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Coherent emission from a stack of long Josephson junctions based on low-temperature superconductors

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The theory of coherent emission of intrinsic Josephson junctions was applied for calculations of IV-characteristics and ac power of emission of a stack of two inductively coupled long junctions with high density of critical currents (10^6 A/m^2) which were based on low-temperature superconductors (MoRe films). Barriers were made of the mixture of Si and W. Barriers had thickness of about 15 nm. Randomly distributed clusters of tungsten in the thick silicon barrier provided weak links between superconducting MoRe films. The critical temperature of the MoRe superconducting films was 9 K. Calculations were made for the system at the temperature 7.7 K. Random spread of critical currents along the junction leaded to the formation of the zero-field step in the IV-curve. The same zero-field step appeared when edges of the homogeneous junction were loaded by the resistance, the capacitance and the inductance. In the stack of two junctions, strong coherent emission appeared at the zero-field step which corresponded to the in-phase mode of oscillations of voltages.

Keywords: Josephson junctions; coherent emission; synchronization; zero-field steps.

Теорію когерентної емісії внутрішніх контактів Джозефсона застосовано до розрахунків вольт - амперних характеристик та потужності емісії пачки з двох індуктивно пов’язаних один з одним довгих контактів з високою густиною критичних струмів (до 10^6 А/м^2) на основі низькотемпературних надпровідників (плівок MoRe). Бар’єри були зроблені з суміші Si та W. Бар’єри мали товщину близько 15 нм. Випадково розподілені кластери вольфраму у товстому бар’єрі кремнію забезпечували слабі зв’язки між надпровідними плівками MoRe. Критична температура надпровідних плівок MoRe була 9 К. Розрахунки були зроблені для системи при температурі 7.7 К. Випадковий розбіг критичних струмів вздовж контакту привів до появи сходинки нульового поля на вольт - амперній характеристиці контакту. Та ж сходинка нульового поля виникає, якщо на краях контакту з однорідним розподілом критичних струмів є навантаження з електричного опору, конденсатора та індуктивності. В пачці з двох контактів сильна когерентна емісія виникла на сходинці нульового поля, яка відповідає синфазній моді осциляцій напруги.

Ключові слова: контакти Джозефсона; когерентна емісія; синхронізація; сходинки нульового поля.

Теория когерентной эмиссии внутренних контактов Джозефсона применена для расчёта вольт - амперных характеристик и мощности эмиссии пачки двух индуктивно взаимодействующих друг с другом длинных контактов с высокой плотностью критического тока (10^6 А/м) на основе низкотемпературных сверхпроводников (плёнок MoRe). Барьеры были сделаны из смеси Si и W. Они имели толщину около 15 нм. Случайно расположенные кластеры вольфрама в толстых барьерах из кремния обеспечивали слабые связи между сверхпроводящими пленками MoRe. Критическая температура сверхпроводящих пленок MoRe была 9 К. Вычисления были сделаны для системы, находящейся при температуре 7.7 К. Случайное распределение критических токов в длинном контакте привело к образованию ступеньки нулевого поля на вольт - амперной характеристике. Та же ступенька возникает, если на краях длинного контакта с однородным распределением критических токов имеется нагрузка из сопротивления, ёмкости и индуктивности. В пачке из двух контактов сильная когерентная эмиссия возникла на ступеньке нулевого поля, которая соответствует синфазной моде осцилляций напряжения.

Ключевые слова: контакты Джозефсона; когерентная эмиссия; синхронизация; ступеньки нулевого поля.

Introduction

The increased attention to mechanisms of synchronization of large number of Josephson junctions is caused by the experimentally found coherent emission from more than six hundred intrinsic Josephson junctions in high-temperature superconductors [1]. The found effect allowed obtaining power of emission up to microwatt in the sub-THz region [2]. Following experimental and theoretical investigations allowed to reveal the new mechanism of synchronization which is supposed to produce in-phase locking of voltage oscillations in stacks of intrinsic junctions in mesa structures of high-temperature superconductors [3-6]. Because in the present paper we will apply this mechanism to another type of superconductors, we describe it in details. The inductive interaction between superconducting layers is possible in the stack of junctions. Due to this interaction, normal modes of electromagnetic waves appear in the system [7, 8]. For example, in the system of two inductively coupled junctions, there appear the in-phase mode and anti-phase mode which have different velocities of propagation [7]. Normal modes can be revealed due to the so-called zero-field steps in IV-characteristics [5, 6]. These steps are formed without applied external magnetic field. It is well known that due to some longitudinal perturbations the standing wave of electromagnetic field can be formed in the long solitary Josephson junction [9]. This wave produces some distribution of ac voltage over the junction. If the distribution of critical currents along the junction is symmetrical, zero-field steps appear in the IV-characteristic of the junction at voltages of even Fiske steps as a result of the interaction between Josephson generation and the standing wave [9]. These voltages are equal to

$$< V_s >= \frac{\Phi_0 \overline{c} s}{D} , \qquad (1)$$

where the sign <…> means averaging over time that is much longer than the period of Josephson oscillations, D is the length of the junction, Φ_0 is the quantum of magnetic flux, $\overline{c}$ is the velocity of light in the junction and s =1,2... is an integer. In the system of two inductively coupled junctions there are two zero-field steps in the IV-characteristic which correspond to two velocities of the propagation of light for different normal modes. In the system of K inductively coupled junctions there are K normal modes and therefore, there is the bunch of K zero-field steps. Among these steps there is the zero-field step that corresponds to the in-phase mode in which all junctions oscillate coherently (the step at highest voltage in the bunch [7]). Thus, to obtain in-phase synchronization of junctions in the stack it is necessary to induce the standing wave in the stack and to measure the zero-field step at highest voltage in the bunch.

The described above mechanism of synchronization can be applied also to the stack of junctions made of low-temperature superconductors. The application of underdamping junctions with high values of the McCumber parameter is not effective for our aim because the subgap steps in the IV-curve can not be revealed properly in ranges of the current-biased scheme which is usually applied in calculations. We consider here the stack of overdamped long junctions. In the present paper we calculated IV-characteristics and power of ac emission for the separate long Josephson junction and the stack of two Josephson junctions. Parameters of calculations were taken for junctions with high density of critical currents (up to 10^6 A/m^2) made of MoRe films with 45% Re and the barrier made of the mixture of Si and W with the concentration of W up to 10% [10]. Clusters of tungsten provide weak links in the barrier. It was proven that at temperatures near the critical temperature ($T_c \approx 9$ K for the given system) and if the length of the weak link is smaller than the length of coherence, dynamics of the weak link can be described by the resistively-shunted model of the Josephson junction (RSJ-model) [9, 11]. We modeled the mentioned system at 7.7 K in the ranges of RSJ-model taking into account capacitances of junctions. We calculated IV-curves for these systems and ac power of emission into the load and discussed obtained results.

The model

The model of calculations is described in details in Refs. [6, 12 - 15]. Here we give only the brief description of the model. Each of the K=2 wide junctions with the index i = 1,2 is divided into n segments. Segments are numbered by the index j=1…n. It is supposed that the 'elementary junction' is placed in the center of each segment. These 'elementary junctions' are divided by the distance $\zeta = \sqrt{\overline{c}CL}$, where $\overline{c}$ is the velocity of light in the junction, L is the inductance of the segment and C is the capacitance of the segment (we suppose all the capacitances are equal to each other). The system of equations which describes the high-frequency scheme of the stack of junctions includes current conservation conditions for 'elementary junctions' and flux quantization conditions:

$$\begin{aligned} &\frac{\Phi_0 C}{2\pi}\frac{d^2\phi_{i,j}}{dt^2} + \frac{\Phi_0}{2\pi R}\frac{d\phi_{i,j}}{dt} + \\ &+ I_{ci,j}\sin\phi_{i,j} = I_b - I^R_{i,j-1,j} + I^R_{i,j,j+1} \end{aligned} , \qquad (2)$$

where $i = 1,2,\ j = 2...n-1$,

$$LI^R_{1,j-1,j} - L_f I^R_{2,j-1,j} + \frac{\Phi_0}{2\pi}\left(\phi_{1,j-1} - \phi_{1,j}\right) = 0 , \qquad (3)$$

where $j = 2...n$,

$$-L_f I^R_{1,j-1,j} + LI^R_{2,j-1,j} + \frac{\Phi_0}{2\pi}\left(\phi_{2,j-1} - \phi_{2,j}\right) = 0 , \qquad (4)$$

where $j = 2...n$, $I^R_{i,j-1,j}$ is the current in the loop between two segments with indices j-1 and j, $I_{ci,j}$ and R are the critical current and the resistance of the segment (we suppose that R = *const*), $\varphi_{i,j}$ is the difference of the phase of the order parameter across the junction which is contained in the segment, L_f is the mutual inductance between two adjacent cells of the 'elementary stack', t is time. Equations (2)-(4) can be solved by means of the method of Runge-Kutta. The result of calculations in this case is the IV-characteristic of the system. We can also attach additional contours containing the resistance, the inductance and the capacitance to the edges of junctions. In this case we can calculate power of ac emission extracted to the load. In the following consideration we will use both the system with loads at edges and the system without loads. To take into account loads, we mark them as fictive segments with indices j=0 and j=n+1 added to edges of junctions. Kirchhoff rules for these segments are as follows:

$$L_{ej}\frac{d^2q_j}{dt^2}+R_{ej}\frac{dq_j}{dt}+\frac{q_j}{C_{ej}}=\mp\frac{\Phi_0}{2\pi}\sum_{i=1}^{2}\frac{d\phi_{i,j\pm1}}{dt}\ , \quad (5)$$

where $j=0,n+1,$ and q_j is the charge flowing through the inductance L_{ej} , C_{ej} and R_{ej} are the capacitance and the resistance of the additional contour. In the present paper we assume $C_{ej}=C$, $L_{ej}=L$ and $R_{ej}=R$. The value of mutual inductance between 'elementary junctions' in the stack was defined as $L_f=\alpha L$, where α is dimensionless parameter. Eqs. (2)-(5) were solved for different bias currents. IV-characteristics were obtained in calculations. The voltage over the system of two junctions was calculated as

$$\left\langle V_{system}\right\rangle=\frac{\Phi_0}{2\pi n}\left\langle\sum_{i=1}^{2}\sum_{j=1}^{n}\frac{d\phi_{i,j}}{dt}\right\rangle . \quad (6)$$

For the comparison of IV-curves for the stack of two junctions and those for one separate junction we will use the value $\left\langle V\right\rangle=\left\langle V_{system}\right\rangle/K$, where K is the quantity of long junctions in the system, i.e. K=1 for the separate junction and K=2 for the stack.

The value of emitted ac power at the left end of the system was calculated as follows:

$$P_l=\frac{1}{KR}\left\langle\left\{\sum_{i=1}^{K}\left[\frac{\Phi_0}{2\pi}\left(\frac{d\phi_{i,1}}{dt}-\left\langle\frac{d\phi_{i,1}}{dt}\right\rangle\right)\right]\right\}^2\right\rangle . \quad (7)$$

The same expression with j=n was used for the calculation of emitted power from the right end of the system.

For calculations we used values of parameters for superconducting layers made of MoRe films with 45% Re and the barrier made of the mixture of Si and W with the concentration of W up to 10% [10]. The critical temperature of this system is 9 K. To satisfy conditions of the application of the RSJ model to this system, we calculated parameters for the temperature 7.7 K. At first, we stated values of critical currents at temperatures $T<<T_c$ which we defined from experimental data [10]: $I_c(T<<T_c)\approx$ 10 mA, $V_c(T<<T_c)\approx$ 3 mV and density of critical currents was equal to J_c=10^6 A/m^2. Dimensions of long layers were 250x40x0.05 cubic micrometers and the thickness of each of the barrier was 15 nm. Then we divided the long junction to n=30 segments and calculated the critical current of the segment and its resistance R. We supposed that the velocity of light in the junction was $\bar{c}(T< \ T_c)=c/\sqrt{\varepsilon}$ where c is the light velocity in vacuum and $\varepsilon\approx12$ is permittivity of silicon, so $\bar{c}\left(T<<T_c\right)\approx8.87\cdot10^7$ m/s. For the calculation of dependences of parameters on the temperature we used the method developed in Ref. 6. For the determination of the value of the critical current at the given temperature we used the plot of the dependence of critical current on the reduced temperature for the weal link in the dirty limit [9].

For the temperature T = 7.7 K parameters of the long junction were as follows: I_c = 2.5 mA, V_c = 0.75 mV, $\bar{c}\left(T=7.7K\right)=6.14\cdot10^7$ m/s, $\beta_c=10.24$. The Josephson depth of penetration of magnetic field was $\lambda_J=86.2\cdot10^{-6}$ m. After the definition of parameters we calculated IV-curves and ac power of emission for one separate long junction and for the stack of two inductively coupled junctions with $\alpha=0.3$. We would like to note that such parameters of our model as the velocity of the propagation of electromagnetic waves in the long junction and the depth of penetration of magnetic field in the junction and temperature dependences of their values were calculated on the base of plausible assumptions and it is of great interest to investigate them experimentally.

Results and Discussion

We discuss at first the electrical properties of the separate long junction without loads at edges. The IV-characteristic of the separate homogeneous long junction is shown in Fig. 1a. It is the typical hysteretic curve which is characteristic for the junction with the finite value of the McCumber parameter β_C. The switch from the hysteretic branch to the zero-current branch appears at 0.38×I_c that corresponds exactly to the switch in the solitary junction of the negligible size with $\beta_C=10.24$ in the range of the RSJ-model [16]. The IV-curve in Fig. 1a does not contain any particularities connected with geometrical dimensions of the system. Analogous results were obtained in Ref. 15 for intrinsic junctions.

Now we consider the IV-curve of the inhomogeneous long junction without loads at edges (Fig. 1b, crosses). Inhomogeneity is created by spread of critical currents of about 10^{-2}%. There is a step in the IV-curve in the hysteretic region at $\left\langle V'_{s=1}\right\rangle\approx0.43$ mV. After the step there is the jump of voltage to the value 0.55 mV. It is shown in Ref. 15 that this behaviour of the IV-curve is caused by the resonant interaction of Josephson generation with standing wave that appears in the inhomogeneous junction. Due to inhomogeneity of critical currents along the junction, there arise longitudinal excitations [15]. They reflect from edges of the junction (it is the so-called Fulton-Dynes mechanism of reflection [17]). The standing wave appears when the even number of halves of wavelengths of the excitation becomes equal to the length of the junction. Just this condition is written in Eq. (1). The standing wave interacts with Josephson generation the same way as the external periodical signal, so zero-field steps appear in the IV-curve.

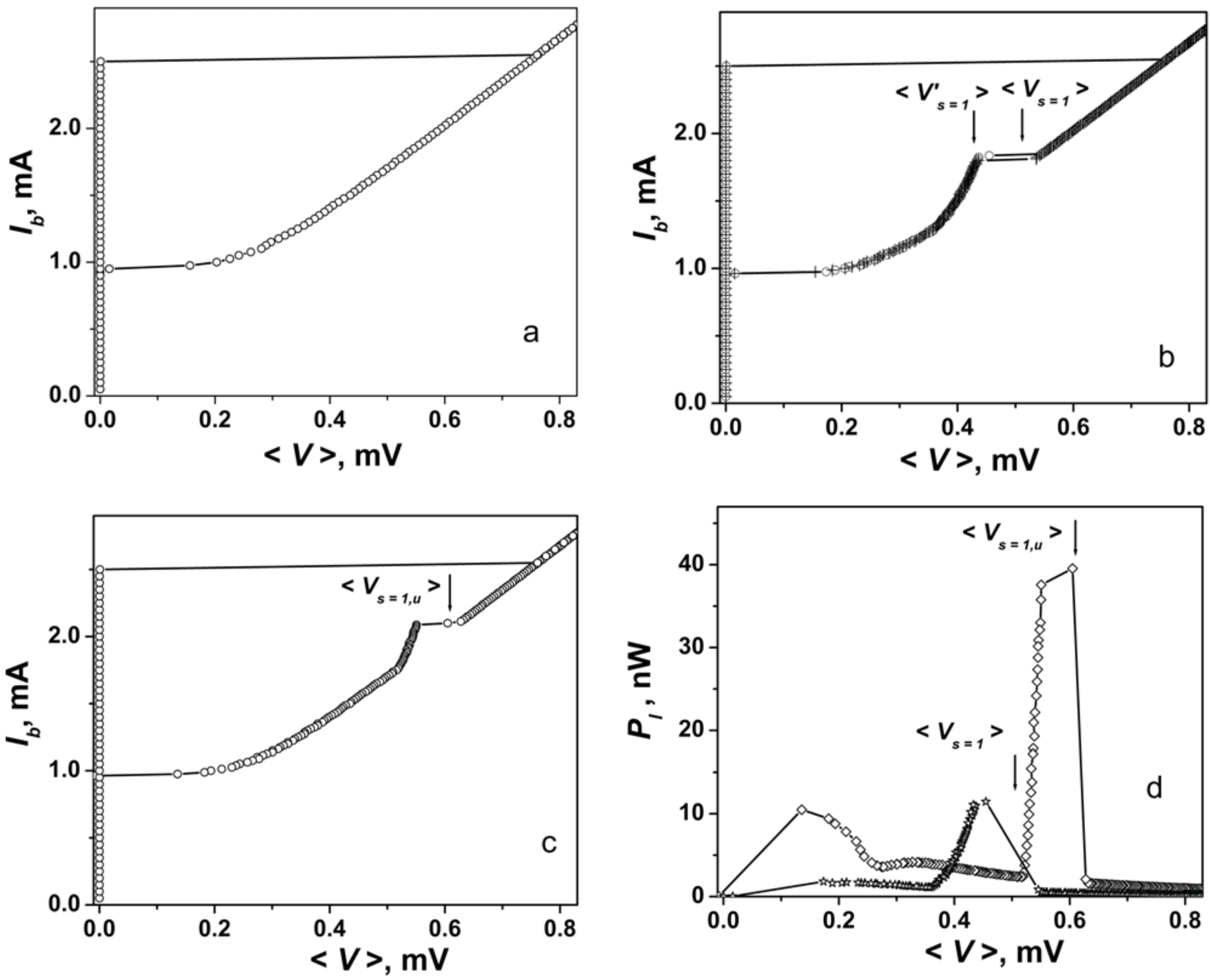

Fig. 1. (a)- the IV-characteristic of one separate long homogeneous junctions without loads at edges. (b)- IV-characteristics of one separate long junction with spread of critical currents 10^{-2}% without loads at edges (crosses) and the same for the homogeneous long junction with loads at edges (circles). Positions of voltages $< V_{s=1} >$ and $< V'_{s=1} >$ are marked by arrows. (c)- the IV-characteristic of the stack of two inductively coupled long junctions with loads at edges. The position of voltage $< V_{s=1,u} >$ is shown by an arrow. (d)- dependences $P_l(\langle V \rangle)$ for the solitary long junction with the load at edges (stars) and for the stack of two inductively coupled long junctions with loads at edges. Positions of voltages $< V_{s=1} >$ and $< V_{s=1,u} >$ are shown by arrows.

Just this step appears in the IV-curve shown in Fig. 1b. According to Eq. (1), the value of $\langle V_{s=1} \rangle = 0.51$ mV. It is shown by an arrow in Fig. 1b. Due to the ambiguity of the IV-curve in the region of the hysteresis, the full height of the step can not be obtained in the range of the current-biased scheme, so the step is interrupted at the value $\langle V'_{s=1} \rangle \approx 0.43$ mV.

Let us consider now the IV-curve of the fully homogeneous junction with loads at edges (Fig. 1b, circles). It is seen that the zero-field step is reproduced in full despite the junction now is homogeneous. Standing waves in this case are excited due to the influence of the loads at edges. [15]. Loads violate homogeneity of the junction, so perturbations propagate along the junction and at some frequencies produce standing waves.

The IV-characteristic of the stack of two long junctions with loads at edges is shown in Fig. 1c. It is known that due to the inductive coupling of layers the zero-field step is split into two steps at voltages $< V_{s=1,d} >=< V_{s=1} > / \sqrt{1+\alpha}$ and $< V_{s=1,u} >=< V_{s=1} > / \sqrt{1-\alpha}$ [6, 13-15]. However, in Fig. 1c there is only one step near $< V_{s=1,u} > \approx 0.51$ mV. The step at $< V_{s=1,d} > \approx 0.45$ mV is not seen. The split appears due to the formation of normal vibrations in the system of coupled layers. At $< V_{s=1,d} >$ voltages over junctions in the 'elementary stack' oscillate anti-phase, and at $< V_{s=1,u} >$ there are in-phase oscillations of voltages over junctions in the 'elementary stack'. Just this mode of in-phase oscillations is used for producing of coherent emission from the stack. To prove this we calculated the dependence of averaged over time ac power emitted in the load at the left end of the solitary junction P_l on voltage (Fig. 1d, stars) as well as the dependence $P_l(< V >)$ for the stack (Fig. 1d, diamonds). The dependence $P_l(< V >)$ for the solitary junction has the maximum at the voltage $\langle V'_{s=1} \rangle \approx 0.45$ mV. The dependence $P_l(< V >)$ for the stack has the maximum at the voltage $\langle V'_{s=1,u} \rangle \approx 0.60$ mV. This value corresponds to the in-phase mode of oscillations of voltages over junctions. The maximal value of emitted ac power P_l at $\langle V'_{s=1,u} \rangle$ for the stack of two junctions is equal to 39.54 nW whereas the maximal value of P_l at $\langle V'_{s=1} \rangle$ for the solitary junction is equal to 11.46 nW. The relation of these values is 3.45 that means nearly full phase locking with the constant phase shift (the relation is equal to 4 for the zero phase shift [16]). This result proves that the zero-field step at $\langle V'_{s=1,u} \rangle \approx 0.60$ mV in the stack of two long junctions corresponds to the in-phase normal mode.

Finishing the discussion we note that in the junctions MoRe with the barrier made of silicon and tungsten phase slip phenomena can appear [10]. Zero-field steps appear often in junctions with phase slip processes [9]. Investigations of phase locking including phase slip processes becomes of great interest for the theory of synchronization.

Summary

In the present paper for synchronization of emission from a stack of long Josephson junctions we applied the mechanism of synchronization which was earlier used for the explanation of phase locking of intrinsic junctions in high-temperature superconductors. Parameters of calculations were taken for low-temperature junctions with high density of critical currents (up to 10^6 A/m^2) made of MoRe films with 45% Re and the barrier made of the mixture of Si and W with the concentration of W up to 10%. The layers had dimensions 250x40x0.05 cubic micrometers and the thickness of the barrier was 15 nm. The main advantage of this system is the small McCumber parameter (it is about 10.24 at the given temperature). We calculated IV-curves and emission to the RLC-load for one long junction and for the stack of two inductively coupled long junctions at the temperature 7.7 K that is close to the critical temperature. We showed that standing waves could be excited in such a system if it had the inhomogeneous distribution of critical currents along the junctions or if there were loads attached to edges of the system. We obtained zero-field steps in IV-curves of long junctions with standing waves and showed that these steps were produced by the resonant interaction of standing wave with Josephson generation. We proved that the zero-field step in the IV-curve of the stack was split and obtained strong coherent emission at the upper zero-field step which corresponds to the in-phase mode of oscillations of voltages.

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