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# An alternative explanation for the nonlinear behavior of the oscillating tuning fork immersed in He II

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Experiments have been carried out on the excitation of hydrodynamic flows in superfluid helium under forced vibrations of a quartz tuning fork immersed in a liquid. Nonlinear oscillations that arise with an increase in the driving force are investigated and are manifested by distortion of the shape of the resonant amplitude-frequency characteristic in comparison with Lorentz curves typical for an extremely small force. Nonlinear resonance curves are described using the Duffing equation, the parameters of which are established by comparing the theoretical calculation with the experimental data. Dependence of the velocity of vibrations of the tuning fork legs on the driving force established with the use of the Duffing equation, is close to that previously obtained for the quasi-laminar flow of He II and containing a cubic velocity contribution due to the mutual friction caused by scattering of phonons by quantized vortices in a turbulent flow.
**Keywords:** quartz tuning fork; turbulence in liquid helium; scattering of phonons by quantized vortices.

## І. Гриценко, Т. Дубчак, К. Михайленко, Г. Шешин, С. Соколов

Проведено експерименти по порушенню гідродинамічних потоків у надплинному гелії при змушених коливаннях кварцового камертона, зануреного в рідину. Досліджено нелінійні коливання, які виникають при збільшенні сили, що змушує, і проявляються перекручуванням форми резонансної амплітудно-частотної характеристики в порівнянні з лоренцевими кривими, типовими для гранично малої сили. Нелінійні резонансні криві описані з використанням рівняння Дуффінга, параметри якого встановлені при порівнянні теоретичного розрахунку з експериментальними даними. Залежність швидкості коливань ніжок камертона від сили, що змушує, установлена з використанням рівняння Дуффінга, виявляється близької до залежності, раніше отриманої для квазіламінарного плину He II і утримуючої кубічний по швидкості внесок у силу взаємного тертя, обумовленої розсіюванням фононів на квантованих вихрах у турбулентному потоці.
**Ключові слова:** кварцовий камертон; турбулентність в рідкоу гелії; розсіювання фононів на квантованих вихорах.

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Проведены эксперименты по возбуждению гидродинамических потоков в сверхтекучем гелии при вынужденных колебаниях кварцевого камертона, погруженного в жидкость. Исследованы нелинейные колебания, которые возникают при увеличении вынуждающей силы и проявляются искажением формы резонансной амплитудно-частотной характеристики по сравнению с лоренцевыми кривыми, типичными для предельно малой силы. Нелинейные резонансные кривые описаны с использованием уравнения Дуффинга, параметры которого установлены при сравнении теоретического расчета с экспериментальными данными. Зависимость скорости колебаний ножек камертона от вынуждающей силы. установленная с использованием уравнения Дуффинга, оказывается близкой к зависимости, ранее полученной для квазиламинарного течения He II и содержащей кубический по скорости вклад в силу взаимного трения, обусловленной рассеянием фононов на квантованных вихрях в турбулентном потоке.
**Ключевые слова:** кварцевый камертон; турбулентность в жидком гелии; рассеяние фононов на квантованных вихрях.

### Introduction and task statement

One of the mostly used methods of studying the laminar and turbulent flow regimes in superfluid helium is the method of a quartz tuning fork immersed in a liquid. The quartz tuning fork differs favorably from the bodies of other geometry, first of all with high quality factor which attains ~ $10^6$. Also essential is the availability of quartz tuning fork (they are manufactured in industry), as well as their high durability [1].

When working with tuning forks with the prongs of different sizes, one can change the frequency of the resonances and the form of the amplitude-frequency characteristic (AFC) [2]. Moreover, as established during an experimental study of the appearance and development of superfluid turbulence in the temperature range down to ~ *20 mK*, under increasing the velocity of movement of the tuning prongs up to *0.02 m/s* the shape of the resonant AFC starts to deform. This deformation is explained in [2,3] as a result of the nonlinearity of tuning fork oscillations. Turbulent flows in He II at higher temperatures, *140* and *150 mK*, were investigated in Ref. 3 by the quartz tuning fork method. It was shown that the deviation from the linear dependence of the tuning fork velocity on the exciting force was observed at oscillation velocities exceeding *0.04 m/s*. The physical reason for the nonlinearity observed in [2,4] is, probably, the effect of an attached mass associated with quantum superfluid fluid vortices located in a thin layer of a liquid near the surface of a tuning fork, and there are the arguments [3] in favor of the fact that nonlinear deformations of AFC are connected with the appearance of an additional, nonlinear force of mutual friction due to the scattering of thermal excitations on the quantized vortices.

The observation of nonlinear effects at the excitation of the motion of He II by a quartz tuning fork calls an attention to the adequate description of the fork nonlinear resonance. The possibility of such a description appears when one uses the equation of a nonlinear oscillator [4,5] in the presence of an excitatory force. A separate case of the equation proposed in [4] is the Duffing equation [5], in which, unlike [4], the coefficient is set to zero with a quadratic displacement of the x term in the left-hand side and only the cubic term is available:

$$\frac{d^2x(t)}{dt^2}+\gamma\frac{dx(t)}{dt}+\omega_0^2x(t)+\mu x^3=\frac{F(t)}{m}\ ; \quad (1)$$

here x – deviation of the tuning fork leg from equilibrium position in presence of the excitatory force $F(t)=F_0\cos\omega t$, $\omega_0$ is resonance frequency of the tuning fork, $\gamma = 2\pi\Delta f$ is attenuation and $\Delta f$ is the width of the resonance line. Here *m* being effective mass of tuning prongs and $\mu x^3$ accounts the nonlinear behavior of the oscillator with $\mu$ being the coefficient of nonlinearity. This term leads to a resonant frequency shift compared to $\omega_0$. Moreover, depending on the sign of $\mu$, the resonance frequency of the oscillations is shifted toward higher or lower frequencies.

In this paper, for the analysis and adjusting of experimental data we apply the Eq. (1), which provides almost the same results as the more general equation [5], but at the same time is more convenient in calculations. The aim of the analysis is finding out the connection between the nonlinear mode of oscillation of the tuning fork prongs and the change in the dependence of the velocity of oscillation on the excitatory force. The aim of actual work is the establishment and research of such a connection, as well as the clarification of the possible influence of the nonlinear force of mutual friction in the superfluid fluid on the nonlinear behavior of the resonator - quartz tuning fork.

**Measurement procedure and experimental results**

We used a miniature quartz tuning fork, kindly provided to us by the laboratory of Lancaster University, with a resonant frequency in the vacuum of *24983.72 Hz*, length of the leg is $1{,}8\cdot10^{-3}$ *m*, thickness and width of the legs are *75* and *90 mkm*, respectively. The cell and the measurement procedure were previously described in detail [3,6,7]. The studies were made with the solution fridge working at two operating regimes. In one of them we pumped out $^4He$ from a one-Kelvin bath whereas a working solution was condensed in the solution refrigerator. This mode was used to determine the constant of the tuning fork in the experimental cell cooled down to *T = 1.4 K*. In other measuring mode the solution fridge worked providing the temperature of the cell and the test fluid of *140 ± 1 m*K.

The resistance thermometers of $RuO_2$ were used to determine the temperature. They were placed on the plate of the dissolution chamber and directly in the fluid under study. The thermometers were calibrated using a crystallization thermometer based on the pressure measurement along the $^3He$ melting curve. The accuracy of the measurement and temperature stabilization was ±*1 mK* being provided by the heater connected by the feedback with the resistance sensor CryoBridge S72A.

In the beginning of the experiment, we measured the quartz tuning fork frequency in a vacuum under different excitatory forces and *T = 1.4 K*. Sine-wave constant amplitude *U*, which is fed from the generator to one of the electrodes of the tuning legs, set the magnitude of the excitatory force, which was determined as $F_0 = aU/2$. On the other electrode, the frequency dependence of the amplitude of the ac current *I* was measured. This quantity is connected with the oscillation velocity of the tuning legs $\upsilon$ as $\upsilon = I/a$. The piezoelectric constant of the tuning fork was determined from the AFC measured in a vacuum [6].

In Fig. 1 we show typical AFC for a tuning fork in a vacuum obtained with different excitatory forces.

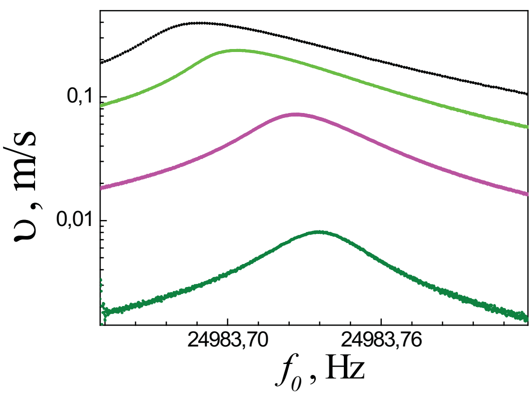


*Fig. 1.* Velocity of the oscillation of tuning legs in vacuum at different excitatory forces, bottom-up: $1{,}51\cdot10^{-11}$, $1{,}51\cdot10^{-10}$; $6{,}05\cdot10^{-10}$; $1{,}21\cdot10^{-9}$ N. [6].

It is clearly seen in Fig. 1 that at high excitatory forces and, consequently, high voltage amplitudes *U* one observes a nonlinear oscillation regime that manifests itself in the deformation of the form of the frequency response. Also the resonance frequency decreases with an increase in the excitatory force. Maximum excitatory force in Fig. 1 is $1,29•10^{-9}$ N, while the oscillation velocity in the resonance maximum was *0.4 m/s*, and the resonance frequency was decreased by *0.048 Hz* comparing with the value at the minimum excitory force. After measuring in vacuum, the solution refrigerator was cooled down to $T < 1\ K$, to study the flows in $^4He$. Passing through the nitrogen trap, helium traps and filling capillary, helium attained the experimental cell and condensed there.

Experimental dependences of the oscillation velocity of the tuning fork legs in presence of the excitatory force at temperature of *140 mK*, obtained in various experiments, are shown in Fig. 2. As is seen in Fig. 2, at oscillation velocities $\upsilon \geq 0.046\ m/s$ one observes a noticeable deviation from the linear dependence $\upsilon(F_0)$ shown by the solid line. As was suggested in Ref. 6, this deviation may be explained by the appearance of an additional frictional force that arises due to an increase of the density of quantum vortices and the scattering of thermal excitations - phonons - on their cores (mutual friction) [8]. The flow of helium characterized by the deviation the dependence $\upsilon(F_0)$ from the linear one was called a quasi-laminar in the work [6]. This flow is characterized by the above-mentioned new dissipative mechanism [8].

The force of mutual friction is proportional to the cube of the velocity of the legs: $F_0 \sim \upsilon^3$, which is typical for a turbulent flow (dotted line in Fig. 2). As a result, the total friction force has the form $F_0 = \lambda_q \upsilon + n\upsilon^3$ (solid and dotted lines) [3, 6] where $\lambda_q = 1.32•10^{-9}$ *kg/s* and $n = 4,62•10^{-8}$ $kg{\cdot}s/m^2$, and well describes the experimental data. At the experimental temperature of *140 mK*, the first term, as shown in Ref. 3, is completely determined by the force of friction in the quartz tuning fork material and is due to the bending oscillations of its prongs.

All the amplitudes of the oscillation velocity were measured at the maximum of the resonance curves. At the same time, the resonance curves, at increase in the excitatory force, are deformed due to a nonlinear additional frictional force. In this connection, in [3, 6], we were to analyze the types of AFC curves in the quasi-laminar flow regime. It was shown that the dependence $\upsilon(F_0)$ of Fig. 2 may be conveniently divided into five ranges characterized by a specific type of AFC (characteristic AFCs for each range are given in the works [3, 6]): (I) – region of laminar potential flow He II. Characteristic AFC of this region is shown in Fig. 1 of Ref. 3 and is approximated by Lorentzian. AFC for region II is shown in Fig. 2a of Ref. 3. As was noted, this region is characterized by spontaneous jumps between laminar potential and turbulent currents. Region III was previously depicted in Ref. 6 in Fig. 3 and is characterized by the fact that the AFC starts to be asymmetric relatively the maximum of the resonance curve, and there is a "collapse" towards the lower frequencies. The asymmetry of the AFC curve increases with increasing applied excitatory force until the instability does appear on the resonance curve, being the characteristic feature of the nonlinear behavior of the oscillating body. It should be emphasized that in the region III, regardless of the measurement conditions, one observes both quasi-laminar and turbulent flows. Fig. 3 of present paper and work [6] shows the AFC, measured at a stable quasi-laminar flow regime without the transition to turbulence.

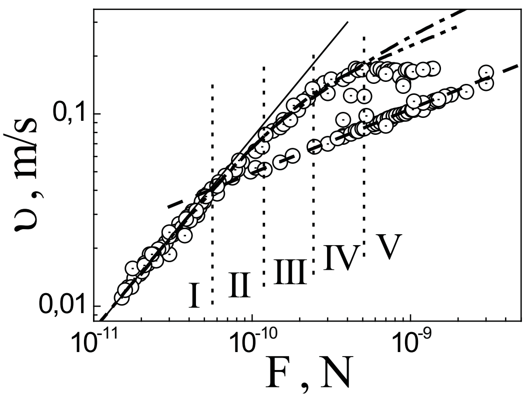


*Fig. 2.* Dependence of the oscillation velocity of the tuning fork legs on the excitatory force at $T = 140$ *мК*. Solid line is linear dependence $\upsilon \sim F_0$. Dotted line is for turbulent flow mode ($\upsilon^3 \sim F_0$), dot-dash line is calculation accounting the force of mutual friction [3]. Bar-dashed dotted line is the calculation based on the solution of the non-linear Duffing equation, described below in the text.

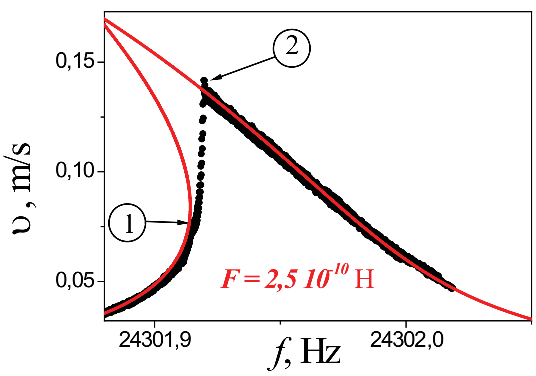


*Fig. 3.* Amplitude-frequency characteristic of tuning fork for region III with excitatory force $F_0 = 2,5•10^{-10}$ *N*. Solid curve - calculation using non-linear Duffing equation. 1 and 2 are the points of the beginning and end of the instability on the resonance curve.

In the regions IV and V, the shapes of the resonance curves are qualitatively identical, and in these regions the breakdown in the turbulent flow was observed, in each of the experiments carried out, in the form of a sharp decrease - a jump from the quasi-laminar to turbulent flow. For region IV the curve is shown in Fig. 2b of Ref. 3 and for area V - in Fig. 4 of Ref. 6. As can be seen in Fig. 4 of Ref. 6, in the region V the shape of the resonance curve is strongly deformed in comparison with Lorentzian, and in Fig. 2 it is evident that at the maximum of AFC, the velocity of oscillation of the tuning legs ceases to depend on the applied force.

The fact that the nonlinearity of the oscillation of the tuning fork legs in the regions II-V arises, probably, because of the appearance of an additional nonlinear force of mutual friction in He II, is supported by the measurements made in vacuum. When measured in a vacuum, the amplitude of the velocity was almost three times higher than that at the maximum amplitude of oscillation in He II (see Fig. 1), but there was no markedly expressed nonlinearities of oscillations (deformation of the form of AFC). Thus, it can be argued that the nonlinearity of the oscillations of the tuning fork legs observed in He II, is due to the nonlinear friction force in the liquid, in which the tuning fork is immersed. A similar conclusion was made in the work [2].

**Results and discussion**

As was noted above and as was shown in Fig. 2 of Ref. 3, as well as in Figs. 3 and 4 of this work, an increase in the excitatory force causing the oscillation of the legs of the tuning fork, leads to the nonlinearity of oscillations, which manifests itself in the deformation of the shape of the AFC until the appearance of instability of the oscillations and reduction of their resonance frequency. To describe these effects, we solve the equation (1) with respect to the modulus of amplitude of the oscillation velocity $\upsilon$. The result is

$$\upsilon = \frac{F_0}{m}\frac{\omega}{\sqrt{\left(\omega_V^2 - \omega^2 - \omega b \upsilon^2\right)^2 + \omega^2\gamma^2}}; \quad (2)$$

where $\omega_V$ and $\omega$ are the resonance frequency of the tuning fork in the vacuum and the current frequency, respectively, $b\upsilon^2$ is the factor which, according to [5], is proportional to the square of the amplitude of the oscillation velocity and the coefficient $b$ is connected with the coefficient of nonlinearity in Eq. (1) by the relation $\mu = \frac{2}{3}\omega_0^3 b$.

The dependence of the velocity on the frequency of nonlinear oscillations calculated by Eq. (2), is shown in Fig. 4 using a constant value $b = 40\ s/cm^2$, which, as will be shown later, is close to the average value in all the experiments carried out. It can be seen that even for low excitatory forces the frequency dependence of velocity demonstrates a slight asymmetry caused by the nonlinearity of oscillations (dashed line, $F_0 = 1{,}5\bullet10^{-10}\ N$). With the increase in the excitatory force, the velocity also increases, and the nonlinearity of oscillations is expressed more and more (a dashed-and dotted curve for $F_0 = 3\bullet10^{-10}\ N$), which leads to a decrease in the resonance frequency and the appearance of instability (points 1 and 3). Instability appears at point 1 if one moves from the left to the right towards to point 1, further movement in frequency continues to the right from the point 2. When moving in frequency in the opposite direction, i. e. from the right to the left, instability, as one might expect, should appear at point 3, with further motion towards lower frequencies from point 4 (hysteresis). However as it was shown in Ref. 2, the measurement of AFC when moving from high frequencies to lower ones and back, give practically identical result. The reason for this is unclear and additional research is needed to clarify the problem. It can be assumed that the nonlinear behavior of the system tuning fork - superfluid is described by the nonlinear term in (2), which origin is mainly connected with the fluid and processes in it. If the nonlinear behavior is related with the properties of the tuning fork itself, then instability at point 3 of Fig. 4 with decreasing frequency would be observed.

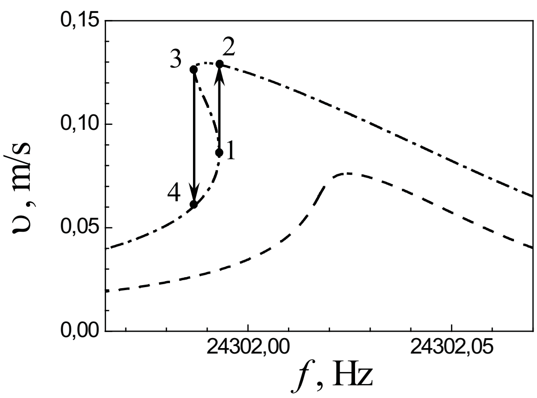


*Fig. 4.* Amplitude-frequency characteristics, calculated using Eq. (2): dashed line - calculation for $F_0 = 1{,}5\bullet10^{-10}\ N$ and $b = 40\ s/cm^2$; dashed-and-dotted line - for $F_0 = 3\bullet10^{-10}\ N$ and $b = 40\ s/sm^2$. Arrows show the jumps of the amplitude of oscillations in the event of instability.

It should also be noted that the value of $b$ in Eq. (2) strongly affects the form of the frequency response of velocity, which is determined by this equation. Value of $b$ was estimated by comparing the calculated dependence with the experimental data for the AFC, measured at different excitatory forces for the corresponding experimental data $\omega_V$, $\omega$, $\gamma$, $m$ and $F_0$. The $b$ is the only adjustable parameter. Thus, selecting the value of the coefficient $b$ one can attach the agreement with experimentally obtained resonance curves. Solid lines in Fig. 3 is the result of such calculations.

Note also that at excitatory forces corresponding to the regions II, III, and IV, the resonance curves are well described completely, and for the excitatory forces of the region V, the coefficient *b* was determined from the part of the curve to the left of point 1. In this frequency range, when the instability finished at point 1 of Fig. 4 of Ref. 6, the velocity value was always below than that at point 2 of Fig. 4 and did not coincide with the values corresponding to the right side of the calculated resonance curve. One should remember that, as it follows from the Fig. 2, the velocity at point 2 of region V is practically constant being and does not depending on the force.

The obtained values of *b* are shown in Fig. 5 for AFCs which are the result of all measurements. Interval of the excitatory force in Fig. 6 corresponds to the range of values of the excitatory forces in Fig. 2. The figure clearly shows that there is a huge scatter of the values of *b*. The solid line corresponds to the root-mean-squared value in the studied range, the mean value of the coefficient of nonlinearity coefficient $\mu$ is $9.2 \cdot 10^{16}\ s^{-2}m^{-2}$. At the same time the measurement accuracy of the frequency strongly affects the value of *b*. The nonlinearity coefficient can also be determined from the data of Ref. 3 presented as $\mu = n\left(\omega_0^3/m\right)$. In this case $\mu = 2{,}5 \cdot 10^{16}\ s^{-2}m^{-2}$ which is more than three times less than the above value obtained in the actual article. Such a noticeable difference between our values of $\mu$ and those of Ref. 3 may be attributed to the fact that the dependence of the damping coefficient $\gamma$ on the geometry of the problem was not taken into account in Ref. 6.

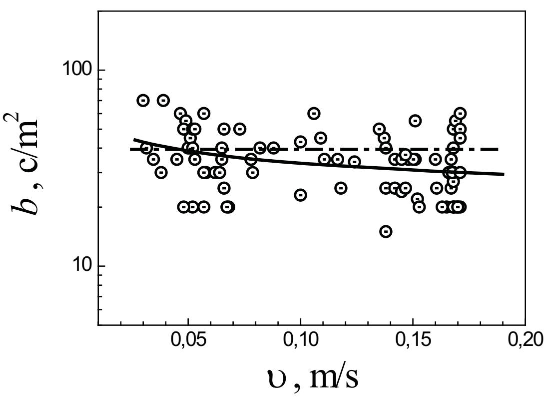


*Fig. 5.* The values of *b*, determined from the experimental data on the dependence of the velocity of oscillations on the excitatory force using the non-linear Duffing equation. The solid line is the average value throughout the range of exciting forces. The dotted line is done for $b = 50\ s/m^2$.

To determine the effect of the excitatory force on the amplitude of the velocity of the tuning fork prongs in the nonlinear regime, using the Eq. (2), the frequency dependences of velocity were calculated for different excitatory stresses and forces. The value of the velocity corresponding to the end of the instability was determined - point 2 in Figs. 3 and 4, corresponding to the maximum velocity. For low excitatory forces, if the instability was absent (regions I and II), the velocity was taken at the maximum at the resonance frequency of the frequency dependence of velocity. Thus, the dependence $\upsilon(F_0)$ was obtained allowing comparison with experimental data. The best agreement between the estimated and experimental data was achieved at $b = 50\ s/m^2$, calculation is shown in Fig. 2 by bar-dashed-and-dotted line. Dot-dash line on Fig. 2 shows the dependence accounting the contribution of mutual friction force, cubic in velocity, in addition to the linear contribution [3], the dotted line corresponds to the turbulent flow when $F_0 \sim \upsilon^3$. As is seen from the figure, when considering the nonlinearity of oscillations (deformation of the shape of the resonance curve), the amplitude of the velocity is a nonlinear function of the applied force. One observes also a rather good agreement between experimental data and the calculation made using the Duffing equation (bar-dashed dotted line in Fig. 2). Note that mean value is within the scatter of the values of *b*. Thus, one concludes that the experimental data in Fig. 2 can be described both with the solution of the Duffing equation (1), and with the consideration of the cubic term in the expression for the force of mutual friction.

Experimental data indicate that the velocity does not depend on the excitatory force in the region V with relatively high these forces (see Fig. 2). The frequency dependence using the Duffing equation can be described only to the left from the point 1 of the beginning of instability (see Fig. 4).

## Conclusions

In present paper, the study is carried out of nonlinear phenomena accompanying the oscillations of a quartz tuning fork, submerged in superfluid helium. The nonlinearity of the oscillations of the tuning fork legs is manifested by the deformation of the shape of the resonance curve for the amplitude-frequency characteristic of the tuning fork. It is shown that the nonlinear frequency response is well described using the Duffing equation for a nonlinear oscillator, by which the dependence of the oscillation velocity of the legs on the excitatory force is treated. It is shown that the same dependence can be obtained by adding a term, cubic in velocity, to the expression for the mutual friction force in the quasi-laminar flow regime. This term is due to the scattering of phonons by quantized vortices of He II, whose density increases with increasing velocity of oscillations. In addition, such a behavior may also indicate an increase in the attached mass or a decrease in the plasticity of the tuning fork due to the appearance of quantum vortices fixed to the surface of the quartz tuning fork.

Thus, the results of our research indicate that the nonlinearity of the tuning fork oscillations is mainly due to the dissipative processes in the superfluid fluid, in which the tuning fork oscillated, which is accompanied by the appearance of a nonlinear term in the dependence of the velocity of oscillations on the excitatory force.

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