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Emission from Josephson junctions with Gaussian distribution of critical currents

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The model which allows to obtain the spectrum of emission of systems of Josephson junctions with the inhomogeneous distributions of critical currents along junctions is developed. With the use of this model we study electrical properties of systems in which junctions have the Gaussian distributions of critical currents. In particular, IV-characteristics and power of emission from inhomogeneous junctions with dimensions smaller than the Josephson depth of penetration of magnetic field have been investigated. We showed that for such junctions the dependence of emitted power on voltage (i.e. the spectrum of emission) had maxima at voltages corresponded to Fiske steps in the whole range of voltages, though in the IV-characteristics particularities (nuclei of zero-field steps) were not seen and they could be revealed only in derivatives of these curves. The comparison of our results with similar results which we obtained earlier for long junctions allows to suppose that the investigated mechanism of the formation of zero-field steps is general and it is valid for both long and short junctions. We investigated the averaged on random realizations height of some maximum of emitted power at different values of the Gaussian standard deviation of critical currents and found the square dependence of this height on the dimensionless parameter which characterizes the standard deviation. This result was in agreement with the theory of zero-field steps. We also considered electrical properties and power of emission from the stack of two long interacting with each other Josephson junctions in magnetic field. Each of the junctions had small (about 10^{-3} %) Gaussian distribution of critical currents. We found that if magnetic field was absent then there were only normal modes in the system (namely, the in-phase mode and the anti-phase mode). Zero-field steps were formed at voltages corresponded to the split even Fiske step. There were only normal modes in the system also when the relation of magnetic field to the value of the magnetic field at which the critical current becomes zero was more than 0.6. When this relation was smaller, other modes existed as well. We supposed that some normal modes could be destroyed because due to magnetic field standing waves were formed at both odd and even Fiske steps, so some modes could be locked with standing waves.
Keywords: Josephson junctions, power of emission, zero-field steps, Fiske steps.

Емісія контактів Джозефсона з гаусовм розподілом критичних струмів

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Розроблена модель, яка дозволяє отримувати спектри емісії систем з контактами Джозефсона з неоднорідним розподілом критичних струмів вздовж контактів. За допомогою цієї моделі вивчено електричні властивості систем, в яких контакти мають гаусів розподіл критичних струмів. Зокрема, досліджені вольт-амперні характеристики та потужність випромінювання від неоднорідних контактів, розміри яких менші за джозефсонівську глибину проникнення магнітного поля. Показано, що в таких контактах залежності потужності випромінювання від напруги (тобто, спектри емісії) мають максимуми при напругах,

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які відповідають сходинкам Фіске, в усьому діапазоні напруги, хоча на вольт-амперній характеристиці особливості (зародки сходинок Фіске) слабо проявлені, і можуть бути виявлені тільки на похідних цих кривих. Порівняння отриманих результатів з подібними результатами, які були раніше розраховані для довгих контактів, дозволяє припустити, що досліджений механізм формування сходинок нульового поля є загальним, і що він діє як у довгих, так і в коротких контактах. Досліджено усереднену по реалізаціям висоту одного з максимумів потужності випромінювання при різних значеннях гаусівського стандартного відхилення критичних струмів. Знайдено квадратичну залежність цієї висоти від безрозмірного параметру, який пропорціональний стандартному відхиленню. Цей результат узгоджується з теорією сходинок нульового поля. Розглянуто також електричні властивості та потужність емісії пачки з двох довгих контактів Джозефсона, які взаємодіяли один з одним та знаходилися у магнітному полі. Кожний контакт мав невеликий (порядку 10-3 %) гаусівський розподіл критичних струмів. Знайдено, що при відсутності магнітного поля в системі є тільки нормальні моди електромагнітних коливань, (а саме, синфазна та протифазна моди). Сходинки нульового поля були сформовані при напругах розщепленої парної сходинки Фіске. Тільки нормальні моди спостерігаються в системі тоді, коли відношення магнітного поля до тієї величини поля, при якій критичний струм дорівнює нулю, перевищувало величину 0,6. Коли це відношення ставало меншим, було виявлено існування інших мод. Ми вважаємо, що деякі нормальні моди могли бути зруйновані, тому що завдяки магнітному полю формувалися стоячі хвилі на сходинках Фіске, і деякі моди зчепилися зі стоячими хвилями.

Ключові слова: контакти Джозефсона, потужність емісії, сходинки нульового поля, сходинки Фіске

INTRODUCTION.

The spectrum of emission from Josephson junctions and systems containing junctions is very informative for fundamental investigations of phase dynamics. It can be obtained by means of the measurement of the emitted power at different averaged voltages over the junction. According to the Josephson relation $n=V/F_0$ between the frequency n and the averaged voltage V (F_0 is the quantum of magnetic flux), one can obtain the dependence of emitted power on the frequency. One can also calculate such a spectrum for the model system and compare it with the experiment. Such calculations become actual recently because of investigations of the coherent emission from high-temperature superconductors which reveal the intrinsic Josephson effect [1-3]. Strong coherent emission without applied magnetic field was observed at resonant steps of IV-characteristic of mesa-structures made of these superconductors. Obviously, such an effect relates to emission at so-called zero-field steps [1, 4]. Zero-field steps appear in IV-characteristics in the absence of external magnetic field if there is random spread or some distribution of critical currents along the junction. If two adjacent segments of the junction have different critical currents, the circulation screening ac current appears between these segments. This current produces additional high-frequency voltages across segments, and due to the interaction between adjacent segments, this excitation propagates along the junction. If the magnetic field is applied to the junction, positions of the current steps (the so-called Fiske steps) in the IV-characteristic are determined by the well-known expression [5, 6]:

$$V_p = \frac{\Phi_0 \bar{c} p}{2D}, \quad p=1, 2, 3, \dots , \tag{1}$$

where D is the length of the junction and $\bar{c}$ is the velocity of light in the junction. For zero-field steps the value of p can be only even (p=2, 4, 6, …). Such choice of the even integer number is connected with the specific mechanism of the movement of electromagnetic excitations in the junction. Electromagnetic excitations which produce zero-field steps in long junctions are not standing electromagnetic waves but Josephson vortices which move along the junctions and are reflected at ends of junctions (the so-called Fulton-Dynes mechanism of the movement [5]). However, in some cases zero-field steps can be caused by standing waves like usual Fiske steps. It was obtained theoretically for the specific kind of the random distribution of critical currents with the exponential autocorrelation function that the height of the dc current step (the zero-field step) in the limit of the small quality factor at any resonant frequency is proportional to the square of the relative amplitude of fluctuations [7]:

$$I_p = \frac{1}{2} I_c z_p \gamma_1^2, \tag{2}$$

where I_c is the averaged critical current, $\gamma_1=I_f/I_c$ with I_f is the amplitude of the fluctuation of the critical current, $z_p = \frac{Q_p}{\pi^2 p^2}\left(\frac{D}{\langle\lambda_J\rangle}\right)$, Q_p is the quality factor and $\langle\lambda_J\rangle$ is the averaged Josephson length of the penetration of magnetic flux. In the present paper by means of the analyze of the calculated spectrum of emission we check the validity of this expression for the short ($D<\lambda_J$) junction with the Gaussian distribution of critical currents (namely, we check the proportionality $P\sim I_p\sim\gamma^2$, where P is the emission power and γ is the dimensionless value which is proportional to the standard deviation). We also consider the stack of two inductively interacting long junctions with the Gaussian distribution of critical currents in applied magnetic field (we use dimensional units of the normalized magnetic flux $\frac{\Phi}{\Phi_0}$ instead of magnetic field). By means of the analyzes of the spectrum of emission we show that the pure normal modes (namely, the in-phase mode and the anti-phase mode) in such a system exist only when there is strong magnetic flux through the system or when the magnetic flux through the system is equal to zero. When the magnetic flux is small, there appears some amount of modes of electromagnetic waves like those in the non-interacting (autonomous) junctions.

The model of the junction

The main idea of our calculations of the emission power is to present the short Josephson junction with the inhomogeneous distribution of critical currents as a set of segments with the homogeneous critical current for each of the segments. For this we can use the model which we used earlier for the description of long Josephson junctions [8-10]. Therefore, here we provide only the very short description of the model. We divide the junction to n segments. Each of the k=1, 2, …n segments has the critical current I_{ck}, the resistance R_k and the capacitance C_k (we assume $C_k=C$ for all k). Critical currents I_{ck} have random values with some standard deviation. The characteristic voltage for all segments is equal to $V_c=I_{ck}\times R_k$, so values of resistances of segments are equal to $R_k=V_c/I_{ck}$. Upper superconducting electrodes of neighbor segments of the junction are connected with each other and form the upper superconducting electrode. Similarly, lower electrodes of neighbor segments are connected with each other and form the lower superconducting electrode. Each loop between segments has the inductance L, so the total inductance of the junction is $(n-1)\times L$. The condition of the propagation of electromagnetic waves along the junction is $\bar{c} = \xi/\sqrt{LC}$ with $\bar{c}$ is the velocity of light in the junction and ξ is the length of the segment. Each of the segments is fed by a dc bias current I_b, so the total bias current through the junction is $I=n\times I_b$. Currents between loops of neighbor segments with indices k and k+1 are equal to $I_{k,k+1}$. For the boundary conditions one adds usually one fictive loops at both ends of the junction. These loops does not contain segments of the junction but have resistances, capacitances and inductances which define currents at ends. For boundary conditions of the transmission lines with open ends parameters of the circuits should satisfy conditions of the impedance of vacuum (about 300 Ohm). Then dynamic equations for phase differences j_k across segments, conditions of the conservation of magnetic flux and boundary conditions are as follows:

$$\frac{\Phi_0 C}{2\pi}\frac{d^2\varphi_k}{d\tau^2} + \frac{\Phi_0}{2\pi R_k}\frac{d\varphi_k}{d\tau} + $$
$$+I_{ck}\sin(\varphi_k) = I_b - I_{k-1,k} + I_{k,k+1},$$
$$k = 2 \ldots n-1, \qquad (3)$$

$$I_{k-1,k} = -\frac{1}{L}(\varphi_{k-1} - \varphi_k),\ k = 2, \ldots n, \qquad (4)$$

$$\frac{\Phi_0 C}{2\pi}\frac{d^2\varphi_1}{d\tau^2} + \frac{\Phi_0}{2\pi R_1}\frac{d\varphi_1}{d\tau} + $$
$$+I_{c1}\sin(\varphi_1) = I_b - \frac{dq_a}{dt} + I_{1,2}, \qquad (5)$$

$$\frac{\Phi_0 C}{2\pi}\frac{d^2\varphi_n}{dt^2} + \frac{\Phi_0}{2\pi R_n}\frac{d\varphi_n}{d\tau} + $$
$$+I_{cn}\sin(\varphi_n) = I_b + \frac{dq_b}{dt} - I_{n-1,n}, \qquad (6)$$

$$L_e\frac{d^2q_a}{d\tau^2} + R_e\frac{dq_a}{dt} + \frac{q_a}{C_e} = -\frac{\Phi_0}{2\pi}\frac{d\varphi_1}{d\tau}, \qquad (7)$$

$$L_e\frac{d^2q_b}{d\tau^2} + R_e\frac{dq_b}{d\tau} + \frac{q_b}{C_e} = \frac{\Phi_0}{2\pi}\frac{d\varphi_n}{d\tau}, \qquad (8)$$

where q_a and q_b are charges flowing through contours at the left and right sides of the junction, correspondingly, L_e, C_e and R_e – the inductance, the capacitance and the resistance in the contours, correspondingly (we assume that these parameters have the same values at the left side and at the right side of the junction). Eqs. (3)-(8) were solved by the method of Runge-Kutta. Parameters of external contours were L_e= 10^{-6} H, R_e=300 Ohm, C_e=9.6·10^{-14}F. The voltage across the segment is equal to $V_k = \frac{\Phi_0}{2\pi}\langle\frac{d\varphi_k}{dt}\rangle$ with angle brackets is the averaging on time. The voltage over the whole junction is equal to $V = \left(\frac{1}{n}\right)\sum_{k=1}^{n} V_k$. The power of emission is equal to $P = \langle\left[\sum_{k=1}^{n}\left(\frac{\Phi_0}{2\pi}\frac{d\varphi_k}{d\tau} - V_k\right)^2\right]\rangle$. We will plot the IV-characteristic and the power of emission in normalized units V/V_c and P/P_c with $P_c=V_c\times I_c$ and I_c is the averaged critical current of the junction, correspondingly.

RESULTS AND DISCUSSION

THE SHORT JUNCTION WITH THE GAUSSIAN DISORDER OF CRITICAL CURRENTS

The Josephson junction had the square form with dimensions 10x10 micrometers. It had the averaged critical current I_c=4×10^{-4} A, the characteristic voltage 2×10^{-3} V, the averaged resistance R=5 Ohm, the McCumber parameter about 90 and the quality factor about 137 at the voltage 0.76×V_c which corresponds to the first Fiske step. The Josephson length of penetration was about λ_J»2.3×10^{-5} m, so the condition $D < \lambda_J$ was satisfied. The junction was divided to 60 segments. The velocity of light in the junction was 1.47×10^7 m/s.

We set the Gaussian distribution of critical currents of segments I_{ck} with the standard deviation $\sigma = \gamma \cdot I_c$ with I_c is the averaged over all segments value of the critical current, so $\gamma = \frac{\sigma}{I_c}$ is the dimensionless parameter which shows the relative deviation of the critical current from its averaged value I_c like the parameter γ_1 in Eq. (2).

The example of the IV-characteristic for one of the realizations of the Gaussian distributions of critical currents with γ=0.3 is shown in Fig. 1a. It is seen that the IV-curve does not contain any particularities. However, the dependence of the emitted power P on the normalized voltage over the junction V/V_c reveals the set of maxima. These maxima appear at voltages which correspond to

Fiske steps $V_p = 0.76 \cdot p$ with p=1, 2, … (see Eq. (1)). Values of p are written above maxima in Fig. 1b. Particularities at Fiske steps are seen only in the derivative of the IV-characteristic (Fig. 1c). All found features of the IV-curves for short junctions ($D<\lambda_J$) with the Gaussian spread of critical currents (namely, the existence of some "nuclei" of Fiske steps in the IV-curves and the existence of the distinct maxima of emitted power at corresponding voltages) are similar to those which were found in Ref. [11] for long inhomogeneous junctions ($D>\lambda_J$). We can conclude that these features are the same for both long and short junctions.

Our method of the investigation of electrical properties of Josephson junctions with the disordered critical currents can be applied to the check of the validity of Eq. (2) for junctions with the Gaussian disorder of critical currents. The expression (2) was obtained in Ref. [7] for the specific exponential distribution function. It is necessary to check it for the Gaussian disorder which is more common in Josephson junctions. According to Eq. (2), the height of the Fiske step is proportional to the square of the value of γ_1 which characterized the degree of the disorder. This proportionality can be checked with the use of our method. However, straightforward calculations of heights of Fiske steps are impossible because steps are negligibly small and they only manifest themselves by means of particularities in derivatives of the IV-curves (see Fig. 1a, Fig. 1c). However, one can calculate heights of maxima of the

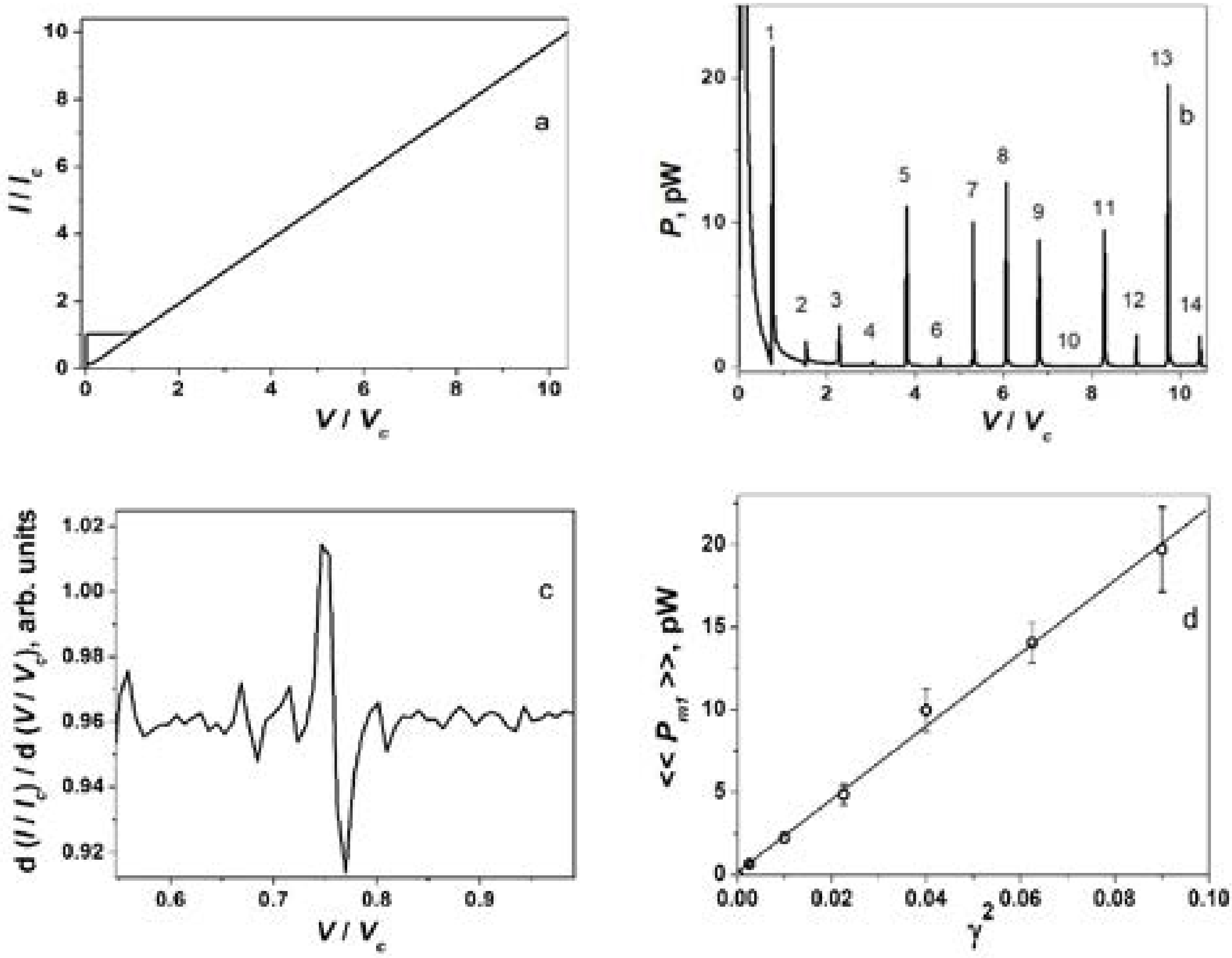

Fig. 1. (a) - the IV-characteristic of the short Josephson junction. (b) – the dependence of the emitted power on the normalized voltage over the junction. Numbers above maxima correspond to values of p for Fiske steps. (c) – the dependence of the derivative of the IV-characteristic on the normalized voltage in the vicinity of the first Fiske step at $0.76\times V_c$. (d) – the dependence of the averaged over realizations maximal value of emission power at the first Fiske step $\langle\langle P_{m1}\rangle\rangle$ on the square of the parameter γ which is proportional to the standard deviation (circles). The line is the approximation of data by the method of least squares.

emitted power from dependences $P=f(V/V_c)$ (see Fig. 1b). Emitted power is proportional to the value $I_p \times V_p$, so the averaged over many realizations of critical currents value of maximal emitted power at the given Fiske step p is equal to $\langle\langle P_{mp} \rangle\rangle = I_p V_p$ (the sign $\langle\langle \ldots \rangle\rangle$ means averaging on realizations). We check the dependence of this value on γ. We chose the maximum P_{m1} of the dependence $P=f(V/V_c)$ at the voltage of the first Fiske step $V_1=0.76 \times V_c$ (see Fig. 1b). We calculated this maximum for about 150-200 realizations of the Gaussian distribution of critical currents for each of the chosen values of γ and obtained the averaged value $\langle\langle P_{m1} \rangle\rangle$. Then we repeated calculations for the new value of γ etc. The time of calculations of the averaging over realization value $\langle\langle P_{m1} \rangle\rangle$ for the certain value of γ was about 12 hours. The obtained values of $\langle\langle P_{m1} \rangle\rangle$ are plotted against values of γ^2 in Fig. 1d. It is seen that the dependence $\langle\langle P_{m1} \rangle\rangle = f(\gamma^2)$ is linear. Thus, we proved that $\langle\langle P_{m1} \rangle\rangle \sim \gamma^2$ that is agreement with Eq. (2).

STACK OF TWO JUNCTIONS WITH THE GAUSSIAN DISTRIBUTION OF CRITICAL CURRENTS IN MAGNETIC FIELD

The second application of the developed approach of the spectrum of emission is the calculation of resonant steps in the stack of two long Josephson junctions with the inductive interaction between junctions. This interaction is characterized by the coefficient α that is the relation of the mutual inductance of two neighbor superconducting layers to their self-inductance. It was shown that each of the Fiske steps at the voltage V_p in such a system is split to two steps at the voltage $V_{p,\alpha} = \frac{V_p}{\sqrt{1+\alpha}}$ which corresponds

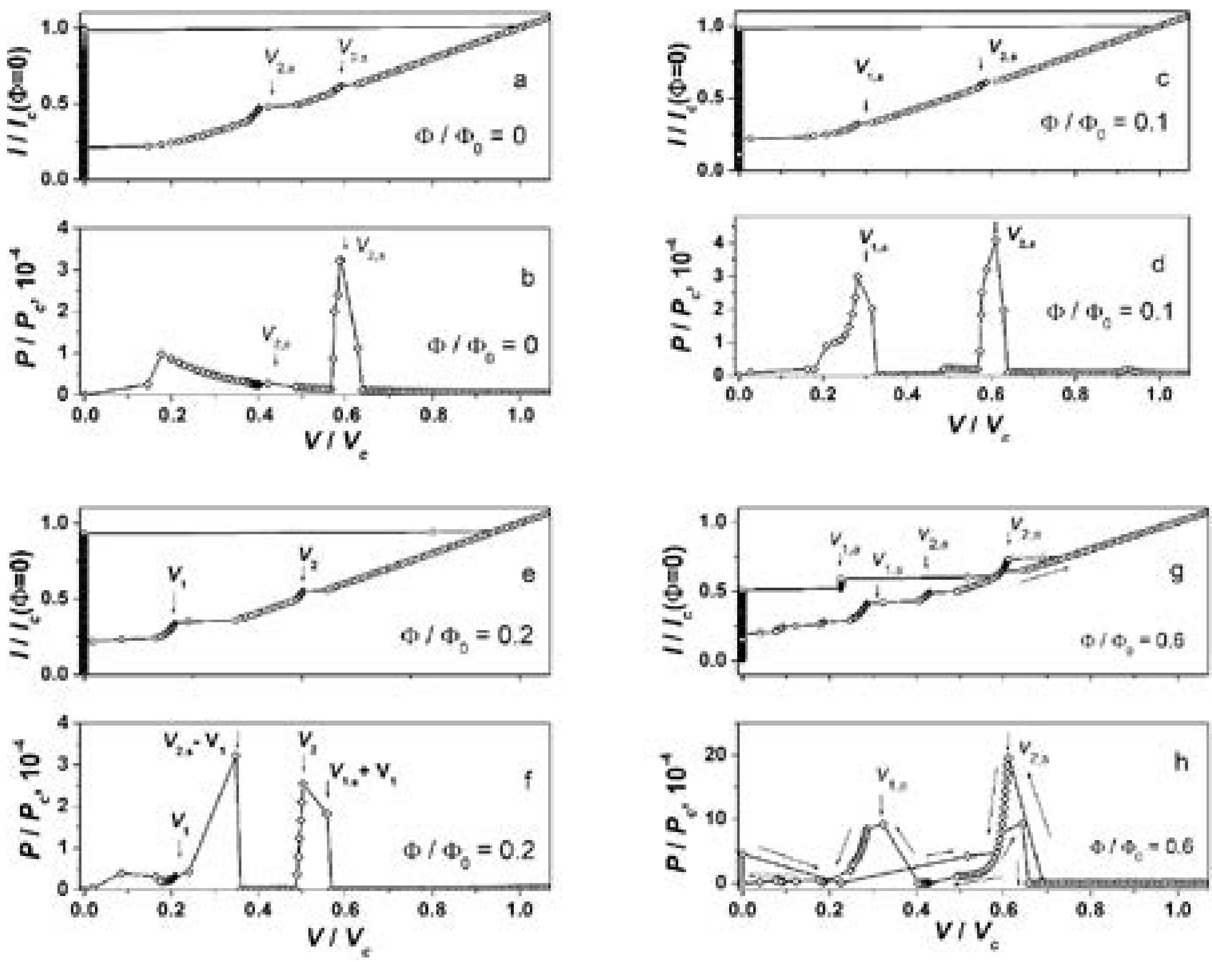

Fig. 2. (a), (b) -IV-characteristics of the stack of two Josephson junctions for $\Phi = 0$ (upper layer) and the dependence of power of emission on normalized voltage (lower layer). Arrows show positions of split Fiske steps. Corresponding normalized voltages of split Fiske steps are written above arrows. (c), (d) – the same for $\Phi = 0.1 \cdot \Phi_0$. (e), (f) – the same for $\Phi = 0.2 \cdot \Phi_0$, (g), (h) – the same for $\Phi = 0.6 \cdot \Phi_0$. Long arrows show the increase and the decrease of the bias current.

to the Fiske step for the anti-phase mode and the voltage $V_{p,s} = \frac{v_p}{\sqrt{1-\alpha}}$ which corresponds to the Fiske step for the in-phase mode. If the distribution of critical currents in the long junctions ($D > 2\lambda_J$) is inhomogeneous, the so-called zero-field steps appear in the IV-characteristic without applied magnetic field.

For the obtaining of the IV-characteristics and the dependence of emitted power on voltage (the emission spectrum) we used the model of two long junctions [10]. In the following consideration we will use the normalized values of magnetic flux through the loop between adjacent segments $\frac{\Phi}{\Phi_0}$ as a measure of magnetic field. Further we will also discuss values of magnetic field obtained from data of magnetic flux.

Each of two junctions had the length D=6×10^{-5} m, the width 3×10^{-4} m and the thickness 10^{-8} m. The averaged critical current of each of the junctions was I_c=1.447×10^{-3} A, and the characteristic voltage was V_c=4.2×10^{-3} V. Each of the junctions was divided to 100 segments. Both junctions had Gaussian distributions of critical currents with the standard deviation 10^{-5}×I_c, i.e. 10^{-3}%. The velocity of light in both junctions was 6.272×10^{7} m/s. The London depth of the penetration of magnetic field was equal to 10^{-7} m. The thickness of the insulating barrier with ε=4 is equal to 10^{-9} m. The Josephson depth of penetration of magnetic flux was λ_J=2.8 ×10^{-5} m, so the relation D>2×λ_J was valid. The parameter of the inductive interaction between superconducting layers was α=0.3. We chose parameters of junctions close (but not equal) to those values of high temperature superconductors.

With the use of these parameters one can obtain values of voltages corresponding to zero-field steps of separate (autonomous) junctions: v_p = (V_p/V_c)»0.26×p, p=1, 2, … that gives values v_1=V_1/V_c»0.26 and v_2 =V_2/V_c»0.52. Due to the inductive interaction between junctions in the stack, each of these steps is split to two steps which correspond to the anti-phase mode at $V_{p,a}$ and the in-phase mode at $V_{p,s}$, so for the first Fiske step we obtain $v_{1,a}$ = $V_{1,a}/V_c$»0.23, $v_{1,s}$ =$V_{1,s}/V_c$»0.31, and for the second Fiske step we obtain $v_{2,a}$ =$V_{2,a}/Vc$»0.45, $v_{2,s}$ =$V_{2,s}/Vc$»0.60.

The IV-characteristic of the stack calculate without applied magnetic field is shown in Fig. 2a and the dependence of emitted power on the normalized voltage is shown in Fig. 2b. It is seen from Figs. 2a, b that there are only two collective modes in the system, namely, the in-phase mode and the anti-phase mode. There is only one even split zero-field step which corresponds to p=2 and there is the maximum of emitted power at $v_{2,s}$»0.60 (Fig. 2b). Note that the zero-field step which corresponds to the anti-phase mode exists in the IV-characteristic at $v_{2,a}$»0.45 (Fig. 2a) but there is not emitted power at this step (Fig. 2b).

Now we consider the behavior of the IV-characteristic of the stack and its emitted power when magnetic field is applied. In Figs. 2c, d the IV-curve and the dependence of the normalized emitted power are shown for the case when the magnetic flux F through the system reaches the value 0.1×F$_0$. One can see from Fig. 2c that the step which correspond to the anti-phase mode at $v_{2,a}$»0.45 vanishes but the step at $v_{1,s}$»0.31 appears. The step at $v_{2,s}$ »0.60 remains at this plot. There are distinct maxima of emitted power at these steps (Fig. 2d). When the magnetic flux reaches the value 0.2×F$_0$, the steps in the IV-characteristic at voltages which correspond to the non-split Fiske steps at v_1»0.26 and v_2»0.52 appear (Fig. 2e). There are maxima of emitted power at voltages v_2, $v_{2,s} - v_1$ and $v_{1,s} + v_1$ (Fig. 2f). The value of magnetic field which correspond to 0.2×F$_0$ is equal to 0.066 T. This value is large and it is comparable with typical values of magnetic fields for Fraunhofer-like dependences of critical currents on the magnetic field for stacks of intrinsic junctions in high-temperature superconductors [12].

The increase of the magnetic flux up to 0.6×F$_0$ leads to the appearance of steps at $v_{1,a}$, $v_{1,s}$, $v_{2,a}$, $v_{2,s}$ in the IV-curve (see Fig. 2g) and very large maxima of emitted power at $v_{1,s}$ and $v_{2,s}$ (see the scale of emitted power in Fig. 2h). Despite of the complicated and hysteretic form of the IV-characteristic, only these steps appear.

It follows from Figs. 2a, b that there are only the anti-phase mode and the in-phase mode of oscillations of voltage in the stack when the magnetic field is absent. This means that voltage over one of the junctions in the stack oscillate in-phase with voltage over another junction or these voltages oscillate anti-phase. The same result is obtained when the stack is placed in the strong magnetic field (Figs. 2g, h). But in relatively small magnetic fields (i.e. when neither the Fulton-Dynes mechanism of oscillations dominates nor the formation of standing waves dominates) there appear other modes in the stack (Figs. 2e, f). This means that some quantity of normal modes were destroyed. Due to magnetic field standing waves were formed at both odd and even Fiske steps, so some modes could be locked with standing waves.

CONCLUSIONS

We investigated numerically IV-characteristics and power of emission from a short Josephson junction with the Gaussian distribution of critical currents. Dimensions of the junction were smaller than the Josephson depth of penetration of magnetic field. We found sharp maxima of emitted power at voltages corresponded to positions of Fiske steps up to large values of voltages. These maxima appeared without applied magnetic field. Irregularities of the IV-characteristic at these voltages are extremely small and can be revealed only in the derivative of the IV-curve. Obtained results are explained in ranges of the theory of Fiske steps for junctions with the disorder of critical currents [7]. We showed that the found particularities are the same as the found earlier particularities of electrical properties of long junctions [11]. We analyzed also the dependence of the averaged over realizations emitted power on the parameter γ which is proportional to the standard deviation of the Gaussian distribution and found the square dependence. Using the calculation of emitted power, we analyzed dynamics of the stack of two long inductively interacting junctions with the Gaussian distribution of critical currents in the external magnetic field. We found that when the magnetic field is absent, only normal modes contributed to emitted power. The same result we obtained when the magnetic field is strong. To the contrary, in small magnetic fields some new maxima appeared at voltages which correspond to frequencies of mods of non-interacting junctions and combination frequencies.

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