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Two-dimensional vortex pair interaction with the wedge

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Рассмотрена задача о взаимодействии двухмерных локализованных вихревых диполей с острым клином, двигающихся с начальный момент времени перпендикулярно к одной из поверхностей клина. Экспериментальные исследования показали, что вихревые диполи при приближении к твердой поверхности разделяются и двигаются в противоположные стороны. Вихревая структура при взаимодействии с острой кромкой генерирует вторичные вихри, которые могут образовывать новые вихревые диполи. Сформирована численная модель взаимодействия вихревых диполей с острым клином, основанная на модели точечных вихрей в приближении идеальной несжимаемой жидкости. Для избегания бесконечных значений скорости на острой кромке используется условие Кутта-Жуковского, которое адекватно описывает процесс формирования вторичной завихренности около острой кромки. Сравнительный анализ процессов переноса жидкости, формирующей в начальный момент "атмосферу вихря", свидетельствует о хорошем соответствии численных результатов и экспериментальных данных.

Ключевые слова: *вихревой диполь, двухмерное течение, острая кромка, условие Кутта-Жуковского, численное моделирование, экспериментальные исследования.*

Розглянуто задачу про взаємодію двомірних локалізованих вихрових диполів з гострим клином, що рухаються з початковий момент часу перпендикулярно до однієї з поверхонь клина. Експериментальні дослідження показали, що вихрові диполі при наближенні до твердої поверхні розділяються і рухаються в протилежні сторони. Вихрова структура при взаємодії з гострою кромкою генерує вторинні вихори, які можуть утворювати нові вихрові диполі. Сформована чисельна модель взаємодії вихрових диполів з гострим клином, заснована на моделі точкових вихорів в наближенні ідеальної нестисливої рідини. Для уникнення нескінченних значень швидкості на гострій кромці використовується умова Кутта-Жуковського, яка адекватно описує процес формування вторинної завихренности біля гострої кромки. Порівняльний аналіз процесів переносу рідини, яка формує в початковий момент "атмосферу вихора", свідчить про гарну відповідність чисельних результатів і експериментальних даних.

Ключові слова: *вихровий диполь, двомірна течія, гостра кромка, умова Кутта-Жуковського. чисельне моделювання, експериментальні дослідження.*

The problem of interaction of two-dimensional localized vortex dipole with an edge of the wedge, which moving perpendicular to one of the wedge surfaces at the initial moment is considered. Experimental studies have shown that the vortex dipoles at the approach to the solid surface are separated, and vortices moved in opposite directions. Vortex structure, when interacting with a sharp edge, generates secondary vortices that may form new vortex dipoles. The numerical model for the interaction of vortex dipole with an edge of the wedge, based on the model of point vortices in the approximation of an ideal incompressible fluid is formed. To avoid infinite velocities at the sharp edge model used Kutta-Zhukovsky condition, which adequately describes generating process of the secondary vorticity near the sharp edge. Comparative analysis of transferring processes of fluid forming at the initial moment "vortex atmosphere" shows good agreement of numerical results and experimental data.

Key words: *vortex dipole, two-dimensional flow, sharp edge, Kutta-Zhukovsky condition, numerical modelling, experimental investigation.*

1. Introduction

Recently, in the world literature on fluid mechanics have been formed quite clearly a tendency associated with an increasing in the practical interest of many researchers to solve problems that have a real practical application [1,2]. Among solution methods for these problems one can be identified experimental studies, and analytical or numerical-analytical methods. In many cases, experimental researches meets a number of difficulties caused by the complexity both of experimental setting, result processing, and the high cost of experimental equipment for investigation. Researchers often use analytical methods or direct numerical methods that deal with introducing a certain number of assumptions, forming mathematical models of the flow, which can significantly limit the application ranges of the achieved results [1,3,4].

From this point of view, the problem of large-scale vortex interaction with solid surfaces is the most significant. It is known that the kinetic energy of the fluid flow near a solid surface generates wall vortex motion of various sizes [5-7]. Subsequent energy dissipation of this motion in the cascade of wave-number leads to the formation of large-scale vortex structures in the boundary layer. This process leads both to an intensification in vortex motion and eventually to the separation from the solid surface. In the absence of the stabilizing effects these large-scale vortex structures can intensify heat and mass transferring and destruct the boundary layer [5,7-9]. That is why, the analysis of vortex generation processes, vortex interaction with solid surfaces, as well as processes of heat and mass transfer in vortex flows has a certain practical and scientific interest.

It is known that two vortices with opposite intensities can form a vortex pair, which is called dipolar vortex [10-12]. This structure can move in fluid translationally with fixed self-induced velocity. If two vortices have not equal intensities, then vortices start the rotational motion with constant angular velocity relative to one another. There is an interesting problem deals with an explanation of the dipolar vortex behaviour near a solid boundary. This type of interaction occurs when large-scale atmospheric vortices approaching to mountain ranges, oceanic vortices move near the peninsula, dams, seawalls, etc. Preliminary discussion of this problem we can found, for example, in [13,14] and references therein.

Solution of the evolution problem of large-scale vortex flows is reduced to the calculation of the velocity field distribution in time. It is very difficult to achieve an analytical solution of the vorticity field in real flows [15]. This way meets a number of difficulties, which in most cases deals with insuperable difficulties (details of the problem and discussion we can found, for example, in [10-12,16]).

One of the more widely used methods for solving the problem of large-scale vortex motion is a numerical method based on the direct numerical simulation of generating and transferring of vorticity field [17-20]. Despite the universality of certain numerical schemes and methods for solving problems of vortex dynamics, the direct numerical simulation of the vorticity transport equation requires considerable computing resources, and analysis of the results (actually, that is a data filed) becomes problematic. Another method, which is also often used in the studying of coherent structure evolution, deals with the separation of small volume of fluid in which the vorticity field is concentrated. Physical parameters in these elementary volumes are chosen from the condition that the induced velocity field is equal to the velocity field induced by distributed vorticity in the space around the point in the consideration [10-

12,21,22]. Such methods of solution called in the modern literature as discrete vortex methods in many cases of practical importance is quite effective both to solve problems of vortex dynamics, and to analyze processes of heat and mass transfer in real fluid flows. It is important that these numerical methods for the solution do not require large computational capacities, and achieved results are quite simple for the interpretation and subsequent analysis [11,23,24].

However, the transition from the continuous distribution of vorticity, observed in the real flow, to a discrete analogue is not always equivalent, especially during an analysis of small-scale fluctuations of the velocity field. Moreover, in some cases, discrete analogues during computation can lead to numerical instability elimination which requires using of special numerical algorithms or computational methods. A detailed analysis of this problem we can found in [25-28]. Nevertheless, the question on ranges of applicability of discretization methods of distributed vorticity field remains open for today and requires further detailed comparison between numerical results and experimental data.

Often the method of conformal mappings [22,25,29] applied for analytic solutions of the problem mentioned before. In this case the construction of the solution is based on the transformations, which allow satisfying the boundary conditions on the solid surface or on some part of this surface. Despite the fact that the method conformal transformation is not always possible to adapt for different geometries of the flows, which has some practical interest, this method has curtain flexibility and gives a number of advantages both for achieving an analytical solution and for following analysis of results.

Forming analytical and numerical solutions of the problem on the interaction of coherent vortex structures with wedge solid surface we assume that the vorticity field is a number of localized vortex structures coming into the external flow. To describe the separation process the discrete methods of vortex dynamics analyse the motion of the single vortex structure, the intensity of which suppose equal to the integral of the vorticity coming to the flow for some time interval [23,25,30]. The number of these vortex structures is not fixed in this problem.

General scale analysis shows that it is necessary to take into account only the inertia of the fluid flow near edges of surfaces, viscosity effect has much more small effect on the process of vortex generation at the edges [8,9,31]. In particular, parameters of discrete vortices in the flow near the sharp edges are determined from Kutta-Zhukovsky condition, which does not allow the formation of an infinitely large velocity of the flow at the edges themselves. This condition allows us to determine the intensity of the corner vortex, which describes in the first approximation the processes of vortex generation at the sharp edge in the real viscous flows. Sometimes the mapping function of conformal transformations has a singularity on a sharp edge. In this case, it is necessary to apply Kutta-Zhukovsky conditions on the image plane. The adequacy problem of such a solution to the real flow is the question that still remains controversial problem among many researchers [1,2]. To find the answer for this question we would like to carry out a detailed comparison of the numerical simulation results and data of a laboratory experiment.

It is known [12,32,33], that the vortex pair during own motion involves in own motion the part of surround fluid, forming a so-called “vortex cloud” or “atmosphere”.

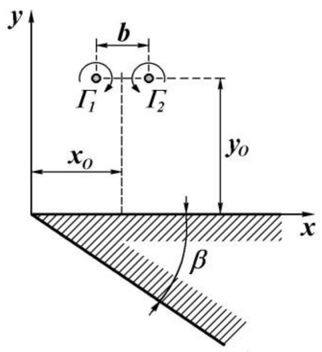

Fig.1. Geometry of the problem

The form of this fluid region is very close to an ellipse, and a stationary vortex pair (vortex dipole) does not change its shape and, therefore, the volume. The interaction of vortices with a wedge brokes the stationary motion of the vortex pair. As a result, the distance between vortices changes in the pair. This process leads to changing the size of the vortex cloud that results in more intensive regimes of stirring and mixing processes of various scalar fields in the flow [33-37].

The main purpose of this paper is identifying the main peculiarities of motion of the vortex pair near the edge of the wedge, and determining of general characteristics of the mixing process of passive fluid during an interaction of vortex pair with the solid surface.

2. Mathematical method

Consider the motion associated with a point vortex pair near a wedge, as indicated in the definition sketch of fig.1. The initial vortex dipole with intensities $\Gamma_1 = -\Gamma_2 = \Gamma$ and distance b between vortices moves perpendicular to the half-plane. We suppose that the geometrical centre of the dipole placed at the point (x_0, y_0).

We define the z-plane as the physical plane. The mapping

$$z = \zeta^{\tau} \tag{1}$$

with $\tau = 2\beta/\pi$, maps the exterior region of a wedge in the z-plane onto the upper half-plane of the ζ-plane (fig.2).

The complex potential $w(\zeta)$ in the z-plane due to N vortices is

$$w(\zeta) = -i\sum_{j=1}^{N}\Gamma_j \ln\left(\frac{\zeta-\zeta_j}{\zeta-\zeta_j^*}\right), \tag{2}$$

where $\zeta_j = \xi_j + i\,\eta_j$ represents the position of vortex number j with constant intensity Γ_j, and * denotes the complex conjugation. The flow defined by the complex potential $w(\zeta)$ in the ζ-plane corresponds with a flow in the physical z-plane given by the complex potential $W(z) = w(\zeta(z))$.

Hence, the complex velocity $u - iv$ in the physical z-plane is

$$u - iv = \frac{dW}{dz} = -\frac{i}{(dz/d\zeta)}\sum_{j=1}^{N}\Gamma_j\left(\frac{1}{\zeta-\zeta_j} - \frac{1}{\zeta-\zeta_j^*}\right) \tag{3}$$

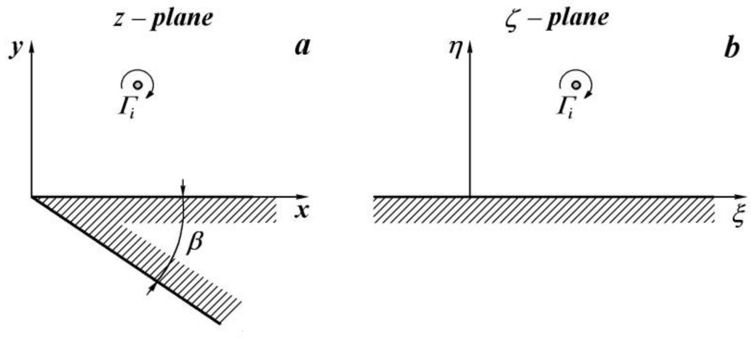

Fig.2. Conformal mapping: physical z-plane (a) and transformed ζ-plane

The indices j =1, 2 are pertinent to the vortex pair, while j > 2 refers to other vortices that can be present in the flow field. Index $j = N$ refers to a vortex whose intensity depends on time and which is determined by satisfying the Kutta-Zhukovsky condition

$$\frac{dw}{d\zeta} = 0 \qquad \text{at} \qquad \zeta = 0. \tag{4}$$

The governing equation is normalized by using as reference length the distance b between the vortex pair vortices at initial time t = 0, and as reference velocity the translation speed $U = \gamma/(2\pi b)$ of the vortex pair, where γ is (absolute) strength of the individual vortices. Hence, the reference time is b/U. Thus, in equations (2), (3) the strengths are $\Gamma_1 = -\Gamma_2 = 1.0$.

The velocity of a vortex with constant circulation Γ_j ($j \neq N$) is given by:

$$\frac{d\zeta_j^*}{dt} = \left[-i \sum_{l=1, l\neq j}^{N} \Gamma_l \frac{1}{\zeta_j - \zeta_l} + i \sum_{l=1}^{N} \Gamma_l \frac{1}{\zeta_j - \zeta_l^*} + \frac{i\Gamma_j}{2} \frac{d}{d\zeta}\left(\ln \frac{dz}{d\zeta} \right) \right] \frac{1}{\left| dz / d\zeta \right|^2}, \tag{5}$$

while, according to the zero force model the vortex with time dependent circulation Γ_N ($j = N$) moves with velocity

$$\frac{d\zeta_N^*}{dt} = \left[-i \sum_{l=1, l\neq j}^{N} \frac{\Gamma_l}{\zeta_N - \zeta_l} + i \sum_{l=1}^{N} \frac{\Gamma_l}{\zeta_N - \zeta_l^*} + \frac{i\Gamma_N}{2} \frac{d}{d\zeta}\left(\ln \frac{dz}{d\zeta} \right) \right] \frac{1}{\left| dz / d\zeta \right|^2} - , \tag{6}$$

$$-z^*(\zeta_N) \frac{1}{\Gamma_N} \frac{d\Gamma}{dt} \frac{1}{\left(dz / d\zeta \right)^*}$$

with
$$\Gamma_N = -\left(\sum_{j=1}^{N-1} \Gamma_j \frac{\eta_j}{\varsigma_j^2 + \eta_j^2} \right) / \left(\frac{\eta_N}{\varsigma_N^2 + \eta_N^2} \right)$$

and
$$\frac{1}{\Gamma_N} \frac{d\Gamma_N}{dt} = \frac{\xi_N^2 + \eta_N^2}{\Gamma_N \eta_N} \sum_{j=1}^{N-1} \frac{\Gamma_j}{(\xi_j^2 + \eta_j^2)^2} \left[2\eta_j \xi_j \frac{d\xi_j}{dt} - (\xi_j^2 - \eta_j^2) \frac{d\eta_j}{dt} \right] -$$

$$- \frac{1}{\eta_N (\xi_N^2 + \eta_N^2)^2} \left[-2\eta_N \xi_N \frac{d\xi_N}{dt} + (\xi_N^2 - \eta_N^2) \frac{d\eta_N}{dt} \right] \tag{7}$$

Equations (5), (6) are integrated by a fourth-order Runge-Kutta algorithm. The simulation is started with N = 3, that is the vortex pair in its initial position and a

Kutta-satisfying vortex placed close to the tip of the wedge, in $z = -0.02$. The *Kutta vortex* is released when its circulation is $|\Gamma_N| > 0.01$. When $|\Gamma_N|$ reaches close to the tip of the wedge a maximum, the $j = N$ vortex is set as a constant circulation vortex, N is increased to $N + 1$ and a new *Kutta vortex* is placed in $z = -0.02$.

It is well known [33] that vortex structures may entrain ambient fluid while they move around. For example, a vortex pair consisting of two point vortices with equal but oppositely-signed circulations translates steadily along a straight line and carries a certain amount of fluid within the separatrix region. The shape of “vortex atmosphere” is closely approximated by an ellipse with axes 1.73b and 2.09b, where b is the distance between the vortices for a given moment. For the case of different vortex intensities (in absolute sense), the pair translates steadily along a circular trajectory, again carrying an “atmosphere” of trapped fluid along.

When approaching a solid boundary, the distance between the vortices changes, leading to changes in the size and the shape of the atmosphere. This usually results in entrainment of ambient fluid into the atmosphere and/or detrainment of atmosphere fluid. The ratio of the amounts of entrained and detrained fluid depends on the type of vortex interaction. The advection characteristics of non-stationary point vortex constellations can be adequately studied by applying the so-called “contour kinematics” technique [34,39]. According to this method, a contour is described by a large set of passive fluid particles (markers). Each marker moves with the locally induced fluid velocity, and the spatial position of the specific contour at any time is determined by the positions of its markers, connected appropriately. In order to calculate their displacements in time, each marker can be interpreted as a point vortex with zero circulation. Hence, its change in position can be obtained directly from equation (5), namely

$$\frac{d\zeta_m^*}{dt} = \left[i\sum_{l=1}^{N} \frac{\Gamma_l}{\zeta_m - \zeta_l^*} - i\sum_{l=1}^{N} \frac{\Gamma_l}{\zeta_m - \zeta_l} \right] \frac{1}{|dz/d\zeta|^2}, \quad (8)$$

where $\zeta_m = \xi_m + i\eta_m$ is the position of the m-th marker, $m = 1..., M$, and M is the number of markers necessary to describe the contour with a given precision.

Locally, contours may show strong deformations in time. While some segments of the contour may undergo substantial stretching and deformation, other parts may hardly deform at the same time. The description of stretched and deformed segments of the contour by a fixed number of markers obviously causes difficulties and implies locally a poor resolution. Just increasing the initial number of markers on the contour does not solve this problem adequately, because the exact locations of intensively deformed contour segments are not known in advance.

To overcome this difficulty, we can apply the PSI (piece spline-interpolation) method [38], which uses a variable number of markers to describe the spatial position of the contour. If the distance between two adjacent markers becomes larger than some critical value, then the stretched part of the contour is interpolated by functions $\xi(L)$ and $\eta(L)$, where L is the length of the contour, starting from some marker with index, for example, $m = 1$. Then the position of an additional marker (or markers) of the contour is defined by this interpolation. Hence, the total number of markers M used in calculations varies in time. Likewise, if the distance between two markers

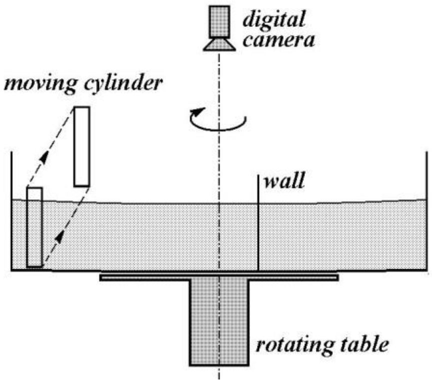

Fig.3. Schematic drawing of the experimental set-up

becomes smaller than some specified minimum value, the number of markers can be reduced.

This numerical "contour kinematics" method has been used successfully in studies of vortex-induced advection of tracer material for a number of different vortex-vortex interactions [39].

3. Experimental set-up and dye visualisation

Laboratory experiments were carried out in a rectangular container (horizontal dimensions 100 *cm* x 150 *cm*) filled with tap water to a depth H = 20 *cm*. This tank is mounted on a turntable that rotates at a constant angular velocity Ω = 0.70 *rad/s*. Prior to each experiment, the fluid was allowed to reach a state of solid-body rotation during at least 45 *min*, which is much longer than the Ekman spin-up timescale $T_E = H/(\nu\Omega)^{1/2} \approx 4$ *min*. In order to avoid any topographic effects associated with the parabolic free-surface shape [35], a specially designed parabolic bottom plate was mounted in the tank [40].

A dipolar vortex is conveniently created by dragging an open, thin-walled cylinder (diameter 6 cm) horizontally along a straight line through the fluid while simultaneously lifting it slowly out of the fluid. For this purpose a guiding rail was mounted above the rotating tank (fig.3).

For properly chosen translation speed, the flow in the wake of the cylinder was observed to become organized in a columnar dipolar vortex – its axial alignment conform the Taylor-Proudman theorem. This dipole generation technique was also applied by [35]. A solid obstacle was placed beforehand at some specified position in the fluid. In the experiment described here, a flat plate with a sharp edge was used, i.e. a sharp wedge with apex angle $\beta = 0^0$.

In a number of experiments the flow evolution was visualized by adding fluorescent dye to the fluid, both to the fluid inside the translating cylinder and to the fluid adjacent to the plate. Quantitative information about the flow was obtained by using High-resolution Particle Velocimetry. The working fluid was seeded with small tracer particles with a diameter of 250 μm, with a density somewhat smaller than that of the fluid. The floating particles were illuminated by four slide projectors, and their motion was recorded by a corotating digital camera mounted above the container.

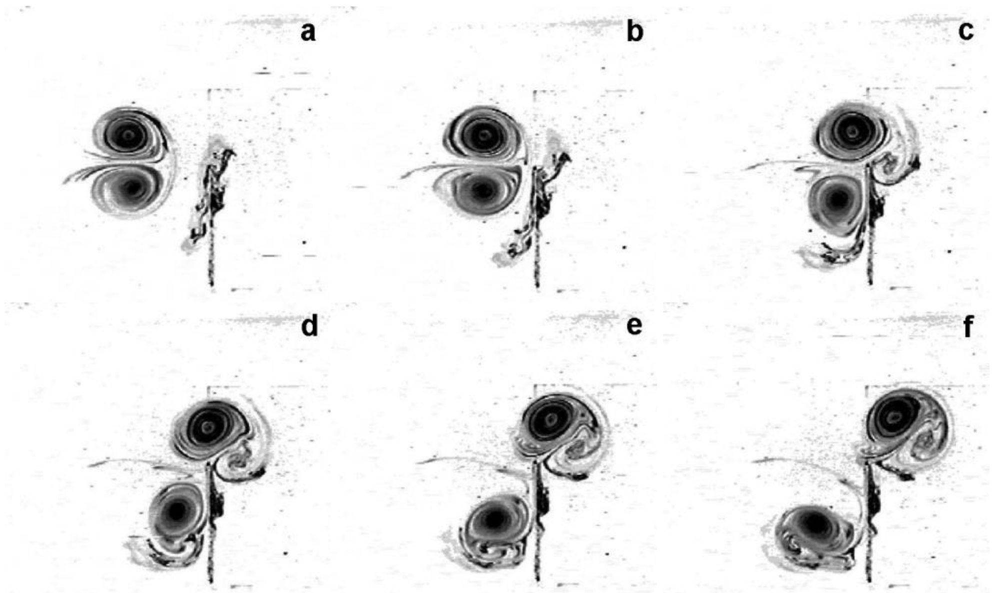

Fig.4. Dye-visualisation of a dipolar vortex colliding against a flat plate with a sharp edge, with off-set

An example of a dye experiment is shown in fig.4. In this experiment, the dipole was initially aligned with the edge of the plate, i.e. with an off-set distance $\delta \approx 0$ (fig.4). In the photographs the dye-visualised dipole is seen to approach the plate from the left (fig.4b) and the irregular dye patches on either side of the plate have been introduced to visualize the formation of any secondary vortices near the plate. As the dipole gets closer to the plate, it is observed to split. A secondary vortex (with negative circulation) forms behind the plate edge (fig.4c), which subsequently combines with the positive-vorticity part of the dipole, thus forming a new dipole that moves away. Because this dipole is a-symmetric (the vortex produced at the edge is weaker than the original dipole half) it moves along a curved trajectory.

On the "frontal" side of the plate, the flow associated with the negative vorticity part of the dipole induces a viscous boundary layer, owing to the no-slip condition at the wall. The vorticity in the boundary layer is of opposite sign, i.e. positive (fig.4d). This positive vorticity patch is advected by the flow induced by the negative half of the dipole (fig.4e), and is seen to roll up into a single positive vorticity patch that pairs with the negative half of the initial dipole (fig.4f). This newly formed dipole is a-symmetric too, and also moves along a curved path. This latter behaviour is very similar to what has been found by [14] in his study of a vortex dipole colliding against a flat solid wall.

4. Results

In this section we will discuss results obtained with the point-vortex model of the vortex dipole moving close to the tip of the solid wedge, showing the vortex trajectories for different initial off-set values. Also, the strengths and paths of the vortices generated at the wedge tip will be considered. Next, experimental results will be discussed and a direct comparison with the point-vortex results will be made.

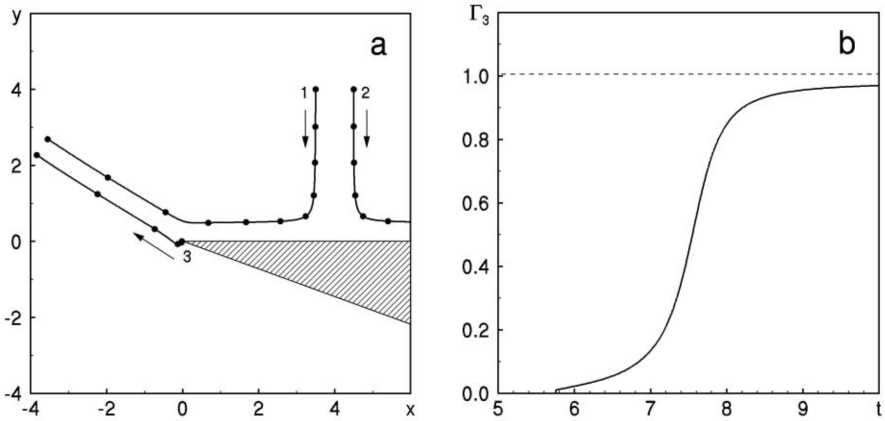

Fig.5. Interaction of the vortex pair with a wedge, x_0 = 4.0:
a) trajectories of vortices, b) the circulation of the third vortex

4.1. Vortex trajectories

Let us consider interaction of a vortex pair with a wedge of $\beta = 20^o$. There are two vortices with circulation $\Gamma_1 = -\Gamma_2 = -1.0$, which have initial coordinates $x_1^0 = x_0 - b/2$, $x_2^0 = x_0 + b/2$ and $y_1^0 = y_2^0 = y_0$ according to fig.1. The sign of circulations is chosen in such a way that the self-induced velocity of vortex pair be directed to the solid surface. We can analyse features of vortex pair interaction with wedge for values x_0 changing in a sufficiently large range and for fixed value y_0. Let b = 1.0 and y_0 = 4.0. It means that vortex pair at the initial moment is far enough from wedge surfaces, and it is possible to suppose that surfaces do not essentially influence the initial phase of vortex motion.

First we consider the case x_0 = 4.0. The vortex trajectories is shown in fig.5,a. Circles in the figure show spatial positions of vortices at equidistant moments $t_n = n\Delta t$, where Δt = 1.0. Arrows specify directions of vortex motion. At the initial stage vortices *1* and *2* move to the negative values of axis *Oy* perpendicularly to the solid surface. When $t \approx 4.0$, vortices have come close enough to the surface that begins to influence vortex trajectories: distance between vortices increases. As vortex pair is far enough from the vertex of wedge, vortex trajectories develop symmetrically. Then vortex *1* moves to the wedge, while vortex *2* moves in the positive values of axis *Ox*. The symmetry in trajectories has losed.

When vortex *1* achieves the vertex of wedge, t ≈ 7.0-8.0. The vortex moving near the vertex generates an angular vortex. Fig.5,b shows changes in circulation of new vortex in time. It is shown that the circulation of vortex *3* achieves value $|\Gamma_1|$. It results in formation of a new vortex pair with equal circulation (by module). Vortex *2* moves together with vortex *3* in the negative direction of axis *Ox*, forming a vortex pair leaving the wedge surfaces.

The analogous interaction of vortex pair with the wedge can be seen in fig.6 with x_0 = 0.0. Here and later we use analogous notations in figures. In this case vortex *1*

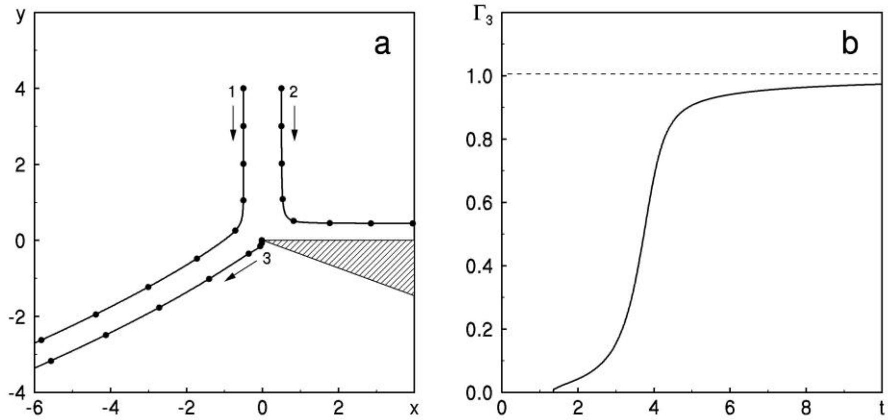

Fig.6. Interaction of the vortex pair with a wedge, x_0 = 0.0:
a) trajectories of vortices, b) the circulation of the third vortex

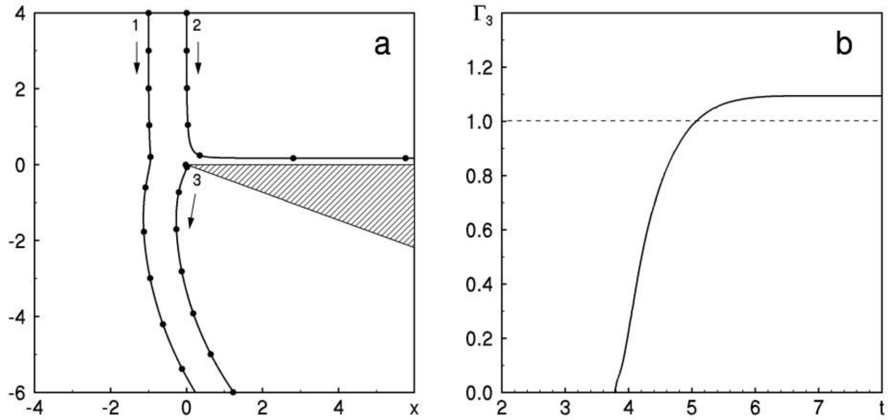

Fig.7. Interaction of the vortex pair with a wedge, x_0 = –0.5:
a) trajectories of vortices, b) the circulation of the third vortex

comes closer to the vertex wedge compared to the case considered before. This position results in fast growth of circulation of the angular vortex *3* (fig.6,b). When the formation process is over, new vortex pair with vortices *1* and *3* leaves the sharp edge but with another direction in comparison with case considered before, while the trajectory of vortex *2* is similar one. Note that the circulation of vortex *3* increases in both cases at the moments when the initial vortex *2* moves close to the wedge.

The case x_0 = –0.5 is shown in fig.7. Vortex *1* passes in greater distance from the wedge compared to the previous cases. Therefore vortex trajectories are barely different. On the other hand vortex *1* goes in the direction to the vertex of the wedge and causes the occurrence of a new vortex *3*, its circulation growing quickly enough (see fig.7,b) and achieving the value $|\Gamma_3| > |\Gamma_1|$. The following evolution of vortex system has an analogous tendency. It is necessary to note that new pair has vortices with non-equal circulation. Vortex pair *1+3* has a slight rotation in counter-clockwise

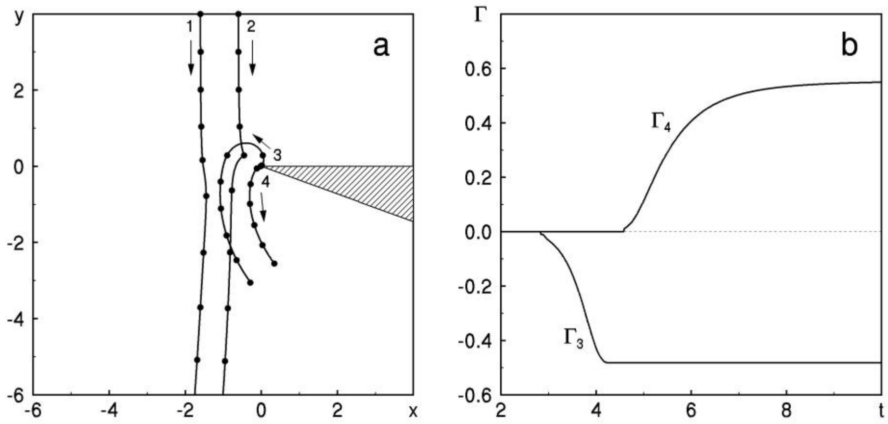

Fig.8. Interaction of the vortex pair with a wedge, x_0 = –1.1:
a) trajectories of vortices, b) the circulation of the third vortex

direction and, therefore, moves closer to the bottom surface of the wedge (fig.7,a).

We can find more differences in the interaction of vortex pair with the wedge at x_0 = –1.1, presented in fig8. In this case vortex *2* strongly influences the formation process of angular vortex, vortex *2* gets to the vertex at $t \approx 4.0$. The circulation of vortex *3* grows quickly enough. However, in this case the sign of circulation coincides with circulation of vortex *1* (fig.8,b). Distance between new vortices and wedge increasing, and vortices moves to the initial vortex pair. New vortex induces a velocity field, which even stops vortex *2*. The existence of two vortices (with numbers *2* and *3*) near the vertex a wedge results in generation of another vortex *4*), its circulation having the opposite sign to that of vortex *3*. Thus, there are four vortices in considered system. New vortices push out vortex *2* in the direction of vortex *1*, while vortices *3* and *4* form new vortex pair, their circulations being a little bit smaller compared to the initial vortex pair.

Finally, the further displacement, x_0 = –2.0, leads to the case, then the initial vortex pair passes the wedge at large enough distance (fig.9). At the moment when vortex *2* is situated close to the vertex of wedge, induced velocity results in formation of vortex *3* with negative circulation. Now this vortex, leaving the wedge, forms vortex *4*. Fig.9,b shows changes of vortex circulations in time. New vortex pair has two vortices with small enough circulations, therefore this vortex pair moves away from the wedge with small enough self-induced velocity.

The analysis of vortex interactions for other parameters x_0 shows that there is some critical value x_{cr} at which the type of point vortex interaction changes. The exchange interaction occurs at $x_0 > x_{cr}$. In this case vortex *1* when approaching the wedge forms a vortex pair together with the angular vortex and this pair leaves the wedge region. The vortex *2* goes along a solid surface in the direction of positive values of *x*-axis. If $x_0 < x_{cr}$ the initial vortex pair generates two angular vortices, which move independently from the wedge. The shorter the distance between initial vortex pair and

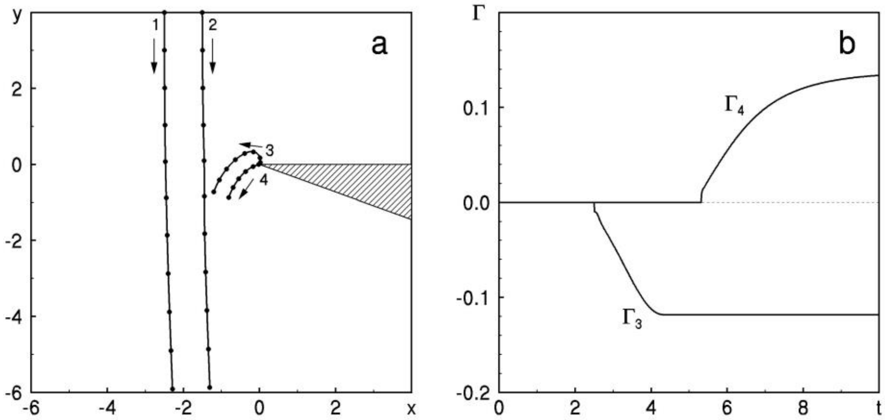

Fig.9. Interaction of the vortex pair with a wedge, $x_0 = –2.0$:
a) trajectories of vortices, b) the circulation of the third vortex

wedge vertex, the lower the intensity of new vortex pair. Research shows that the critical parameter x_{cr} is in range $–1.0 < x_{cr} < –0.9$ and depends on the value β.

4.2. Comparison point vortex model with an experimental data

The experimental research was carried out for interaction of vortex pair with a plate, $\beta = 0^o$. Vortex pair was formed at a sufficient distance from the plate, therefore the influence of solid surface on a dipole generation and on an initial motion are minimal. Fig.10 shows contour plots during interaction with a wedge for case x_0 = –1.4. The time interval between figures is Δt = 5.0 *s*. Initial distance between vortices in the dipole is b = 100 *mm*, and vortex intensities have $|\Gamma_1| = |\Gamma_2| = 1.8 \cdot 10^4$ mm^2/s. In this case self-induced velocity of vortex dipole is $U_s \approx 30$ *mm*/*s*. During generation process the intensity of vortices is not equal and the dipole is not moving perpendicularly to the wall. Therefore initial offset x_0 is defined at the following moment of approaching of vortex dipole to the plate when intensity of vortices become approximately equal and dipolar vortex goes perpendicularly to the solid surface.

The initial vorticity distribution is shown in fig.10,a. This moment corresponds to t = 0 *s*. Hereinafter contours for negative vorticity are plotted by dashed lines, and contours for positive volumes are shown by continuous lines. Moving nearer to the edge the dipolar vortex generates an angular vortex with positive vorticity, t = 5.0 *s*. The angular vortex begins to leave the wedge, however it appears that intensity of angular vortex is not enough to influence the forward motion of the initial vortex dipole (fig.10,c). As a result, the vortex dipole has passed the region of the edge without essential changes in the trajectory, and the angular vortex stopped at a distance of about 100 *mm* from the vertex of the wedge. The vorticity distribution for the moment t = 15 *s* is shown in fig.10,d.

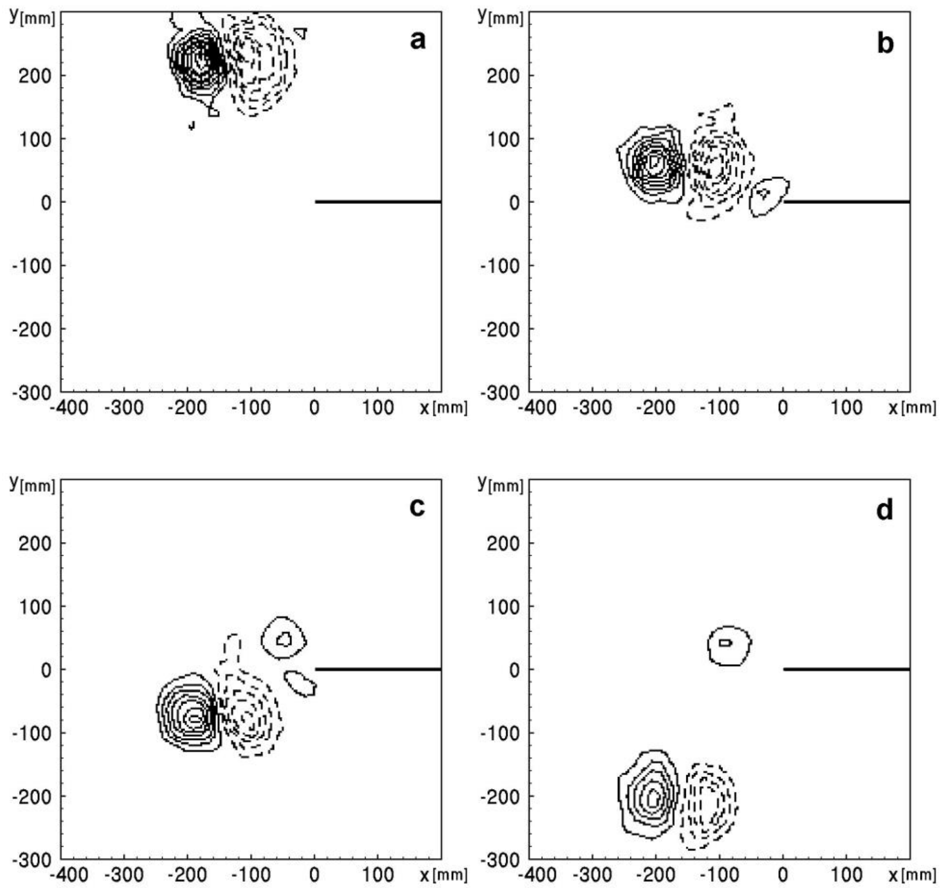

Fig.10. Interaction of the dipolar vortex with a wedge, $x_0 = -1.4$ (experiment) at:
a) t = 0 s, b)t = 5.0 s, c) t = 10.0 s, d) t = 15.0 s.

Consider an advection process of a passive fluid in a velocity field of vortex pair interacting with a wedge for case considered before, $x_0 = -1.4$. Let vortex pair at the initial moment is inside the marked circular region of radius $r_0 = 1.1$ with the centre in a point (x_0, y_0). Here we apply notation in fig.1. The size of circular region exceeds a little the dimension of a vortex atmosphere.

Initial vortex position and the region of passive fluid under investigation is shown in fig.11. Initial distance between vortices in the pair $b = 1.0$, the vortex intensity are $|\Gamma_1| = |\Gamma_2| = 1.0$. As before $y_0 = 4.0$. Filled circles in figures indicates vortex position through equidistant moments $\Delta t = 1.0$. When vortex pair passes the wedge peak, the angular vortex *3* is formed. The vortex positions and atmosphere of vortex pair is shown in fig.11,b for moment $t = 3.0$. When initial vortex pair passes the edge, a new angular vortex is generated. As a result, the angular vortex pair is formed. This moment is shown in fig.11,c. New pair has no influence on the trajectory of initial vortex pair, but angular vortices have certain effect on the structure of vortex atmosphere. Final vortex positions that correspond to fig.10,d, is shown in fig.11,d. It

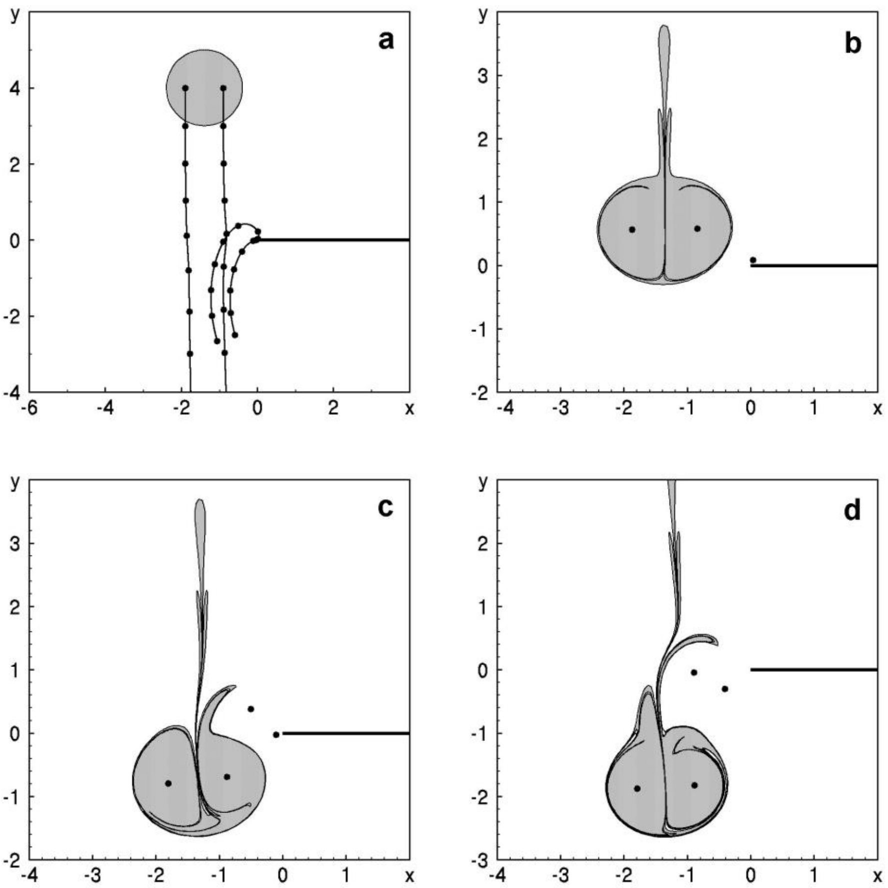

Fig.11. Interaction of the vortex pair with a wedge, $x_0 = -1.4$:
a) trajectories of vortices at moments $t_n = n\Delta t$, $\Delta t = 1.0$, b) atmosphere at $t = 3.0$,
c) atmosphere at $t = 6.0$, c) atmosphere at $t = 9.0$.

is visible that vortex pair continues its own motion, while the angular vortex pair moves with smaller self-induced velocity. The comparison fig.10 and fig.11 allows to conclude that numerical and experimental data agree.

Exchange interaction of initial vortex pair with an angular vortex takes place for $x_0 = -0.5$. This case is shown in fig.12 with notations introduced above. Approaching of vortex pair to the wedge results in forming of an angular vortex with rather high intensity (fig.12,b). As a result, the vortex pair, in which one vortex is replaced by an angular vortex, is formed. Then the vortex pair leaves the region of the edge. Vorticity distribution for typical moment $t = 15$ s is shown in fig.12,c. On the other hand, the right vortex from dipole with positive vorticity meets solid surface. The influence of a border results in displacement of the vortex in the positive direction of x-axis. Note that the close solid border results in generation of a secondary (negative) vorticity, which surrounds a moving initial vortex. The final distribution of vorticity field is shown in figure with index “d”.

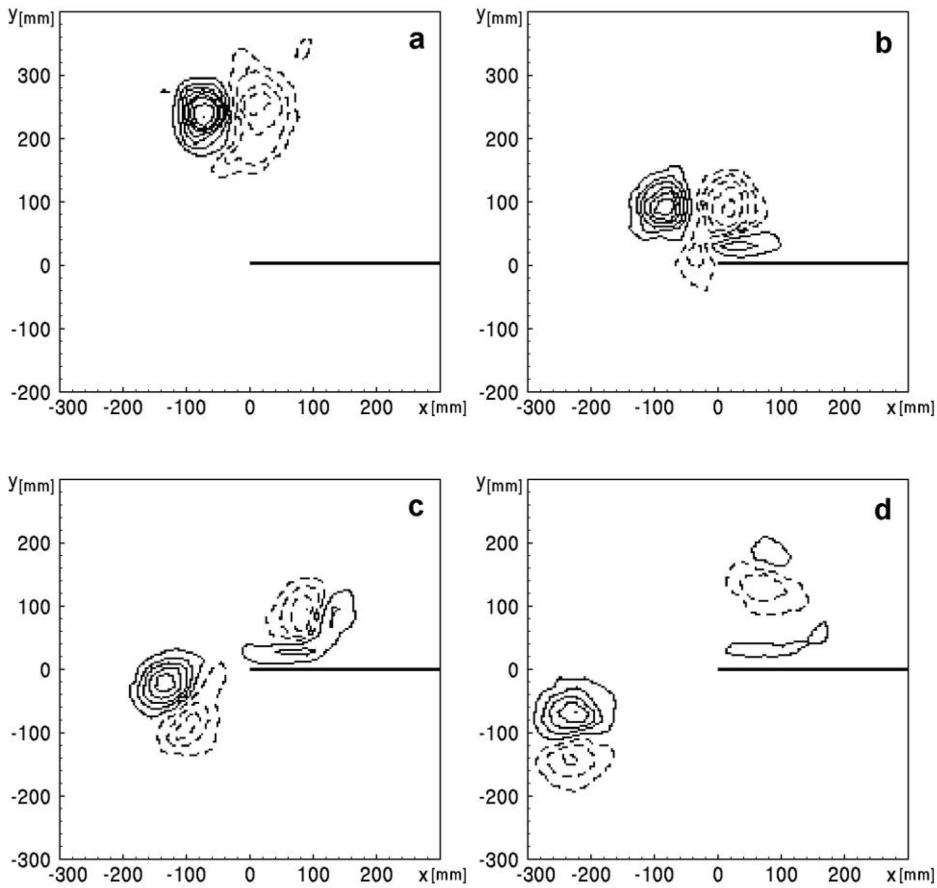

Fig.12. Interaction of the dipolar vortex with a wedge, $x_0 = -0.5$ (experiment) at: a) t = 0 s, b)t = 5.0 s, c) t = 10.0 s, d) t = 15.0 s.

Advection of a passive fluid is shown in fig.13 for case $x_0 = -0.5$. The size and shape of vortex atmosphere at the initial stage of approaching of vortex pair to the solid surface does not practically differ from the case considered before. The intensive generation of the angular vortex results in intensive rotation of the contour, which is involved in a velocity field of angular vortex (fig.13,c). Then, new vortex pair is formed with the vortex atmosphere consisting as from both dyed and fresh fluid accompanied by angular vortex. The comparison of advection processes allows to conclude that the growth rate of intensity of an angular vortex have strong influence on the advection process of passive fluid.

Let's now consider interaction of the vortex dipole with the wedge for $x_0 = 1.0$, which is shown in fig.14. At the initial moment both vortices in dipole interact with the flat surface. The induced velocity field results in division of vortices in dipole and they start motion in the opposite directions. It is interesting to note that both vortices also form region of secondary vortices positioned between initial vortices and solid surface. In time, the left vortex gets to the wedge vertex and generates a new angular

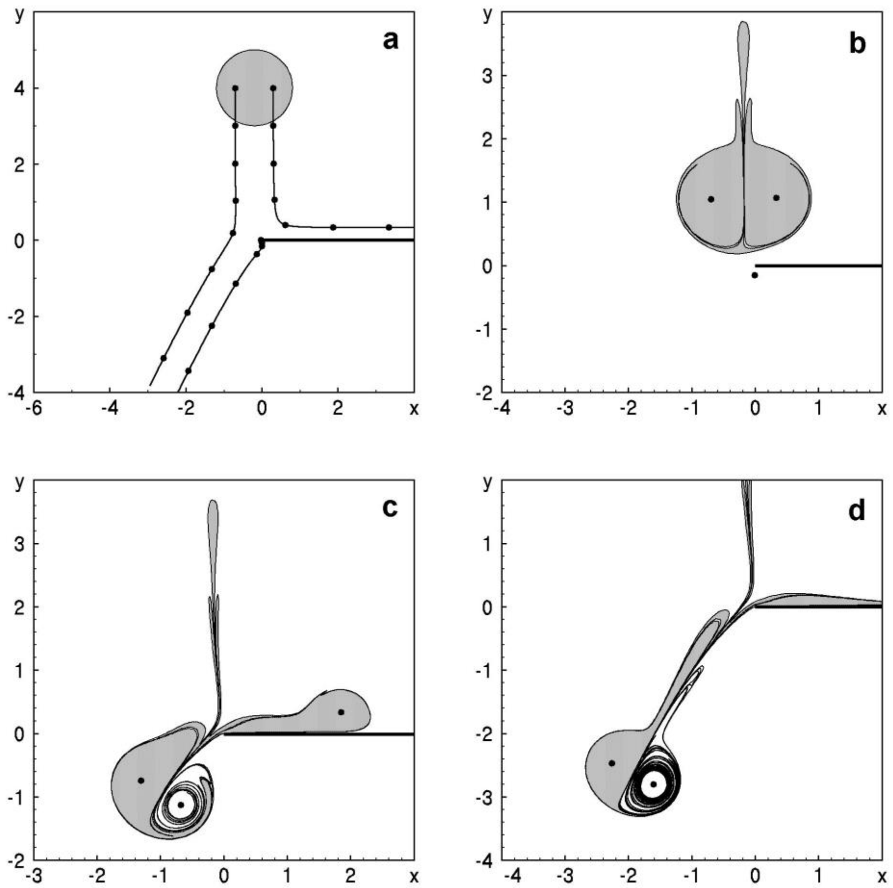

Fig.13. Interaction of the vortex pair with a wedge, $x_0 = -0.5$:
a) trajectories of vortices at moments $t_n = n\Delta t$, $\Delta t = 1.0$, b) atmosphere at $t = 3.0$,
c) atmosphere at $t = 6.0$, c) atmosphere at $t = 9.0$.

vortex. As a result, a new vortex pair moving away from the wedge is formed. As the intensity of the initial vortex appears a little bit less compared to intensity of the angular vortex, the pair goes under some angle to the surface. The final vortex distribution is shown in fig.14,d.

If $x_0 = 1.0$, the vortex pair interacts at the initial stage with one of the flat surfaces of the wedge (see also fig.4, then vortex *1* passes near the vertex forming an intensive angular vortex). The advection process in this case is presented in fig.15. By analogy with previous cases the initial position of vortices, circular dyed region of passive fluid, and vortex trajectories are shown in figure with an index “a”. Small filled circles show positions of vortices in equidistant time intervals $t_n = n\Delta t$, where $\Delta t = 1.0$.

Position of the marked fluid region under investigation at the moment $t = 3.0$ is shown in fig.15,b. We can see that vortex pair approaches a flat surface and forms a vortex atmosphere, the shape of which represents an ellipse with major axis in horizontal direction. As the initial marked region slightly exceeds sizes of vortex

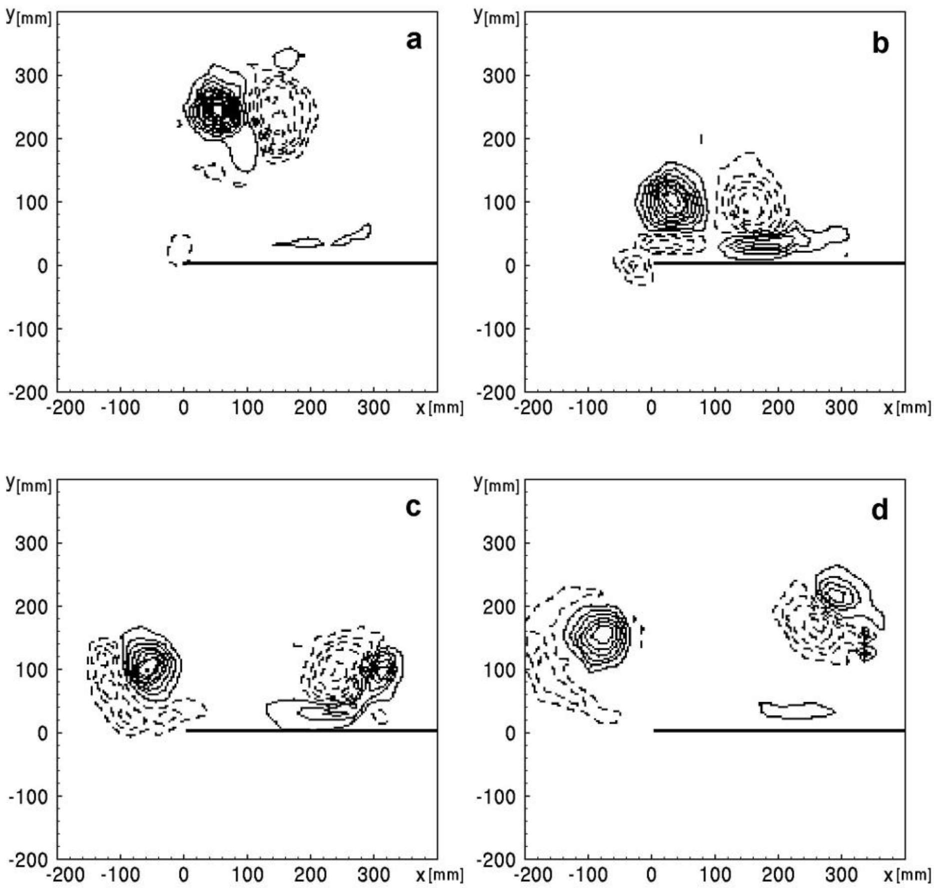

Fig.14. Interaction of the dipolar vortex with a wedge, x_0 = 1.0 (experiment) at: a) t = 0 s, b)t = 5.0 s, c) t = 10.0 s, d) t = 15.0 s.

atmosphere, “excess” fluid leaves the vortex pair from the stern part, forming a typical vortex tail.

The further motion of vortices along the surface results in division of the vortex atmosphere on two approximately equal parts. If evolution of vortex *2* does not undergo any changes, vortex *1* coming nearer to a vertex of the wedge, forms an intensive angular vortex, which partially involves a part of the dyed fluid. The formation process of vortex pair atmosphere is well shown in fig.15,d. Note, that the left part of this atmosphere rotates around vortex *1* and contains mainly the dyed fluid, while other part rotating around the vortex *3*, consists of fresh fluid. Here the dyed fluid occupies only a thin part of a peripheral zone of vortex atmosphere. It is interesting to note that there is a typical tail behind the new vortex pair.

Dependencies of relative change of contour length in time for different interactions of vortex pair with the wedge are shown in fig.16 for a typical value x_0. It is shown that intensive stretching of investigated contour takes place in cases when the right vortex in vortex pair passes in minimal distance from the wedge. In these cases two additional angular vortices are formed. In some cases, vortex generation occurs inside

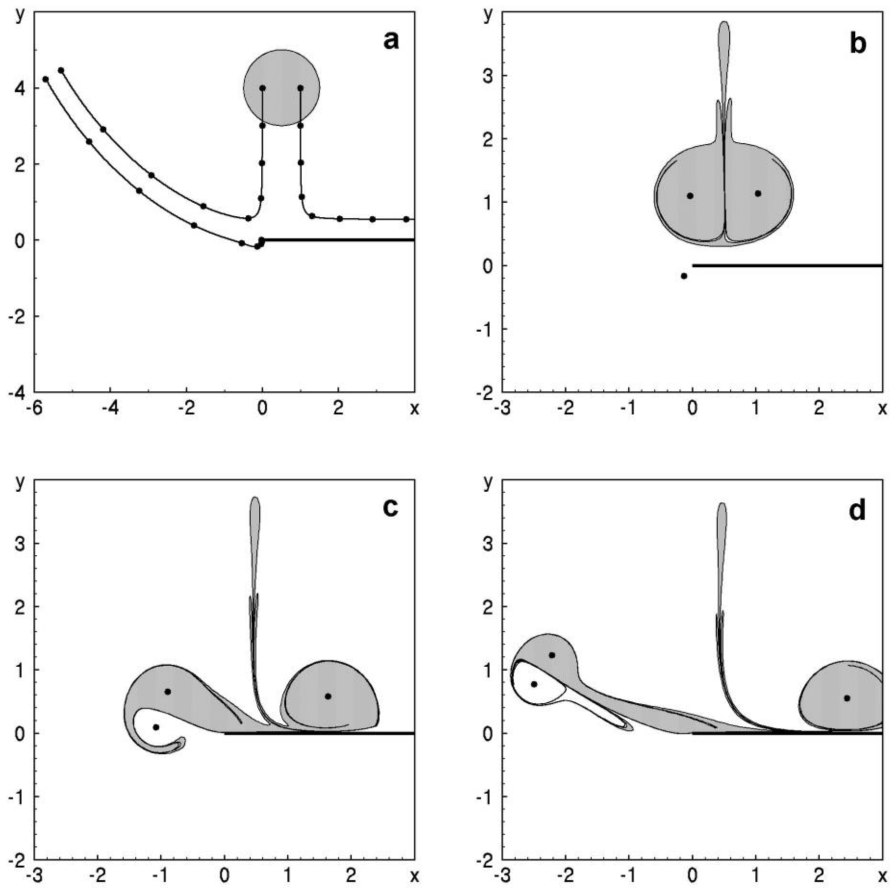

Fig.15. Interaction of the vortex pair with a wedge, $x_0 = 1.0$:
a) trajectories of vortices at moments $t_n = n\Delta t$, $\Delta t = 1.0$, b) atmosphere at $t = 3.0$,
c) atmosphere at $t = 6.0$, c) atmosphere at $t = 9.0$.

the initial atmosphere of vortex pair. This fact, probably, is the main reason of intensive stirring process of passive fluid during interaction of vortex pair with the wedge.

6. Conclusions

The interaction of the two-dimensional vortex pair with a wedge formed by two half-planes was investigated. The main type of evolution of large-scale vortex structures and advection processes in the velocity field induced by vortices in the region adjacent to the edge of the wedge was studied both theoretically and experimentally. The numerical model based on the dynamics of point vortices is formed, which are tested according to the results of a laboratory experiment.

In the experiment, a dipolar vortex was generated by raising a circular cylinder, which has been initially filled with fluid, from the tank rotating at a constant angular velocity. Studies have shown that there are at least three possible types of interaction between the vortices. If the dipolar vortex moves at a sufficiently large (in relation to

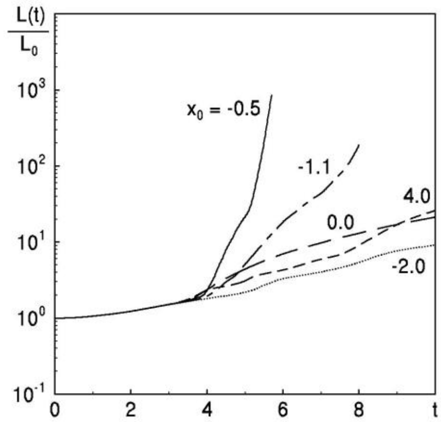

Fig.16. Relative contour length changes in time for various x_0

the distance between the initial vortices) distance, vortex dipole remain in an undisturbed state and does not interact with a sharp edge of the wedge. This interaction occurs at $x_0 > 1.5$ (fig.1). Another type of vortex motion associated with the interaction in which the closest vortex to the edge loses some intensity during generation of secondary vorticity field that is formed near the top solid surface. This process leads to a curvature of the trajectory of dipolar vortex after passing through a region containing the edge. This interaction is observed in the experiment for values $0.5 < x_0 < 1.5$. When the displacement is $-1.0 < x_0 < 0.5$, the initial dipolar vortex generates two new dipoles. These vortices move away from the wall along asymmetric trajectories. Finally, if $x_0 < -2.0$, then two vortices in the formed dipolar structure are symmetrical.

Analysis of the interaction of two point vortices with a sharp wedge was carried out numerically in the approximation of an ideal incompressible fluid. To eliminate the infinite velocity at the sharp edge and to satisfy Kutta-Zhukovsky condition we add the corner point vortex to the system under consideration. Its intensity is determined by the boundary conditions at the top of the two-dimensional angle. However, in contrast to the classical method [8,23,29] in our research we propose to use only one corner vortex, located near the edge of the surface. If the influence of the induced velocity field from vortex pair is small, then corner vortex does not leave the region with the edge. In this case, the corner cortex has the sufficiently small intensity and does not have an influence both on the flow near the solid surfaces, and on the moving vortex pair. However, the vortex pair approached to the edge can increase (by module) of the intensity of the corner vortex. In this case, we need to take into account the variable intensity of the corner vortex and to determine the value of its intensity for each moment in time. When, for various reasons, the additional vortex leaves the angular region, we have to introduce again another corner vortex to the system under consideration. Intensity of this vortex is determined by Kutta-Zhukovsky condition. At this moment, the intensity of the previous corner vortex is fixed. During the calculation of the amount of angular vortices introduced into the system is not limited.

During an evolution the vortex pair engages in own motion the part of the nearby fluid, forming so-called “vortex atmosphere”. This cloud at the steady motion of vortex pair has an elliptical shape, and the size of the cloud does not change over time. The interaction of vortices with the corner region disturbs the stationary motion of the vortex pair. As a result, the fluid mixing process occurs with different modes in fluid velocity field induced system point vortices near the edge of the solid surface.

The exchange interaction of the vortex pairs with a corner vortex leads to quite intense mixing process of passive fluid moved initially inside the vortex cloud. Studies show that the corner vortex stimulates more intensive mixing process compared with same process induced by formed vortex pairs. These results are confirmed by laboratory experiment.

These studies are dedicated to the memory of our colleague (G.J.F. van Heijst, L.Zannetti) and teacher (A.Gourjii) prof. Slava Meleshko (Taras Shevchenko National University of Kyiv, Ukraine) tragically died in 2011. He was the initiator of this research, carried out a significant part of a coordination work and actively participated in the writing initial versions of this paper.

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