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Discrete singularities method in problems of liquid vibrations in spherical tanks

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This paper presents an analysis of low-frequency liquid vibrations in rigid partially filled spherical containers with baffles. The liquid is supposed to be an ideal and incompressible one and its flow is irrotational. A shell of revolution is considered as the container model. For evaluating a velocity potential, the system of singular boundary integral equations has been obtained. The method of discrete singularities as well as the multi-domain boundary element method are used for its numerical simulation.

Key words: *ideal incompressible liquid, sloshing, spherical shell, baffle, singular integral equations, boundary element method*

В роботі надано аналіз низькочастотних коливань рідини в жорсткому частково заповненому рідиною сферичному контейнері з перегородкою. Припускається, що рідина є ідеальною нестисливою, а її рух є безвихровим. Як модель контейнера обрано оболонку обертання. Для обчислення потенціалу швидкостей отримано систему сингулярних інтегральних рівнянь. Для її чисельного розв’язання застосовані метод дискретних особливостей та метод граничних супер-елементів.

Ключові слова*: ідеальна нестислива рідина, плескання, сферична оболонка, перегородка, сингулярні інтегральні рівняння, метод граничних елементів*

В работе проведен анализ низкочастотных колебаний жидкости в жестком частично заполненном жидкостью сферическом контейнере с перегородкой. Предполагается, что жидкость идеальная, несжимаемая, а ее движение является безвихревым. В качестве модели контейнера выбрана оболочка вращения. Для вычисления потенциала скоростей получена система сингулярных интегральных уравнений. Для ее численного решения применены метод дискетных особенностей и метод граничных суперэлементов.

Ключевые слова*: идеальная несжимаемая жидкость, плескания, сферическая оболочка, сингулярные интегральные уравнения, метод граничных элементов.*

1. Problem statement and basic relations

The intensive movement of liquid in reservoirs has been a scientific research subject for several decades. The problem is of great interest because of the extreme importance of sloshing control in fuel tanks of launch vehicles. The proximity of the frequency of fluid vibrations to the frequencies of regulating mechanisms leads repeatedly to stability losses, immediate deorbits, destructions of aircrafts [1]. Liquid spattering and sloshing in spherical tanks was studied in the papers [2,3]. A characteristic feature of spherical tanks is the change in radius of a free surface according to changes in a filling level. There exist known analytical solutions for almost completely filled tanks with small radii of the free surface, the so-called "ice fishing problems" formulation. The effect of baffles on sloshing frequencies was studied by Biswal et al. [4]. The numerical method using a finite element formulation was developed by Kumar and Sinhamahapatra [5]. Sloshing in spherical tanks for liquefied natural gas carriers was studied by Faltinsen and Timokha [6] and for water supply towers by Curadelli et al. [7]. Various approximate methods for solving the natural sloshing problem, starting with the famous works by Budiansky [8] and McIver [9] and, recently, by Patkas & Karamanos [10] have been proposed.

In this paper we consider the problem of fluid vibrations in the spherical shell. To reduce the sloshing in the shell, an internal baffle is installed, Fig. 1.

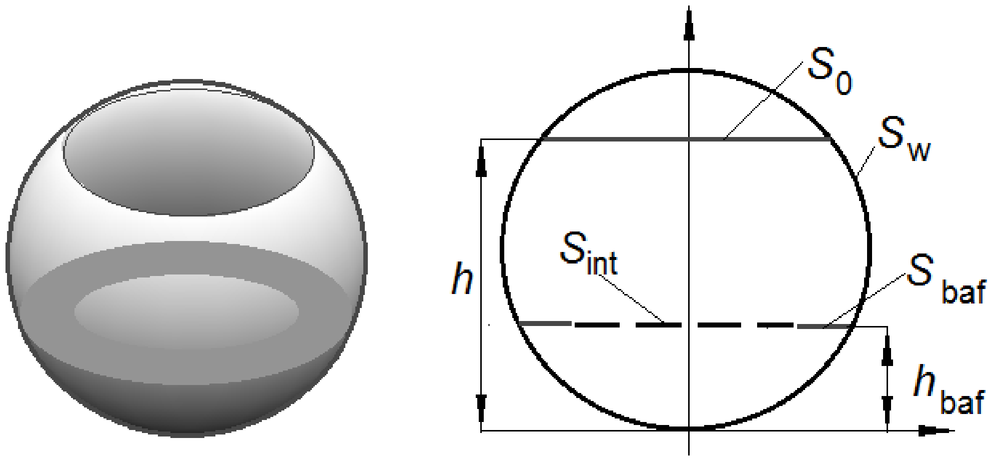


Fig. 1. A spherical fuel tank with an internal baffle

We denote a wetted surface of the shell as S_w, and a free surface as S_0. Let h be the filling level, h_{baf} be a height where the baffle is located. We also denote the baffle surface as S_{baf}., and as S_{int} an interface surface [11]. It is assumed here that a liquid is an ideal and incompressible one, and its motion, beginning from the state of rest, is irrotational. For these conditions, there exists a fluid velocity potential Φ

$$V_x = \frac{\partial\Phi}{\partial x}; V_y = \frac{\partial\Phi}{\partial y}; V_z = \frac{\partial\Phi}{\partial z},$$

that satisfies the Laplace equation. A liquid pressure p upon the shell walls is determined from the linearized Cauchy-Lagrange integral by the formula

$$p = -\rho_l\left(\frac{\partial\Phi}{\partial t} + gz\right) + p_0,$$

where Φ is the velocity potential, g is the acceleration of gravity, z is the vertical fluid point coordinate, ρ_l is a fluid density, p_0 is an atmospheric pressure. On the wetted surfaces of the shell and baffle the non-penetration boundary condition is set, on the free surface the following dynamic and kinematic boundaries are given

$$\left.\frac{\partial\Phi}{\partial n}\right|_{S_0} = \frac{\partial\zeta}{\partial t}; \quad \left.p - p_0\right|_{S_0} = 0,$$

where the function ζ describes the shape and position of the free surface. Thus for the velocity potential we have the following boundary-value problem

$$\nabla^2\Phi = 0\,;\ \left.\frac{\partial\Phi}{\partial\mathbf{n}}\right|_{S_w \cup S_{\mathrm{baf}}} = 0\,;\left.\frac{\partial\Phi}{\partial n}\right|_{S_0} = \frac{\partial\zeta}{\partial t};\quad \left.p - p_0\right|_{S_0} = 0\,;\left.\frac{\partial\Phi}{\partial t} + g\zeta\right|_{s_0} = 0\,. \quad (1)$$

To calculate the liquid vibrations in the presence of the baffle we use the multi-domain method (boundary super-element method). In doing so, we introduce an "artificial" interface surface S_{int} [12], then divide the region filled with the liquid into two parts $\Sigma_1;\Sigma_2$, bounded by the surfaces S_{w}, S_{baf}, S_{int} and S_{w}, S_{baf}, S_{int}, S_0. On the interface surface, the following boundary conditions are set:

$$\left.\Phi\right|_{S_{\mathrm{int}}\cap\partial\Sigma_1} = \left.\Phi\right|_{S_{\mathrm{int}}\cap\partial\Sigma_2};\quad \left.\frac{\partial\Phi}{\partial\mathbf{n}}\right|_{S_{\mathrm{int}}\cap\partial\Sigma_1} = -\left.\frac{\partial\Phi}{\partial\mathbf{n}}\right|_{S_{\mathrm{int}}\cap\partial\Sigma_2} \quad (2)$$

The boundary value problem (1), (2) is reduced to a system of singular integral equations in the form [5]

$$\begin{array}{l} A_{11}\varphi_1 + A_{12}\varphi_{1i} = B_{12}q_1;\quad P_0 \in S_1; \\ A_{21}\varphi_1 + A_{22}\varphi_{1i} = B_{22}q_1;\quad P_0 \in S_{\mathrm{int}}; \end{array} \quad (3)$$

$$\begin{array}{l} A_{32}\varphi_{1i} + A_{33}\varphi_2 + A_{34}\varphi_0 - \omega^2 B_{34}\varphi_0 = -B_{32}q_1;\quad P_0 \in S_2; \\ A_{22}\varphi_{1i} + A_{23}\varphi_2 + A_{24}\varphi_0 - \omega^2 B_{24}\varphi_0 = -B_{22}q_1;\quad P_0 \in S_{\mathrm{int}}; \\ A_{42}\varphi_{1i} + A_{43}\varphi_2 + A_{44}\varphi_0 - \omega^2 B_{44}\varphi_0 = -B_{42}q_1;\quad P_0 \in S_0. \end{array}$$

Here we introduce the following notations

$$A_{ij} = A(S_i, S_j);\quad B_{ij} = B(S_i, S_j),\quad i,j = \overline{1,4};$$

$$A(S,\sigma)\psi = \iint_S \psi \frac{\partial}{\partial\mathbf{n}}\frac{1}{|P - P_0|}dS;\quad B(S,\sigma)\psi = \iint_S \psi \frac{1}{|P - P_0|}dS;\quad P_0 \in \sigma,$$

$$\varphi_1 = \left.\Phi\right|_{S_1};\quad \varphi_2 = \left.\Phi\right|_{S_2};\quad \varphi_{ji} = \left.\Phi\right|_{S_{\mathrm{int}}\cap\partial\Sigma_j};\quad \varphi_0 = \left.\Phi\right|_{S_0};\quad q = \frac{\partial\Phi}{\partial\mathbf{n}},$$

where $S_1 = S_{w1} \bigcup S_{\mathrm{baf}}$; $S_2 = S_{\mathrm{int}}$; $S_3 = S_{w2} \bigcup S_{\mathrm{baf}}$; $S_4 = S_0$, S_{w1}, S_{w2} are wetted shell surfaces in the subregions $\Sigma_1; \Sigma_2$.

The numerical solution of the system of integral equations (6) is carried out by the method of discrete singularities (MDS) [13]. The transformation to one-dimensional boundary integral equations is preliminary carried out by transition to cylindrical coordinates and representation of unknown functions in the form

$$\varphi_k(r,z,\theta) = \varphi_k(r,z)\cos\alpha\theta, \quad (4)$$

where α is a harmonic number.

2. Analysis of numerical results

2.1. Low-frequency oscillations of a spherical shell without baffles

Consider the spherical shell of radius R = 1 m, partially filled with the ideal incompressible fluid, with the filling level h. The numerical analysis is carried out for

$(0.2 < h/R < 1.99)$ and various $\alpha\left(\alpha = \overline{0,3}\right)$. The MDS and the multi-domain method of boundary elements (MDBE) are used [12, 13].

Both MDS and MDBE are applied here. The boundary elements with constant approximation of unknowns inside the elements are used. In MDS there are 200 elements along the spherical surface and 150 elements along the free surface. In MBEM we divide the computational domain into two parts by the artificial interface surface at $h_{\text{int}} = 0.5h_1$ using 100 boundary elements in each sub-domain along the spherical surface and 150 elements along the free surface. We use practically the same mesh to find a numerical approximation of low eigenvalues for the so called "ice-fishing problem". In this problem, formally, we should consider an infinitely wide and deep ocean covered with ice, with a small round fishing hole. Sloshing in such "containers" was studied by McIver [9]. We approximate this infinite case using the spherical tank with the small round hole on its top. It allows us to compare our numerical results with those obtained in the papers [3],[9].

In Tables 1-2 we compare results obtained by using MDS and MBEM with those obtained in [3],[9] for axisymmetric (α=0) and non-axisymmetric (α=1) modes. Four first frequencies ($n = \overline{1,4}$) are evaluated for each α. Here we consider different filling levels h_1. The value h_1/R_1=1.99 corresponds to the ice-fishing problem.

Table 1: Frequency of axisymmetric oscillations of the fluid-filled spherical shell.

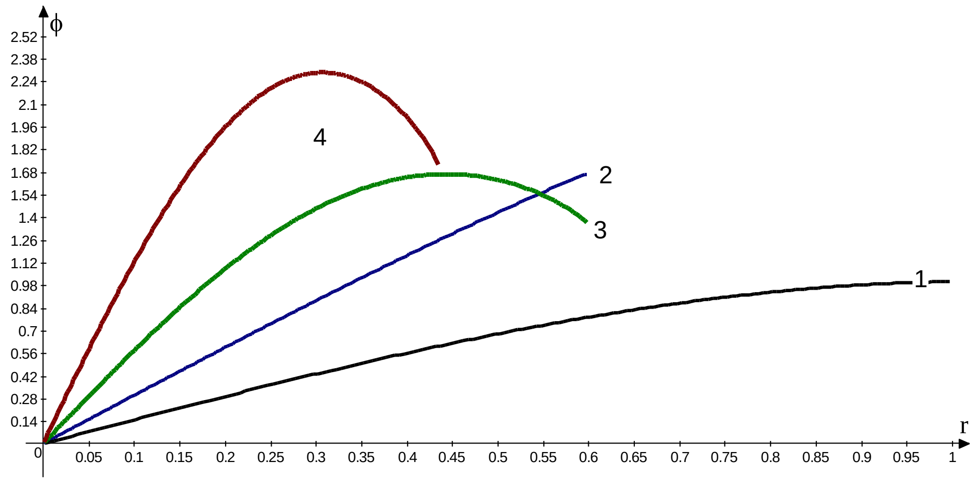
n	Method	Filling level *h, m*				
		h_1=0.2	h_1=0.6	h_1=1.0	h_1=1.8	h_1=1.99
1	[3]	3.8261	3.6501	3.7451	6.7641	29.0500
	[9]	3.8261	3.6501	3.7451	6.7641	29.2151
	MDBE	3.4034	3.5455	3.7294	6.6098	30.7081
	MDS	3.8314	3.6510	3.7456	6.7665	29.1811
2	[3]	9.2561	7.2659	6.9763	12.1139	51.8122
	[9]	9.2561	7.2659	6.9763	12.1139	52.0467
	MDBE	9.2636	7.2893	6.9796	12.0008	52.9393
	MDS	9.2686	7.2684	6.9780	12.1205	52.0255

Table 2: Non-axisymmetric slosh frequencies of the fluid-filled spherical shell, Hz.

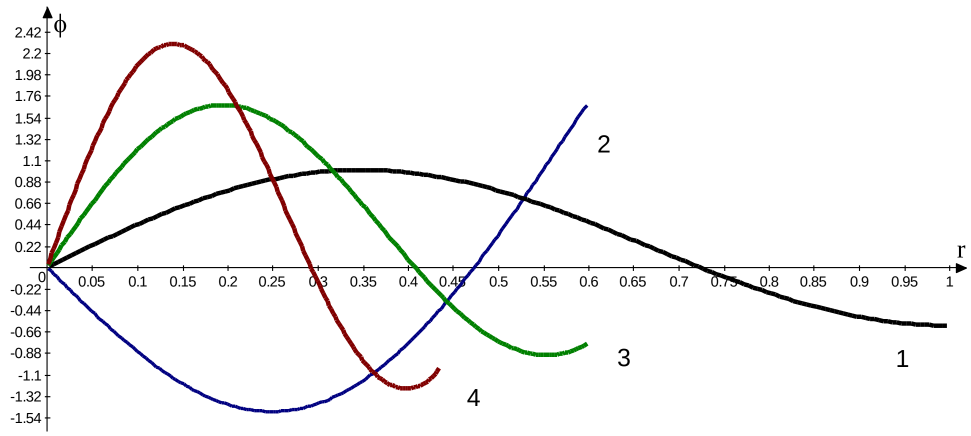
n	method	Filling level h_1, m				
		h_1=0.2	h_1=0.6	h_1=1.0	h_1=1.8	h_1=1.99
1	[3],	1.0723	1.2625	1.5601	3.9593	18.9838
	[9]	1.0723	1.2625	1.5601	3.9593	19.1582
	MDBE	1.1034	1.2777	1.5638	3.9606	19.1603
	MDS	1.0723	1.2626	1.5603	3.9508	19.1130
2	[3],	6.2008	5.3860	5.2755	9.4534	41.3491
	[9]	6.2008	5.3860	5.2755	9.4534	41.7683
	MDBE	6.1227	5.3534	5.2749	9.4582	41.5327
	MDS	6.2090	5.3697	5.2764	9.4538	41.5333

Different levels of fluid filling are considered, including h_1 = 1.99, that corresponds to «ice-fishing problem», [9]. The results of calculations are close but in some cases MDS gives more accuracy.

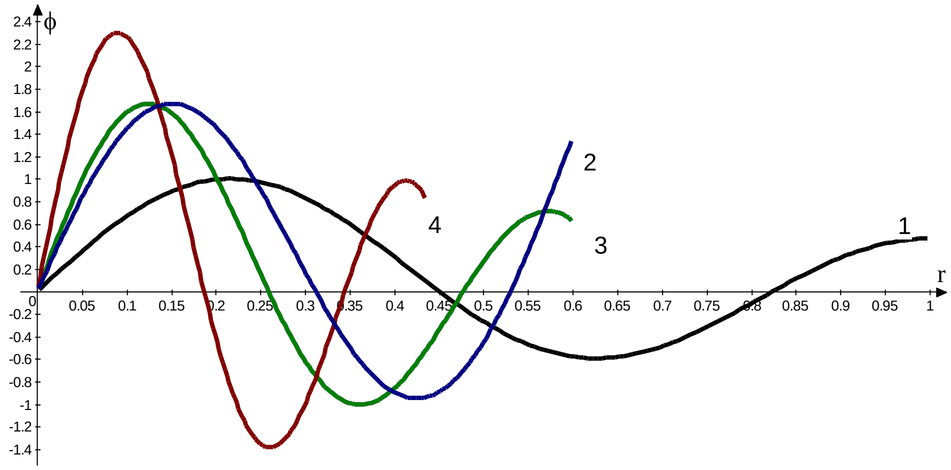
Considering our approximate natural sloshing modes one can observe how free surface profiles change with the liquid depth. These results are illustrated in Fig. 2 for the three lowest eigenvalues of the mode α = 1, and for α = 2 in Fig.3. Here numbers 1,2,3,4 correspond to the different non-dimensional filling levels: h_1 /R_1=1.0; 0.2; 1.8; 1.9, respectively.



a) α = 1, m=1

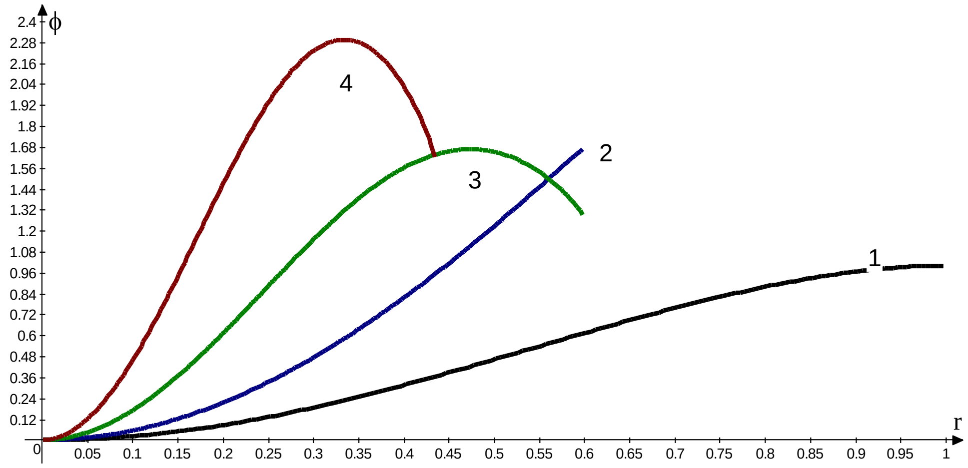


b) α = 1, m=2

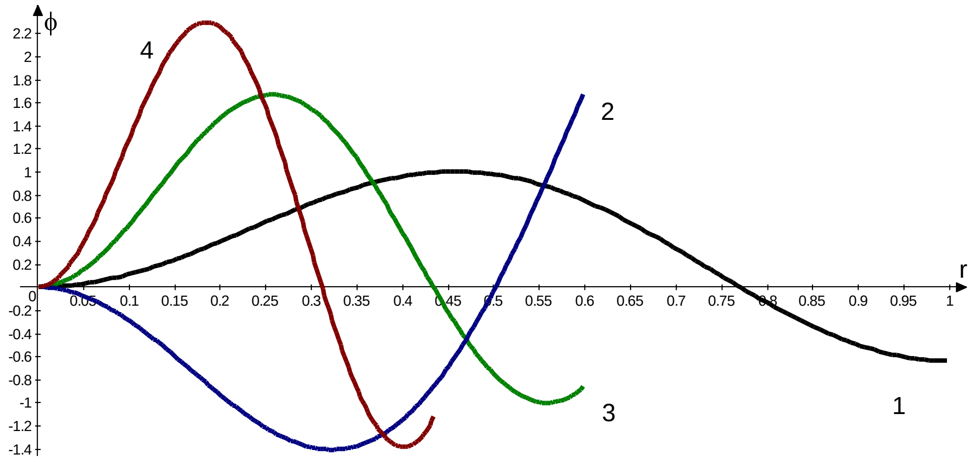


c) $\alpha = 1.m=3$

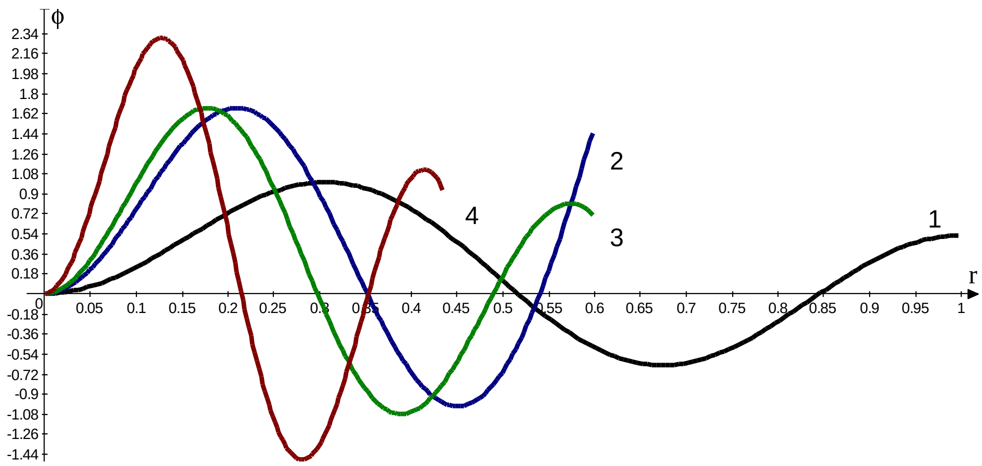
Fig. 2. The radial wave profiles n=1,2,3 for different non-dimensional liquid depths h_1/R_1.



a) $\alpha = 2$, $m=1$



b) $\alpha = 2$, $m=2$



c) $\alpha = 2$, m=3

Fig. 3: The radial wave profiles n=1,2,3 for different non-dimensional liquid depths h_1/R_1.

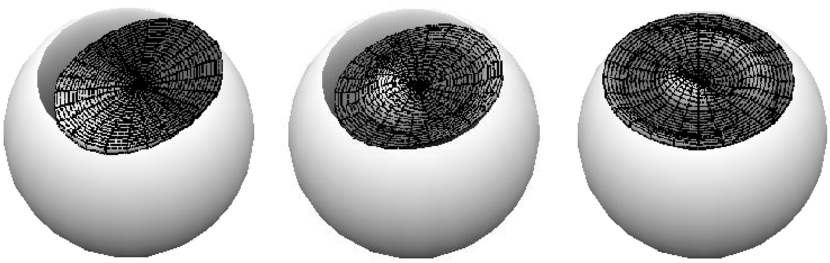


Fig. 4. Spatial wave patterns for α=1; m=1,2,3.

In the spherical tank with $0 < h_1/R_1 < 0.5$ the lowest mode presents a spatial wave pattern that looks like inclination of an almost flat free surface. Increasing the liquid depth yields more complicated free surface profiles. Fig.4 demonstrates the spatial wave patterns for α=1, n=1,2,3 at h_1/R_1=1.8.

2.2. Low-frequency oscillations of the baffled spherical shell

The rigid spherical tank of radius R_1=1m filled to the depth h_1=1.4m is considered. The inner periphery of the tank contains a thin rigid-ring baffle. The baffle position is h_{baf}=1m. The different annular orifices in the baffle are considered. Radii of these orifices are radii R_{int} of the interface surfaces. The first four frequencies for the mode α=1 are evaluated for radii R_{int} =1.0m, R_{int}=0.7m, and R_{int}=0.2m. Note that R_{int} =1.0m corresponds to the un-baffled tank. These frequencies are presented in Table 3.

Table 2: Vibrations of the tank with a baffle, Hz

m	ω^2/g		
	R_{int} =1.0 m	R_{int} = 0.7 m	R_{int} = 0.2 m
1	2.1232	2.0435	1.4234
2	5.9800	5.9723	5.8405
3	9.4789	9.4785	9.4567

Figure 5 shows the first three forms of fluid vibrations in the spherical shell at α=1 with baffles.

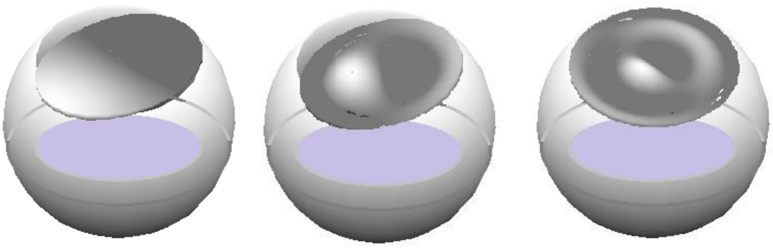


Fig. 5. Modes of liquid vibrations in the baffled spherical shell.

When the baffle is installed, the mode shape becomes almost flat.

3. Conclusion

Sloshing in the tank may be controlled by installing baffles, and the effectiveness highly depends on the shape, the location, and the number of baffles inside the tank. But in practice, the effect of baffles usually can be seen after the baffle has already been installed. Also, the visual inspection of the sloshing event inside the tank is not adequate for baffles design validation. Due to the complexities associated with the sloshing phenomenon, the numerical simulation is an effective method to meet the design intent, and shorten the development time. The proposed method makes it possible to determine a suitable place with a proper height for installing the baffles in tanks by using the numerical simulation. Estimating the frequency of fluid oscillations in spherical tanks on the basis of the proposed method will allow detuning from the operating frequency range of the regulating mechanisms.

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