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Review of theory of mesocopic systems

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As part of this work, the theory of mesoscopic systems was substantiated. The main effects of mesoscopic systems are provided; it is determined that the macroscopic characteristics of the system are significantly fluctuating within the mesoscopic level. The basic indicators of coherence of the quantum phase are determined and the mechanisms of influence are outlined. Six effects of mesoscopic systems with detailed justification are characterized. The theory of mesoscopic systems is based on the following mesoscopic effects: the Aaronov-Bohm effect; integral quantum output effect; fractional quantum Hall effect; Universal fluctuations of conduction; quantization of conductivity of a quantum point contact; direct currents in mesoscopic rings.

Small scales of time and/or length and low temperatures are characteristic for a mesoscopic regime. Under the conditions where the temperature is reduced, the time/length of the coherence of the phase increases, and the mesoscopic regime extends over larger scales of time/length. At Kelvin temperatures, the time and length scales in semiconductor samples are respectively picoseconds and micrometers.

Prospects for further developments in this area of research are based on a detailed study of mesoscopic effects, based on the growing trend for the production and research of materials containing the smallest structures and having low-dimensional features, that leads to the mesoscopic regime.

Keywords: mesoscopic systems; fluctuation; quantum phase; coherence; nanostructured system; macroscopic characteristics

У рамках даної роботи зроблений огляд теорії мезоскопічних систем. Зазначено основні ефекти мезоскопічних систем, визначено, що макроскопічні характеристики системи значно флуктуюють, у рамках мезоскопічного рівня. Визначено основні показники когерентності квантової фази та окреслено механізми впливу. Охарактеризовано шість ефектів мезоскопічних систем з детальним обґрунтуванням. Теорія мезоскопічних систем ґрунтується на наступних мезоскопічних ефектах: ефект Ааронова-Бома; ефект інтегрального квантового виходу; дробовий квантовий ефект Холла; універсальні флуктуації кондактанса; квантування провідності квантового точкового контакту; постійні струми у мезоскопічних кільцях.

Перспективи подальших розробок у даному напрямку дослідження ґрунтуються на детальному вивченні мезоскопічних ефектів виходячи зі зростаючої тенденції до виготовлення та дослідження матеріалів, що містять найменші структури та мають низькорозмірні риси, що призводить до мезоскопічного режиму.

Ключові слова: мезоскопічні системи; флуктація; квантова фаза; когерентність; наноструктурована система; макроскопічні характеристики.

Аннотация. В рамках данной работы сделан обзор теории мезоскопических систем. Указаны основные эффекты мезоскопических систем, определено, что макроскопические характеристики системы значительно флуктуирует, в рамках мезоскопических уровня. Определены основные показатели когерентности квантовой фазы и обозначены механизмы воздействия. Охарактеризованы шесть эффектов мезоскопических систем с подробным обоснованием. Теория мезоскопических систем основывается на следующих мезоскопических эффектах: эффект Ааронова-Бома; эффект интегрального квантового выхода; дробный квантовый эффект Холла; универсальные флуктуации кондактанса; квантования проводимости квантового точечного контакта; постоянные токи в мезоскопических кольцах.

Перспективы дальнейших разработок в данном направлении исследования основываются на детальном изучении мезоскопических эффектов исходя из растущей тенденции к изготовлению и исследования материалов, содержащих самые структуры и имеют низкоразмерные черты, что приводит к мезоскопических режима.

Ключевые слова: мезоскопические системы; флуктация; квантовая фаза; когерентность; наноструктурированная система; макроскопические характеристики.

Introduction and research problem statement

The constant trend in modern material science is to offer and explore systems that contain the smallest structures. The obtained systems are suitable for a mesoscopic regime, in which the coherence of the quantum phase leads to a change in the electronic states of quantum devices. At the same time, microscopic details of the sample, such as precise configurations of impurities in disordered systems, determine some quantitative behaviour patterns. This may lead to the expressed fluctuations of the quantity measured in different samples, which are macroscopically equivalent.

At the end of the 20^{th} century, the apparent trend towards nanostructured systems appears in physical science, which include semiconductor structures and magnetic materials, as well as internally nanostructured systems, such as biomaterials and macromolecules. These smallest structures are suited to the so-called mesoscopic regime in which quantum effects become relevant to the behaviour of materials. At the same time, significant progress in the controlled production of submicron solid-state structures, as well as the general availability of low-temperature plants, allowed to systematically investigate artificially created structures with electronic properties having been modified or even prevailing over the effect of quantum interferences. This allows conducting experiments in a mesoscopic regime that directly investigate the quantum properties of phase coherent systems of many bodies.

Research paper's objective. Make an overview of the theory of mesoscopic systems. Describe the basic indicators of the quantum phase coherence and outline the mechanisms of influence.

Analysis of recent research and publications. Good starting points for the study of mesoscopic physics are the recent scientific papers on this subject presented by M.A. Ivanov [1], S.M. Shevchenko [2], M.V. Denisenko and A.M. Satanin [3]. Klinskikh A.F., H. T.T. Nguyen, P.A. Meleshenko [4], in the annex to a series of fundamental "secondary" macroscopic quantum effects, as well as quantum-dimensional effects in mesoscopic systems, present some modern methods of quantum mechanics that have not found any consistent coverage in the academic literature.

Article [5] deals with the investigation of the conductivity of impurities of weakly doped (N <1,017 cm^{-3}) noncompensated (K <10^{-3}) silicon from the electric (E) and magnetic (H) fields at temperatures corresponding to the saturation of the 03 conductivity.

Khalilov V.R. [6] presents the relativistic quantum Aaronov-Bohm effect for a free (in the availability of a three-vector Coulomb potential of Lorentz) and bound fermion states. The author obtained the general scattering amplitude in the combination of three-vector Coulomb potentials of Aaronov-Bohm and Lorentz as a sum of two scattering amplitudes.

However, despite the scale of scientific research on the subject of this paper, the issue of substantiation of the theory of mesoscopic systems remains open and requires detailed elaboration.

Research findings

The mesoscopic regime is an intermediate between the quantum world of microscopic systems (atoms or small molecules) and the classical world of macroscopic systems, such as large fragments of a condensed matter. Mesoscopic systems, as a rule, consist of a large number of atoms, but their behaviour is significantly influenced by the effects of quantum transitions. This is mesoscopic physics on the verge of statistical physics and quantum physics.

The coherence of the quantum phase, which is required for the appearance of interference effects, is maintained only for a finite time τ_{φ}, which is called the phase separation period. The phase coherence is lost when the system or its components being studied interact with its medium, for example through electron-phonon scattering. In electronic conductors, the time of separation of the final phase corresponds to the length of phase separation L_{φ}.

Mesoscopic quantum effects appear when the typical time scales or system lengths are less than the time or length of phase separation. In many cases this means that the corresponding size of the system L must be less than the phase coherence length [2]

$$L < L_{\varphi} \tag{1}$$

For an electron, the time/length of the coherent phase is limited to electron-electron and electron-phonon scattering. These processes are important at high temperatures, but both types of scattering are suppressed at low temperatures, the reason for this is the dependence of the coherence of the phases on the temperature.

It is important to note that only the processes of scattering, in which the excitation (phonon, electron excitation, etc.) of the environment is created or destroyed, result in the loss of phase coherence. Such scattering processes leave a trace inside the environment, which in principle can be observed, and resembles the measurement of the particle trajectory. These processes are usually inelastic and associated with the transfer of energy. However, processes that change the environment without transferring energy can also lead to decoherence.

In contrast, scattering of electrons from static impurities is always elastic. Despite the fact that the phase of electrons could be modified during the scattering process, this occurs in a clearly defined way and does not destroy the effects of coherence of the phases.

Therefore, the mesoscopic regime is characterized by small scales of time and/or length and low temperatures. When the temperature drops, the time/length of phase coherence increases, and the mesoscopic regime extends over larger scales of time/length. At Kelvin temperatures, the scale of time and length in semiconductor samples are respectively picoseconds and micrometers.

Since small finite systems at low temperatures are found in mesoscopic physics, the interlayer interval Δ of the discrete electron spectrum may become larger than the product of the Boltzmann constant and temperature. Then, the electronic and thermodynamic properties of the sample are determined not only by global values, such as the average density of states, but also by the spectrum details. However, the exact spectrum depends on the configuration of impurities, which leads to fluctuations of the observed values between macroscopically indistinguishable samples.

These fluctuations are interesting for study, because qualitative effects are often universal in the sense that they are not dependent on microscopic details.

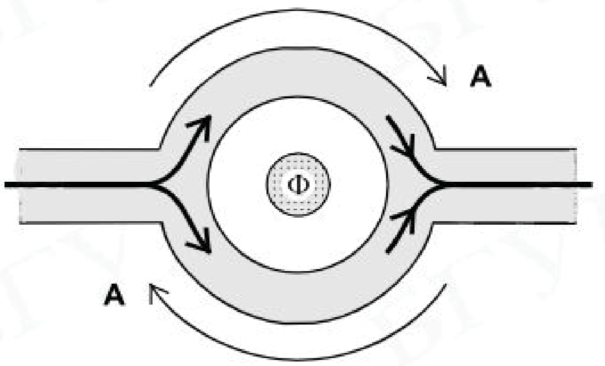
The theory of mesoscopic systems is based on mesoscopic effects:

– Aaronov-Bohm effect;
– Integral quantum output effect;
– Fractional quantum Hall effect;
– Universal conductance fluctuations;
– Quantization of conductivity of a quantum dot contact;
– Direct currents in mesoscopic rings.

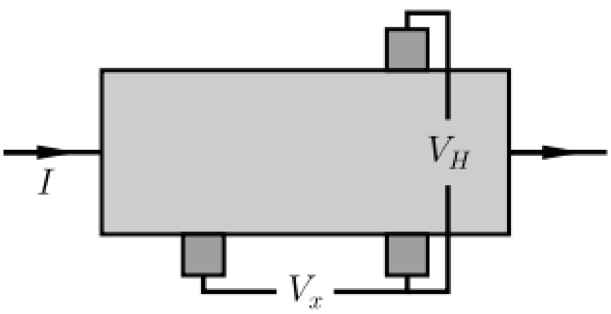
Aaron-Bohm effect

One of the most striking effects of phase coherence is the ability to observe the Aaronov-Bohm oscillations in the conductivity of mesoscopic structures containing small normal metal rings [6]. At low temperatures, when the coherence length of the phase is greater than the length of the ring, the interference of the electron amplitudes is important, which can pass through both one, and through another part of the ring. It is necessary to add to the internal difference of the phases of the two paths the effect of the magnetic field, which leads to the phase shift set as

$$\varphi_{\mathrm{B}} = {}^{2\pi e}/_{h} \oint d\vec{s}\vec{A} = {}^{2\pi e}/_{h}\, \Phi \qquad (2)$$



a



b

Fig. 1 Aaron-Bohm effect: a) distribution of magnetic flux; b) geometry of the Aaronov-Bohm effect. *According to [2,6].*

The integral of a closed loop of a ring from a vector potential $\vec{A}$ gives a phase shift proportional to the magnetic flux Φ through a ring set as the area of the ring multiplied by the (constant) magnetic field strength B perpendicular to the plane of the ring. The conductivity component (the ratio between the current through the sample and the applied voltage) is proportional to $cos\ (\varphi_0 + \varphi_B)$, which leads to observations h/e- of periodic oscillations of conductivity of the device as a function of the magnetic flux penetrating the ring, as shown in Figure 1a.

The longitudinal voltage V_x is measured between two points along one edge of the sample, whereas the Hall voltage is measured between the points on the opposite edges of the samples.

Integral quantum output effect

One of the first and most striking observations of the macroscopic effects of phase coherence in the electronic properties of solid-state devices was the discovery of the integral quantum Hall effect [6] by Klaus von Klitsinger in 1980, awarded the Nobel Prize in 1985.

When measuring the Hall effect in the inverse layer of a silicon MOS (metal-oxide-semiconductor) transistor at low temperatures (T ~ 1 K) and in strong magnetic fields (B> 1 T), the linear dependence of the Hall resistance turns into a number of degrees (plateaus). The value of the resistance on these plateaus is equal to the combination of fundamental physical constants divided by an integer.

When, the plateau is observed on the Hall resistance R_H, longitudinal electrical resistance becomes very small. At low temperatures, the current in the sample can proceed without dissipation (scattering). In the course of research, Klaus von Klitzing used two-dimensional electron gas.

The Hall effect provides that when a conductor is placed in a magnetic field B, it creates a transverse voltage between the opposite sides of the sample, proportional to the longitudinal current I. This dependence can be written through the so-called Hall resistance

$$V_H = R_H I \qquad (3)$$

Classically, using Drude's formula, we get Hall's resistance

$$R_H = {}^{B}/_{en_s} \qquad (4)$$

with two-dimensional electron density n_s. The magnetic field does not affect the longitudinal resistance R_x calculated from the ratio of the voltage drop between the two points on the same side of the sample to the current I within the Drude's theory.

The longitudinal resistance is reduced to zero, with the exception of some values of the magnetic field, where peaks appear.

Fractional quantum Hall effect

The transition to stronger magnetic fields and the decrease of temperatures in two-dimensional electron gases allows observing additional Hall resistance plateaus at fractional filling factors, such as ν = 1/3. This so-called fractional quantum Hall effect was discovered in 1982 [4]. Particularities in case of fractional filling can be traced to the existence of correlated collective quasiparticle excitations [3]. Thus, unlike the integer quantum Hall effect, the Coulomb interaction between electrons is necessary to explain the fractional quantum Hall effect. Quasiparticles have a fractional charge (for example, e / 3 with ν = 1/3). From the shock noise measurements [1], it has recently been confirmed that the charge carriers at ν = 1/3 in the regime of the fractional quantum Hall effect actually have a charge e/3.

Universal fluctuations of conductance

The use of disordered wires in a mesoscopic regime has expressed fluctuations as a function of external parameters such as magnetic field or Fermi energy. These fluctuations were detected by [3] in low temperature (below 1 K) conductivity of the inverse layer in a disordered silicon transistor. Fluctuations are reproduced and reflect the imprint of the sample. The origin of oscillations is the interference of the various paths that electrons can take during passage through the sample, as shown in Figure 2.

In a macroscopically equivalent sample with a microscopically different configuration of impurities, fluctuations are qualitatively similar, but their exact characteristics can be completely different. The most striking feature of the conductivity oscillations is that their typical amplitude is universal in diffusion regime [2].

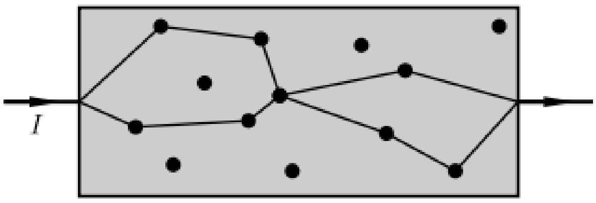


Fig. 2. Possible paths of an electron through disordered wire, with processes of elastic scattering on impurities. The paths of an electron are influenced by the magnetic field or the value of the Fermi wave vector, which leads to oscillations of conductivity in the mesoscopic regime. *According to [6]*

Regardless of the mean conductivity value, the oscillations always form the order of quantum conductivity e^2/h and depend only on the basic symmetries (for example, the symmetry of inversion time) of the system [4]. This can cause the repulsion of the eigenvalues of the matrices of random transmissions. Quantization of conductivity of a quantum dot contact A point contact is a bond between two leading materials. Such a link can be formed by imposing a restrictive narrowing on the wire or by inducing the electrons to pass through a narrow channel determined electrostatically when they lead from one two- or three-dimensional region of the sample to another. In the case of a very narrow width, narrower than the average free path and the length of the coherence of the phases (), which is called ballistic quantum dot contact. Constant currents in mesoscopic rings

Quantization of conductivity of a quantum dot contact

A point contact is a bond between two conductive materials. Such a link can be formed by imposing a restrictive narrowing on the wire or by inducing the electrons to pass through a narrow channel determined electrostatically when they lead from one two- or three-dimensional area of the sample to another. In case of a very narrow width W, narrower than the average free path and the length of the coherence of the phases ($W \ll l, L_\varphi$), which is called ballistic quantum point contact.

Constant currents in mesoscopic rings. The electrons in the mesoscopic rings can support the current around the ring in a thermodynamic equilibrium, even at zero temperature. This current depends on the magnetic flux and cannot dissipate dissipative. Therefore, it flows forever even in ordinary conductors, and that is why it is called a steady current.

Direct currents in mesoscopic rings. The electrons in the mesoscopic rings can maintain the current around the ring in a thermodynamic equilibrium, even at zero temperature. This current depends on the magnetic flux Φ and cannot scatter dissipatively. Therefore, it flows forever even in ordinary conductors, and that is why it is called a steady current.

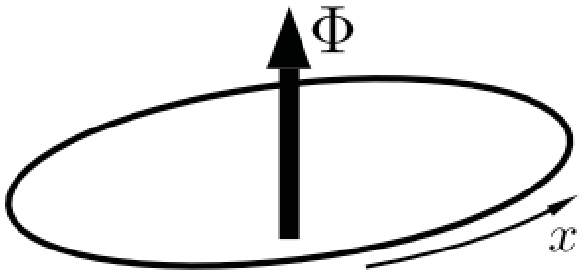


Fig. 3. An ideal one-dimensional ring run through by a magnetic flux Φ. *According to [2,4].*

Figure 3 shows an ideal one-dimensional circle ring $L \ll L_\varphi$. It is well known that a magnetic field cannot affect the behavior of one-dimensional systems. This, however, does not occur when the one-dimensional system is closed on the ring. In this topology, the flux Φ connecting the ring leads to a phase shift $2\pi\Phi / \Phi_0$

accumulated by an electron moving around the ring, $\Phi_0 = h / e$ is a quantum of the flux. Using a calibration transformation, this phase shift can be given by [2] in the boundary state, eliminating the magnetic vector potential from the Schrödinger equation for electrons and leading to generalized periodic boundary conditions

$$\psi(x = 0) = exp(i2\pi\Phi / \Phi_0)\psi(x = L) \quad (5)$$

for single-particle wave functions $\psi\ (x)$. It follows that all the electronic properties of the rings must be periodic in a magnetic flux, the period of which is a quantum of the flux Φ_0, similar to the Aaronov-Bohm effect.

The wave function of non-interacting electrons in a pure ring are flat waves

$$\psi(x) \propto exp(ikx) \quad (6)$$

The boundary state of equation (5) limits the possible wave vectors k to values

$$k_n = {}^{2\pi}/_L \left(n - {}^{\Phi}/_{\Phi_0}\right)$$

where $n = \{0, \pm1, \pm2, \pm3, \dots\}$. Flux dependence of the corresponding one-particle energies

$$E_n = {}^{\hbar^2 k_n^2}/_{2m} = {}^1/_{2m}\left[{}^h/_L\left(n - \frac{\Phi}{\Phi_0}\right)\right]^2 \quad (8)$$

which is shown in Figure 4. The direct current at zero temperature is set as the sum of currents $e\hbar k_n/mL$ from the lowest levels in the ring. The direct current can be written as

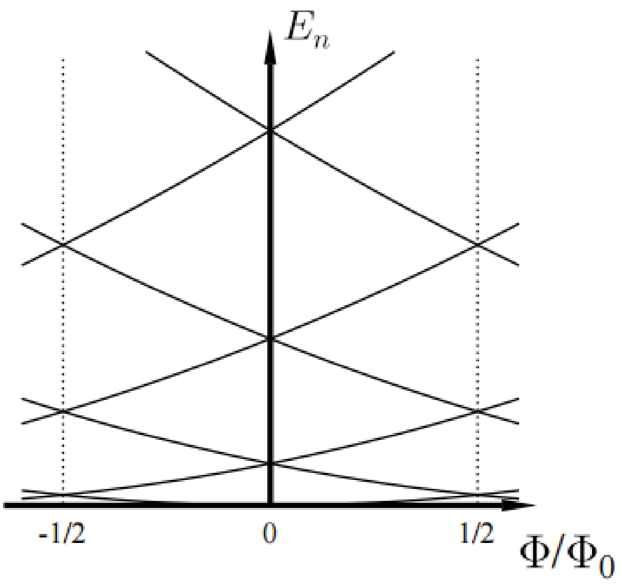


Fig. 4. The dependence of the flux on the lowest energies of one particle in the ring, for -3 ≤ n ≤ 3.*According to [2,4].*

$$I_p = -{}^{dE}/_{d\Phi} \quad (9)$$

with the total electron energy E. Since, at a given value of Φ, the sign of the derivative one-particle energies relative to the magnetic flux fluctuates with a quantum number n, the total steady current decreases by eliminating adjacent levels. The resulting current with a large number of particles dominating over the last electron (at the Fermi level) and order

$$I_p^{ld} \sim {}^{ev_F}/_L \quad (10)$$

with the Fermi speed v_F.

In disordered rings of finite width with elastic free path length $l \ll L$, the theoretical value is more difficult to obtain even for non-interacting electrons. In a diffusion regime a steady current of the following order is expected

$$I_p^{diff} \sim {}^{ev_F}/_L \frac{l}{L} \quad (11)$$

decreasing in the ratio $l\ /\ L$.

The experimental value of the direct current in diffusion rings [2, 3] is much larger (at least for an order) than this theoretical prediction. It is believed that the discrepancy is due to the electronic interaction, which was neglected when deriving the equation (11). Despite the fact that electronic interaction seems to play an important role, it is also important to assert that interactions cannot affect the steady current in pure rotary-invariant 1d rings [2, 6], and the non-binding result (10) is consistent with an experimental one for a pure semiconductor ring in a ballistic regime [3].

This led to a large theoretical activity associated with the combined effect of interactions and disorders for increasing the steady currents in the mesoscopic rings. Despite the fact that different theoretical approaches indicate an increase in steady current in disordered samples due to repulsion of Coulomb interactions, there is still no quantitative understanding of experiments.

Conclusions from this study and prospects for further developments in this area

As part of this research paper, the theory of mesoscopic systems was reviewed. In a mesoscopic regime there are many interesting, sometimes unexpected effects due to phase coherence of electronic wave functions. Some of these effects are very promising for use in nanoelectronic devices or quantum standards in metrology.

The most outstanding example, the quantum Hall effect, is already used as the standard of resistance. On the other hand, mesoscopic systems provide an opportunity to study the basic features of quantum mechanics. They also allow studying directly the features of interacting correlated quantum systems of many bodies. Examples are the fractional quantum Hall effect and transport spectroscopy of interacting electrons at quantum dots.

Prospects for further developments in this area of research are based on a detailed study of mesoscopic effects coming from the growing trend for the production and research of materials containing the smallest structures and having low-dimensional features, which leads to the mesoscopic regime.

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