Вісник ХНУ імені В.Н. Каразіна, серія «Фізика», вип. 34, 2021. с. 33-35

PACS: 43.00.00

UDC: 534.8

Sound resonances in supercritical and superfluid helium

N.O. Herashchenko, K.E. Nemchenko, T.G. Vikhtinskaya, S.Yu. Rogova

Education and Research Institute of Computational Physics and Energy, V.N. Karazin Kharkiv National University, Svobody Sq. 4, 61077, Kharkiv, Ukraine

ORCID: 0000-0001-6580-5991

DOI:10.26565/2222-5617-2021-34-05

For twenty years of research, the processes of radiation and dissipation occurring during oscillations of quartz tuning forks in superfluid helium and its mixtures have turned from an object of research into a tool for studying the properties of helium. Quartz tuning forks are used to study various properties of helium - viscosity, thermal conductivity, radiation of the first and second sounds, and also as a precision temperature sensor. Experimental observations of these phenomena were carried out in a wide range of temperatures and pressures, but the results of observations have not yet been exhaustively described theoretically.

The aim of this work is to study density and pressure oscillations to determine the conditions under which oscillations of a solid wall excite the first sound in superfluid helium and sound in supercritical helium, and to calculate the contributions of these processes to the formation of resonances during oscillations of closed tuning forks. In particular, the experimentally observed excitation of standing waves of pressure oscillations by an oscillating closed tuning fork, the appearance and properties of resonances depending on the temperature and pressure of helium are considered.

As a result of the work, a model was built that described the physical features of the experimentally observed resonance phenomena.

Keywords: acoustic resonance, superfluid helium, supercritical helium, standing wave, density fluctuations.

Звукові резонанси в надкритичному і надрідкому гелії

Н.О. Геращенко, К.Е. Немченко, Т.Г. Віхтинська, С.Ю.Рогова

ННІ компютерної фізики та енергетики, Харківський національний університет імені В.Н. Каразіна, м. Свободи 4, 61022, Харків, Україна

За двадцять років досліджень, процеси випромінювання та дисипації, які відбуваються при коливаннях кварцових камертонів у надплинному гелії та його розчинах, із об'єкта дослідження перетворилися в інструмент для дослідження властивостей гелію. Кварцові камертони використовуються для дослідження різних властивостей гелію – в'язкості, теплопровідності, випромінювання першого та другого звуку, а також як прецизійний датчик температури. Експериментальні спостереження цих явищ проводилися у широкому діапазоні температур та тиску, але результати спостережень ще не отримали вичерпного теоретичного опису.

Метою даної роботи є дослідження коливань густини та тиску для визначення умов, за яких коливання твердої стінки збуджують перший звук у надплинному гелії та звук у надкритичному гелії, і обчислення внесків цих процесів до формування резонансів під час коливань закритих камертонів. Зокрема, розглянуто експериментально спостережуване збудження стоячих хвиль коливань тиску камертоном, що коливається, поява і властивості резонансів в залежності від температури та тиску гелію.

В результаті роботи побудована модель, що описує фізичні властивості резонансних явищ, які спостерігалися експериментально.

Ключові слова: акустичний резонанс, надплинний гелій, надкритичний гелій, стояча хвиля, коливання густини.

Звуковые резонансы в сверхкритическом и сверхтекучем гелии

Н.О. Геращенко, К.Е. Немченко, Т.Г. Вихтинская, С.Ю.Рогова

УНИ компьютерной физики и энергетики, Харьковский национальный университет имени В.Н. Каразина, м. Свободы 4, 61022, Харьков, Украина

За двадцать лет исследований процессы излучения и диссипации, происходящие при колебаниях кварцевых камертонов в сверхтекучем гелии и его растворах, из объекта исследования превратились в инструмент для изучения свойств гелия. Кварцевые камертоны используются для исследования различных свойств гелия – вязкости, теплопроводности, излучения первого и второго звука, а также в качестве прецизионного датчика температуры. Экспериментальные наблюдения этих явлений проводились в широком диапазоне температур и давлений, но результаты наблюдений еще не получили исчерпывающего теоретического описания.

Целью данной работы является исследование колебаний плотности и давления для определения условий, при которых колебания твердой стенки возбуждают первый звук в сверхтекучем гелии и звук в сверхкритическом гелии, и вычисление вкладов этих процессов в формирование резонансов во время колебаний закрытых камертонов. В частности, рассмотрены экспериментально наблюдаемое возбуждение стоячих волн колебаний давления колеблющимся закрытым камертоном,

возникновение и свойства резонансов в зависимости от температуры и давления гелия.

В результате работы была построена модель, описывающая физические особенности экспериментально наблюдаемых резонансных явлений.

Ключевые слова: акустический резонанс, сверхтекучий гелий, сверхкритический гелий, стоячая волна, колебания плотности.

For the last twenty years, the processes of radiation and dissipation by oscillating quartz tuning forks in superfluid helium and in the mixtures of helium isotopes have turned from an object of research into a tool for studying the properties of helium [1, 2].

The experiments, on which our theoretical research is based, were provided in Ref.3, in which the dependences of resonances on temperature and pressure, as well as the overlap of resonances were observed. These phenomena have a very obvious explanation, but have not yet received a quantitative description.

In Ref.3 the acoustic resonances were studied in liquid helium and its ^{3}He – ^{4}He superfluid mixtures during three experiments, in each of which quartz tuning forks were immersed in a cell filled with liquid, the main resonant frequency of which in vacuum is about 32.768 Hz. These piezoelectric tuning forks were excited by alternating voltages at electrodes located on a quartz crystal.

Due to the rather complex geometry of the experimental setup, the resonance spectrum can be determined with certain assumptions. That is, the properties of tuning forks and their acoustic modes do not provide an accurate quantitative determination of the speed of sound. Therefore, as a result in [3] only approximate and qualitative conclusions are given.

In order to describe the phenomena that manifested themselves during experiments with a quartz tuning fork oscillating in superfluid ^{3}He – ^{4}He solutions of various concentrations, this paper proposes the use of a certain model that can help to describe the obtained results with definite accuracy.

Therefore, in order to consider the conditions under which the resonances of the first and second sounds can be observed, we consider a one-dimensional model of a bounded vessel in which there is a fixed wall on one side and an oscillating one on the opposite side. This model focuses on the study of simultaneous generation of collective modes, as well as the dependence of the intensity distribution between these modes depending on the temperature and concentration of solutions. At the same time the proposed model does not take into account the specific shape of the quartz tuning fork and the cylindrical shape of the flask.

To solve this problem, we consider the oscillations of the pressure, density and velocity, and the perturbation of entropy, temperature and concentration will not be taken into account. In this case the complete system [4] of the hydrodynamic equations for ^{3}He – ^{4}He superfluid mixtures is reduced to that describe the relationship between pressure and density, when entropy and concentration are constant.

$$\begin{cases} \dfrac{1}{c_1^2}\dfrac{\partial P}{\partial t} + \rho_0 \dfrac{\partial V}{\partial x} = 0 \\ \dfrac{\partial V}{\partial t} + \dfrac{1}{\rho_0}\dfrac{\partial P}{\partial x} = \nu_{eff}\dfrac{\partial^2 V}{\partial x^2} \end{cases} \tag{1}$$

Here $c_1 = \sqrt{\left(\partial P / \partial \rho\right)_{\sigma,c}}$ is the velocity of sound in supercritical helium, or the velocity of the first sound in superfluid helium, taken at the constant entropy and concentrations, ν_{eff} is the effective viscosity coefficient.

To study the qualitative features of the generating and the overlapping of the resonances, consider a model system of a narrow vessel with helium, in which one wall is stationary, and instead of another is an oscillator with a given mass, stiffness, natural frequency and quality factor. This model does not take into account the geometric features of the oscillations of the tuning forks in a closed flask, but can be used to study the physical features of the resonances.

The equation of motion for the moving wall can be written as follows:

$$M\ddot{x} = -k(x-L) - \tilde{\gamma}\dot{x} + AP(x=L)e^{-i\omega t} + F_0 e^{-i\omega t}, \tag{2}$$

where M is the mass, k is the stiffness, $\tilde{\gamma}$ if the friction coefficient, A is the cross section of the vessel, $P(x = L)$ is the pressure of the mixture at the coordinates $x = L$ of oscillating wall, F_0 is the amplitude and ω is the frequency of the external force. After division by mass we get the equation that complements the system (1)

$$\ddot{x} = -\omega_0^2(x-L) - \gamma\omega_0\dot{x} + \alpha P_0(x=L)e^{-i\omega t} + a_0 e^{-i\omega t} \tag{3}$$

Here $\omega_0 = \sqrt{k/M}$ and γ are the natural frequency and the quality of the oscillator, $\alpha = A / M$, and $a_0 = F_0 / M$ is the acceleration amplitude.

The solution of the system of the equations (1) and (3) gives the results for the squared velocity amplitude of the oscillating wall in the presence of the liquid in the vessel:

$$|V_0|^2 = \frac{\omega^2 |a_0|^2}{\left(\begin{array}{l}\left(\left(\omega^2-\omega_0^2\right)\sin\left(\omega L/c_1\right)-\lambda\omega\omega_0\right)^2+\\+\left(\omega\omega_0\gamma\sin\left(\omega L/c_1\right)+y\left(\omega^2-\omega_0^2\right)\right)^2\end{array}\right)}. \quad (4)$$

Here $\lambda = \alpha\rho_0 c_1/\omega_0$ is the dimensionless coefficient of resonances "interaction" that describes the overlapping, and the coefficient $y = \omega^2 L v_{eff}/2c_1^3$ describes the effective width of the sound resonances.

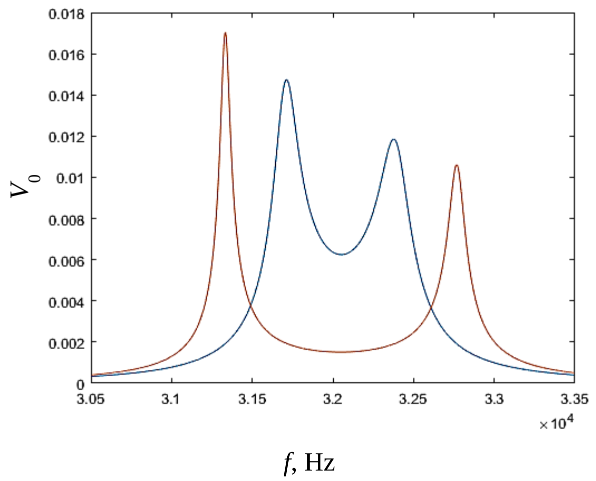

Fig. 1. Resonant sound curves in supercritical helium. The blue line indicates the result taking into account the coefficient of "overlap" of resonances, and the orange - the value of this coefficient is zero

Expression (3) solves the problem about the resonances which are caused by vibrations of quartz tuning forks, as well as sound vibrations in helium. This expression allows one to describe the overlap of these resonances and to give a quantitative description of the phenomena observed in the experiments from Ref. 3.

Figure 1 shows an example of close resonances of driving and acoustic vibrations in the absence of overlap (orange line) and in the presence of overlap (blue line). The graphs presented are calculated using formula (3) and qualitatively describe the dependences observed in the experiment (see Fig. 3 of the Ref. 3.)

For the case of liquid helium, expression (3) describes the resonances of the first sound in the superfluidity region and the ordinary sound in the normal region. The temperature dependence of these resonances is shown in Fig. 2 and covers the temperature range from 1.25 K to 4.25, which includes the lambda–transition to the superfluid state. The calculated dependences qualitatively and quantitatively coincide with those observed in the experiment (see Fig. 5 of the Ref. 3.)).

Finally, the paper carried out a theoretical study on the energy dissipation of body, which oscillates in supercritical and superfluid helium and its solutions. During the work, the physical principles that cause the appearance and overlap of resonances caused by oscillations of quartz tuning forks as well as hydrodynamic modes in helium are considered.

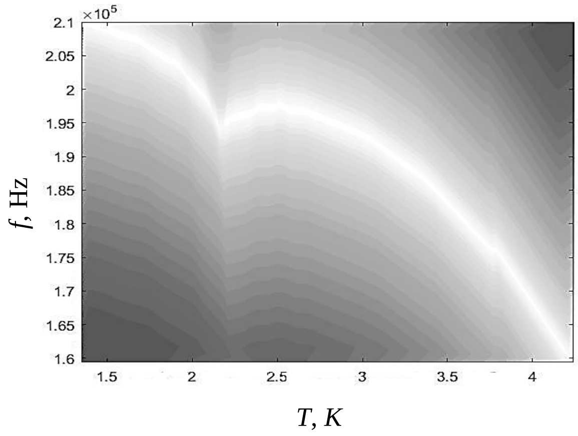

Fig. 2. Resonances of the first sound in superfluid helium depending on temperature and sound in the normal helium. The presence of resonant frequencies is indicated by a light color

A model that can explain the physical properties of resonant phenomena observed experimentally has been developed.

Explicit analytical expression (3) for resonance curves, which are caused by the first sound in superfluid helium, are obtained, and corresponding graphs are constructed.

Comparison with the experiment revealed that the results obtained with some accuracy coincide with the experimentally obtained data.

A method for calculating the parameters of resonant curves is proposed, which allows developing algorithms for further study of different types of interaction of tuning forks with helium.

References/Literature

1. D.O. Clubb, O.V.L. Buu, R.M. Bowley, R. Nyman, and J.R. Owers-Bradley. J. Low Temp. Phys., **136**, 1 (2004). https://doi.org/10.1023/B:JOLT.0000035368.63197.16
2. D. Schmoranzer, M. La Mantia, G. Sheshin, I. Gritsenko, A. Zadorozhko, M. Rotter, and L. Skrbek. J. Low Temp. Phys., **163**, 317 (2011). https:/doi.org/10.1007/s10909-011-0353-1
3. A. Salmela, J. Tuoriniemi, J. Rysti. J. Low Temp. Phys., 162, 678 (2011). https://doi.org/10.1007/s10909-010-0246-8
4. I.M. Khalatnikov, Theory of Superfluidity, (Nauka, Moscow, 1970), 160 p. (И.М. Халатников, *Теория сверхтекучести*, (Наука, Москва, 1970), 160 с.). [In Russian]
5. K. Nemchenko, S. Rogova, T. Vikhtinskaya. J. Low Temp. Phys., **187**, 324, (2017). https://doi.org/10.1007/s10909-017-1761-7
6. E.Ya. Rudavsky, V.K. Chagovets, G.A. Sheshin, V.A. Vrakina. Low Temperature Physics, **46**, 49, (2020). (Э.Я. Рудавский, В.К. Чаговец, Г.А. Шешин, В.А. Вракина. Физика низких температур, **46**, 49, (2020)). [In Russian]