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Evaluation of accuracy of the methods for determining spatial characteristics of electron radiation depth-dose distribution

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In the work, the numerical methods of determining the standard characteristics of the electron radiation depth-dose distribution by processing the measurement results are compared with the empirical formulas linking the same characteristics with the electrons energy. The comparison results allow authors to estimate the accuracy of methods used in radiation technologies to determine the characteristics of electron radiation energy.

Key words: *electron radiation, depth-dose distribution, numerical methods, semi-empirical model, empirical formulas.*

У роботі проведено порівняння чисельних методів обробки результатів вимірювань для визначення стандартних характеристик глибинних розподілів дози електронного випромінювання і емпіричних формул, що зв'язують ці характеристики з енергією електронів. Результати порівняння дозволяють оцінювати точність визначення характеристик енергії електронного випромінювання, методами, які використовуються в радіаційних технологіях.

Ключові слова: *випромінювання електронів, розподіл дози по глибіні , чисельні методи, напів-імперична модель, імперичні формули.*

В работе проведено сравнение численных методов обработки результатов измерений для определения стандартных характеристик глубинных распределений дозы электронного излучения и эмпирических формул связывающих эти характеристики с энергией электронов. Результаты сравнения позволяют оценивать точность определения характеристик энергии электронного излучения, методами которые используются в радиационных технологиях.

Ключевые слова: *излучение электронов, распределение дозы по глубине, численные методы, полу-импирическая модель, импирические формулы.*

Introduction

In radiation technologies, computer simulation of dose distributions in the irradiated objects is the main stage of the irradiation process type and regime selection [1-4]. To be correct, the simulation needs reliable data representing characteristics of the irradiated target and the type of radiation as well the actual parameters of radiation installation itself. One of the problems concerning the radiation facilities that use the electron beams (EB) is determination of energy parameters of their beams. The standard methods for EB energy determining currently used in radiation-technological centers are based on measurements of the depth-dose distribution with the help of dosimetry wedge or stack.

Based on the measurement results, the spatial characteristics of electron radiation dose distribution are determined such as the practical range $R_{\mathbf{p}}$ and half-value depth $R_{\mathbf{50}}$. In turn, the values R_p and R_{50} are used in empirical formulas for calculation of EB energy characteristics, namely the most probable energy of electron $E_{\mathbf{p}}$ and its average energy $E_{\mathbf{av}}$.

The standards [2, 3] describe the formal procedures for determining the spatial characteristics of the dose distribution. However, because these measurements result in the huge set of discrete data, the said formal procedures for determining the practical range R_p and half-value depth R_{50} are reduced to solving incorrect mathematical problems.

Approximations of various types are used to obtain quasi solutions of incorrect mathematical problems form tabular data. It is clear that the accuracy of determined practical range R_p and half-value depth R_{50} depends on methods and functions, which are used to approximate measurements.

In addition, the standards [2, 3] do not tell which of approximation methods serve as the background for above empirical formulas correlating electron beam energy characteristics with the spatial characteristics of the depth-dose distribution.

Therefore, realization of methods recommended by the international standards, has the actual subtask to compare the empirical relations, which correlate the most probable energy of electron beam E_p and the practical range R_p, with the practical methods of determining the measured depth-dose distribution approximations, which are used then to find the practical range R_p and half-value depth R_{50}. The results of this comparison allow estimating the accuracy of methods, by which the spatial characteristics of depth-dose distribution of electron radiation are determined.

The R_p value definition is given in [1] – «The *practical range* R_p is defined as point where the tangent at the steepest point (the inflection point) on the almost straight descending portion of the depth versus absorbed dose curve meets the extrapolated bremsstrahlung background.» In accordance with this definition, the value R_p should be calculated from relations:

$$D_{rad} = D(x_p) + D'(x_p) \cdot (R_p - x_p) \tag{1}$$

$$x_p = \arg\max_{x \in [Rm, Ro]} (-D'(x)) \tag{2}$$

where

D_{rad} – value of the extrapolated bremsstrahlung background,

$D(x)$ – the depth-dose distribution of electron radiation,

$D'(x)$ – the first derivative of the depth-dose distribution of electron radiation,

x_p – the inflection point of the depth-dose distribution,

R_m – the depth at which dose distribution has the maximum value,

R_0 – continuous slowing-down approximation range of electrons.

The value x_p can be determined using the second derivative of the depth-dose distribution of electron radiation $D''(x)$ either as solution of equation

$$D''(x_p) = 0, \tag{3}$$

or by numerical methods of determining the position of an unconditional minimum of the function $D'(x)$ on the almost straight descending portion of the depth.

The method of the value R_p determining represented by the expressions (1) - (3) involves the first and second derivatives of the depth-dose distributions. The results of dose distribution measurements have the tabular form and the corresponding derivatives can be approximated by several different methods.

The common approximation method applied in practice of radiation sterilization centers is to use the linear approximation of data in the field of deep recession depending on the dose. Some known works have proposed and tested the methods, which approximate measurements in the area where the depth-dose distribution descends, utilizing polynomials of various degrees [5], in particular, the 4th degree polynomial [6].

However, the traditional as well as the proposed methods of polynomial approximation can use only a small part of the whole amount of measurements. For example, for linear approximation of the depth-dose distribution decreasing, only that depth range is used, in which the relative value of the dose varies from 0.8 to 0.2 of the maximum one.

The more complete information contained in the measurements of the depth-dose distribution is possible to obtain using computational methods based on physical models of electron radiation transfer in matter.

In this connection, an interesting approach was proposed by V.T. Lazurik [5]: to approximate the results of measurements of the depth-dose distribution by software EMID [7], which implements a semi-empirical model of the dose distribution of a monoenergetic electrons beam incident normally upon a semi-infinite target. In this approach, an approximation of the measurements results is performed by fitting the model parameters [8, 9,] with use the least squares method.

This computational method was successfully verified and validated using the dosimetry wedges on electron beam radiation facility in the Institute of Nuclear Chemistry and Technology, Warsaw, Poland [5, 12, 13].

The considered semi-empirical model provides good description of the dose distribution in the target depth. Therefore, it is expected that the half-value depth R_{50} (defined as depth at which the absorbed dose decrease equals 50% of its maximum value [1],) can be calculated with sufficient accuracy by PFSEM method.

However, according to the expressions (1) and (2), to calculate the values of practical range Rp, it is necessary to know the first and second derivatives. Note that, as a rule, the model built by approximating the empirical data dependencies does not provide a correct description of derivatives of these dependencies.

That is why, possibility to estimate the modeling accuracy of the dose distribution spatial characteristics R_p and R_{50} with the help of the semi-empirical model of electron energy decrease is of interest for the radiation technology. This stimulates further development of new methods for determining the energy of absorbed electrons.

In this study the empirical formulas linking the characteristics of the electron beam energy (most probable energy E_p and the average energy of electrons $E_{\mathbf{av}}$) with the spatial characteristics of depth-dose distribution (the practical range R_p and half-value depth R_{50}) are compared. The formulas are determined in the standards [1 - 3] and the tabular data are presented in [2]. As an indicator of the accuracy of empirical formula, it is proposed to use the relative deviation of the electron energy, for which the values

of the practical range R_p and half-value depth R_{50} are known, from the electron energy "restored" by this formula using this known R_p or R_{50}.

A series of calculations of the first and second derivatives of the functions used to approximate the results of measurements and computer realization of the semi-empirical model of electron radiation depth-dose distribution [4, 5] was carried out. Based on obtained results, the values of the practical range Rp were calculated according to the definition given in the standards.

Comparison of the results determined by processing the same measurements with different numerical methods allows estimating the accuracy of these methods and making recommendations for computer dosimetry development.

1. Comparison of accuracy of the standard empirical relations for dependence of electron energy E and the spatial characteristics of dose distribution R_p and R_{50} for electrons in aluminum.

For this comparison, three groups of empirical relations were selected:
the data from ICRU Report 35, ASTM Standard: E 1649-94, ISO/ASTM 51649

$$E_p=5{,}09*R_p+0.2 \qquad 5\ \text{MeV} < Ep < 25\ \text{MeV} \qquad (4)$$
$$Eav=6{,}20*R_{50} \qquad 10\ \text{MeV} < Ep < 25\ \text{MeV};$$

the data from ISO/ASTM Standard 51649

$$E =0.423+4.69*Rp+0.0532*Rp^2 \qquad 2{,}5\ \text{MeV} < Ep < 25\ \text{MeV} \qquad (5)$$
$$E =0.734+5.78*R_{50}+0.0504*R_{50}^2 \qquad 2{,}5\ \text{MeV} < Ep < 25\ \text{MeV};$$

and the data from ISO/ASTM Standard E 1649-94

$$E =0.069+7.44*Rp-8.56*Rp^2 \qquad 0{,}1\ \text{MeV} < E < 1\ \text{MeV} \qquad (6)$$
$$E =0.068+10.72*R_{50}-21.05*R_{50}^2 \qquad 0{,}1\ \text{MeV} < E < 1\ \text{MeV}$$
$$E =0.297+6.61*R_{50}-0.325*R_{50}^2 \qquad 1\ \text{MeV} < E < 10\ \text{MeV}$$
$$E =1.27+4.33*Rp+0.0596*Rp^2 \qquad 10\ \text{MeV} < E < 50\ \text{MeV}$$
$$E =2.15+4.65*Rp+0.223*Rp^2 \qquad 10\ \text{MeV} < E < 50\ \text{MeV}$$

As an indicator of error $K_{j,i}(E)$ of empirical formula, it is used the relative deviation of the electron energy E, for which was taken the value of spatial parameter Ri (either the practical range R_p, or half-value depth R_{50}) from the electron energy Ej(Ri). The last is the energy "restored" based on this Ri with the help of empirical formula from the j-th group:

$$K_{j,i}(E) = [E_j(R_i(E)) -E] / E, \qquad (7)$$

where:

- Ri(E) is the dependence of the spatial parameter of type i (i = p for practical range Rp and I = 50 for half-value depth R_{50}) on electron energy E,
- Ej(Ri) is determined by empirical formula from group j reflecting the dependence of electron energy on spatial parameter of type i,
- index j is the number of the above formula group, so it takes one of the values 4,5, 6.

The calculated $K_{j,i}(E)$ values are shown in the Tables 1 and 2. The first column of both tables contains the values of electron energy E. The second column contains the corresponding values of R_p (Table 1) and R_{50} (Table 2) given in [2]. The values R_{50} and R_p were used to calculate the electrons energy in accordance with the empirical formulas (4) - (6).

The calculated values of $K_{j,i}(E)$ are presented in the rest of Table columns. Two numbers in one cell (slash separated) show the calculation results obtained by two empirical formulas for energy values of 1 MeV and 10 MeV. The first value is obtained by the formula, which corresponds to energy domain below the point, the second value is the result of formula for domain above this boundary energy. Italics in the tables highlights the calculated values for the boundary energies (bold), and outside the scope of applicability of the empirical formulas.

It should be noted that for the energies of 1 MeV and 10 MeV, the first number is positive while the second one is negative. This shows that even the most complete and accurate approximation of the empirical data described in the standard ASTM E 1649-94 does not allow us to choose definitely the empirical formula from the provided set to perform calculations in domains near the boundary energy values of 1MeV and 10 MeV.

Table 1. – Errors of empirical formulas for the dependence of the electron energy $E(R_p)$ on the practical range R_p in aluminum.

E, MeV	**R_p**	**$K_{1,p}(E)$**	**$K_{2,p}(E)$**	**$K_{3,p}(E)$**
0,2	0,0161	-	-	**-6.7**
0,5	0,063	-	-	**0.7**
1	0,152	-	-	**0.2/0.2**
2	0,356	*0.6*	*5.0*	**0.04**
5	0,971	**2.8**	**0.5**	**0.00**
10	2,00	**3.8**	**0.2**	**-0.2/1.7**
20	4,04	**3.8**	**1.2**	**-1.3**
50	9.89	*1.1*	*4.0*	**-0.2**

Table 2. – Errors of empirical formulas for $E(R_{50})$ – the dependence of the electron energy on the half-value depth R_{50} in aluminum.

E, MeV	**R_{50}**	**$K_{1,50}(E)$**	**$K_{2,50}(E)$**	**$K_{3,50}(E)$**
0,2	0,0116	-	-	**-5.2**
0,5	0,0448	-	-	**1.2**
1	0,111	-	-	**-0.1/2.7**
2	0,259	-	*11.7*	**-0.6**
5	0,741	*-8.1*	**0.9**	**0.3**
10	1,59	**-1.4**	**0.5**	**-0.1/1.1**
20	3,28	**1.7**	**1.2**	**-1.0**
50	*7.56*	*-6.2*	*-5.4*	**0.1**

As it follows from the data presented in Tables 1-2, the differences between the electron energy estimations obtained with empirical formulas may amount to several percent and substantially depends on the energy values domain. This conclusion is consistent with general estimations of accuracy of empirical formulas presented in the official documents (standards).

The errors of empirical dependences of electron energy on the spatial parameters of the depth-dose distribution are caused by the following factors:

1. Uncertainty in selection of the method and function type to apply for approximation of measurements of the depth-dose distribution, which is required to determine the values of the spatial characteristics of the electron radiation dose distribution such as practical range Rp and half-value depth R_{50}.

2. Uncertainty in selection of the function type to apply for approximation of relations between the dose distribution spatial characteristics, such as practical range Rp(E) and half-value depth $R_{50}(E)$, and electron energy E.

3. The dependence of the results on the amount of data and the approximation domain size.

2. Comparison of methods for determining the spatial characteristics of electron radiation depth-dose distribution

The comparison was made of the data sets representing the depth-dose distributions calculated by Monte-Carlo method for detailed physical model and semi-empirical model of electron radiation transfer in matter.

The depth-dose distributions for monoenergetic electrons with energy E in the semi-infinite target were calculated on the basis of semi-empirical model according to [8] and using the Monte Carlo method. For these calculations were used the computing blocks "Analytics" and “Monte Carlo” of software package ModeRTL [4,5]. As a result, the values of dose $D_e(x, E)$ were obtained for 50 basic space points, which uniformly covered the interval of depths x, from the surface of the target up to $R_0(E)$ -continuous slowing-down approximation range of electrons.

Depth-dose distributions in the aluminum target were calculated for electron energies of 2, 5 and 10 MeV. Depth-dose distributions in the carbon, polystyrene, water and wood targets were calculated for electrons with energies of 10 MeV. The results of depth-dose distribution simulation by method Monte-Carlo have a relative statistical uncertainty not greater than 10^{-4}.

The data for depth-dose distributions obtained by both method Monte-Carlo and with semi-empirical model were approximated by the 3d and 4-th degree polynomials and linear function. The approximation parameters were adjusted by the least squares method.

For the polynomial approximations of discrete data, the domain of dose recession from its maximum value in the target was selected. For the linear approximation, the domain of dose recession was selected in accordance with the recommendations of standards, i.e., where the dose changes between 0.2 and 0.8 of the maximum value in the target. The number of spatial points, at which the approximation was carried out, was in the range 10 – 14 points.

The numerical calculations of the first and second derivatives were performed for the depth-dose distributions obtained by the Monte Carlo method and with the semi-

empirical model. The expressions for the same order derivatives of the approximating functions were obtained analytically.

The depth-dose distributions in the targets irradiated by electrons and their first and second derivatives obtained by various approximating methods are presented in Figures 1-6. The results obtained on the basis of the depth-dose distribution modeling by the Monte-Carlo method are marked as follows: "circles" correspond to values of depth-dose distribution, "diamonds" – to the first derivative of depth-dose distribution, triangles - its second derivative.

The vertical dotted line marks the spatial point x_p, which is determined according to equation (3) $D''(x_p) = 0$.

The simulation results obtained on the basis of calculations with semi-empirical model shown in Figures 1-4 are marked by continuous curves. The dashed curves shows the dependences of the first and the second derivatives on the depth-dose distributions obtained by data approximation with the of polynomial.

Dependencies of the first and second derivatives on the depth-dose distribution obtained by data approximation with 3d degree polynomial are shown in Figures 5-6 by continuous curves. The dashed curves shows the above dependencies (depth-dose distribution, the first derivative and the second derivative) obtained on the base of data approximation by linear function.

Polynomials approximating the data, which were obtained by method Monte-Carlo, are presented in Figures 1-6. The values that are necessary for the calculation of R_p in accordance with (1) are shown in the figures as the points of intersection of the vertical dashed line and the corresponding curve.

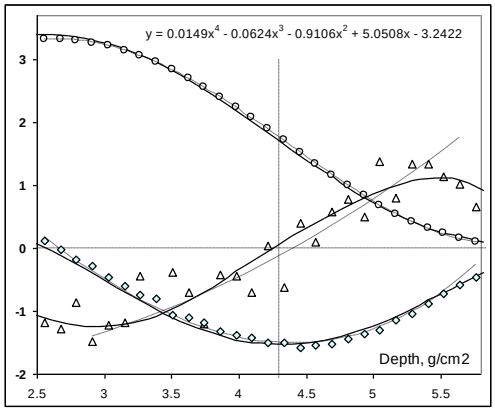

Fig. 1. Depth-dose distribution in the aluminum target irradiated by electrons with energy 10 MeV and its first and second derivatives obtained using semi-empirical models and approximation of data by a polynomia of the 4th degreel.

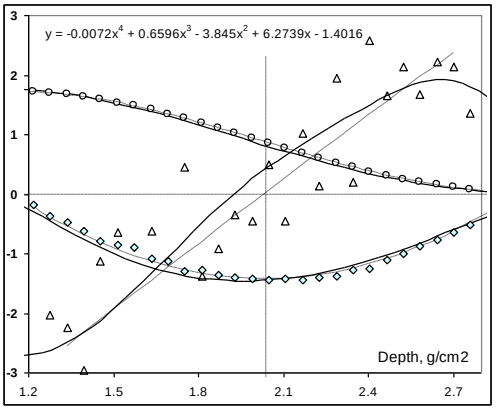

Fig. 2. Depth-dose distribution in the aluminum target irradiated by electrons with energy 5 MeV and its first and second derivatives obtained using semi-empirical models and approximation of data by a polynomial of the 4th degreel.

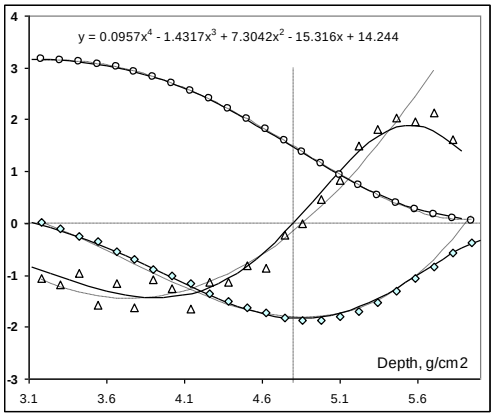

Fig. 3. Depth-dose distribution in the carbon target irradiated by electrons with energy 10MeV and its first and second derivatives obtained using semi-empirical models and approximation of data by a polynomial of the 4th degree.

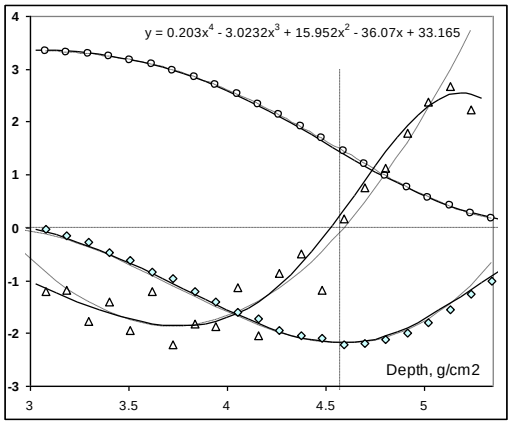

Fig. 4. Depth-dose distribution in the polystyrene target irradiated by electrons with energy 10 MeV and its first and second derivatives obtained using semi-empirical models and approxi- mation of data by a polynomial of the 4th degree.

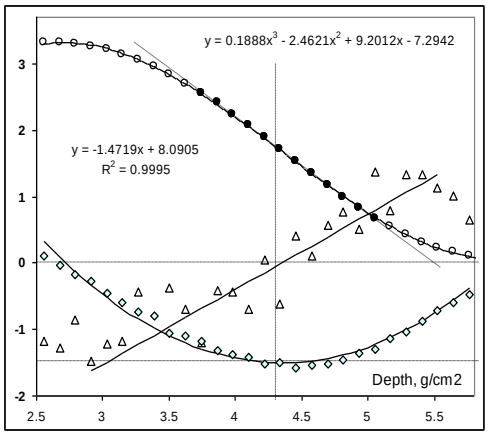

Fig. 5. Depth-dose distribution in the aluminum target irradiated by electrons with energy 10 MeV and its first and second derivatives obtained using approximation of data by a linear function and a polynomial of the 3d degree.

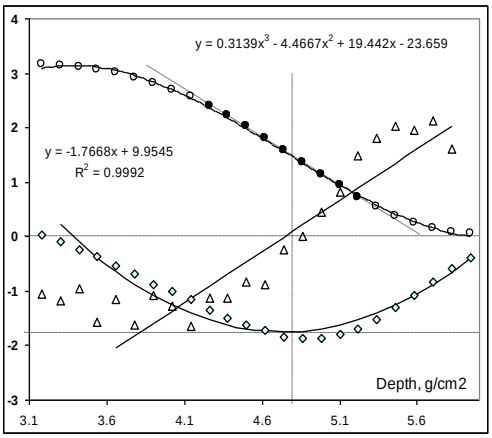

Fig. 6. Depth-dose distribution in the carbon target irradiated by electrons with energy 10 MeV and its first and second derivatives obtained using approximation of data by a linear function and a polynomial of the 3d degree.

As can be seen from the presented Figures, the semi-empirical model and polynomial approximation methods (using 3d degree and 4th degree polynomials) allows calculating the values of depth-dose distribution and its first derivatives with satisfactory accuracy.

The accuracy estimates for methods to determine the spatial characteristics of depth-dose distributions of electron radiation are performed on the base of practical range calculated using various methods to determine derivatives of the depth-dose distributions.

Comparison of dependencies for the second derivative of the depth-dose distributions, shown in Figures 1 - 6, leads to the conclusion, that the position of the point, which is determined by the second derivative of the depth-dose distribution, can have quite a large error.

Therefore, it is interesting how the error in practical range R_p value depends on the error in determined point x_p.

Let us consider the following function $R(x)$

$$R(x) = x - \frac{D(x) - D_{rad}}{D'(x)} \tag{8}$$

where x is the target thickness.

Here we use the notations introduced in the description of (1).

Obviously, $R_p = R(x_p)$.

The derivative $R'(x)$ is

$$R'(x) = \frac{(D(x) - D_{rad}) \cdot D''(x)}{(D'(x))^2} \tag{9}$$

$$\text{and } D''(x_p) = 0 \text{ gives } R'(x_p) = 0 \tag{10}$$

It means that the practical range R_p is an extreme value of function (8), which corresponds to the value of the argument $x = x_p$. The procedure of finding the value of practical range R_p described in [1] allows concluding that it is a local minimum.

It should be note, that equation (10) was obtained without significant assumptions about the function $D(x)$ and, therefore, is valid for different methods of calculating the practical range R_p. Therefore, we should expect a weak dependence of the magnitude of the practical range R_p error in determining the position of the point x_p, all of the numerical methods for processing the results of depth-dose distribution measurements, discussed in this paper.

The calculation results of dependencies $R(x)$ according to (9), with using of various computational methods for various materials are presented in Figures 7-10. Markers (circles) present the results obtained on the basis of simulation the depth-dose distribution with method Monte-Carlo.

Solid curves correspond to calculations using a semi-empirical model. The dashed curves represent calculations based on the data received by method Monte-Carlo for the case of the 3d and 4th degree polynomials approximation. The dashed strait lines are the linear approximation of data obtained by method Monte-Carlo in the area where the dose changes between 0.2 and 0.8 of its maximum value in the target. The vertical dashed line corresponds to the spatial point x_p found using a semi-empirical model.

As can be seen from the figures, the dependencies $R(x)$ have minimum in the points x_p that is in accordance with equation (10). Note that the position of minimum can shift significantly depending on the method of discrete data processing. However,

the uncertainty in point position x_p does not cause big changes in calculated value of practical range R_p.

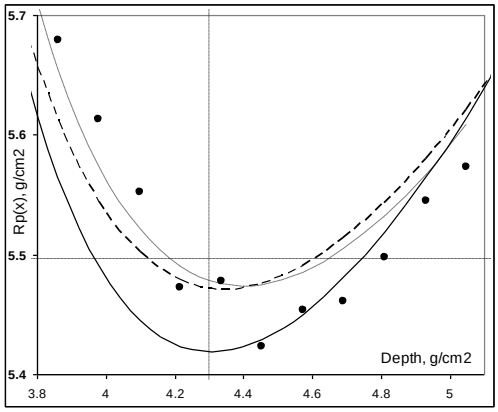

Fig. 7. The dependences $R(x)$ in the aluminum target irradiated by electrons with energy 10 MeV obtained using different methods of approximation of depth-dose distribution.

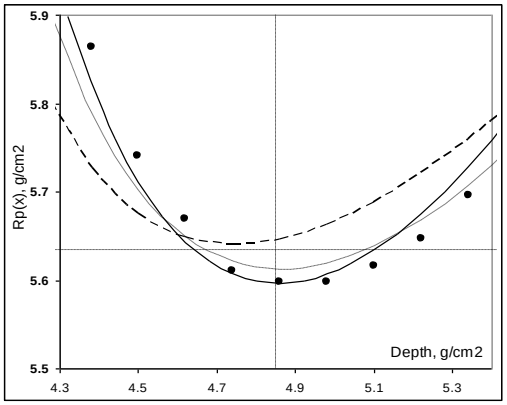

Fig. 8. The dependences $R(x)$ in the carbon target irradiated by electrons with energy 10 MeV obtained using different methods of approximation of depth-dose distribution.

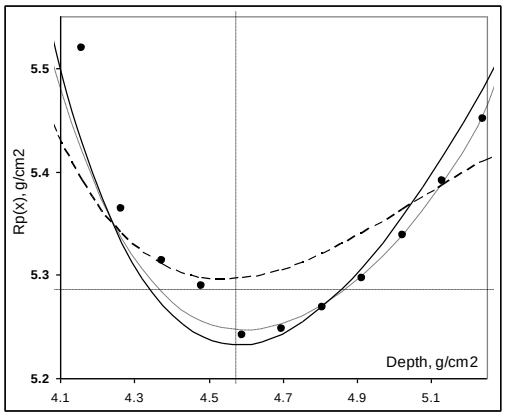

Fig.9. The dependences $R(x)$ in the polystyrene target irradiated by electrons with energy 10 MeV obtained using different methods of approximation of depth-dose distribution.

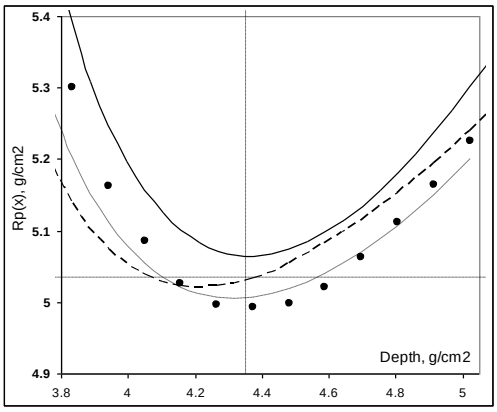

Fig. 10. The dependences $R(x)$ in the water target irradiated by electrons with energy 10 MeV obtained using different methods of approximation of depth-dose distribution.

It is interesting to note, that existence of the absolute minimum in dependence $R(x)$ causes systematic shift of calculation results for values of practical range R_p when the linear approximation is used for depth-dose distribution in domain of depths close to the point x_p.

The calculations results of the practical range values R_p obtained by various computational methods for various materials are presented in Tables 3-5.

Table 3. The results of calculation values for practical range obtained with different computational methods.

	MC	SM	M1	M3	M4	Av	σ	Unc., %
Al, 10MeV	5.424	5.423	5.497	5.471	5.475	5.458	0.033	0.60
C, 10MeV	5.598	5.6	5.634	5.64	5.613	5.617	0.019	0.34
PS, 10MeV	5.244	5.236	5.286	5.296	5.246	5.262	0.027	0.52
Water, 10MeV	4.995	5.068	5.034	5.023	5.006	5.025	0.028	0.56
Wood, 10MeV	5.254	5.332	5.295	5.302	5.256	5.288	0.033	0.62
Al, 5MeV	2.643	2.598	2.669	2.653	2.652	2.643	0.027	1.02
Al, 2MeV	0.968	0.95	0.978	0.968	0.968	0.966	0.010	1.05

Table 4. Deviations from the average for practical range values calculated with different computational methods.

	MC, %	SM, %	M1, %	M3, %	M4, %
Al, 10MeV	-0.62	-0.64	0.71	0.24	0.31
C, 10MeV	-0.34	-0.30	0.30	0.41	-0.07
PS, 10MeV	-0.33	-0.49	0.46	0.65	-0.30
Water, 10MeV	-0.60	0.85	0.18	-0.04	-0.38
Wood, 10MeV	-0.64	0.84	0.14	0.27	-0.60
Al, 5 MeV	0	-1.70	0.98	0.38	0.34
Al, 2 MeV	0.17	-1.70	1.20	0.16	0.16

To identify the columns of Tables 3-4 the following notation were used:

MC - data are obtained by Monte Carlo method, derivatives are calculated by the methods of numerical differentiation with minimal number of nodes, the result in g/cm^2.

SM - data are obtained from semi-empirical model, derivatives are calculated by the methods of numerical differentiation with minimal number of nodes, the result in g/cm^2.

M1 - data are obtained by Monte Carlo method, derivatives are calculated using linear approximation of data in a restricted range of values, the result in g/cm^2.

M3 - data are obtained by Monte Carlo method, derivatives are calculated using data approximation with a cubic polynomial in the range of depth where there is observed the maximum dose up to $R_0(E)$ range of the electrons, which was calculated in the approximation of continuous deceleration, the result in g/cm^2.

M4 - data are obtained by Monte Carlo method, derivatives are calculated using data approximation with a quadric polynomial in the range of the descending portion of the depth-dose distribution, the result in g/cm^2.

Av – the average value of practical range R_p obtained by variety of computational methods, the result in g/cm^2.

σ - standard deviation for values of practical range obtained by variety of computational methods, the result in g/cm^2.

Unc. - The uncertainty of calculating results for values of practical range, the result in %.

As it follows from the Tables, the practical range R_p calculated by various computational methods have relatively little uncertainty (see columns σ and Unc. In Table 3). It should be noted, that the highest values of uncertainty is observed in the case of the aluminum target irradiated by electrons with energies of 2 and 5 MeV. This is due to a significant deviation of the calculation results obtained using semi-empirical model (see. SM column in Table 4) with respect to the average value of practical range R_p.

This means, that the semi-empirical model does not provide the error of less than 1% for a wide range of electron radiation energy values. Comparison of the data presented in the Table 4, using the method of linear approximation of the data in a limited area values (column M1) leads to a systematic overestimation of calculating the value of practical range R_p. The reason for this overestimation has already been noted in this work at study the absolute minimum in dependency $R(x)$ at the point x_p.

The results obtained for the aluminum target irradiated by electrons with energies of 2,5 and 10 MeV allows to compare them with those given in [2]. These results already have been used in this study to evaluate the accuracy of standard empirical relations for the dependence of the electron energy E of the spatial characteristics of R_p and R_{50} dose distributions of electrons in aluminum.

Comparison results are presented in Table 5. The values of R_p practical range of [2] (see. similar column in Table 1) are presented in Table 5, the data column Av and σ are taken from Table 3, but are presented in units of g/cm2. Column ΔRp/σ shows the relative deviations for practical range values obtained in this work (column Av) from those given in [2], in terms of mean-square deviations (column σ).

Table 5. Comparison of calculation results for the practical range of electrons with various energies in aluminum.

E, MeV	R_p	Av	σ	$\Delta R_p/\sigma$
2	0,356	0,358	0,0038	0,526
5	0,971	0,979	0,0099	0,808
10	2,00	2,021	0,0122	1,721

As shown in the Table 5, the obtained results are consistent with standard [2] data at a level which does not exceed 2σ.

Conclusions

The paper compares and discusses the empirical formulas linking the characteristics of the electron beam energy (most probable energy E_p and the average energy E_{av} electrons) with the spatial characteristics of depth-dose distribution (the practical range R_p and half-value depth R_{50}), presented in international standards ICRU Report 35, ASTM Standard: E 1649-94, ISO/ASTM 51649.

It was shown that the difference in estimations of electron energy obtained by different empirical formulas can amount to several percent and substantially depends on the range of electron energies.

The errors of empirical formulas are associated with the following:

• uncertain choice of method and type of function for approximation of the depth-dose distribution measurements, which are needed to determine the values of the spatial characteristics R_p and R_{50} of the depth-dose distribution;

• uncertain choice of method and type of function needed to obtaining the empirical dependency R_p (E) and R_{50} (E) of spatial characteristics R_p and R_{50} on the electron energy E.

A series of calculations of the first and second derivatives of the functions used for approximation of the results of measurements as well as computer implementation of the semi-empirical model of depth-dose distribution of electron radiation were performed.

It is shown that semi-empirical model and polynomial (3d-degree and 4th-degree) approximation methods allow calculating the depth-dose distribution and its first derivative with satisfactory accuracy.

Comparison of the second derivative values of the depth-dose distribution approximated by various methods leads to the conclusion that the inflection point x_p, which is necessary to calculate the value the practical range R_p, cannot be determined without significant errors.

The error in the value of practical range R_p was investigated as function of error in position of the point x_p. It was shown that function $R(x)$, which represents the dependence of practical range on point x selected from the depth-dose distribution, has a minimum at $x = x_p$ for all numerical methods of processing of measured depth-dose distributions.

On this base the conclusion can be made that dependence of the error of practical range R_p on error in position of the point x_p is weak.

It was shown that existence of the absolute minimum of function $R(x)$ is the cause of systematic shift in results of practical range R_p calculation when linear approximation of the depth-dose distribution is used.

Based on the numerical differentiation the measurement results, the values of practical range Rp were calculated in accordance with the definition given in the standards. The calculation results are presented in graphical and tabular forms and illustrate the conclusions.

Comparison of results obtained by different numerical methods, which can be used for processing of measurement results, allowed us to estimate the accuracy of methods applied to determine characteristics of electron energy.

The relative uncertainty in practical range R_p calculated by different computational methods was estimated as not exceeding 1%.

The greatest deviations was obtained in the case of an aluminum target irradiated by electrons with energies of 2 and 5 MeV. This was linked with uncertainty in the values of depth-dose distributions calculated using a semi-empirical model.

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