІ ІДЕАЛЬНО ПРОВОДЯТЬ

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У цій статті проаналізовано двовимірний ефект Казимира одного тіла на прикладі наносот квадратної форми. У класичному одновимірному ефекті Казимира двох тіл сила Казимира між двома пластинами виникає як різниця електромагнітних тисків квантово-вакуумних флуктуацій нульової точки по різні боки кожної з пластин. Пластини штовхаються одна до одної зовнішніми полями квантово-вакуумних осциляцій, щільність яких в класичній конфігурації перевищує щільність внутрішніх. Можна спробувати створити різницю електромагнітних тисків квантово-вакуумних осциляцій по різні боки однієї пластини за рахунок різниці геометрії вакуумних резонаторів на різних сторонах пластини. Для цього необхідно виростити нанокомірки на одній з поверхонь гладкої металевої пластини. В результаті було виявлено, що формула для сили на одиницю площі дуже схожа на формулу класичного ефекту Казимира, за винятком значення коефіцієнта пропорційності.

Силу, прикладену до ідеально провідних сот на пластині в результаті різниці питомої густини енергії на різних її сторонах, можна інтерпретувати як тиск електромагнітних флуктуацій нульової точки. Згідно з формулою, представленою в цій роботі, для золотих наносот розміром близько 2 мкм сила має дорівнювати 8,55 дин на квадратний метр панелі, що є цілком прийнятним значенням для практичного використання очікуваного ефекту для корекції орбіт супутників.

Хоча ефект невеликий, експериментальне підтвердження могло б слугувати вирішальним доказом існування віртуальних квантових фотонів Казимира.

Ключові слова: *Двовимірний ефект Казимира одного тіла, нанокомірки, наносоти, тяга Казимира.*