

PACS: 52.35.Qz, 52.35.Ra, 94.20.wf

UDC: 533.9

Drift wave turbulence and anomalous plasma diffusion in lower hybrid cavities observed in the ionosphere

D.V. Chibisov

*Physics Department, V.N. Karazin Kharkiv National University, Svobody sq. 4, 61022 Kharkiv, Ukraine
dmitriychibisov@karazin.ua*

ORCID: 0000-0002-6154-9772

DOI:10.26565/2222-5617-2019-31-08

In plasma of the Earth's upper ionosphere, regions with a depleted plasma density and an increased level of oscillations with a lower hybrid frequency compared to the environment were found. It was established that such plasma density cavities have cylindrical symmetry and are elongated along the geomagnetic field, so that the longitudinal dimensions significantly exceed the transverse ones. Such structures, called lower hybrid cavities, are quite stable, so that during the passage through them spacecraft do not observe significant changes in the parameters of the cavities. Thus, the nature of the change in cavities over time remains unclear. In this paper, we theoretically investigate the temporal evolution of a cavity in plasma of ionosphere. Since depletion of the plasma density is a cylindrically symmetric region, it creates a radial inhomogeneity in the plasma. In turn, the inhomogeneity of plasma leads to the development of low-frequency drift instability and a turbulent state of plasma. The anomalous plasma diffusion across the geomagnetic field resulting from the development of turbulence of drift waves of inhomogeneous plasma is considered as a mechanism for changing the cavity. In this paper the equation of plasma diffusion in cavity is solved, where the initial radial distribution of plasma density is the upside-down Gaussian. Plasma diffusion occurs radially towards the center, since the plasma density increases with increasing radial coordinate. Obtained solution of the diffusion equation gives the rate of decrease in the depth of cavity. In addition to reducing the depth of the cavity, its expansion also occurs, however, the expansion of the cavity is slower than the decrease in depth. The paper gives plots of the plasma density distribution over the radius for several time values, which show the temporal evolution of the cavity. These dependences show that in a time of the order of 1 second the cavity changes significantly, but does not completely disappear.

Keywords: ionosphere, lower hybrid cavities, drift instability, turbulence of plasma, anomalous plasma diffusion.

Турбулентність дрейфових хвиль і аномальна дифузія плазми в нижньогібридних порожнинах, що спостерігаються в земній іоносфері

Д.В. Чібісов

Харківський національний університет імені В.Н. Каразіна, м. Свободи 4, 61022, Харків, Україна

В плазмі верхньої іоносфери Землі були виявлені області з збідненою щільністю плазми і підвищеним рівнем в порівнянні з навколишнім середовищем коливань з нижньогібридною частотою. Встановлено, що такі порожнини щільності плазми мають циліндричну симетрію і витягнуті уздовж геомагнітного поля, так що поздовжні розміри значно перевищують поперечні. Такі структури названі нижньогібридними порожнинами, досить стійкі, так що при проходженні через них космічні апарати не спостерігають значних змін параметрів порожнин. Таким чином, характер зміни порожнин з плином часу залишається неясним. В цій статті ми теоретично досліджуємо часову еволюцію порожнини в плазмі іоносфери. Оскільки збіднення щільності плазми є циліндрично-симетричною областю, воно створює радіальну неоднорідність в плазмі. У свою чергу, неоднорідність плазми призводить до розвитку низькочастотної дрейфової нестійкості і турбулентного стану плазми. Аномальна дифузія плазми поперек геомагнітного поля в результаті розвитку турбулентності дрейфових хвиль неоднорідної плазми розглядається як механізм зміни порожнини. У даній роботі вирішується рівняння дифузії плазми в порожнині, де початкове радіальний розподіл щільності плазми є перевернутим гаусовим. Дифузія плазми відбувається радіально до центра, оскільки щільність плазми збільшується зі збільшенням радіальної координати. Отриманий розв'язок рівняння дифузії дає швидкість зменшення глибини порожнини. Крім зменшення глибини порожнини, також відбувається її розширення, однак розширення порожнини відбувається повільніше, ніж зменшення глибини. В роботі наведені графіки розподілу щільності плазми по радіусу для кількох значень часу, які показують часову еволюцію порожнини. Ці залежності показують, що за час порядку 1 секунди порожнина істотно змінюється, але не зникає повністю.

Ключові слова: іоносфера, нижньогібридні порожнини, дрейфова нестійкість, турбулентність плазми, аномальна дифузія плазми.

Турбулентность дрейфовых волн и аномальная диффузия плазмы в нижнегибридных пустотах наблюдающихся в земной ионосфере

Д.В.Чибисов

Харьковский национальный университет имени В.Н. Каразина, м. Свободы 4, 61022, Харьков, Украина

В плазме верхней ионосферы Земли были обнаружены области с обедненной плотностью плазмы и повышенным уровнем по сравнению с окружающей средой колебаний с нижнегибридной частотой. Установлено, что такие полости плотности плазмы имеют цилиндрическую симметрию и вытянуты вдоль геомагнитного поля, так что продольные размеры значительно превышают поперечные. Такие структуры, называемые нижнегибридными полостями, достаточно устойчивы, так что за время прохождения через них космические аппараты не наблюдают значительных изменений параметров полостей. Таким образом, характер изменения полостей с течением времени остается неясным. В этой статье мы теоретически исследуем временную эволюцию полости в плазме ионосферы. Поскольку обеднение плотности плазмы является цилиндрически симметричной областью, оно создает радиальную неоднородность в плазме. В свою очередь, неоднородность плазмы приводит к развитию низкочастотной дрейфовой неустойчивости и турбулентного состояния плазмы. Аномальная диффузия плазмы поперек геомагнитного поля в результате развития турбулентности дрейфовых волн неоднородной плазмы рассматривается как механизм изменения полости. В данной работе решается уравнение диффузии плазмы в полости, где начальное радиальное распределение плотности плазмы является перевернутым гауссовым. Диффузия плазмы происходит радиально к центру, поскольку плотность плазмы увеличивается с увеличением радиальной координаты. Полученное решение уравнения диффузии дает скорость уменьшения глубины полости. Помимо уменьшения глубины полости, также происходит ее расширение, однако расширение полости происходит медленнее, чем уменьшение глубины. В работе приведены графики распределения плотности плазмы по радиусу для нескольких значений времени, которые показывают временную эволюцию полости. Эти зависимости показывают, что за время порядка 1 секунды полость существенно меняется, но не исчезает полностью.

Ключевые слова: ионосфера, нижние гибридные полости, дрейфовая неустойчивость, турбулентность плазмы, аномальная диффузия плазмы.

Introduction

Studies of ionosphere by satellites and sounding rockets have established that in the plasma of the ionosphere there are regions with a higher level of lower hybrid oscillations compared to the surrounding plasma, which correlate with a depletion of plasma density. These regions are elongated along the geomagnetic field, so that the longitudinal dimensions much exceed the transverse ones were called lower hybrid solitary structures (LHSS), or lower hybrid cavities (LHC). Since of the high velocities of the spacecraft and small transverse dimensions of LHC, which are of the order of tens of meters, the measurement time is no more than tens of milliseconds. However, during this time, the structure has not changed much, that is the LHC is a relatively stable formation. Despite that there is a number of works to explain this phenomenon, the references to which are given in the review [1], the mechanisms of the appearance of cavities, as well as their stability, are not entirely clear. There are also no explanations for their disappearance and estimates of their lifetime.

This article discusses the problem of the time evolution and disappearance of LHC due to drift turbulence and anomalous transport of inhomogeneous plasma across the magnetic field. Drift turbulence develops due to the growth of unstable drift waves in inhomogeneous magnetized plasma, the frequency of which is much lower than the ion cyclotron frequency. This turbulence has long been a serious problem in plasma confinement in studies of controlled thermonuclear fusion, since it led to anomalous plasma

diffusion across the magnetic field. We consider the same turbulence as a candidate for the role of the mechanism leading to the diffusion of plasma in the cavity and filling it with plasma.

Measurements showed that the cavities have cylindrical symmetry, and therefore, an analysis of turbulence, as well as diffusion processes in the LHC plasma, should be carried out using the model of cylindrical waves. Such a model developed in our works [2-5] was used in [6] for LHC conditions in plasma. In [6], we considered both linear and nonlinear stages of drift instability for cavities, found the level of turbulence in the LHC, and previously investigated the temporal evolution of plasma density in the cavity due to anomalous transport through the magnetic field caused by drift turbulence.

In this work, the anomalous plasma diffusion caused by drift turbulence is studied in detail. We give here the procedure for solving the diffusion equation for the characteristic distribution of the plasma density over the radius and give a graphic illustration of the evolution of the distribution of the plasma density of the cavity.

Drift wave turbulence in lower hybrid cavities

We first briefly consider the conditions in LHC under which anomalous plasma diffusion across the magnetic field occurs. Observations showed that the plasma density distribution in LHC has the form of an upside-down Gaussian distribution [1], which can be described by the equation

$$n(r) = n_0 \left(1 - a \exp\left(-\frac{r^2}{2r_0^2}\right) \right), \quad (1)$$

In (1) n_0 is the plasma density in the environment, a is the depth of the cavity, r_0 is the length of the plasma density inhomogeneity. The main plasma parameters in the cavity are [6]: electron temperature $T_e \sim 0.3eV$, non-uniformity parameter $r_0/\rho_s \geq 3$, where $\rho_s^2 = \rho_{Ti}^2 (T_e/T_i)$ and ρ_{Ti} is the thermal Larmor radius of ions, $r_0 \geq 50m$, geomagnetic field magnetic induction $B_0 \sim 0.2Gs$. Inhomogeneity of the plasma along the radius in the cavity leads to the appearance of a low-frequency drift instability, the frequency and growth rate of which is [6]:

$$\begin{aligned} \omega_m(k) &= \frac{m\omega_{e*}}{1 + k_{\perp}^2 \rho_s^2}, \\ \gamma_m(k) &= \omega_m(k) \frac{\sqrt{\pi} z_{e0} \exp(-z_{e0}^2)}{1 + k_{\perp}^2 \rho_s^2}. \end{aligned} \quad (2)$$

Evaluation shows [6] that at altitudes of up to 1000 km where the main ion component is the singly ionized oxygen, the frequency of the drift oscillations in LHC is of the order of 3-5 Hz, whereas at the altitudes of 1500-2000 km, where the main ion component is protons, the frequency is of the order of 7-9 Hz. The growth time of instability, which is equal to $\gamma_m^{-1}(k)$, is estimated as 0.5–1.5 s.

An analysis of the nonlinear stage of drift instability showed [6] that in LHC the short wavelength part of the drift oscillating spectrum with $k_{\perp}^2 \rho_s^2 > 1$ damped nonlinearly and disappears, whereas long wavelength part of the spectrum at $k_{\perp}^2 \rho_s^2 < 1$ grows nonlinearly. The saturation level of the drift turbulence was estimated as [6]

$$\frac{W}{n_0 T_i} \approx \frac{\rho_s^2}{r_0^2}. \quad (3)$$

Anomalous diffusion of plasma in lower hybrid cavities

Due to drift turbulence in the plasma of LHC, the distribution of the density of the plasma components along the radius changes, and is governed by the diffusion equation

$$\frac{\partial n}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r D_{\perp} \frac{\partial n}{\partial r} \right), \quad (4)$$

where D_{\perp} is the diffusion coefficient across the magnetic field. Note that the diffusion coefficients of electrons and ions are equal, $D_{e\perp} = D_{i\perp} = D_{\perp}$, so that the diffusion is ambipolar. For the level of turbulence of (3) we have [6]

$$D_{\perp} = \frac{c T_e \rho_s}{e B_0 r_0}. \quad (5)$$

The equation (4) gives diffusion of plasma in the direction of axis of LHC, since the plasma density increases from the center.

To solve the equation (4) we introduce the notation $4D_{\perp} = D$ and rewrite them as

$$\frac{\partial n}{\partial t} = \frac{\partial}{\partial r^2} \left(r^2 D \frac{\partial n}{\partial r^2} \right). \quad (6)$$

Take the Laplace transform of (6) by multiplying both sides by $\exp(-pr^2)$ and integrate over r^2

$$\frac{\partial}{\partial t} \left(\int_0^{\infty} n(r,t) e^{-pr^2} dr^2 \right) = \int_0^{\infty} e^{-pr^2} \frac{\partial}{\partial r^2} \left(r^2 D \frac{\partial n(r,t)}{\partial r^2} \right) dr^2.$$

Denote

$$N = N(p,t) = \int_0^{\infty} n(r,t) e^{-pr^2} dr^2 \quad (7)$$

which is the Laplace transform for plasma density by the squared radial coordinate. Then we get the equation

$$\frac{\partial N}{\partial t} = \int_0^{\infty} e^{-pr^2} \frac{\partial}{\partial r^2} \left(r^2 D \frac{\partial n}{\partial r^2} \right) dr^2. \quad (8)$$

Now we integrate the right side of (8) by parts twice and obtain:

$$\frac{\partial N}{\partial t} = -p \int_0^{\infty} n \left(-p e^{-pr^2} r^2 D + e^{-pr^2} \frac{\partial}{\partial r^2} (r^2 D) \right) dr^2.$$

Assume that the diffusion coefficient does not depend on the radius, and then this equation is simplified

$$\frac{\partial N}{\partial t} = p^2 D \int_0^{\infty} n e^{-pr^2} r^2 dr^2 - p D N. \quad (9)$$

For the first term in (9), we use differentiation by parameter

$$p^2 D \int_0^\infty n e^{-pr^2} r^2 dr^2 = -p^2 D \frac{\partial}{\partial p} \left(\int_0^\infty n e^{-pr^2} dr^2 \right) = -p^2 D \frac{\partial N}{\partial p} \quad (10)$$

$$N(p, t) = \frac{1}{p} g \left(t + \frac{1}{Dp} \right). \quad (17)$$

As a result, the eq. (9) for $N(p, t)$ becomes

$$\frac{\partial N}{\partial t} = -p^2 D \frac{\partial N}{\partial p} - pDN,$$

or

$$\frac{\partial N}{\partial t} + p^2 D \frac{\partial N}{\partial p} = -pDN. \quad (11)$$

To solve the partial differential equation (11), we use the method of characteristics. First integral of the eq. (11) is found from the characteristic equation

$$\frac{dt}{1} = \frac{dp}{Dp^2}. \quad (12)$$

The solution of eq. (12), i.e. the first integral of eq. (11) is

$$u = t + \frac{1}{Dp}. \quad (13)$$

To obtain the second integral of (11) rewrite it as

$$\frac{\partial N}{\partial t} + p^2 D \frac{\partial N}{\partial p} = -pDN \frac{\partial N}{\partial N}.$$

Then the second characteristic equation for (11) is

$$\frac{dp}{p} = -\frac{dN}{N}. \quad (14)$$

Thus the second integral is

$$v = Np. \quad (15)$$

First and second integrals yield a general solution of (11):

$$v = g(u), \quad (16)$$

where g is an arbitrary function. Substituting (13) and (15) into (16) we obtain

$$Np = g \left(t + \frac{1}{Dp} \right),$$

or

Now we take into account the initial dependence (i. e. at $t = 0$) of plasma density on the radius (1). Solving the diffusion equation (11), we consider only second term in (1), since the first does not depend on either time or coordinate. Thus we study the evolution of the distribution

$$n(r) = n_0 a e^{-\frac{r^2}{2r_0^2}}. \quad (18)$$

Find the Laplace transform of (18)

$$N_0(p) = n_0 a \int_0^\infty e^{-\frac{r^2}{2r_0^2}} e^{-pr^2} dr^2 = \frac{n_0 a}{p + \frac{1}{2r_0^2}}, \quad (19)$$

and substitute it into (17) where we assume $t = 0$. Thereby we find the function $N_0(p)$ for the initial moment of time

$$N_0(p) = \frac{n_0 a}{p + \frac{1}{2r_0^2}} = \frac{1}{p} g \left(\frac{1}{Dp} \right). \quad (20)$$

Thus we found the explicit form of the function g (16):

$$g(x) = \frac{n_0 a}{1 + \frac{D}{2r_0^2} x}. \quad (21)$$

Substitute into (21) instead of x the value

$$x = t + \frac{1}{Dp},$$

that yields time dependence of $N(p, t)$:

$$N(p, t) = \frac{1}{p} \frac{n_0 a}{1 + \frac{D}{2r_0^2} \left(t + \frac{1}{Dp} \right)},$$

or, otherwise

$$N(p, t) = \frac{n_0 a}{\left(1 + \frac{D}{2r_0^2} t \right) p + \frac{1}{2r_0^2} \left(1 + \frac{D}{2r_0^2} t \right)}. \quad (22)$$

To obtain the dependency $n(r,t)$, we take the inverse Laplace transform for $N(p,t)$ (22):

$$n(r,t) = \frac{1}{2\pi i} \int_{s-i\infty}^{s+i\infty} N(p,t) e^{pr^2} dp = \frac{n_0 a}{\left(1 + \frac{D}{2r_0^2} t\right)} e^{-\frac{r^2}{2r_0^2} \left(1 + \frac{D}{2r_0^2} t\right)^{-1}}. \quad (23)$$

Now in (23) we back to the original diffusion coefficient $D = 4D_{\perp}$ and obtain

$$n(r,t) = \frac{n_0 a}{\left(1 + \frac{2D_{\perp}}{r_0^2} t\right)} e^{-\frac{r^2}{2r_0^2} \left(1 + \frac{2D_{\perp}}{r_0^2} t\right)^{-1}}. \quad (24)$$

Finally, we get the dependence of plasma density in the cavity on the radius and on time as [6]

$$n(r,t) = n_0 \left(1 - \frac{a}{\left(1 + \frac{2D_{\perp}}{r_0^2} t\right)} e^{-\frac{r^2}{2r_0^2} \left(1 + \frac{2D_{\perp}}{r_0^2} t\right)^{-1}} \right). \quad (25)$$

Equation (25) determines the temporary change in the depth of the cavity

$$a(t) = \frac{a}{\left(1 + \frac{2D_{\perp}}{r_0^2} t\right)}, \quad (26)$$

as well as its root mean square radial size

$$\sigma(t) = r_0 \sqrt{1 + \frac{2D_{\perp}}{r_0^2} t}. \quad (27)$$

From equation (26) we obtain the time to decrease the depth of the cavity by a factor of $k = a / a(t)$ [6]:

$$t_k = \frac{(k-1)}{2D_{\perp}} r_0^2 \approx 0.25(k-1) \quad (28)$$

as well as from equation (27) the root mean square radial size of the cavity over the time t_k :

$$\sigma(t) = r_0 \sqrt{k} \quad (29)$$

Thus, if the depth of the cavity decreases by a factor of k , then its transverse dimension increases by a factor of \sqrt{k} , so that anomalous diffusion leads not only to a

decrease in the depth of the cavity, but also to its expansion, however at a lower rate.

Figure 1 shows the temporal evolution of the plasma density distribution over the radius, where dependence (25) is plotted for several time values: $t_0 = 0s, t_1 = 0.25s, t_2 = 0.5s, t_3 = 0.75s, t_4 = 1s$ at initial cavity depth $a = 0.6$. The diffusion coefficient for the plasma parameters given in the first section is $D_{\perp} = 5 \cdot 10^7 \text{ cm}^2 \cdot \text{s}^{-1}$. Figure 1 shows that the cavity disappears in a time longer than 1 second.

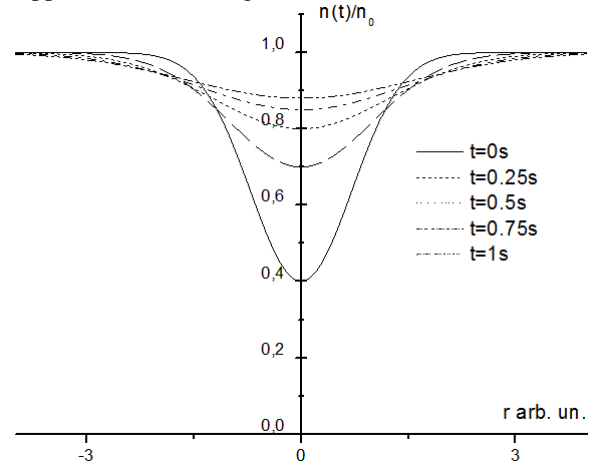


Fig. 1. The dependence of plasma density in cavity on the radius for several moments of time

Conclusions

Radial inhomogeneity of the plasma density and temperature in the plasma of LHC leads to the development of the drift instability and drift turbulence of plasma in the cavity. In turn, drift turbulence causes an anomalous diffusion of the plasma across the magnetic field, which leads to the disappearance of the cavity, and the cavity disappearance time exceeds 1 s.

References

1. P.W. Schuck J. W. Bonnell, P. M. Kintner IEEE Trans. Plasma Sci. 2003. vol. 31, №6, p. 1125-1177.
2. D.V. Chibisov, V.S. Mikhailenko, K.N. Stepanov. Phys. Lett. A. 1991. Vol.157. P.141-145.
3. V.S. Mikhailenko, K.N. Stepanov, D.V. Chibisov. Soviet Journal of Plasma Physics. 1991, vol.17. №10, p.710-716.
4. V.S. Mikhailenko, K.N. Stepanov, D.V. Chibisov. Plasma Phys. Control. Fusion. 1992, v.34, №1, p. 95-117.
5. V.S. Mikhailenko, K.N. Stepanov, D.V. Chibisov. Plasma Phys. Rep. 1995, vol.21. №12, p.141-150.
6. N.A. Azarenkov, D.V. Chibisov, Problems of Atomic Science and Technology. – 2018. – №6. Pp. 117-120