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Mathematical modeling of hydraulic networks of structurally complex technological schema

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The paper analyzes some methods and means of hydraulic network simulation. A method for calculating parameters of such a network is developed. The method is based on the system of differential equations written for the network nodes. The proposed model can be applied for both designing the new piping systems (hydraulic networks) and virtual testing the existing systems under "stress" or emergency conditions.

Keywords: *hydraulic network modeling, system of equations, coefficient of hydraulic resistance, pressure, node.*

Анализируются методы и средства моделирования гидравлических сетей. Предлагается метод расчета параметров гидравлических сетей, разработанный на основе системы дифференциальный уравнений узлов. Предложенная модель позволяет, как проектировать новые системы трубопроводов (гидравлические сети), так и производить оценку работы уже существующих систем в «стрессовых» или аварийных условиях работы.

Ключевые слова: *гидравлическая сеть, моделирование, система уравнений, коэффициент гидравлического сопротивления, давление, узел.*

Аналізуються методи і засоби моделювання гідравлічних мереж. Пропонується метод розрахунку параметрів гідравлічних мереж, розроблений на основі системи диференціальних рівнянь вузлів. Запропонована модель дозволяє як проектувати нові системи трубопроводів (гідравлічні мережі), так і робити оцінку роботи вже існуючих систем в «стресових» або аварійних умовах роботи.

Ключові слова: *гідравлічна мережа, моделювання, система рівнянь , коефіцієнт гідравлічного опору, тиск, вузол.*

1. The problem description and statement

Simulation of hydraulic network (HN) is an important part of the processes of setting up the new as well as operating the existing complex pipeline system. Models that describe a multy-section hydraulic network deal with the high dimension systems of nonlinear equations. Constantly growing complexity of real objects together with problems related to designing the new and to effective controlling the operating ones require to improve existing and develop new modeling and calculating techniques [1].

Pipelines are the complex dynamic systems whose characteristics are not constant, and the law, which governs the changes of these characteristics during the system operation, is not known in advance. Inaccuracies in piping construction embodiment and unsteady conditions of their operation cause significant deviation of actual resistance coefficients from those planned by design. These wrong coefficients are then embedded in equations solving the direct problem of flow distribution. Together with operation loads, which are known only approximately, this leads to unacceptable errors in the mentioned distribution calculations. Hence, the necessity to solve the inverse problems arises. One of such problems is refinement of resistance coefficients of piping network sections. The problems of this kind become even more essential when one has to pass from the pipeline system design to such a system management. The huge amount of hydraulic networks, which already have been built and now are in operation, brings to the fore the task of effective control of existing pipeline networks.

Hydraulic resistance, by definition, is the irreversible loss of fluid energy density (which transforms into heat a cause of the fluid viscous friction [2]) in different parts of hydraulic system (piping, hydraulic drive system, etc.).

To obtain reliable results needed for designing the new as well as for modeling the existing systems, one has to perform the long-term calculations based on measuring of angles between the pipeline sections, their lengths, crossections, etc., but the known models of hydraulic networks have insufficient accuracy while their calculation methods are not as fast as necessary.

The main objective of this study is to develop an efficient calculation method that beats the challenges of design and control of big and complicated pipe systems.

2. A method for hydraulic network modeling

In order to create a mathematical model of a system of pipelines (hydraulic network), it is necessary to build corresponding directed graph together with the matrix of its edges connectedness. Besides that, full information is required on geometric features of used pipes as well as on physicochemical properties of substances and mixtures that flow inside.

The theory of constructing hydraulic networks is presented in [3]. Also, this article proposes a method founded on said theory and an information technology that implements the method. This technology is based on technological schemes that has allowed eliminating any user’s participation in formation of the connectedness matrix for nodes, vertices and edges. When modeling a hydraulic network and its technological process, different computing modules exchange information automatically.

The input data for hydraulic network modeling are the following:

- piping section hydraulic resistances (hydraulic resistance is a coefficient that can be calculated based on pipe geometrical features);
- density of the substance that passes through the piping section;
- the piping section equivalent diameter;
- the degree of opening of the piping section valve;
- the source and the receiver pressure (initial approximation for solving the system of equations);
- connectedness of edges, nodes, and vertices.

The output data (the results of hydraulic network simulation) are:

- the mixture flow rate for each piping section;
- pressure in piping nodes (points of flow mixing/separation).

Evaluation of hydraulic resistance of the piping section depends on mixture flow mode, but in general case it is determined by both the frictional losses and the local losses.

Various local resistances cause changes of flow velocity direction and/or value. This induces (in addition to frictional) the extra energy (head) losses as result of bumps, local turns, etc. (Figure 1).

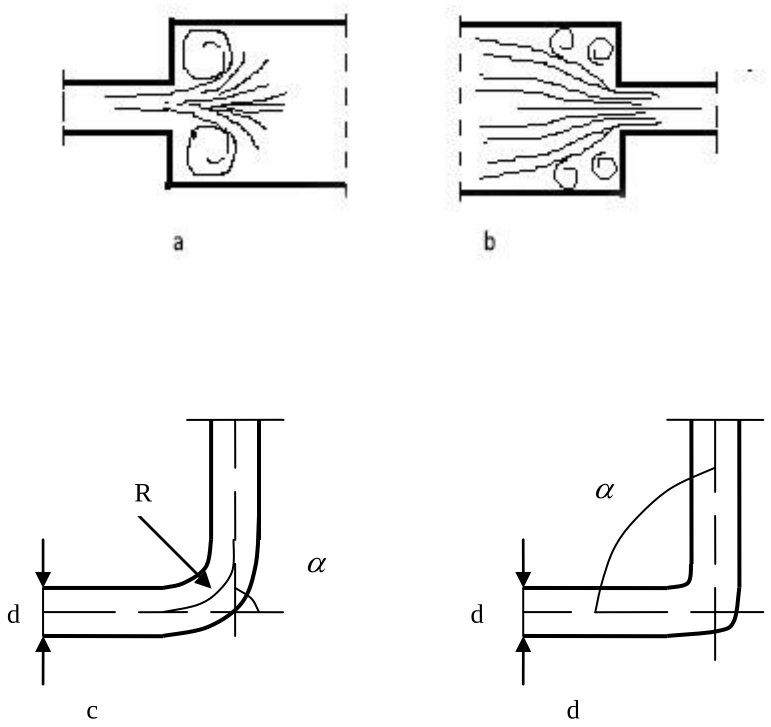

Fig. 1. Some local resistances: a - sudden expansion; b - sudden contraction; c - a smooth 90° turn (a branch); d - a sharp 90° turn (elbow).

The head losses due to local resistances as well as frictional losses are expressed as fractions of velocity head. The ratio of the head drop in a given local resistance $h_{м.с.}$ to the velocity head $\frac{w^2}{2g}$ is called the coefficient of local resistance and is designated as $\xi_{м.с.}$.

So, the expression for each of the local resistances is $h_{м.с.} = \xi_{м.с.} \cdot \frac{w^2}{2g}$, and for the total their sum:

$$h_{м.с.} = \sum \xi_{м.с.} \cdot \frac{w^2}{2g},$$

where $\xi_{м.с.}$ is an experimental value that could be found in handbooks.

The head loss h_n is:

$$h_n = (\lambda \cdot \frac{L}{d} + \sum \xi_{м.с.}) \cdot \frac{w^2}{2g} = (\zeta_{тр} + \sum \xi_{м.с.}) \frac{w^2}{2g}.$$

Consequently, the pressure loss across a pipeline section is:

$$\Delta P_n = h_n \rho g = (\lambda \cdot \frac{L}{d} + \sum \xi_{м.с.}) \cdot \frac{\rho w^2}{2} = (\zeta_{тр} + \sum \xi_{м.с.}) \frac{\rho w^2}{2}, \text{ N/m}^2 \text{ [4]}.$$

From the viewpoint of mathematical modeling, a network scheme is a graph. In other words, it is a structure comprised of a number of points (nodes) and connecting them segments (arcs, edges). In terms of hydraulic networks, the graph elements have a very clear meaning: the nodes represent feeders, consumers, wells, etc., while edges are pipes. In some cases, it is necessary to introduce additional nodes in those points where the measurement equipment is installed, as it can change the pipeline diameter, or influence the flow in other way. Since the direction of mixture movement through the network is determined, the nodes winding order at a given time is strictly determined too. Thus, the above graph is oriented.

The Fig. 1 illustrates the principle of nodes and flow rates naming. All nodes are numbered (named) randomly. Then the flow rate in the pipeline connecting the nodes i and j is denoted as Q_{ij}. Withdrawal from the node i is denoted as q_i. Any of hydraulic calculations deal with equations of two types: the equations reflecting the balance of mixture rates (conservation) in the network nodes and those for mechanical energy balance in network sections (edges) [5].

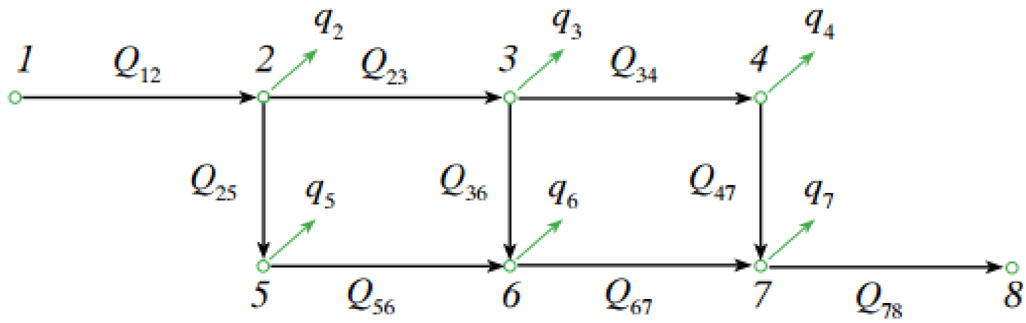

Fig. 2. The principle of naming the hydraulic network graph elements

In terms of hydraulic network, this means that the sum of mixture flow rates that enter a node equals the sum of the effluent mixture flow rates. For example, for node № 3 in Fig. 2, we have:

$$Q_{23} = Q_{36} + Q_{34} + q_3.$$

Condition of mechanical energy balance must be written for each pipeline. According to the Bernoulli's equation for the pipeline, wherein the mixture moves from node i to node j, neglecting the velocity heads, one has:

$$\frac{p_i}{pg} + z_i = \frac{p_j}{pg} + z_j + \Delta h_{w,ij},$$

where $\Delta h_{w,ij}$ is the head loss between the nodes i and j; $z_{j,i}$ are the vertical coordinates of the central points of crossections that are perpendicular to the flow velocity vectors; $\frac{p_i}{pg}$ is the hydraulic head.

The total number of equations that describe a network operation can reach several hundreds or even thousands. The system of these equations is nonlinear in flow rates, and this creates the major difficulties when solving it.

In the case of an arbitrary law of hydraulic resistance and unknown steady flow rates in pipes, the system of corresponding equations looks as follows:

$$
\begin{aligned}
&c_{11}x_1 + \ldots + c_{1n}x_n = q_1,\\
&\ldots\\
&c_{k1}x_1 + \ldots + c_{kn}x_n = q_k,\\
&c_{k+11}s_1\left|x_1\right|^{\beta_1 -1}x_1 + \ldots + c_{k+1n}s_n\left|x_n\right|^{\beta_n -1}x_n = h_1,\\
&\ldots\\
&c_{n1}s_1\left|x_1\right|^{\beta_1 -1}x_1 + \ldots + c_{nn}s_n\left|x_n\right|^{\beta_n -1}x_n = h_{n-k},
\end{aligned}
\tag{1}
$$

where

x_i is the flow rate in the pipe i;

coefficients $c_{ij} = 1,-1,0$ must be found from the first or second Kirchhoff's laws. In the first case, the influent flow in some point determines the coefficient value equal to 1, and the value -1 corresponds to the effluent flow. In the case of the second Kirchhoff's law and nonlinear equations, the values of coefficients c_{ij} can be either +1,-1(depends on the winding order) if the section j is included into the calculation cycle, which corresponds to the i th nonlinear equation, or $c_{ij} = 0$;

h_i is the sum of all productive heads in all edges of i th contour;

q_i - inflow into (withdrawal from) a node;

β_j is the degree in the law that determines dependence of the head drop on discharge.

The matrix A comprised of coefficients of equations listed in (1) is the matrix of the hydraulic network graph. The system (1) can be represented in vector form:

$$Ax = Q. \tag{2}$$

Adding the matrix of circuits B, we obtain the compact form of second Kirchhoff's law for the whole scheme:

$$BSXx = H, \tag{3}$$

where

$$S=\begin{bmatrix} s_1 & \ddots & 0 \\ 0 & & s_n \end{bmatrix} \text{ and } X=\begin{bmatrix} |x_1|^{\beta_1-1} & \ddots & 0 \\ 0 & & |x_n|^{\beta_n-1} \end{bmatrix}. \tag{4}$$

Let us introduce new variables to refer to the matrix and vector in the right part of the nonlinear system of equations:

$$F(X)=\begin{bmatrix} A \\ BSX \end{bmatrix}, G=\begin{bmatrix} Q \\ H \end{bmatrix}, \tag{5}$$

then the system of equations (1) in vector form looks like

$$F(X)x=G. \tag{6}$$

From the fact that (1) is built in accordance with Kirchhoff's laws for the hydraulic circuits, the following theorem follows.

Theorem: *Let x is a vector solution of the system of nonlinear equation (1) composed for the flow rates calculation problem, let x is such that each its component* x_i *is not zero. If* $F(x)$ *is the system matrix then this x is the unique solution of the equation* $F(x)x=G$.

The new method to solve the system (6) was developed based on modified successive approximation method proposed by R.T. Faizullin together with K.V. Loginov.

Solution to the problem is sought as the limit of iterations of the following form:

$$\begin{gathered} x^{(k+\frac{1}{2})}=\alpha x^{(k)}+(1-\alpha)x^{(k-\frac{1}{2})} \\ F(x^{(k+\frac{1}{2})})x^{(k+1)}=G, \end{gathered} \tag{7}$$

where F is the system (6) matrix; $x^{(k)}$, $x^{(k+1)}$ are the approximate solutions obtained in the steps k and $k+1$, correspondently; $x^{(k-\frac{1}{2})}$, $x^{(k+\frac{1}{2})}$ are some intermediate solutions; $\alpha\in(0,1)$. For example, in the case of quadratic law of resistance, $\alpha=0.5$. The node pressure values were taken as the initial approximation.

This is such a modification of simple iteration method, each step of which is preceded by averaging of some extremum values and followed by finding new extremums based on approximation obtained at this step. This is necessary because the simple iteration method does not converge: at some iteration, the approximation oscillates in several areas.

$$\sum Q_{ik}<\varepsilon_q, \tag{8}$$

$$|x_{ik}-x_{ik-1}|<\varepsilon_x, \tag{9}$$

$$x_i=\frac{x_{i_{\min}}+x_{i_{\max}}}{2}. \tag{10}$$

The aim of iteration process is to approximate the values of pressure in network nodes to use them further as parameters in the system of equations. The flow material balance expressions written for nodes (8) and the minimum difference (with a given accuracy) between pressure values in each node i obtained at iterations k and $k-1$ (9) are used as conditions to obtain solution. The method algorithm is the following. The maximum pressure $x_{i_{\max}}$ for a node i is obtained as the highest pressure in the set of all nodes connected to i^{th} one; correspondently, the minimum pressure $x_{i_{\min}}$ is the lowest one. Before the next iteration, the averaging (10) is performed for each node. Then goes the iteration calculation. This done, new maximum and minimum values are obtained.

One of advantages of the proposed method is that it converges in a broad region. Also, the rate of this convergence is close to the square one, as is implied by the following statement.

Statement: *If, upon the iteration method (7) application, the Degree of Nonlinearity in the law of hydraulic resistance for each section is β, the parameter that determines the next iteration approximation $\alpha = \frac{1}{\beta}$, and the linear part residual at each iteration is equal to 0, then the modified method of successive approximation is equivalent to the Newton's method.*

This fact allows stating that the method converges provided there is a sufficiently good initial approximation. Unfortunately, the strong evidence of convergence without good initial approximation has not been achieved for general case. The proof is complicated by the fact that in general the operator $F^{-1}(x)H$ is not limited. However, analysis of the problem when its dimension was small (2 and 3) has shown that we can prove the convergence of our method under certain conditions. This as well as numerous experiments and practical application of the method confirm that the convergence takes place without requirement of good initial approximation [6-11].

3. Algorithms and data exchange in hydraulic networks simulation

Prior to the numerical modeling of hydraulic networks, one needs to prepare the input data. An algorithm for this preparation is shown in the Fig. 3.

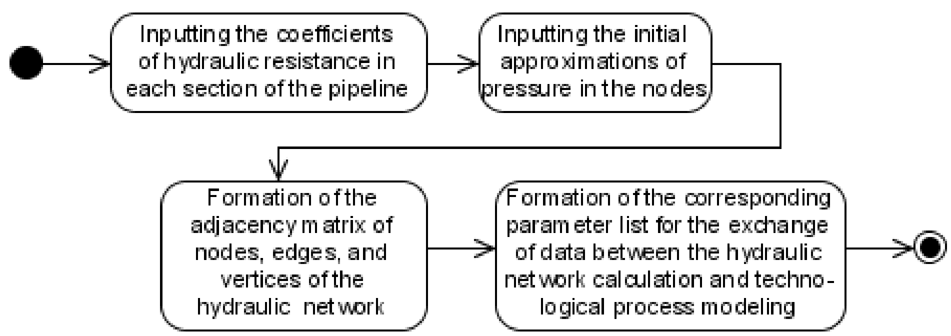

Fig. 3. An algorithm of input data preparation for hydraulic net simulation

To form the system of equations, one must have the connectedness matrix for edges, nodes, and vertices. The method to fill such matrix is described in [3].

Modeling of Hydraulic network and physicochemical processes inside it includes the dynamic analysis of processes in structurally complex systems. This requires to create a table of parameter compliance necessary for information exchange between calculations of two types. The data needed for numerical simulation of hydraulic networks (physical and chemical properties of the mixtures and the state of opening of valves along the flow way) are transferred from the technological model to the model of hydraulic network, and the data needed for numerical modeling of physicochemical processes (pressures and flow rates in the piping system sections obtained as solution of described above system) are passed in the opposite direction.

So, the data transfer is carried out by means of parameter compliance table. Then the cycle starts to process the hydraulic network nodes, which are pre-determined and are present in the connectedness matrix. The total number of nodes is n. Each node i connects k edges. In a loop over the edges of a given node, the equations reflecting the balance of flows in it are solved. The equations have been solved and balance achieved, the results are transferred to the module that simulates physicochemical processes. Then the iteration is repeated. In this way the continuous sequential modeling of the hydraulic network and internal physicochemical processes proceeds taking into account the external effects (valves and dampers control, changes in boundary conditions, etc.).

4. The numerical simulation results

Numerical simulation of a hydraulic network assumes solving a system of equations under condition of the balance convergence for each network node. When the proposed method is applied in the case of a new piping system, the needed resistance coefficients are calculated based on geometry of pipes and type of mixture that will run inside. In the case of a training system creation or testing some known network under altered boundary conditions (the system stress test), one can use the simplified formulas (sometimes even fitting method) because the sum of pressure drops along the edges is already known. In the last case (the coefficients are fitted based on pressure drop), the computation results would not be that accurate, but still they would quantitatively reflect the system behavior under some external impact.

The Fig. 4 shows the results of computation of pressure values in the hydraulic network nodes, which, in their turn, are the elements of devices of some technological scheme.

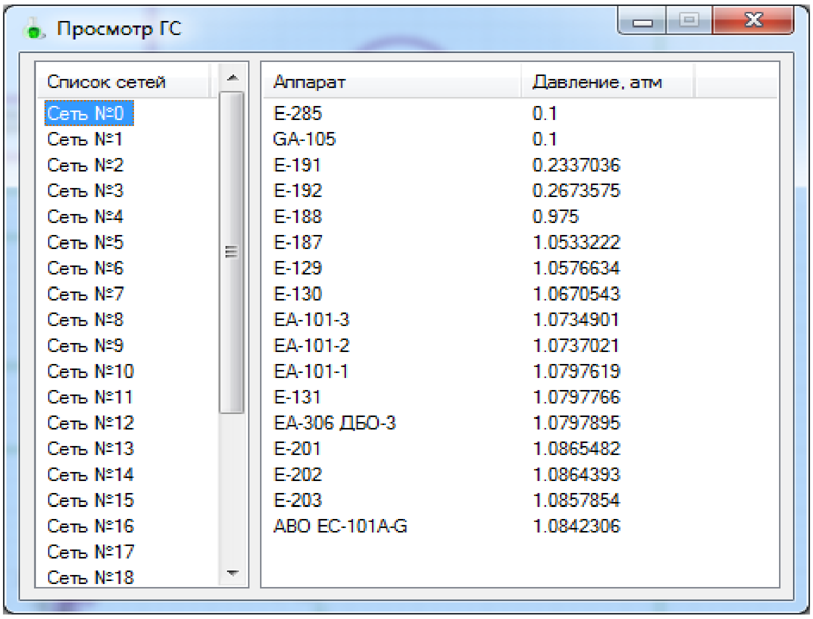

Fig. 4. The results of computation of pressure values in the hydraulic network nodes

The Fig. 5 represents the information window, which outputs one of hydraulic network calculation results, namely the mixture rate along the flow, which can be shown in different units depending on the mixture physicochemical properties.

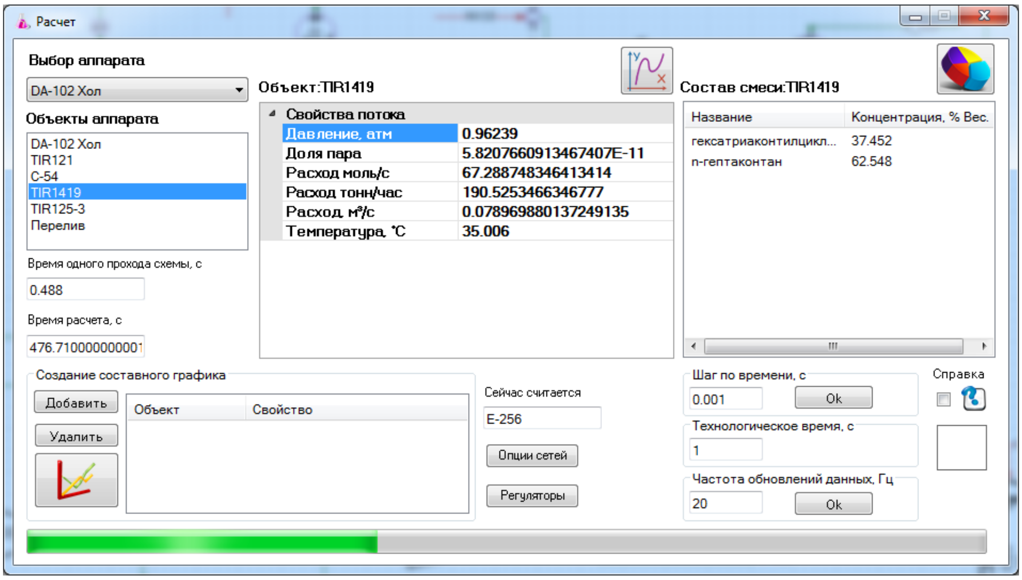

Fig. 5 Flow rate

5. Results and conclusions

Production technologies evolution and safety requirements stimulate the development of computer modeling of hydraulic networks and technological processes.

The method and process have been developed that allow computing the mixture flow rate in each of the pipeline sections and pressure in its nodes (points of flow mixing/separation), including those cases when the system has some defects.

Calculations performed using this technique have shown simulation results close to the real manufacturing parameters in both cases: when hydraulic resistance coefficients of the pipeline sections are known exactly or fitted.

In the future, the authors plan to improve and automate the process of fitting the coefficients for the case of existing pipeline network, as well as to optimize the algorithm of solving systems of equations due to reduction of the number of internal iterations for systems with the large number of nodes.

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