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Investigation of the influence of the relaxation parameter on the viscous fluid flow over circular cylinder modeling process with the lattice Boltzmann method

G. Bulanchuk, A. Ostapenko
Pryazovsky State Technical University, Ukraine

In this work we investigate the influence of the relaxation parameter for the lattice Boltzmann method on the flow modeling process for the viscous fluid. The relaxation parameter influence on the other method parameters, the simulation time and the numerical solution stability has been considered by example of the fluid flow around circular cylinder modeling in a plane channel. Modeling has been performed at moderate Reynolds numbers. The flow pattern, the drag coefficient of the cylinder and the calculation time for the different Reynolds numbers have been shown. The results have been compared with the known experimental data and the other numerical solutions.

Key words: *relaxation parameter, lattice, Boltzmann equation, numerical solution*

В роботі досліджується вплив параметра релаксації методу граткових рівнянь Больцмана на процес моделювання течії в'язкої рідини. Досліджено вплив параметра релаксації на інші параметри методу, час моделювання і стійкість чисельного розв’язку на прикладі моделювання обтікання колового циліндра течією в'язкої рідини в плоскому каналі. Моделювання проводилось за помірними числами Рейнольдса. Досліджується характер течії, коефіцієнт лобового супротиву циліндра та час моделювання за різних числах Рейнольдса. Отримані результати порівнюються із відомими експериментальними даними та іншими чисельними рішеннями.

Ключові слова: *параметр релаксації, решітка, рівняння Больцмана, чисельний розв’язок*

В работе исследуется влияние параметра релаксации метода решеточных уравнений Больцмана на процесс моделирования течения вязкой жидкости. Рассмотрено влияние параметра релаксации на другие параметры метода, время моделирования и устойчивость численного решения на примере моделирования обтекания кругового цилиндра в плоском канале. Моделирование проводилось при умеренных числах Рейнольдса. Исследуется характер течения, коэффициент лобового сопротивления цилиндра и время моделирования при различных числах Рейнольдса. Полученные результаты сравниваются с известными экспериментальными данными и другими численными решениями.

Ключевые слова: *параметр релаксации, решетка, уравнение Больцмана, численное решение.*

1. Introduction

Lattice Boltzmann method (LBM) is one of the new promising approaches in the computational fluid dynamics (CFD) based on the kinetic theory of gases [1]. Although there are many traditional widely used methods in the CFD, such as the finite element method [2], the diffusion velocity method [3], the spectral method [4] and others. LBM is rapidly growing in popularity due to lots of opportunities and advantages. In recently published works LBM has already been used for the modeling of multicomponent flows [5], multiphase flows [6], flows with free boundaries [7], flows with heat transfer [8], flows with moving boundaries [9], drag and lift coefficient calculations [10-12]. There are such advantages of the method as easy programming, the simplicity in setting the complex boundary conditions and wide opportunities in the parallel computing on CPU or GPU, in particular, the usage of the CUDA technology that gives the significant increase in speed of computation [13].

But in spite of these advantages, it should be noted that there is the disadvantage – the conditionally stability [14-16] that complicates the flow modeling at high Reynolds numbers [15, 17-20].

The relaxation parameter is one of the parameters that influences the method`s stability. It is usually assumed that $\tau=1$ [11-15, 17-20] which is the most safe value for the modeling, as it does not cause the instability.

The aim of this work is to investigate the relaxation parameter influence on the simulation time, the accuracy of numerical results and to define the limits at which the solutions remain stable.

2. The Lattice Boltzmann Method: D2Q9 – BGK model

For modeling the fluid dynamics with the lattice Boltzmann method, the pseudo-particles described by the discrete particle densities distribution function f_k [15] are used. Each value of the function f_k describes the probability of a particle movement in one of the *k* directions. Let us note, that according to the kinetic theory of gases the particle density distribution function defines the probability density of finding a particle around a point in a six-dimensional phase space (coordinates and velocities) [22].

Let us divide the computational domain into square cells with the sides *d*. Each cell will contain nine values of the particle density distribution function. Thus, the particles can move to one of the eight possible directions or remain at rest (figure 1). Such model is called a two-dimensional nine-vectors model of the lattice Boltzmann method (D2Q9) [14].

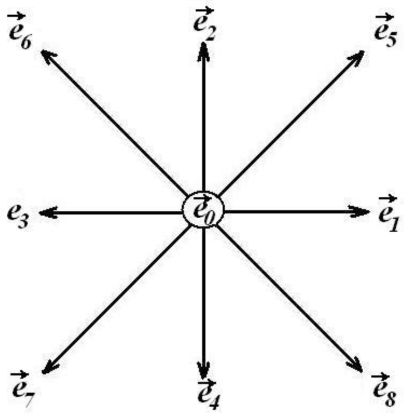

Fig.1. Possible directions of the particle movement according to D2Q9 model

Firstly, we should specify the kinematic viscosity of the fluid and the number of cells per unit length *N* that will determine the cell`s size *d*. After that the time step can be calculated according to the equation [14]:

$$\Delta t = \frac{1}{3}\frac{d^2}{\nu}\left(\tau - \frac{1}{2}\right) \tag{1}$$

where τ – nondimensional relaxation parameter [14,15]. Based on (1) and condition $\Delta t > 0$ we can get the limit: $\tau > 0.5$. Usually it is set as $\tau=1$ [11-15, 17-20]. But in this work we will use the relaxation parameter value from the range $0.5 < \tau \le 1$.

Let us define such modeling parameters as the lattice speed c and the lattice speed of sound c_s according to the expression [14]:

$$c_s = \frac{1}{\sqrt{3}}c = \frac{1}{\sqrt{3}}\frac{d}{\Delta t} \tag{2}$$

The discrete system of kinetic equations which describes the dynamics of the pseudo-particles is following [14]:

$$f_k\left(\vec{r} + \vec{e}_k d, t + \Delta t\right) = f_k\left(\vec{r}, t\right) + \Omega_k,\ k = \overline{0,8}. \tag{3}$$

where Ω_k – collision operator [14] (approximation of the collision integral from the integral Boltzmann equation);

$\vec{r} = (x, y)$ – vector of coordinates;

t – time.

Let us use the model of the collision integral in the form of BGK (Bhatnagar-Gross-Krook) approximation [14], which is a linear relaxation to the local Maxwell equilibrium [4,14,15]:

$$\Omega_k = \frac{f_k^{eq}\left(\vec{r}, t\right) - f_k\left(\vec{r}, t\right)}{\tau} \tag{4}$$

For modeling isothermal fluid flows under the LBM we are using the expansion of the Maxwell equilibrium distribution function by the powers of the velocity vector [23]:

$$f_k^{eq}\left(\vec{r}, t\right) = w_k \rho\left(\vec{r}, t\right)\left(1 + \frac{\left(c\vec{e}_k, \vec{u}\left(\vec{r}, t\right)\right)}{c_s^2} + \frac{1}{2}\frac{\left(c\vec{e}_k, \vec{u}\left(\vec{r}, t\right)\right)^2}{c_s^4} - \frac{1}{2}\frac{\vec{u}\left(\vec{r}, t\right)^2}{c_s^2}\right) \tag{5}$$

where w_k – weights;

ρ – density;

$\vec{u}$ – velocity vector.

The weights for the D2Q9 model are: $w_0 = \frac{4}{9}; w_{1-4} = \frac{1}{9}; w_{5-8} = \frac{1}{36}$ [15].

The conversion from the particle densities distribution function to the real fluid parameters such as density ρ, velocity $\vec{u}$ and pressure p can be done according to the equations [15]:

$$\rho(\vec{r},t)=\sum_{k=0}^{8} f_k(\vec{r},t); \qquad \vec{u}(\vec{r},t)=\frac{1}{\rho(\vec{r},t)}\sum_{k=0}^{8} c\vec{e}_k f_k(\vec{r},t); \quad p(\vec{r},t)=c_s^2\rho(\vec{r},t) \tag{6}$$

To calculate the drag coefficient of the bodies in the flow let us use the following equation [10]:

$$C_d=\frac{|F_x|}{\rho U_{in}^2 S} \tag{7}$$

where F_x – x – component of the total force, acting on the body in the fluid;
U_{in} – inlet velocity (fig. 2);
S – area of the body.

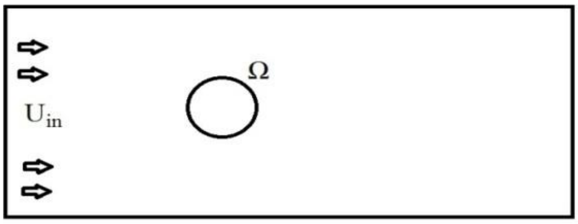

Fig. 2. The scheme of the problem of the flow around a circular cylinder

To calculate the total force acting on a body in the flow, use the formula [10]:

$$\vec{F}=\sum_{\Omega}\sum_{k=0}^{8} i\cdot c\vec{e}_{\bar{k}}\left[f_k(\vec{x}_b,t)+f_{\bar{k}}(\vec{x}_b+\vec{e}_{\bar{k}}\Delta x,t)\right] \tag{8}$$

where **Ω** – boundary layer of the body (fig. 2)
$\vec{x}_b$ – boundary cell of the body ($\vec{x}_b=(x,y)\in\Omega$);
i – indicator; *i=1*, if the cell $\vec{x}_b+\vec{e}_k$ is a body cell and *i=0* if it is the fluid cell;
$\vec{e}_{\bar{k}}$ – opposite to $\vec{e}_k$ direction of the particle movement (fig. 1)

As in this work in contrast to [10-12] the lattice speed (*c≠1)* but is defined by the equation (2) and the relaxation parameter is varying within the range $0.5<\tau\le 1$ we should add the additional multipliers to the equation (8). Thus, the expression for the calculation of the total force acting on a body in the flow will be:

$$\vec{F}=\nu\frac{1}{2\tau-1}\sum_{\Omega}\sum_{k=0}^{8} i\cdot c\vec{e}_{\bar{k}}\left[f_k(\vec{x}_b,t)+f_{\bar{k}}(\vec{x}_b+\vec{e}_{\bar{k}}\Delta x,t)\right] \tag{9}$$

As mentioned above, the LBM disadvantage is its conditional stability [14-16]. The stability of the solution is affected by:

- c_s – the lattice speed of sound; as shown in [14], the method remains stable when $c_s < \sqrt{1-U_{\max}^2}$, where $U_{\max}$ is the maximum speed value in the computational domain;
- τ – the relaxation parameter; to avoid the negative influence of the relaxation parameter it is usually set as $\tau = 1$ [15];
- c – the lattice speed; as it is shown in [14,24,25], the method remains stable when $M << 1$, where M is the lattice Mach number, which can be calculated by the equation:

$$M = \frac{U_{\max}}{c} \tag{10}$$

3. Numerical results

Let us consider a rectangular domain which size is *3x1* m (fig. 3). The domain is filled with a liquid with a kinematic viscosity ν. The no-slip boundary conditions are set on the upper and lower boundaries of the domain. The fluid enters through the left boundary with the velocity U_{in}=0.1 m/s. The outlet of the liquid is set on the right boundary using the constant pressure condition: $P_{out} = \text{const}$. There is a circular cylinder with the radius *R*=0.0625 m in the channel. The cylinder`s center has coordinates (0.6; 0.5), taking into account the location of the coordinate axes, as shown in fig. 3. So the blockage ratio is $B = \frac{H}{D} = \frac{1}{2 \cdot 0.0625} = 8$. This value is not a random choice, but the most common in modeling such flows [11,12].

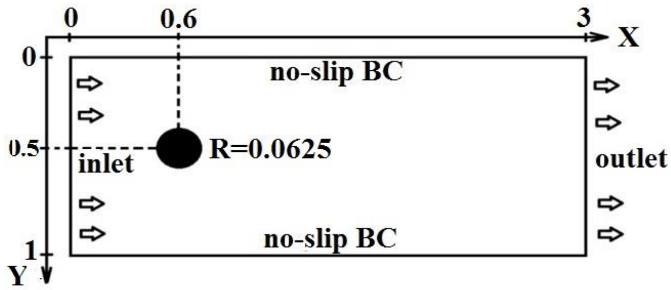

Fig. 3. The flow around the circular cylinder in a plane channel problem

The modeling has been performed using the original program written in C ++ in MS Visual Community 2015 development tool on a computer with quad-core Intel Core i3 processor, 4GB RAM, 2 GHz frequency and using OpenMP technology for CPU parallel computing.

The series of calculations have been carried out with different relaxation parameters and Reynolds numbers. The dimension of the grid has been adjusted so that the lattice Mach number has been *M* = 0.15. As shown in [25], this value allows us to get the results with high accuracy within the optimum simulation time.

The results of the calculation at Reynolds number Re=40, namely: the number of cells per unit length *N*, which is required to achieve the value *M*=0.15, the drag coefficient C_d, the relative error of simulation ε and the simulation time T_s is presented in tab.1. In order to compare obtained results, the flow simulation with the same parameters by the finite element method (FEM) in the Comsol Multiphysics package has been performed. The results have been compared with the experimental data presented in [26] as well. The calculations have been carried out with the relaxation parameters τ = 1.0, 0.75, 0.6, 0.55, 0.535 till point-in-time T = 100 s. The flow pattern is shown in fig. 4. As you can see, the flow is laminar and vorticity behind the cylinder does not occur.

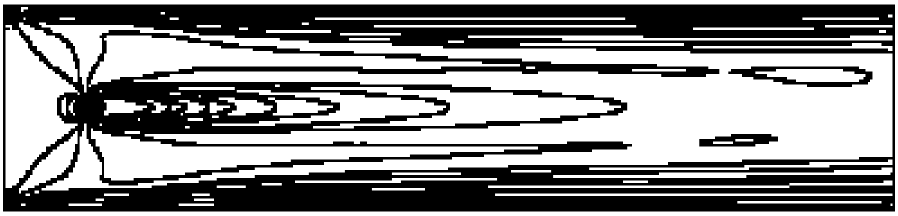

Fig. 4. The lines of equal velocities in the flow around circular cylinder at the Reynolds number Re = 40, received with LBM

Table 1. The results of the simulation of the flow around a circular cylinder at Re = 40 with the various relaxation parameters of LBM

	N	C_d	ε, %	T_s
LBM, τ=1,0	500	1,4678	0,8	14 h 42 min
LBM, τ=0,75	300	1,3827	6,6	4 h 36 min
LBM, τ=0,6	110	1,2946	12,5	16 min
LBM, τ=0,55	70	1,3642	7,8	3 min
FEM, Comsol	100	1,55	4,7	21 min
Experiment, [26]	-	1,48	-	-

It can be seen from Tab.1, that the relaxation parameter decrease can reduce the number of cells per unit and thus substantially reduce simulation time.

Similar calculations have been made for Reynolds numbers Re = 60, 100. The results are showed in tables 2 and 3 respectively. The formation of the vortex street behind the cylinder (figure 5) is typical for these Reynolds numbers (unlike Re = 40) and the frequency of the vortices increases with the increase in Reynolds number.

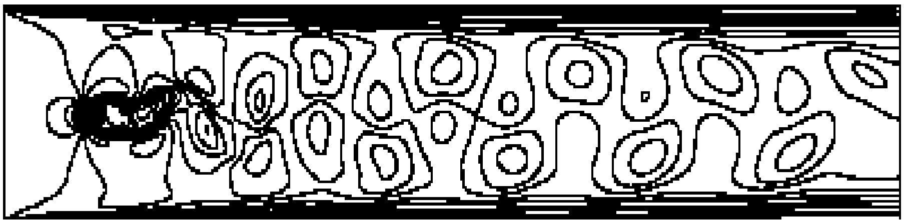

Fig. 5. The lines of equal velocities in the flow around circular cylinder at the Reynolds number Re = 100, received with LBM

Table 2. The results of the simulation of the flow around a circular cylinder at Re = 60 with the various relaxation parameters of LBM

	N	C_d	ε, %	T_s
LBM, τ=1,0	600	1,3325	4,1	19 h 3 min
LBM, τ=0,75	400	1,3116	5,6	7 h 21 min
LBM, τ=0,6	200	1,3663	1,7	1 h 21 min
LBM, τ=0,55	100	1,1405	17,9	12 min
FEM, Comsol	100	1,34	3,6	22 min
Modeling, [11]	-	1,39	-	-

It can be seen from tables 1 and 2, that modeling flows with the lattice Boltzmann method with the parameter τ = 1 consumes significant time. Moreover, if we increase Reynolds number, simulation time significantly increases. Reducing the relaxation parameter significantly reduces simulation time. The obvious advantage of this approach, however, is offset by a possible instability of the solution and the increased possibility of a computing error. As we can see it is extremely problematic to calculate flows at Reynolds numbers Re> 60 with the value τ = 1 because of large simulation time. The next series of calculations for Re = 100, 200, 300, 400 have been made with τ <1 (table 3-6.).

Table 3. The results of the simulation of the flow around a circular cylinder at Re = 100 with various relaxation parameters of LBM

	N	C_d	ε,% [26]	T_s
LBM, τ=0,75	500	1,1865	4,3	14 h 42 min
LBM, τ=0,6	300	1,1929	3,8	9 h 39 min
LBM, τ=0,55	200	1,1922	3,8	2 h 15 min
LBM, τ=0,535	100	1,1833	4,6	23 min
FEM, Comsol	100	1,2875	3,8	19 min
Modeling, [26]	-	1,24	-	-
Modeling, [27]	-	1,33	-	-

It can be seen from table 3, reducing the relaxation parameter for the Re = 100 is fully justified and provides the solution with a relative error ε<5% for small simulation time.

Table 4. The results of the simulation of the flow around a circular cylinder at Re = 200 with various relaxation parameters of LBM

	N	C_d	ε, %	Ts
LBM, τ=0,55	300	1,1836	1,0	4 h 56 min
LBM, τ=0,535	225	1,1229	4,2	2 h 32 min
FEM, Comsol	100	1,1825	1,0	20 min
Modeling, [27]	-	1,172	-	-

Tab. 3-4 shows that the closer the relaxation parameter to its limit (τ = 0.5), the more significant the influence of its change on the calculations. In this case, even a little deviation of the relaxation parameter entails a significant change in the calculations.

The results obtained with LBM at Re = 300, 400 are shown in tab. 5 and 6, respectively.

Table 5. The results of the simulation of the flow around a circular cylinder at Re = 300 with various relaxation parameters of LBM

	N	C_d	*ε*, % FEM	*Ts*
LBM, τ=0,535	300	1,1296	0,5	4 h 58 min
FEM, Comsol	100	1,135	-	22 min

It is obvious, that modeling the flow at Reynolds numbers Re≥300 with the lattice Boltzmann method should be done with the 0.5 <τ≤0.535. With τ> 0.535 simulation time will increase significantly which makes modeling not suitable for a practical application.

Table 6. The results of the simulation of the flow around a circular cylinder at Re = 400 with various relaxation parameters of LBM

	N	*Cd*	*ε*, % FEM	*Ts*
LBM, τ=0,535	400	1,1774	5,4	10 h 58 min
FEM, Comsol	100	1,1175	-	19 min

Further reducing the relaxation parameter, i.e. τ<0.535, results in the instability of the solution, therefore it is still impossible to do simulation with this parameter at high Reynolds numbers.

4. Conclusion

The studies show that reducing the relaxation parameter can reduce the number of cells per unit length of the computational domain and thus significantly reduce simulation time. But it is necessary to control the stability of the solution.

The calculations show that decreasing the relaxation parameter value allows to obtain the solutions with a relative error *ε*<5% for small simulation time. The closer the relaxation parameter to its limit (τ=0.5) the more considerable the influence of its changes on the calculations. In this case, even a small deviation of the relaxation parameter entails significant changes in the calculations.

The research conducted shows that the flow simulation at Reynolds numbers Re≥300 with the lattice Boltzmann method should be carried out using the relaxation parameter within the range 0.5<τ≤0.535.

It should be noted that in the similar works, concerning modeling the fluid flow around bodies in a plane channel with the lattice Boltzmann method, the solutions for Re>150 have not been received [11,12, 15, 17, 19]. In this work, we have obtained the stable solutions for Reynolds numbers up to Re=400. In the future we plan to obtain stable solutions at high Reynolds numbers.

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