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PECULARITIES OF HIRST EXPONENT ESTIMATION FOR NATURAL PHYSICAL PROCESSES

O. V. Lazorenko[1], A. A. Onishchenko[2], I. A. Taranova[1], M. A. Udovenko[1]

[1]V. N. Karazin Kharkiv National University, 4 Svobody Sq., 61022 Kharkiv, Ukraine
[2] Kharkiv National University of Radioelectronics, 14 Nauka Av., 61166 Kharkiv, Ukraine
E-mail: Oleg.V.Lazorenko@karazin.ua

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In bounds of the non-linear and system paradigms, been formulated by L. F. Chernogor in the last 1980th, all processes in open, non-linear, dynamical systems are very complex, non-linear, ultra-wideband or fractal ones.

According to the fractal paradigm put forward in the early 2000s by V. V. Yanovsky, fractality is one of the fundamental properties of the surrounding world. Therefore, the study of fractal characteristics, in particular, of natural physical processes is actual, interesting and useful.

The fractal dimension based on the Hurst exponent is one of the oldest and most famous ones. Based on the study of model fractal signals, it is demonstrated that the dependence between the estimate of the Hurst fractal dimension, obtained by the normalized range method, and its true value is significantly non-linear. To decrease of influence of the errors arising as a result of this, it is proposed to use the method of the corrective function.

The practical effectiveness of the proposed method is demonstrated on the example of the analysis of experimental results obtained in the middle 1960s by H. E. Hurst, which discovered the presence of a somewhat strange grouping of the values of the Hurst fractal dimension around the value of 1.27 for various natural physical processes. A hypothesis about the possibility of explaining this fact precisely by the nonlinearity of the mentioned dependence for R/S-method was proposed.

Keywords: *nonlinear paradigm, natural physical process, fractal paradigm, fractal analysis, fractal dimension, corrective function, rescaled range analysis method, Hurst exponent.*

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INTRODUCTION

It is well known that the term 'fractal' (from the Latin 'fractus', meaning 'broken') has been proposed by great American physicist and mathematician Benoit Mandelbrot in 1975 [1]. After some tens of years accompanied by strong scientific fights between the thousands of supporters and opponents, the fractal ideas had fully won (see, for example, [2 – 12]).

Following so called 'fractal paradigm' proposed in 2003 by famous physicist Prof. V. V. Yanovsky (Kharkiv, Ukraine), fractality (as non-linearity earlier) was appeared to be one of the fundamental properties of the world around us [13, 14]. Fractality has already become a new paradigm of modern science. Moreover, in the non-linear and the system paradigms been formulated in 1980s by Prof. L. F. Chernogor (Kharkiv, Ukraine), it has been claimed that in the open, complex, non-linear, dynamical systems, many natural and artificial processes, in particular, inspired by operation of the powerful, non-stationary sources of energy release can be classified as short-time, ultra-wideband, non-linear and fractal ones [15].

To discover, research, describe and explain the fractal properties of natural and artificial processes, it is necessary to apply different methods of mono-fractal and multi-fractal analysis (see, for example, [16]). Namely these methods give an useful and comfort set of different numerical characteristics been able sufficiently fully reflect all main peculiarities of the signals and processes investigated.

A fractal dimension D is appeared to be one of the most important such numerical characteristics (see, for example, [12]). In general, there are a lot of different fractal dimensions, which can be estimated as for mathematical, as for physical (natural) fractal signals and processes (see, for example, [16]). Each of them has own peculiarities for calculation methods and own set of appropriate cases for application. In this paper, we shall deal with only one of such fractal dimensions, namely, the Hurst dimension D_H and with only one method for its calculation, namely, rescaled range (R/S) analysis method called frequently as simply 'R/S method'.

The purposes of the paper are to investigate the peculiarities of the Hurst dimension estimation with R/S method and to try explaining some seems strange regularities obtained in 1980th by B. Mandelbrot for Hurst dimension D_H (and, of course, for the Hurst exponent H) of many natural physical processes.

R/S METHOD AND HURST FRACTAL DIMENSION

Being as natural, as artificial origin, many real signals and processes in nature have fractal properties and, therefore, are the physical fractals [1 – 9]. It is important, that in the most cases, these properties are understood namely in statistical sense, not in algebraic or geometric ones [1 – 9]. Therefore, to describe them correctly, the statistical numerical characteristics should be used.

The Hurst exponent H been introduced by H. E. Hurst in 1951 (twenty-four years before fractals!) in the paper [17] is appeared to be such the oldest statistical numerical characteristic. In bounds of the Generalized Brownian Motion (GBM) Model [3], the Hurst exponent H and fractal dimension D_H called as the Hurst fractal dimension are connected with the relation $D_H = 2 - H$.

Today, to estimate the Hurst exponent of a signal $X(t)$, there are many different ways. Being proposed by H. E. Hurst in 1965 [18], the oldest way is well known as the Rescaled Range Method or R/S method [3].

According to the new fractal analysis method classification introduced in 2022 [19], all existing mono-fractal analysis methods can be divided at six different groups, namely, methods based on the geometric characteristics, methods based on the algebraic characteristics, methods based on the statistical characteristics, methods based on the frequency and time-frequency characteristics, complex methods and special methods. Being statistical by the origin, R/S method is appeared to be a member of the group of methods based on the statistical characteristics.

As the examples of another ways of the Hurst exponent estimation, the Variogram [20] or Semivariogram [21] Method, the Mandelbrot and Wallis Method [22], the Dispersion Analysis Method [23] known as the Standard Deviation Analysis [24] too, the Autocorrelation Analysis Method [25], the Second Moment Method [26], the Peltier and Levi-Vehel Method [27], the Variance Plot Method [28], the Detrended Fluctuation Analysis Method [29], the Aggregated Dispersion Method [30], the Aggregated Signal Absolut Values Method [30], the Scaled Windowed Variance Method [31], the Detrended Moving Average Method [32], the Signal Summation Conversion Method [33], the Diffusion Entropy Analysis Method [24], the Variational Dimension Method [34], the Adaptive Fractal Analysis Method [35], the Fractal Dimension Algorithm Method [36], the Generalized Variogram Method [37] and many others can be listed.

As well known, the main idea of the R/S method is following (see, for example, [21]). Note, we shall speak about a discrete signal investigated, since in practice, if the digital signal processing methods are applied, all the signals and the processes researched should be represented in discrete form only.

Let's consider a discrete signal s_i containing N points ($i = \overline{1, N}$, that is the variable i varies in bounds from 1 to N with a unit step). For this signal, its partial sums

$$y(n) = \sum_{i=1}^{n} s_i,\ n \geq 1,$$

dispersions (square of the standard deviations $S(n)$)

$$S^2(n) = \frac{1}{n}\sum_{i=1}^{n}\left[s_i - \frac{1}{n}y(n)\right]^2, n \geq 1$$

a so called ranges $R(n)$ should be estimated. As well known, the range $R(n)$ of the signal s_i is given by the relation:

$$R(n) = \max_{0\leq t\leq n}\left(y(t) - \frac{t}{n}y(n)\right) - \min_{0\leq t\leq n}\left(y(t) - \frac{t}{n}y(n)\right).$$

Basing on these values, so called R/S statistics is build:

$$\frac{R}{S}(n) \equiv \frac{R(n)}{S(n)} = \frac{1}{S(n)}\times$$
$$\times\left[\max_{0\leq t\leq n}\left(y(t) - \frac{t}{n}y(n)\right) - \min_{0\leq t\leq n}\left(y(t) - \frac{t}{n}y(n)\right)\right],$$
$$n \geq 1.$$

In 1951 [17], basing on the results of empirical investigations, H. E. Hurst found that the mathematical expectation of such statistics showed a power-law relationship with the size of the observation window length n as:

$$E[R/S(n)] \sim Cn^H,$$

where C is some limited, positive constant, which doesn't depend on n, H is the Hurst exponent, $E[\,]$is the operation of a mathematical expectation calculation.

Therefore, varying the observation window length n, the plot of the logarithms of $E[R/S(n)]$ vs. the logarithm of n can be obtained. If a signal s_i investigated has really the self-affine (and fractal, of course) properties, all points calculated should be appeared to be grouped around some straight line. Being equal to the angle coefficient of this straight line, the Hurst exponent H can be estimated with usage of the least square method.

It is very important to note, that if that points were appeared to be not grouped around any straight line, it can be claimed, that a signal investigated hasn't a self-affine property and, therefore, is not a fractal one. The Hurst exponent H cannot be estimated in such case at all.

On other hand, in most practical cases, a signal investigated is appeared to be fractal in some limited scale range only, not in all range. In such case, the experimental pointes plotted in the double logarithmic coordinates can be successfully approximated with a linear function in some scale range only, not in all range too. Therefore, they should speak about limited scale range fractal properties of the signal or process researched.

Moreover, as it had been found by B. Mandelbrot (see, for example, [1, 3]), for fractals, the Hurst exponent value H should be limited in the range $0 < H < 1$. Otherwise the signal analyzed is appeared to be not self-affine and, therefore, is not fractal [3]. If the condition $0 < H < 1$ was successfully satisfied, then one can believe that the signal investigated has mono-fractal properties in this range. It is quite possible that for the same signal, some different scale ranges with different Hurst exponent values will be obtained [3].

It is necessary to point, that R/S method can be successfully applied for investigations of the functions as time, as space variables. Fortunately, formal replacing of a time variable by a space one doesn't destroy the R/S method correctness.

Meanwhile, in many cases, the real physical processes, specially being in open, non-linear, dynamical systems [15], are appeared to be non-stationary ones. Moreover, it means that their fractal properties can vary with time too. So, the Hurst exponent H should be estimated for some limited, sliding time window $W(t)$, but not for all signal $X(t)$ at once. In this case, the Hurst exponent becomes a function of the time $H = H(t)$ [38]. In our opinion, it is convenient to connect these Hurst exponent values with corresponding time locations of the center of the sliding time window $W(t)$ used. Namely such approach is applied in this paper.

Let's return to the Hurst fractal dimension D_H calculation with the R/S method application for the entire signal $X(t)$. Namely such approach has been most popular in the middle 1960th, when H. E. Hurst obtained some strange results regarding a set of natural processes and objects. He found (see, for example, [3, 39]) that the Hurst exponent H is more or less symmetrically distributed about a mean of 0.73, with a standard deviation of about 0.09. In this case, the Hurst fractal dimension D_H has a mean of 1.27 and the same standard deviation, $D_H = 1.27 \pm 0.09$. Although this was purely empirical result obtained on the base of generalization of huge amount of experimental data having different natural origin, it looks some strange and surprising. But up to current day, any reasonable theoretical explanations of this strange fact haven't been proposed as by H. E. Hurst and B. Mandelbrot, as by other famous specialists in fractals.

Nevertheless, all the results discussed above have the same peculiarity. They have been obtained with R/S method usage. Therefore, it can be done the following assumption: the strange result obtained could be explained by implying of some hidden peculiarity (or disadvantage) of the R/S method. It is necessary to found it only.

CORRECTIVE FUNCTION METHOD

In 2022 in the paper [40], so called 'Corrective Function Method' for mono-fractal analysis has been proposed. The main idea of this universal method is in following.

Let's consider a mono-fractal analysis method, which allows to obtain an estimation D^* of unknown fractal dimension D of a signal investigated. As it was proposed above, this is a discrete signal s_i containing N points ($i = \overline{1, N}$). The estimation D^* is appeared to be an unknown non-linear function of D and N, that is $D^* = f(D,N)$. It is understood, that in an ideal case this function must be linear and simple ($D^* = D$) and must not depend on N. But in practice, this situation is appeared to absolutely impossible.

For the sake of justice, we note that in general, the hints of an idea of existence of the non-linear function $D^* = f(D,N)$ for some methods of mono-fractal analysis were be done by some specialists yet before the paper [40] appearance. But there was no its clear formulation and no ways to improve the situation proposed.

The main purpose of the Corrective Function Method is to compensate the existing non-linearity of the function $D^* = f(D,N)$ in some way and, therefore, to increase an accuracy of the fractal dimension D estimation obtained with given method of mono-fractal analysis.

To reach this purpose, in the paper [40], it was proposed to inverse the non-linear function $D^* = f(D,N)$ on the base of so called 'Corrective Function' (CF). For each given mono-fractal analysis method, the CF should be built on the discrete grid over the plain (D,N) with application of the model fractal signal set with known changing values of the variables D and N. The steps of changing on D and N values can be chosen by each researcher in the way which he like. For example, for our practical aims, we have used D value changing in bounds $1 \le D \le 2$, with the step 0.01, and N value given by $N = 2^k$, $k \in \mathbb{N}$, where $\mathbb{N}$ is a natural number set. It is clean, that the smaller these steps are, the more precise the fractal dimension D estimation will be. But in the same time, the data volume needed be located in computed rises significantly. Therefore, each researcher should choose between the needed accuracy and the data volume allowed for given calculation.

When the CF on the discrete grid $D^*_{ij} = f(D_i, N_j)$, $i = \overline{1,n}$, $j = \overline{1,m}$ has been obtained, the process of the non-linear function $D^* = f(D,N)$ inversion can be started. For fixed N value, $N = N_{sig}$, the function $D^* = f(D, N_{sig})$, as a function of one variable on the interval $1 \le D < 2$, can have an inverse function $D = f^{-1}(D^*, N_{sig})$ only in the case, when the function $D^* = f(D, N_{sig})$ is monotonic there. In this concrete case, the function $D^* = f(D, N_{sig})$ should be a rising function of D in the interval discussed.

For comparatively big values of N_{sig}, there are no problems to satisfy this demand. But when N_{sig} decreases, for a function $D^*_{ij} = f(D_i, N_j)$, the value $N_{\min}$, below of which the monotonicity of the function $D^* = f(D, N_{sig})$ discussed above ($j = \overline{1,(\min - 1)}$) is appeared to be disrupted, occurs. As well as all close explanations of this process appearance causes and the bulky relations for the fractal dimension D and its estimation error ΔD are considered in the paper [40], which, if needed, can be successfully downloaded for free, we avoid to repeat them here.

Nevertheless, we believe (we don't claim, of course) that the algorithm of determination of the N_{sig} value is appeared to be very useful, since it allows to prepare a well-founded and reasonable answer on the question of what exactly is the minimum number of signal points $N_{\min}$ and why should be used in this method of monofractal analysis.

To prove the importantness of such answer existence, we can cite an opinion of the world famous fractalist J. Feder, which has claimed in the book [3], that for the R/S method, minimal allowing $N_{\min}$ value should be equal at least 2500. May be, this opinion would have some reason, but there was no grounded explanation of such point of view. At the same time, as it was found in the paper [40], for the R/S method, the value $N_{\min}$ is appeared to be principally much smaller ($N_{\min} = 32$) due to the reasons described above. But it should be taken into account that when N_{sig} value decreases, the error of the Hurst fractal dimension estimation ΔD, of course, rises.

RESULTS OF MODELING WITH STOCHASTIC MONO-FRACTAL SIGNALS USAGE

Let's consider the non-linear function $D^* = f(D,N)$ created for the R/S method on the basis of the model mono-fractal signals.

All these signals used in this paper are the different realization of the one stochastic mono-fractal signal model with varying D and N values. This model is a well-known model based on the modified cosine Weierstrass – Mandelbrot function [41]:

$$MW(t) = \sum_{n=0}^{+\infty} \lambda^{(D-2)n} \cos(\lambda^n t + \varphi_n),$$

where λ is a numerical parameter ($\lambda > 1$), D is a fractal dimension ($1 \le D \le 2$), φ_n are the stochastic phases having some chosen distribution law at the interval $[0,2\pi]$, t is dimensionless time variable. Two examples of such model signal realizations with different fractal dimension D values are shown at the Fig. 1.

It should be pointed that if we consider $\varphi_n = const$ for all existing n, a deterministic mono-fractal signal model with given fractal dimension D appears.

As it was pointed above, in all the investigations regarding a CF building, we used a discrete grid over the plain (D,N), where D value was changed in bounds $1 \le D \le 2$, with the step 0.01, and N value was given by $N = 2^k$, $k \in \mathbb{N}$. Namely this way is used now too.

As well as the model fractal signals used are principally stochastic by their nature, for successful and correct modelling, it is necessary to use many different realizations with the same combinations of D and N with consequent averaging of the results obtained. In this case, the averaging over 300 stochastic realizations was used.

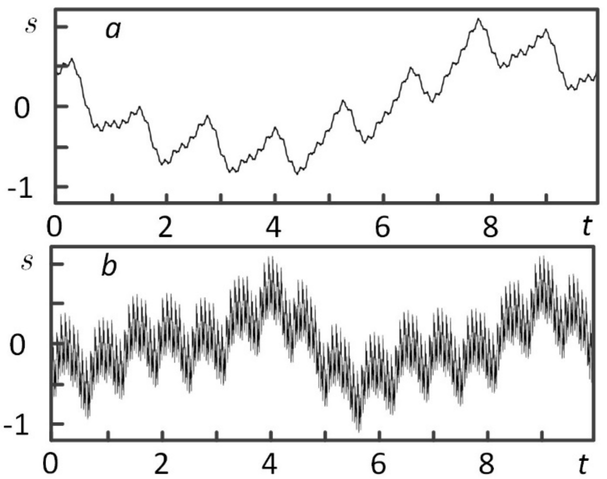

Fig. 1. Model stochastic mono-fractal signals $s(t)$ with different fractal dimension D values: $D = 1.2$ (a) and $D = 1.8$ (b). Here t is a dimensionless time.

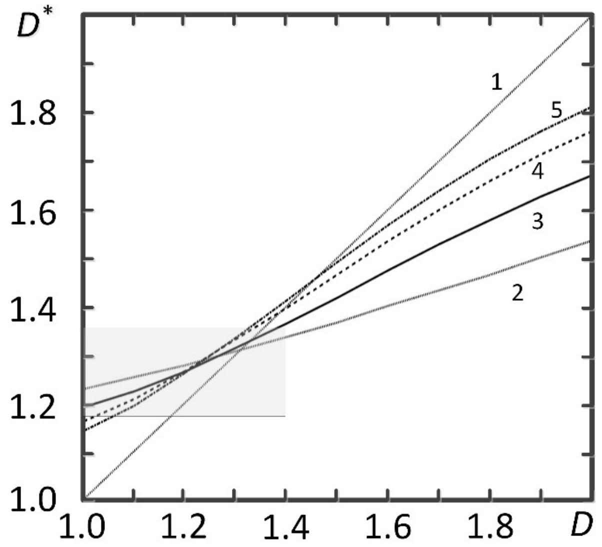

Fig. 2. Comparison of the 'ideal' function $D^* = D$ (1) vs the non-linear function $D^* = f(D, N)$ having fixed N values: $N = 32$ (2), $N = 128$ (3), $N = 512$ (4) and $N = 2048$ (5).

At the table 1, the results of the Hurst fractal dimension D^* estimation for the model stochastic fractal signals with given D and N values obtained with R/S method usage are shown. For each combination of D and N, as the Hurst fractal dimension D^* value calculated, as its error ΔD^* estimated are given. In all calculations performed in this paper, the confidence level was used to be equal to 0.9.

From the table 1, the existing non-linearity of the function $D^* = f(D, N)$ with fixed N values is clearly seen. Moreover, there are some interesting tendencies. First, for fixed D, the more N value is, the less difference between D^* and D values occurs. Second, in the worst case, the error of the D^* value estimation with R/S method usage doesn't exceed approximately 2.5%. Third, for the R/S method, depending on the N value, there is some special bound D_0 value, for which at the interval $1.0 \leq D \leq D_0$ the results of the Hurst fractal dimension D^* estimations turn out to be overestimated, but at the interval $D_0 < D \leq 2.0$ they turn out to be, on the contrary, underestimated. For $N = 32 - 2048$ this D_0 value is found to be slightly increasing in bounds $1.3 < D_0 < 1.4$ with value N rising.

Table 1

Hurst fractal dimension D* estimation for the model stochastic fractal signals with given D and N values obtained with R/S method usage at the interval $1.00 \leq D \leq 2.00$

N	*D*										
	1.00	*1.10*	*1.20*	*1.30*	*1.40*	*1.50*	*1.60*	*1.70*	*1.80*	*1.90*	*2.00*
32	1,23± 0,03	1,25± 0,03	1,28± 0,03	1,31± 0,03	1,34± 0,03	1,37± 0,03	1,40± 0,03	1,43± 0,03	1,46± 0,04	1,50± 0,04	1,53± 0,04
64	1,20± 0,02	1,23± 0,02	1,27± 0,02	1,30± 0,02	1,34± 0,02	1,39± 0,02	1,43± 0,02	1,48± 0,02	1,52± 0,02	1,57± 0,02	1,61± 0,02
128	1,19± 0,01	1,22± 0,02	1,26± 0,02	1,31± 0,02	1,36± 0,02	1,42± 0,02	1,47± 0,01	1,53± 0,01	1,58± 0,01	1,63± 0,01	1,67± 0,01
256	1,18± 0,01	1,22± 0,01	1,26± 0,01	1,32± 0,01	1,38± 0,01	1,44± 0,01	1,50± 0,01	1,56± 0,01	1,62± 0,01	1,67± 0,01	1,72± 0,01
512	1,16± 0,01	1,21± 0,01	1,27± 0,01	1,33± 0,01	1,40± 0,01	1,47± 0,01	1,53± 0,01	1,60± 0,01	1,66± 0,01	1,71± 0,01	1,76± 0,01
1024	1,15± 0,01	1,20± 0,01	1,26± 0,01	1,33± 0,01	1,41± 0,01	1,48± 0,00	1,56± 0,00	1,63± 0,00	1,69± 0,00	1,74± 0,00	1,79± 0,00
2048	1,14± 0,00	1,19± 0,00	1,26± 0,00	1,33± 0,00	1,41± 0,00	1,49± 0,00	1,57± 0,00	1,64± 0,00	1,70± 0,00	1,76± 0,00	1,81± 0,00
4096	1,13± 0,00	1,19± 0,00	1,26± 0,00	1,34± 0,00	1,42± 0,00	1,50± 0,00	1,58± 0,00	1,66± 0,00	1,72± 0,00	1,78± 0,00	1,83± 0,00
8192	1,13± 0,00	1,18± 0,00	1,26± 0,00	1,34± 0,00	1,42± 0,00	1,50± 0,00	1,59± 0,00	1,66± 0,00	1,73± 0,00	1,80± 0,00	1,85± 0,00

Table 2

Hurst fractal dimension D^* estimation and its mean value $\overline{D^*}$ for the model stochastic fractal signals with given D and N values obtained with R/S method usage at the interval $1.00 \le D \le 1.40$

N	*D*					$\overline{D^*}$
	1.00	*1.10*	*1.20*	*1.30*	*1.40*	
32	1,228±0,028	1,253±0,030	1,277±0,031	1,306±0,032	1,336±0,032	1,280±0,031
64	1,203±0,020	1,234±0,021	1,268±0,021	1,303±0,022	1,344±0,022	1,270±0,021
128	1,190±0,015	1,223±0,015	1,264±0,015	1,313±0,016	1,364±0,016	1,271±0,015
256	1,175±0,011	1,218±0,011	1,264±0,011	1,317±0,011	1,379±0,011	1,271±0,011
512	1,161±0,008	1,207±0,008	1,265±0,008	1,331±0,008	1,396±0,007	1,272±0,008
1024	1,147±0,006	1,200±0,006	1,263±0,006	1,334±0,005	1,410±0,005	1,270±0,006
2048	1,141±0,004	1,194±0,004	1,262±0,004	1,334±0,004	1,412±0,004	1,269±0,004
4096	1,134±0,003	1,191±0,003	1,258±0,003	1,336±0,003	1,418±0,003	1,267±0,003
8192	1,129±0,002	1,184±0,002	1,257±0,002	1,336±0,002	1,418±0,002	1,265±0,002

At the Fig. 2, a comparison of the 'ideal' function $D^* = D$ (1) vs the non-linear function $D^* = f(D, N)$ having fixed N values: $N = 32$ (2), $N = 128$ (3), $N = 512$ (4) and $N = 2048$ (5) are shown. The fact of this function non-linearity existence is well confirmed.

A 'strange' result obtained by H. E. Hurst and described above is shown at the Fig. 2 as a small gray rectangle corresponding to the upper and the lower bounds of the confidence interval for the fractal dimension D^*. It is necessary to understand how exactly it is placed relatively all the dependencies shown.

Let's consider closer an interval of fractal dimension D values having such bounds: $1.00 \le D \le 1.40$. As it was pointed above, for R/S method, as a rule, at given interval, the overestimating of the Hurst fractal dimension D^* estimation appears. Assume that there is a set of mono-fractal processes with their own true fractal dimension D values uniformly distributed in this interval. As well known, being persistent ($0.5 < H \le 1$), each of such fractal processes has some long-term dependence (see, for example [1, 3]).

If we would have an 'ideal' estimator, then all right values $D^* = D$ would be obtained. But instead of such 'ideal' estimator, at our disposal, there is a R/S method only. In this case, we should obtain only the D^* values distorted by the non-linear function $D^* = f(D, N)$. The value of these distortions depends on both the D value and the N value. At least, for $N = 32 - 2048$, the interval $1.00 \le D \le 1.40$ is appeared to be non-linearly mapped approximately to the interval $1.15 \le D \le 1.40$. In the table 2, the non-linear function $D^* = f(D, N)$ on the interval discussed is given with more accuracy.

Of course, after the mapping performed, the distribution law of such mono-fractal processes is appeared to be differed from the uniform, but seems it is not so significant now. For simplification, let's suppose that the distribution remains uniform. In such case, for different N values, the mean value of the estimated Hurst fractal dimension $\overline{D^*}$ on the interval $1.00 \le D \le 1.40$ can be simply obtained (table 2).

As for our opinion, these last results are looked to be very surprising. One hand, for an 'ideal' estimator, it is understood that $\overline{D^*} = 1.20$. Other hand, for R/S method, this value is appeared to be significantly shifted up. Nevertheless, at least for $N = 32 - 8192$ the 'strange' result of H. E. Hurst $D = 1.27 \pm 0.09$ is appeared to be excellently agreed in all cases with $\overline{D^*}$ values obtained with R/S method usage. At the Fig. 3, the relative location of the gray rectangle described above and the non-linear function $D^* = f(D, N)$ with its confidence interval for $N = 32$ (Fig. 3, a) and $N = 128$ (Fig. 3, b) is shown.

DISCUSSION

Taking into account the fact that in middle 1960th, when H. E. Hurst had obtained the results discussed above, there were no effective practical ways to process the signals and the processes with big values of points N, the maximal N value should be limited approximately by 1000. Such assumed value is well agreed with the information contained in [3, 39].

It is necessary to ground why in our investigations, namely the interval $1.00 \le D \le 1.40$ is used. It was done not accidentally. It is well known that H. E. Hurst has collected and has investigated the processes having long-term dependences. Today, such processes are well known as persistent ones. They have the Hurst exponent H values changing in the bounds: $0.5 < H \le 1.0$ and, of course, the corresponding Hurst dimension D_H values satisfy the condition: $1.0 \le D_H < 1.5$. At the same time, a value $D_H = 1.5$ corresponds to a delta-correlated process which has no time dependence at all. Of course, it should be avoided and be excluded from a consideration. As well as the Hurst fractal dimension error in the results obtained by H. E. Hurst is $\Delta D_H \approx 0.09$, the upper bound in the condition $1.0 \le D_H < 1.5$ should be decreased approxima-

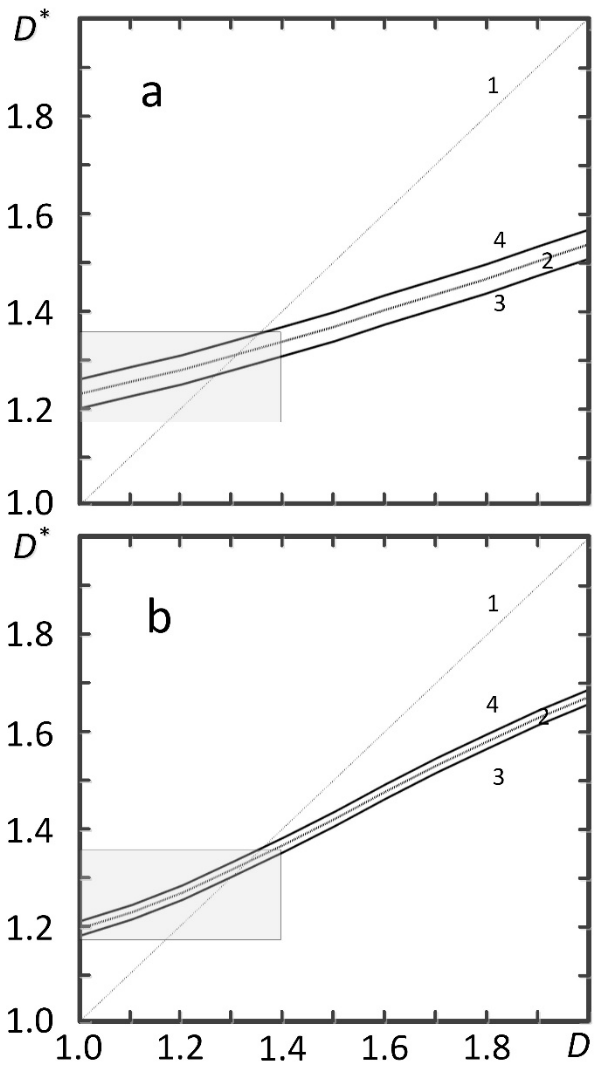

Fig. 3. Comparison of the ideal function $D^* = D$ (1) vs the non-linear function $D^* = f(D, N)$ (2) having fixed $N = 32$ (a) and $N = 128$ (b). The upper (4) and the lower (3) bounds of its confidence interval, a small gray rectangle (the confidence interval bounds) being a graphical view of the 'strange' result obtained by H. E. Hurst are shown.

tely at this value. Thus, we obtain the condition $1.00 \le D \le 1.40$ used is this work. On this reason, the results of modelling described above are looked to be quietly correct and useful.

Thus, we tried to formulate and to ground a hypothesis, which is able to explain the 'strange' results obtained by H. E. Hurst in 1960th yet. On our opinion, an existence of the significant shift for the Hurst fractal dimension D^* estimations observed for persistent natural physical processes can be explained rather by special features of the R/S method applied for processing of the experimental data, than by own really existing properties of these processes.

At the end, it important to point that this a hypothesis only. We suppose, but we don't claim this.

CONCLUSIONS

1. The R/S analysis method is the oldest and the most popular way to estimate a Hurst exponent for any signal or process.
2. In bounds of the Generalized Brownian Motion model, the Hurst fractal dimension D_H and the Hurst exponent H are connected with a simple relation, namely, $D_H = 2 - H$.
3. Grounding on the results of numerical modelling with simultaneous usage of the Corrective Function Method and the set of model stochastic mono-fractal signals based on the modified cosine Weierstrass – Mandelbrot function, for R/S method, it was found that the dependence between a fractal dimension value D^* estimation and a true own fractal dimension value D is appeared to be principally non-linear.
4. The main peculiarities of the R/S method as the oldest and the most popular estimator of Hurst fractal dimension were investigated. The corresponding corrective function was built.
5. A hypothesis, which is able to explain the 'strange' results obtained by H. E. Hurst in 1960th yet, was formulated and grounded on the bases of the numerical modelling results.
6. On our opinion, an existence of the significant shift for the Hurst fractal dimension D^* estimations observed for persistent natural physical processes can be explained rather by special features of the R/S method applied for processing of the experimental data, than by own really existing properties of these processes.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interests.

СПИСОК ВИКОРИСТАНИХ ДЖЕРЕЛ

1. B. B. Mandelbrot. The Fractal Geometry of Nature, San Francisco, CA-Freeman (1982), 460 p. https://doi.org/10.1119/1.13295
2. K. J. Falconer. Fractal Geometry. Mathematical Foundations and Applications, Chichester, Wiley & Sons (1990), 288 p. https://doi.org/10.1002/0470013850
3. J. Feder. Fractals, New York, Plenum Press (1988), 305 p. https://link.springer.com/book/10.1007/978-1-4899-2124-6
4. E. Chandrasekhar, V. P. Dimri, V. M. Gadre, editors. Wavelets and Fractals in Earth System Sciences, CRC Press (2014), 294 p. https://doi.org/10.1201/b16046
5. D. P. Feldman. Chaos and Fractals. An Elementary Introduction, Oxford, University Press (2012), 408 p. https://doi.org/10.1093/acprof:oso/9780199566433.001.0001
6. B. B. Mandelbrot. Fractals and Chaos: The Mandelbrot Set and Beyond, Springer (2005), 400 p. https://doi.org/10.1186/1475-925X-4-30
7. R. M. Crownover. Introduction to Fractals and Chaos, Boston, Jones and Barlett Publishers (1995), 320 p.

https://www.amazon.com/Introduction-Fractals-Chaos-Bartlett-Mathematics/dp/0867204648
8. B. B. Mandelbrot. Multifractals and 1/f Noise, Springer (1999), 442 p. https://doi.org/10.1007/978-1-4612-2150-0
9. D. Harte. Multifractals. Theory and Applications, Boca Raton, Chapman and Hall/CRC Press (2001), 264 p. https://doi.org/10.1201/9781420036008
10. M. Schroeder. Fractals, Chaos, Power Laws. Minutes from Infinite Paradise, New York, W. H. Freeman and Company (1991), 528 p. https://www.amazon.com/Fractals-Chaos-Power-Laws-Infinite/dp/0486472043
11. F. C. Moon. Chaotic Vibrations. An Introduction for Applied Scientists and Engineers, New York, Wiley and Sons (2004), 309 p. https://doi.org/10.1002/3527602844
12. О. В. Лазоренко, Л. Ф. Черногор. Радіофізика та Радіоастрономія, 25 (1), 3 (2020). https://doi.org/10.15407/rpra25.01.003
13. В. В. Яновський. Universitates, 3, 32 (2003).
14. В. В. Яновський. Лекції з нелінійних явищ. Том 1, Харків, Видав. Інституту монокристалів (2006), 456 с.
15. Л. Ф. Чорногор. Про нелінійність у природі та науці, Харків, Харківський національний університет імені В. Н. Каразіна (2008), 528 с.
16. О. В. Лазоренко, Л. Ф. Чорногор. Радіофізика та Радіоастрономія, 28(1), 5 (2023). https://doi.org/10.15407/rpra28.01.005
17. H. E. Hurst. Trans. Amer. Soc. Civ. Eng., 116, 770 (1951).
18. H. E. Hurst, R. P. Black, Y. M. Simaika. Long-term storage: an experimental study, London, Constable (1965), 145 p.
19. L. F. Chernogor, O. V. Lazorenko, A. A. Onishchenko. Low Temperature Physics, 49(4), 459 (2023). https://doi.org/10.1063/10.0017581
20. H. H. Hardy, R. A. Beier. Fractals in Reservoir Engineering. Singapore, New Jersey, London, Hong Kong, World Scientific (1994), 359 p. https://doi.org/10.1142/2574
21. L. Seuront. Fractals and Multifractals in Ecology and Aquatic Science, Boca Raton, London, New York, CRC Press (2010), 344 p. https://doi.org/10.1201/9781420004243
22. B. Mandelbrot, J. R. Wallis. Water Resources Res., 5(1) 228 (1969). https://doi.org/10.1029/WR005I001P00228
23. J. B. Bassingthwaighte. News Physiol. Sci., 3, 5 (1988). https://doi.org/10.1152/physiologyonline.1988.3.1.5
24. N. Scafetta, P. Grigolini. Phys. Rev. E, 66(3), 036130 (2002). https://doi.org/10.1103/physreve.66.036130
25. J. H. Van Beek, S. A. Roger, J. B. Bassingthwaighte. Am. J. Physiol., 257(5), H1670 (1989). https://doi.org/10.1152/ajpheart.1989.257.5.h1670
26. H. M. Hastings, G. Sugihara. Fractals: A User's Guide for the Natural Science, Oxford, Oxford University Press (1993), 248 p. https://www.amazon.com/Fractals-Natural-Sciences-Science-Publications/dp/0198545975
27. R. F. Peltier, J. Lévy-Véhel. Research report, INRIA Rocqencourt (1994). https://hal.inria.fr/inria-00074279
28. J. Beran. Statistics for Long-Memory Processes, Chapman and Hall (1994), 328 p. https://doi.org/10.2307/2983481
29. C.-K. Peng, S. V. Buldyrev, S. Havlin, M. Simons, H. E. Stanley, A. L. Goldberger. Phys. Rev. E., 49, 1685 (1994). https://doi.org/10.1103/PhysRevE.49.1685
30. M. S. Taqqu, V. Teverovsky, W. Willinger. Fractals, 03(04), 785 (1995). https://doi.org/10.1142/s0218348x95000692
31. M. J. Cannon, D. B. Percival, D. C. Caccia, G. M. Raymond, J. B. Bassingthwaighte. Physica A: Statistical Mechanics and Its Applications, 241(3-4), 606 (1997). https://doi.org/10.1016/s0378-4371(97)00252-5
32. N. Vandewalle, M. Ausloos. Phys. Rev. E, 58(5), 6832 (1998). https://doi.org/10.1103/physreve.58.6832
33. A. Eke, P. Hermán, J. Bassingthwaighte, G. Raymond, D. Percival, M. Cannon, … C. Ikrényi. Pflügers Archiv - European Journal of Physiology, 439(4), 403 (2000). https://doi.org/10.1007/s004249900135
34. S. M. Prigarin, K. Hahn, G. Winkler. Numerical Analysis and Applications, 2(4), 352 (2009). https://doi.org/10.1134/s1995423909040077
35. M. A. Riley, S. Bonnette, N. Kuznetsov, S. Wallot, J. Gao. Frontiers in Physiology, 3 (2012). https://doi.org/10.3389/fphys.2012.00371
36. M. J. Sánchez-Granero, M. Fernández-Martínez, J. Trinidad-Segovia. The European Physical Journal B, 85(3), 86 (2012). https://doi.org/10.1140/epjb/e2012-20803-2
37. T. Gneiting, H. Sevcikova and D. B. Percival. Statist. Sci., 27, 247 (2012). https://doi.org/10.1214/11-STS370
38. M. S. Taqqu. Stochastic Processes and Their Applications, 7(1), 55 (1978). https://doi.org/10.1016/0304-4149(78)90037-6
39. H. E. Hurst, R. P. Black, Y. M. Simaika. Long-term storage: an experimental study, London, Constable (1965), 145 p. https://doi.org/10.2307/2982267
40. О. В. Лазоренко, А. А. Онищенко, Л. Ф. Чорногор. Радіотехніка: Всеукр. наук.-тех. збір., 210. 177 (2022). https://doi.org/10.30837/rt.2022.3.210.15.
41. C. Bandt, M. Barnsley, R. Devaney, K. J. Falconer, V. Kannan, P. B. Vinod Kumar, editors. Fractals, Wavelets, and their Applications: Contributions from the Int. Conference and Workshop on Fractals and Wavelets (Springer Proceedings in Mathematics & Statistics), Switzerland, Springer Int. Publ. (2014), 508 p. https://www.amazon.com/Fractals-Wavelets-their-Applications-Contributions-ebook/dp/B00PUM0AQ2

REFERENCES

1. B. B. Mandelbrot. The Fractal Geometry of Nature, San Francisco, CA-Freeman (1982), 460 p. https://doi.org/10.1119/1.13295
2. K. J. Falconer. Fractal Geometry. Mathematical Foundations and Applications, Chichester, Wiley & Sons (1990), 288 p. https://doi.org/10.1002/0470013850
3. J. Feder. Fractals, New York, Plenum Press (1988), 305 p. https://link.springer.com/book/10.1007/978-1-4899-2124-6
4. E. Chandrasekhar, V. P. Dimri, V. M. Gadre, editors. Wavelets and Fractals in Earth System Sciences, CRC Press (2014), 294 p. https://doi.org/10.1201/b16046
5. D. P. Feldman. Chaos and Fractals. An Elementary Introduction, Oxford, University Press (2012), 408 p. https://doi.org/10.1093/acprof:oso/9780199566433.001.0001
6. B. B. Mandelbrot. Fractals and Chaos: The Mandelbrot Set and Beyond, Springer (2005), 400 p. https://doi.org/10.1186/1475-925X-4-30
7. R. M. Crownover. Introduction to Fractals and Chaos, Boston, Jones and Barlett Publishers (1995), 320 p. https://www.amazon.com/Introduction-Fractals-Chaos-Bartlett-Mathematics/dp/0867204648
8. B. B. Mandelbrot. Multifractals and 1/f Noise, Springer (1999),

442 p. https://doi.org/10.1007/978-1-4612-2150-0
9. D. Harte. Multifractals. Theory and Applications, Boca Raton, Chapman and Hall/CRC Press (2001), 264 p. https://doi.org/10.1201/9781420036008
10. M. Schroeder. Fractals, Chaos, Power Laws. Minutes from Infinite Paradise, New York, W. H. Freeman and Company (1991), 528 p. https://www.amazon.com/Fractals-Chaos-Power-Laws-Infinite/dp/0486472043
11. F. C. Moon. Chaotic Vibrations. An Introduction for Applied Scientists and Engineers, New York, Wiley and Sons (2004), 309 p. https://doi.org/10.1002/3527602844
12. O. V. Lazorenko, L. F. Chernogor. Radio Phys. Radio Astron., 25 (1), 3 (2020) (in Russian). https://doi.org/10.15407/rpra25.01.003
13. V. V. Yanovsky. Universitates, 3, 32 (2003) (In Russian).
14. V. V. Yanovsky. Lectures on Nonlinear Phenomena. Volume 1, Kharkiv, Institut monokristallov Publ. (2006), 456 p. (in Russian).
15. L. F. Chernogor. On the Nonlinearity in Nature and Science, Kharkiv, V. N. Karazin Kharkiv National University (2008), 528 p. (In Russian).
16. O. V. Lazorenko, L. F. Chernogor. Radio Phys. Radio Astron., 28(1), 5 (2023) (in Ukrainian). https://doi.org/10.15407/rpra28.01.005
17. H. E. Hurst. Trans. Amer. Soc. Civ. Eng., 116, 770 (1951).
18. H. E. Hurst, R. P. Black, Y. M. Simaika. Long-term storage: an experimental study, London, Constable (1965), 145 p.
19. L. F. Chernogor, O. V. Lazorenko, A. A. Onishchenko. Low Temperature Physics, 49(4), 459 (2023). https://doi.org/10.1063/10.0017581
20. H. H. Hardy, R. A. Beier. Fractals in Reservoir Engineering. Singapore, New Jersey, London, Hong Kong, World Scientific (1994), 359 p. https://doi.org/10.1142/2574
21. L. Seuront. Fractals and Multifractals in Ecology and Aquatic Science, Boca Raton, London, New York, CRC Press (2010), 344 p. https://doi.org/10.1201/9781420004243
22. B. Mandelbrot, J. R. Wallis. Water Resources Res., 5(1) 228 (1969). https://doi.org/10.1029/WR005I001P00228
23. J. B. Bassingthwaighte. News Physiol. Sci., 3, 5 (1988). https://doi.org/10.1152/physiologyonline.1988.3.1.5
24. N. Scafetta, P. Grigolini. Phys. Rev. E, 66(3), 036130 (2002). https://doi.org/10.1103/physreve.66.036130
25. J. H. Van Beek, S. A. Roger, J. B. Bassingthwaighte. Am. J. Physiol., 257(5), H1670 (1989). https://doi.org/10.1152/ajpheart.1989.257.5.h1670
26. H. M. Hastings, G. Sugihara. Fractals: A User's Guide for the Natural Science, Oxford, Oxford University Press (1993), 248 p. https://www.amazon.com/Fractals-Natural-Sciences-Science-Publications/dp/0198545975
27. R. F. Peltier, J. Lévy-Véhel. Research report, INRIA Rocqencourt (1994). https://hal.inria.fr/inria-00074279
28. J. Beran. Statistics for Long-Memory Processes, Chapman and Hall (1994), 328 p. https://doi.org/10.2307/2983481
29. C.-K. Peng, S. V. Buldyrev, S. Havlin, M. Simons, H. E. Stanley, A. L. Goldberger. Phys. Rev. E., 49, 1685 (1994). https://doi.org/10.1103/PhysRevE.49.1685
30. M. S. Taqqu, V. Teverovsky, W. Willinger. Fractals, 03(04), 785 (1995). https://doi.org/10.1142/s0218348x95000692
31. M. J. Cannon, D. B. Percival, D. C. Caccia, G. M. Raymond, J. B. Bassingthwaighte. Physica A: Statistical Mechanics and Its Applications, 241(3-4), 606 (1997). https://doi.org/10.1016/s0378-4371(97)00252-5
32. N. Vandewalle, M. Ausloos. Phys. Rev. E, 58(5), 6832 (1998). https://doi.org/10.1103/physreve.58.6832
33. A. Eke, P. Hermán, J. Bassingthwaighte, G. Raymond, D. Percival, M. Cannon, … C. Ikrényi. Pflügers Archiv - European Journal of Physiology, 439(4), 403 (2000). https://doi.org/10.1007/s004249900135
34. S. M. Prigarin, K. Hahn, G. Winkler. Numerical Analysis and Applications, 2(4), 352 (2009). https://doi.org/10.1134/s1995423909040077
35. M. A. Riley, S. Bonnette, N. Kuznetsov, S. Wallot, J. Gao. Frontiers in Physiology, 3 (2012). https://doi.org/10.3389/fphys.2012.00371
36. M. J. Sánchez-Granero, M. Fernández-Martínez, J. Trinidad-Segovia. The European Physical Journal B, 85(3), 86 (2012). https://doi.org/10.1140/epjb/e2012-20803-2
37. T. Gneiting, H. Sevcikova and D. B. Percival. Statist. Sci., 27, 247 (2012). https://doi.org/10.1214/11-STS370
38. M. S. Taqqu. Stochastic Processes and Their Applications, 7(1), 55 (1978). https://doi.org/10.1016/0304-4149(78)90037-6
39. H. E. Hurst, R. P. Black, Y. M. Simaika. Long-term storage: an experimental study, London, Constable (1965), 145 p. https://doi.org/10.2307/2982267
40. O. V. Lazorenko, A. A. Onishchenko, L. F. Chernogor. Radiotekhnika: All-Ukr. Sci. Inter- dep. Mag., 210. 177 (2022). (in Ukrainian). https://doi.org/10.30837/rt.2022.3.210.15.
41. C. Bandt, M. Barnsley, R. Devaney, K. J. Falconer, V. Kannan, P. B. Vinod Kumar, editors. Fractals, Wavelets, and their Applications: Contributions from the Int. Conference and Workshop on Fractals and Wavelets (Springer Proceedings in Mathematics & Statistics), Switzerland, Springer Int. Publ. (2014), 508 p. https://www.amazon.com/Fractals-Wavelets-their-Applications-Contributions-ebook/dp/B00PUM0AQ2

ОСОБЛИВОСТІ ОЦІНЮВАННЯ ПОКАЗНИКА ХЕРСТА ДЛЯ ПРИРОДНИХ ФІЗИЧНИХ ПРОЦЕСІВ

О. В. Лазоренко[1], А. А. Онищенко[2], І. А. Таранова[1], М. А. Удовенко[1]

[1]*Харківський національний університет імені В. Н. Каразіна, майдан Свободи, 4, 61022 Харків, Україна*
[2]*Харківський національний університет радіоелектроніки, пр. Науки, 14, 61166 Харків, Україна*
E-mail: Oleg.V.Lazorenko@karazin.ua

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У відповідності до нелінійної та системної парадигм, сформульованих Л. Ф. Чорногором наприкінці 1980-х років, всі процеси у відкритих, нелінійних, динамічних системах є дуже складними, нелінійними, надширокосмуговими або фрактальними.

Як стверджує фрактальна парадигма, висунута на початку 2000-х років В. В. Яновським, фрактальність взагалі є однією із фундаментальних властивостей навколишнього світу. Тому вивчення фрактальних характеристик, зокрема, природних фізичних процесів є актуальним, цікавим і корисним.

Фрактальна розмірність, що базується на показникові Херста, є однією із найстаріших і найвідоміших. На основі дослідження модельних фрактальних сигналів продемонстровано, що залежність між оцінкою херстової фрактальної розмірності, що отримується методом нормованого розмаху, та істинним її значенням є істотно нелінійною. Для зменшення впливу похибок, що виникають у результаті цього, запропоновано використовувати метод коригуючої функції.

Продемонстровано практичну ефективність запропонованого метода на прикладі аналізу експериментальних результатів, отриманих ще в середині 1960-х років Г. Е. Херстом, який виявив наявність дещо дивного групування отриманих ним значень херстової фрактальної розмірності навколо величини 1.27 для різних природних фізичних процесів. Висунуто гіпотезу про можливість пояснення цього факту саме нелінійністю згаданої залежності для методу нормованого розмаху.

***Ключові слова**: нелінійна парадигма, природний фізичний процес, фрактальна парадигма, фрактальний аналіз, фрактальна розмірність, коригуюча функція, метод нормованого розмаху, показник Херста*