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To the microscopic theory of the superconductive phase in antiferromagnetic metal compounds

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The theory of the conductivity in antiferromagnetic metal compounds is constructed on the basic of the fluctuational theory of the magnetic superconductors. The superconductor in which there exist localized magnetic moments was considered. It is supposed that magnetic moments are orientated antiferromagnetically in basis plane of the crystal. An estimation for the critical temperature was obtained and necessary and enough conditions of the appearance of high-temperature superconductive phase in rare earth metal compounds were got. The criterion of an appearance of the high-temperature superconductive phase in antiferromagnetic compounds is found.

Keywords: high-temperature superconductors, critical temperature, superconductivity in antiferromagnets, fluctuational theory of magnetic superconductors.

До мікроскопічної теорії надпровідної фази в антиферомагнітних металевих сполуках

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На основі флуктуаційної теорії магнітних надпровідників побудована теорія провідності в антиферромагнітних сполуках металів. Розглянуто надпровідник, в якому існують локалізовані магнітні моменти. Передбачається, що магнітні моменти спрямовані антиферомагнітно в базовій площині кристала. Отримано оцінку критичної температури та отримано необхідні та достатні умови появи високотемпературної надпровідної фази в сполуках рідкісноземельних металів. Знайдено критерій появи високотемпературної надпровідної фази в антиферромагнітних сполуках.

Ключові слова: високотемпературні надпровідники, критична температура, надпровідність в антиферромагнетиках, флуктуаційна теорія магнітних надпровідників.

К микроскопической теории сверхпроводящей фазы в антиферромагнитных металлических соединениях

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На основе флуктуационной теории магнитных сверхпроводников построена теория проводимости в антиферромагнитных соединениях металлов. Рассмотрен сверхпроводник, в котором существуют локализованные магнитные моменты. Предполагается, что магнитные моменты антиферромагнитно ориентированы в базисной плоскости кристалла. Получена оценка критической температуры и получены необходимые и достаточные условия возникновения высокотемпературной сверхпроводящей фазы в соединениях редкоземельных металлов. Найден критерий появления высокотемпературной сверхпроводящей фазы в антиферромагнитных соединениях.

Ключевые слова: высокотемпературные сверхпроводники, критическая температура, сверхпроводимость в антиферромагнетиках, флуктуационная теория магнитных сверхпроводников.

Recently the microscopic theory of the superconductive antiferromagnetic metal compounds was constructed [1-3]. In the paper [3] we also considered the magnetoresistance in a paramagnetic state of this compounds. It was shown, that the magnetoresistance depends on the addition to conductivity of the system, which is defined by the quantum interference of electrons in states with momenta $p_0(\Psi_1)$ and $-p_0(\Psi_2)$. This effect in paramagnetic phase is possible because of the scattering of the conductive electrons on spin density fluctuations of nonlocalized *s*-, *d*-, *f*- electrons and spin fluctuations of localized *d*-, *f*- electrons, if in the system an appearance a long-range magnetic order is possible. The scattering of electrons on spin fluctuations can lead to a jump of an electron from the state $p_0(\Psi_1)$ to the state $-p_0(\Psi_2)$ i.e. the electron trajectory makes a loop. At the entrance we have a momentum p_0 and at the exit we have $-p_0$. An electron spin relocates from the state $\frac{1}{2}$ to the state $-\frac{1}{2}$, in the course of time τ_z [3]. Thus, the magnetoresistance of a paramagnetic phase depends on the spin-spin relaxation processes of a conductive electrons. If a long-range magnetic order in the system is possible (antiferromagnetic, for example), then there exists the temperature of the magnetic phase transition T_N. Near the T_N spin fluctuations anomaly grows and their correlation radius $\langle r_f \rangle \to \infty$, $\langle r_f \rangle = \langle r_{f_0} \rangle \tau^{-\nu}, \nu \cong \frac{1}{2}$ (see [3]). As we consider the superconductive rare earth metal compounds the point T_N may prove to be the transition temperature from the paramagnetic state to the superconductive phase, as magnetic fluctuations suppress the Coulomb repulsion of electrons and stimulate the effective electron-phonon interaction. In the superconductive transition point the resistance is equal to zero, i.e. the conductivity $\sigma \to \infty$, but $\sigma \sim l$, where l is the electron mean free path, and $l = l_0 \tau^{-\nu}$ or $l \simeq \langle r_f \rangle$ where l_0 is the electron mean free path far from antiferromagnetic transition point, at $T \gg T_N$. The resistance in a paramagnetic state depends on the processes of spin-spin relaxation of electrons on fluctuations as well, and it is proportional to

$$R \cong \left(T - T_N\right)^{\frac{1}{2}} \tag{1}$$

How shall we consider the superconductor in which there exist localized magnetic moments (antiferromagnetic ordering)? We suppose that magnetic moments are orientated in basis plane of the crystal. From the papers [1–3] it follows that in the superconductive phase there exists a field from the fluctuating subsystem of spins of nonlocalized *s*-, *d*-, *f*- electrons [1–3]. The system of electron spins, distributed randomly in space has $\mathrm{SO}(3)$-symmetry ($\mathrm{SO}(3) = \mathrm{SU}(2)/\mathbb{Z}_2$). Orientation of the spin in a point with coordinates x_ν is defined by the axis with direction $\mathbf{n}$ and turning angle θ^3 around this axis, i.e. is determined by turning angles $\theta^\alpha, \alpha = 1,2,3$ in the spin space. The state of electron with the spin *s* is given by two own functions $\Psi_{\pm S}$. Now we shall define the wave function of electron in the fluctuating field A_ν

$$\Psi_{\pm S}\left(\mathbf{x}/\Gamma\right) = \frac{1}{N} T \exp\left(\oint_\Gamma dx_\nu A_\nu\right) \Psi_{\pm S}\left(\mathbf{x}\right) \tag{2}$$

where $N \geq 2$ (see [1, 2]), $A_\nu = \sum_{\alpha=1}^{N^2-1} \hat{\tau}^\alpha A_\nu^\alpha$, $\hat{\tau}^\alpha$ are SU(N) group generators, A_ν is the functional of a bent-torsion tensor of the electron spin system and is considered with the Yang-Mills field potential [5]. Now we can write the hamiltonian of a system if there are localized magnetic moments in the system.

$$\begin{aligned}
\mathcal{H}_\Gamma = {} & NTr \int_{\mathbf{x}\in\Gamma} d\mathbf{x} \times \\
& \times \left\{ \left\{ \frac{1}{N} T \exp\left(\oint_{\Gamma_{\mathbf{x},\tau}} dx_\nu A_\nu\right) \times \right.\right. \\
& \times \left[\overset{+}{\Psi}_\alpha(\mathbf{x}) \frac{1}{2\mu_e} p_{\mathbf{x}}^2 \Psi_\alpha(\mathbf{x}) + \right. \\
& + \sum_{k,\varkappa} e_{k\varkappa} \hat{b}^{+}_{k\varkappa}(\mathbf{x}) \hat{\xi}_\varkappa(\mathbf{x}) b_{k\varkappa}(\mathbf{x}) + \\
& + \frac{i}{\sqrt{2}} \sum_{k,\varkappa} g_{ph}(\mathbf{x}) e_{k\varkappa} \eta_\varkappa(\mathbf{x}) \begin{bmatrix} \hat{b}_{k\varkappa}(\mathbf{x}) - \\ -\hat{b}^{+}_{k\varkappa}(\mathbf{x}) \end{bmatrix} \times \\
& \times \overset{+}{\Psi}_\gamma(\mathbf{x}) \Psi_\gamma(\mathbf{x}) + \overset{t}{\Psi}_\alpha(\mathbf{x}) \overset{+}{\Delta}_{\alpha\gamma}(\mathbf{x}) \Psi_\gamma(\mathbf{x}) + \\
& + \overset{+}{\Psi}_\alpha(\mathbf{x}) \Delta_{\alpha\gamma}(\mathbf{x}) \overset{+t}{\Psi}_\gamma(\mathbf{x}) + \\
& \left.\left. + Q_J(\mathbf{x}) \left\langle S^\zeta(\mathbf{x}/\Gamma) \right\rangle \left(S^\zeta_{1\mathbf{x}} + S^\zeta_{2\mathbf{x}} \right) \right] \right\} \times \\
& \left. \times \frac{1}{N} \widehat{T \exp}\left(\oint_{\Gamma_{\mathbf{x},\tau}} dx_\nu \widehat{\overset{+}{A}_\nu}\right) + \left[\left(\frac{\delta}{\delta S_{\mu\nu}} \right)^2 + \hat{I}(\Gamma) \right] \widehat{B(\Gamma)} \right\} + \\
& + NTr \int_{\mathbf{x},\mathbf{x}'\in\Gamma} d\mathbf{x} d\mathbf{x}' \frac{1}{N} \widehat{T \exp}\left(\oint_{\Gamma_{\mathbf{x},\tau}} dx_\nu \widehat{A_\nu}\right) \left\{ \frac{1}{2} \widehat{\overset{+}{\Psi}_\alpha(\mathbf{x})} Tr \times \right. \\
& \times \left[\widehat{\overset{+}{\Psi}_\beta(\mathbf{x}')} V(\mathbf{x}-\mathbf{x}') \widehat{\Psi_\beta(\mathbf{x}')} \right] \widehat{\Psi_\alpha(\mathbf{x})} + \\
& \frac{1}{2} J(\mathbf{x}-\mathbf{x}') \Delta\widehat{S^\alpha_{1\mathbf{x}}} \Delta\widehat{S^\alpha_{2\mathbf{x}'}} + I(\mathbf{x}-\mathbf{x}') \left(\Delta\widehat{S^\beta_{1\mathbf{x}'}} + \Delta\widehat{S^\beta_{2\mathbf{x}'}} \right) \times \\
& \left. \times \widehat{\overset{+}{\Psi}_\alpha(\mathbf{x})} \widehat{\sigma^\beta_{\alpha\gamma\mathbf{x}}} \widehat{\Psi_\gamma(\mathbf{x})} \right\} \frac{1}{N} \widehat{T \exp}\left(\oint_{\Gamma_{\mathbf{x},\tau}} dx_\nu \widehat{\overset{+}{A}_\nu}\right)
\end{aligned} \tag{3}$$

In the expression (3) $p_{x_\nu} = \tau_0 \nabla_{x_\nu} - \left[A_1, \right]$, $e_{k\varkappa}$ are the unity vectors, $g_{ph}(\mathbf{x})$ is an electron-lattice potential. $\hat{b}^{+}_{k\varkappa}$, $\hat{b}_{k\varkappa}$ are phonon operators, $\varkappa$ is the parameter of elementary cell, $\Delta_{\alpha\gamma}(\mathbf{x})$ is a function which defines the superconductive order parameter, $Q_J(\mathbf{x}) = -\int d\mathbf{x}' J(\mathbf{x}-\mathbf{x}')$, $J(\mathbf{x}-\mathbf{x}')$ is the exchange potential between localised moments. $S^{\zeta}_{1\mathbf{x}}$, $S^{\zeta}_{2\mathbf{x}}$ are spin operators of magnetic sublattices delivering to the quantization axis 1ζ and 2ζ, which are orientated in the basis plane along *y*-axis (*x*-axis is orientated along *c*-axis). $\left\langle S^{\zeta}\left(\dfrac{\mathbf{x}}{\Gamma}\right)\right\rangle$ is an average sublattice moment, $\Delta S^{\alpha}_{j\mathbf{x}} = S^{\alpha}_{j\mathbf{x}} - \left\langle S^{\alpha}\left(\dfrac{\mathbf{x}}{\Gamma}\right)\right\rangle$, $\alpha = (X, Y, Z)$. $I(\mathbf{x}-\mathbf{x}')$ is an exchange potential between localised spins and spins of nonlocalized *s*-, *d*-, *f*-electrons. $B(\Gamma) = \dfrac{1}{N} T \exp\left(\oint_{\Gamma} dx_\nu A_\nu\right)$. $\delta S_{\mu\nu}$ is an increment of Γ – contour area.

Using the microscopic theory of the superconductive phase in rare earth metal compounds [1, 2] (see also [4]) we can write the condition of the appearance of a superconductive phase in an antiferromagnetic superconductor.

$$\begin{gathered}
\lim \left\langle \lambda_{eph}\left(2\mathbf{p}_F \left|\Gamma_{\mathbf{x},\tau}\right| \bar{p}_{\perp_0}\right)\right\rangle_{\rho\left(x,\bar{p}_{\perp_0}\right)} = \\
= \ln^{-1}\left[\frac{2\gamma}{\pi}\frac{\langle \omega_D \rangle_{\rho(x)}}{T_N}\right] . \\
\left\langle Tr\, B(\Gamma_{\mathbf{x},\tau})\right\rangle \to 1
\end{gathered} \tag{4}$$

$\gamma = e^{C}$, C is Eiler constant. $\langle \omega_D \rangle_{\rho(x)}$ is an average Debye energy.

$$\begin{gathered}
\left\langle \lambda_{eph}\left(2\mathbf{p}_F \left|\Gamma_{\mathbf{x},\tau}\right| \bar{p}_{\perp_0}\right)\right\rangle_{\rho\left(x,\bar{p}_{\perp_0}\right)} = \\
= \int_{C^{+}_{\infty}} d\zeta V_{ph}\left(\zeta \left|\Gamma_{\mathbf{x},\tau}\right| \bar{p}_{\perp_0}\right)\left\langle \rho^{ph}_{\varkappa_0}(\zeta)\right\rangle_{\rho_{\varkappa_0}} - . \\
-V_C\left(\Gamma_{\mathbf{x},\tau} / 2\mathbf{p}_F\right)
\end{gathered} \tag{5}$$

$$\begin{gathered}
V_{ph}\left(\zeta \left|\Gamma_{\mathbf{x},\tau}\right| \bar{p}_{\perp_0}\right) = \frac{3}{2}\left[\theta(\zeta) + \theta(-\zeta)\right]\left\langle Tr\, B(\Gamma_{\mathbf{x},\tau})\right\rangle \times \\
\times \left\langle \mathrm{ch}\left(\pi \ell_{\varkappa} \bar{p}_{\perp_0}\right)\right\rangle_{\rho\left(x,\bar{p}_{\perp_0}\right)} \int_{\sigma_F} \frac{d\sigma_F}{\left|\nabla_{\mathbf{p}}\varepsilon(\mathbf{p})\right|_{\sigma_F}} g^2_{ph}
\end{gathered} \tag{6}$$

$\left|\nabla_{\mathbf{p}}\varepsilon(\mathbf{p})\right|_{\sigma_F}$ is the electron velocity on the Fermi surface. $\left\langle \mathrm{ch}\left(\pi \ell_{\varkappa} \bar{p}_{\perp_0}\right)\right\rangle_{\rho\left(x,\bar{p}_{\perp_0}\right)} = \dfrac{1}{\pi \ell_{\varkappa} \bar{p}_{\perp_0}} \mathrm{sh}\left(\pi \ell_{\varkappa} \bar{p}_{\perp_0}\right)$ is the electron-phonon interaction intensification parameter. $\left\langle \rho^{ph}_{\varkappa_0}(\zeta)\right\rangle_{\rho_{\varkappa_0}}$ is an average phonon density of states.

$$\begin{gathered}
V_C\left(\frac{\Gamma_{\mathbf{x},\tau}}{2\mathbf{p}_F}\right) = \left\langle Tr\, B(\Gamma_{\mathbf{x},\tau})\right\rangle \times \\
\times \int_{\sigma_F} \frac{d\sigma_F}{\left|\nabla_{\mathbf{p}}\varepsilon(\mathbf{p})\right|_{\sigma_F}} \left\langle \mathcal{V}_{\varkappa_0}(2\mathbf{p}_F)\right\rangle_{p_{\varkappa_0}}
\end{gathered} \tag{7}$$

$\left\langle \mathcal{V}_{\varkappa_0}(2\mathbf{p}_F)\right\rangle$ is an effective Coulomb repulsion parameter renormalized by longwave fluctuations of a localized spins. The condition (4) is nonsufficient for the appearance the superconductive phase in rare earth metal compounds, as it is in the low-temperature antiferromagnets. In order to write the sufficient condition, we must calculate the correction to the longitudinal sound velocity near the superconductive phase transition point. Corresponding calculations lead us to the following result [6,7].

$$\begin{gathered}
\left|\left\langle \frac{S - S_n}{S_n}\right\rangle_{(\Gamma_\nu)}\right| = \\
= \left|\left\langle \begin{gathered} \mathcal{V}(\varepsilon_F)\frac{\left|\Delta_0\left(\Gamma_{\mathbf{x},\tau}\right)\right|^2}{\langle \omega_D \rangle_{\rho(\varkappa)}} \times \\ \left(1 + \ln\left[\frac{2\gamma}{\pi}\sqrt{\frac{\langle \omega_D \rangle^2_{\rho(\varkappa)} - \mu^2_{\varkappa}}{T^2_N - \mu^2_{\varkappa}}}\right]\right) \times \\ \times \int_{C^{+}_{\infty}} d\zeta V_{ph}\left(\zeta \left|\Gamma_{\mathbf{x},\tau}\right| \bar{p}_{\perp_0}\right)\left\langle \rho^{ph}_{\varkappa}(\zeta)\right\rangle_{\rho_{\varkappa_0}} \end{gathered}\right\rangle_{(\Gamma_\nu)}\right|
\end{gathered} \tag{8}$$

$\mathcal{V}(\varepsilon_F)$ is the average density of states on the Fermi surface. $\left\langle \dfrac{S - S_n}{S_n}\right\rangle_{(\Gamma_\nu)}$ is the averaging over space configurations. $\left|\Delta_0\left(\Gamma_{\mathbf{x},\tau}\right)\right|$ is the superconductive order parameter.

$$\mu_\chi = \frac{\rho_f}{2\mu_e} \cdot \frac{\chi^2}{\varkappa \cdot \chi_{0S} + \chi_{0S}^2 \frac{\rho_f^2}{\rho_F^2}} \qquad (9)$$

$$\rho_f = \frac{2\pi}{\langle r_f \rangle},$$

$$\chi_{0S} = \mu_0^2 g^2 \left\langle Tr B(\Gamma_{\mathbf{x},\tau}) \right\rangle^2 \times$$
$$\times \left\langle \mathrm{ch}\left(\pi \ell_\varkappa \bar{p}_{\perp_0}\right) \right\rangle_{\rho(x,\bar{p}_{\perp_0})} \mathcal{V}(\varepsilon_F) f_\varkappa(\varepsilon_F)$$

μ_0 is Bohr magneton, g is Lande factor, μ_e is an electron mass, χ_{0S} is the susceptibility of free electrons in the field $A(x)$ (that is an analog of Pauli susceptibility) (see [3]), $f_\varkappa(\varepsilon)$ is the Fermi distribution function, χ is the paramagnetic susceptibility. Near the transition points $\mu_\varkappa = \varepsilon_F \frac{\rho_f^2}{\rho_F^2} \cdot \frac{\chi}{\chi_{0S}}$. Then, from the expression (8) the condition for the temperature of the antiferromagnetic instability **f**ollows:

$$T_N > \varepsilon_F \frac{\rho_f^2}{\rho_F^2} \cdot \frac{\chi}{\chi_{0S}} \qquad (10)$$

Substituting $\langle r_f \rangle = \langle r_{f_0} \rangle \tau^{-\nu}$ (see the expressions (9), (10) in the paper [3]) and $\frac{\chi}{\chi_{0S}} = \tau^{-2\nu}$, we have

$$T_N > \left(\frac{a}{\langle r_f \rangle}\right)^2 \varepsilon_F \qquad (11)$$

The parameter $\left(\frac{a}{\langle r_f \rangle}\right)^2$ can be estimated and it is of order of 10^{-2}. Then, we get an estimation for the $T_N \to T_C > 10^2\,\mathrm{K}$, i.e. conditions (4), (10) are necessary and enough conditions of the appearance of high-temperature superconductive phase in rare earth metal compounds.

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