DOI: https://doi.org/10.26565/2304-6201-2024-63-03

УДК 539.3+519.6

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Spectral boundary value problem for coaxial shells of revolution

The main objective of this study is to develop an efficient numerical approach using boundary elements to estimate natural frequencies of liquid vibrations in composite tanks. The spectral boundary value problem for liquid tanks is to find the natural frequencies and modes of free surface sloshing. The calculation of hydrodynamic forces on the walls of tanks with liquid is an important problem for ensuring the strength and stability of movement of industrial tanks and vessels. The vibrations of shell structures, including cylindrical and conical shells connected by rings, are analyzed. The area between the shells is filled with an ideal incompressible fluid. Numerical modeling uses the superposition method in combination with the boundary element method. A numerical solution of the spectral boundary value problem regarding fluid vibrations in rigid shell structures has been carried out. Frequencies and modes are determined by solving systems of singular integral equations. For the shells of revolution, these systems are simplified to one-dimensional equations, where the integrals are calculated along curves and line segments. Efficient numerical procedures are used to calculate one-dimensional integrals with logarithmic and Cauchy features. Test calculations confirm the high accuracy and efficiency of the proposed method. The importance and practical significance of the method lies in the ability to study fluid fluctuations in real compound fuel tanks of launch vehicles under different load conditions. This makes it possible to study the movement of liquid in fuel tanks and reservoirs under the action of external loads. The elaborated method will be used in computer modeling the dynamic behavior of liquid tanks and the stability study of liquid movement in compound fuel tanks of launch vehicles. In the future, it is planned to study the vibrations of elastic coaxial shells with liquid, using various composite materials

Keywords: *cylindrical-conical tanks, systems of singular integral equations, boundary element method, liquid sloshing.*

How to quote: Gnitko V. I., Degtyarev K. G., Kriutchenko D. V., and Naumenko V. V. “Spectral boundary value problem for coaxial shells of revolution.” *Bulletin of V.N. Karazin Kharkiv National University, series Mathematical modelling. Information technology. Automated control systems*, vol. 63, pp.29-37, 2024. https://doi.org/10.26565/2304-6201-2024-63-03

Як цитувати: Gnitko V. I., Degtyarev K. G., Kriutchenko D. V., and Naumenko V. V. Spectral boundary value problem for coaxial shells of revolution. *Вісник Харківського національного університету імені В. Н. Каразіна, серія Математичне моделювання. Інформаційні технології. Автоматизовані системи управління*. 2024. вип. 63. С.29-37. https://doi.org/10.26565/2304-6201-2024-63-03

1 Introduction

Shells of revolution are widely used as fluid storage tanks in water supply systems, oil and gas facilities, various industries, and nuclear facilities for storing liquids, chemical substances, and various wastes. When external loads are suddenly applied to partially filled tanks, significant splashing of the free surface occurs. This splashing refers to movements of the liquid's free surface inside the container, which has a substantial impact on the container's dynamic response. The spectral boundary value problem for liquid tanks involves finding the natural frequencies and sloshing modes of the free surface. Calculating hydrodynamic forces on the walls of liquid-filled tanks is crucial for ensuring the strength and stability of industrial tanks and vessels. Key aspects in studying the liquid include assessing the distribution of hydrodynamic pressure, forces, and moments, as well as determining the natural frequencies of the liquid's free surface. These parameters directly affect the dynamic stability and strength of the containers. Although full-scale experiments provide the most accurate assessment of tank strength under dynamic conditions, they are often expensive and dangerous. Therefore, computer modelling has become the forefront of modern scientific research in this area.

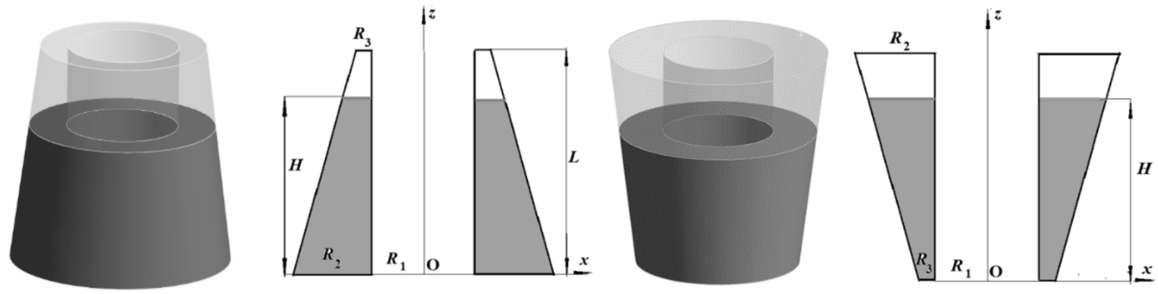
2 Problem formulation and literature review

The problem of fluid vibrations in tanks and reservoirs presents a significant challenge for various industries, including aerospace, chemical, mechanical, and nuclear engineering, as well as a complex task for physicists and mathematicians. Fluctuations of the liquid can lead to catastrophic damage to water and oil storage tanks and deviations from the calculated trajectories of launch vehicles. Due to these potentially dangerous effects, liquid sloshing in tanks has been the focus of many theoretical and experimental studies over the last few decades [1-3]. These studies have addressed many important phenomena, especially the linear and nonlinear sloshing effects in both inviscid and viscous fluids [4-6]. General overviews of existing methods to the problem of fluid oscillations are provided in [7-9]. It is noteworthy that while compound shells of revolution are typical structures in fuel tanks for launch vehicles and tanks used in the automotive, chemical, and agricultural industries, the study of the oscillations of such tanks has received insufficient attention in the scientific literature [10-12]. Therefore, this study, dedicated to solving the spectral boundary value problem concerning fluid oscillations in compound rotational shells, addresses a highly relevant and topical issue.

3 The research aim and problem statement

The purpose of this study is to develop computer technology based on a combination of mode superposition and boundary element methods to estimate the natural frequencies and modes of fluid vibrations in compound shell structures.

The study focuses on compound shells of revolution that are partially filled with liquids, as illustrated in Fig. 1(a-b). Here the wetted surface of the shell structure is denoted as S_1, while S_0 represents the free surface of the liquid. The liquid is assumed to be ideal and incompressible one.



a) b)

Fig. 3.1. Compound shells of revolution and their drafts

The tanks under consideration consist of coaxial cylindrical and conical shells connected by rings that form the bottoms. The liquid is contained between the shells. The surfaces of the shells and bottoms are wetted, and the free surface at rest is at height H, forming a ring described in the polar coordinate system as follows: $\{z = H, R_1 < r < R_0\}$. Here $R_0 = R_3 + (R_2 - R_3)(L - H)/L$ for structure depicted on Fig. 1a), whereas for shell structure 1b) we have $R_0 = R_3 + (R_2 - R_3)H/L$.

Let assume that the movement of the liquid between the shells is vortex-free. The fluid velocity vector is denoted hereinafter as $\mathbf{V}(V_x, V_y, V_z)$. The incompressibility condition takes the form $\mathrm{div}\mathbf{V} = 0$. The condition of the motion potentiality implies that there exists a scalar potential Φ, and $\mathbf{V} = \mathrm{grad}\Phi$. From these assumptions, it follows that the potential Φ satisfies the Laplace equation in the region occupied by the liquid. Let Ω be the area occupied by the liquid, and let **P** be a point inside the area Ω. The liquid pressure p on the wetted surfaces of the shell is determined

$$\frac{p}{\rho_l} = -\frac{\partial\Phi}{\partial t} - gz + \frac{p_0}{\rho_l}, \tag{3.1}$$

where ρ_l is the liquid density, g is the gravity acceleration, and z is the vertical coordinate of the point **P**.

Formulate the boundary conditions for the Laplace equation to evaluate the potential Φ. On the wetted surfaces S_1 the no-flow condition must be fulfilled, while on the free surface S_0 dynamic and kinematic conditions are applied [13]. The non-penetration condition is given by

$$\left.\frac{\partial\Phi}{\partial \boldsymbol{n}}\right|_{S_1} = 0, \tag{3.2}$$

where $\boldsymbol{n}$ is the unit normal vector to the surface S_1.

Kinematic and dynamic conditions are fulfilled on the free surface

$$\left.\frac{\partial\Phi}{\partial \boldsymbol{n}}\right|_{S_0} = \frac{\partial\zeta}{\partial t}; \quad p - p_0|_{S_0} = 0, p - p_0 = -\rho_l\left(\frac{\partial\Phi}{\partial t} + g\zeta\right). \tag{3.3}$$

Here p is the liquid pressure on the wetted surfaces, p_0 is the atmospheric pressure, $\zeta=\zeta(x, y, t)$ is an unknown function describing the motion and location of the free surface.

Thus, the boundary value problem is formulated for the Laplace equation

$$\nabla^2\Phi = 0, \mathbf{P}\in\Omega\,, \frac{\partial\Phi}{\partial \mathbf{n}} = 0,\ \ \mathbf{P}\in S_1,\ \ \frac{\partial\Phi}{\partial \mathbf{n}} = \frac{\partial\zeta}{\partial t}, p - p_0 = 0,\ \ \mathbf{P}\in S_0 \tag{3.4}$$

relatively to the potential Φ, that is connected with the unknown function $\zeta(x, y, t)$ by boundary conditions.

4 Method of mode superposition

Since the shells of revolution are considered and due to the linearity of relations (3.4), let us represent the unknown functions Φ and ζ in cylindrical coordinates as following series:

$$\zeta(r, \theta, t) = \sum_{l=0}^{m} \cos(\, l\theta) \sum_{k=1}^{n} d_{kl}(t)\zeta_k(r), \tag{4.1}$$

$$\Phi(r, \theta, z, t) = \sum_{l=0}^{m} \cos(l\theta) \sum_{k=1}^{n_2} \dot{d}_{kl}\,(t)\varphi_k(r, z) \tag{4.2}$$

In this case, the kinematic condition will be fulfilled if the following relationship exists between the basic functions $\varphi_k(r, z)$ and $\zeta_k(r)$ on the free surface:

$$\left.\frac{\partial\varphi_k(r,z)}{\partial \mathrm{n}}\right|_{z=H} = \zeta_k(r). \tag{4.3}$$

The functions $\psi_{kl} = \varphi_{kl}(r, z)\cos(l\,\theta)$ must satisfy the Laplace equation.

From the dynamic and kinematic conditions, it is follows

$$\frac{\partial\Phi}{\partial t} = -g\zeta, \quad \frac{\partial^2\Phi}{\partial t^2} = -g\frac{\partial\,\zeta}{\partial\, t} = -g\frac{\partial\Phi}{\partial \mathbf{n}}. \tag{4.4}$$

Assuming the harmonic change in coefficients $d_{kl}(t)$ via time as $d_{kl}(t) = D_{kl}\exp(i\omega_{kl}t)$, one can obtain from (4.4)

$$\frac{\partial\psi_{kl}}{\partial\mathbf{n}} = \frac{\omega_{kl}^2}{g}\ \psi_{kl}. \tag{4.5}$$

Thus, the spectral boundary value problem is formulated relatively functions ψ_{kl} [14]

$$\nabla^2\psi_{kl} = 0, \mathbf{P}\in\Omega\,, \frac{\partial\psi_{kl}}{\partial\mathbf{n}} = 0,\ \ \mathbf{P}\in S_1,\ \frac{\partial\psi_{kl}}{\partial\mathbf{n}} = \frac{\omega_{kl}^2}{g}\ \psi_{kl},\ \ \mathbf{P}\in S_0. \tag{4.6}$$

The boundary element method (BEM) is used to solve the spectral boundary value problem. Note that in representations (4.1)-(4.2) the multipliers $\sin(l\theta)$ are also can be involved.

5 Boundary element method

To apply the boundary element method in its direct formulation, we use Green's third formula [15]

$$2\pi\psi_{kl}(\mathbf{P}_0) = \iint_S \frac{\partial\psi_{kl}}{\partial\mathbf{n}}\frac{1}{|\mathbf{P}-\mathbf{P}_0|}dS - \iint_S \psi_{kl}\frac{\partial}{\partial\mathbf{n}}\frac{1}{|\mathbf{P}-\mathbf{P}_0|}dS, \tag{5.1}$$

where $|\mathbf{P}-\mathbf{P}_0|$ is the Cartesion distance between points $\mathbf{P},\mathbf{P}_0$, and $S = S_1 \cup S_0$. Using the boundary conditions of the spectral problem (4.6), we obtain

$$2\pi\psi_{kl} + \iint_{S_1}\psi_{kl}\frac{\partial}{\partial\mathbf{n}}\left(\frac{1}{|\mathbf{P}-\mathbf{P}_0|}\right)dS_1 - \frac{\omega_{kl}^2}{g}\iint_{S_0}\frac{\psi_{kl}}{|\mathbf{P}-\mathbf{P}_0|}dS_0 + \iint_{S_0}\psi_{kl}\frac{\partial}{\partial\mathbf{n}}\left(\frac{1}{|\mathbf{P}-\mathbf{P}_0|}\right)dS_0 = 0, \mathbf{P}_0\in S_1,$$

$$2\pi\psi_{kl} + \iint_{S_1}\psi_{kl}\frac{\partial}{\partial\mathbf{n}}\left(\frac{1}{|\mathbf{P}-\mathbf{P}_0|}\right)dS_1 + \frac{\omega_{kl}^2}{g}\iint_{S_0}\frac{\psi_{kl}}{|\mathbf{P}-\mathbf{P}_0|}dS_0 = 0,\ \mathbf{P}_0\in S_0. \tag{5.2}$$

For shells of revolution, using the representation $\psi_{kl} = \varphi_{kl}(r,z)\cos(l\,\theta)$, we arrive at the one-dimensional system of singular integral equations in the form

$$2\pi\varphi_{kl}(r_0,z_0) + \int_\Gamma \varphi_{kl}(r(z),z)\Theta\,(z,z_0)r(z)d\Gamma - \frac{\omega_{kl}^2}{g}\int_0^R \varphi_{kl}(\rho,H)\Xi(\mathbf{P},\mathbf{P}_0)\rho d\rho = 0, \mathbf{P}_0 \in S_1, \tag{5.3}$$

$$2\pi\varphi_{kl}(r_0,H) + \int_\Gamma \varphi_{kl}(r(z),z)\Theta\,(z,z_0)r(z)d\Gamma - \frac{\omega_{kl}^2}{g}\int_0^R \varphi_{kl}(\rho,H)\ \Xi(\mathbf{P},\mathbf{P}_0)\rho d\rho = 0,\ \mathbf{P}_0 \in S_0.$$

$$\Theta(z,z_0) = \frac{4}{\sqrt{a+b}}\left\{\frac{1}{2r}\left[\frac{r^2-r_0^2+(z_0-z)^2}{a-b}E_l(k) - F_l(k)\right]n_r + \frac{z_0-z}{a-b}E_l(k)n_z\right\},$$

$$\Xi(P,P_0) = \frac{4}{\sqrt{a+b}}F_l(k),\ \ a = r^2 + r_0^2 + (z-z_0)^2,\ b = 2rr_0.$$

In (5.3) the generalized elliptical integrals are introduced

$$E_l(k) = (-1)^l(1-4l^2)\int_0^{\pi/2}\cos 2\,b_1 l\theta\sqrt{1-k^2\sin^2\theta}\,d\theta, \tag{5.4}$$

$$F_l(k) = (-1)^l\int_0^{\pi/2}\frac{\cos 2b_1 l\theta d\theta}{\sqrt{1-k^2\sin^2\theta}}d\theta,\quad k^2 = 2b/(a+b). \tag{5.5}$$

To calculate the integrals in equations (5.4) and (5.5), the method proposed in [16] is used, which is based on the arithmetic-geometric mean. To solve the system of singular equations in (5.3), the boundary element method with a constant approximation of the density along the element is imp;oyed, as described in [13] and [17].

6 Numerical results

To validate the proposed method, numerical results were compared with data provided in [18]. Both V-shaped and Λ-shaped conical tanks with R_1 = 1м і α = π/3, filled with liquid, are considered, where R_2 represents the smaller radius of the cone. We focused on the first frequencies with wave numbers l= 0, 1, 2, as these are the lowest natural frequencies governing hydrodynamic loading. The comparison results are summarized in Table 6.1 for various values of R_2,

During the numerical computations, 120 boundary elements were consideed along the conical section, 100 elements along the radius of the free surface, and another 100 elements along the radius of the structure bottom. Increasing the number of elements did not significantly alter the results.

The outcomes obtained using the proposed one-dimensional arithmetic-geometric mean method (MGE) closely match those of [18]. Some discrepancies were noted particularly at R_2=0.2м for Λ-shaped conical tanks, consistent with [18]'s observation of the semi-analytical method's reduced accuracy in this specific scenario.

In what following, exactly 120 boundary elements are used along both cylindrical and conical surfaces to investigate fluid vibrations in coaxial shells

Table 6.1. Sloshing frequencies in cone tanks

	V-shapei					Λ-shape				
R_2,м	0.2	0.4	0.6	0.8	0.9	0.2	0.4	0.6	0.8	0.9
l =0, k =1										
[18]	3.386	3.386	3.382	3.139	2.187	24.153	10.014	6.665	4.550	2.683
МГЕ	3.389	3.390	3.391	3.192	2.200	20.027	10.034	6.669	4.545	2.678
l =1, k =1										
[18]	1.304	1.302	1.254	0.934	0.542	11.332	5.629	3.515	1.661	0.726
МГЕ	1.305	1.307	1.259	0.954	0.574	11.303	5.626	3.481	1.651	0.732
l =2, k =1										
[18]	2.263	2.263	2.255	2.015	1.361	17.760	8.967	5.941	3.724	1.923
МГЕ	2.265	2.270	2.269	2.048	1.394	17.939	8.965	5.941	3.726	1.951

Next, the spectral boundary value problem (4.6) has been solved, which made it possible to find the modes $\varphi_k(r, \theta, z)$ and their corresponding fundamental frequencies for the structures shown in Fig. 1. The following geometric dimensions were chosen: L = 1 m, R_1 = 0.5 m, R_2 = 0.5 m, R_3 = 0.25 m, filling level H is 1.25 m.

The specificity of these structures is that the free surface has the shape of a ring. The free surface has the same shape when considering toroidal shells, sloshing of liquids in such shells was studied in [11].

The values of the lower eight sloshing frequencies of coaxial cylindrical-conical shell structures are given in table 6.2.

Table 6.2. Sloshing frequencies in co-axial shells, Hz

№	1	2	3	4	5	6	7	8
Shell structure, Fig.1a)								
частота	0,6277	0,6277	0,8892	0,8892	1,0779	1,0779	1,2355	1,2355
Shell structure, Fig.1b)								
частота	0,5512	0,5512	0,8153	0,8153	1,0027	1,0027	1,1548	1,1548

Note that there are multiple sloshing frequencies. They correspond to the factor $\sin(l\theta)$ in equations (4.1)-(4.2). The corresponding sloshing modes of the free surface are shown in Fig. 6.1-6.2. The sloshing frequencies of both structures differ slightly, but for structures with a smaller radius of the free surface, they are higher. This difference decreases with increasing wave number.

The lowest frequencies correspond to the first, second and third wave numbers. This corresponds to the calculation data given in [5, 7, 10] regarding sloshing of liquid in conical and cylindrical shells.

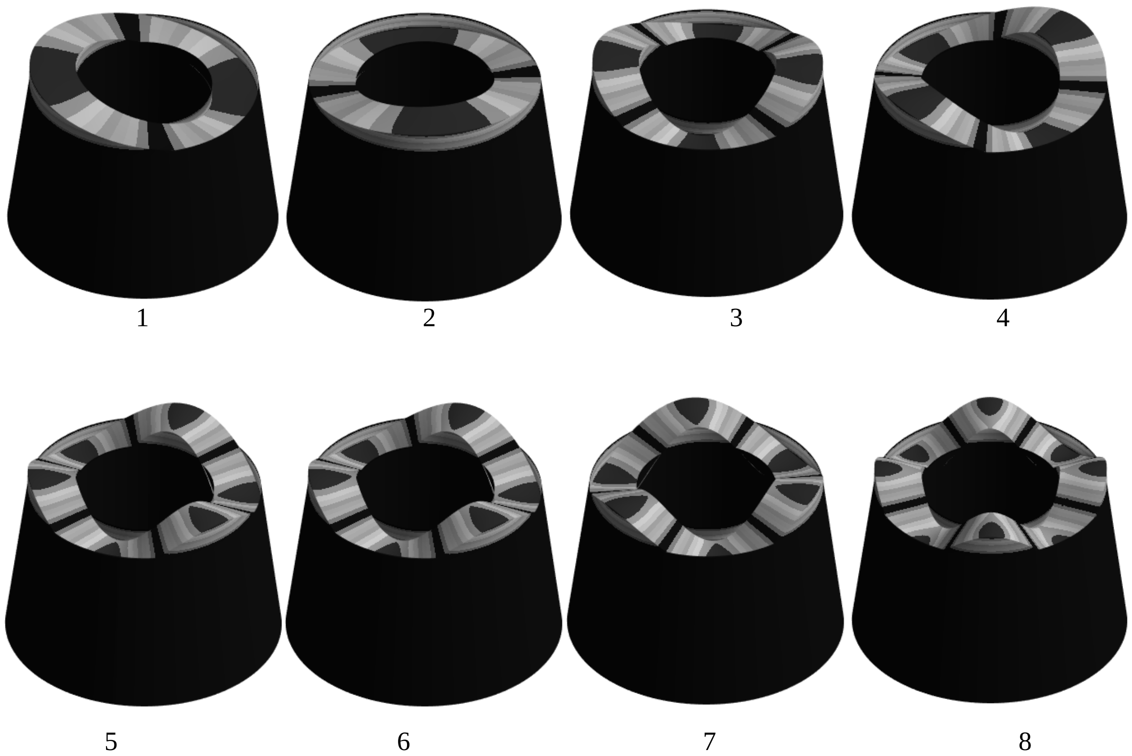


Fig. 6.2 Vibration modes $\varphi_k(r,\theta,H)$ of the free surface in shell structure 1a).

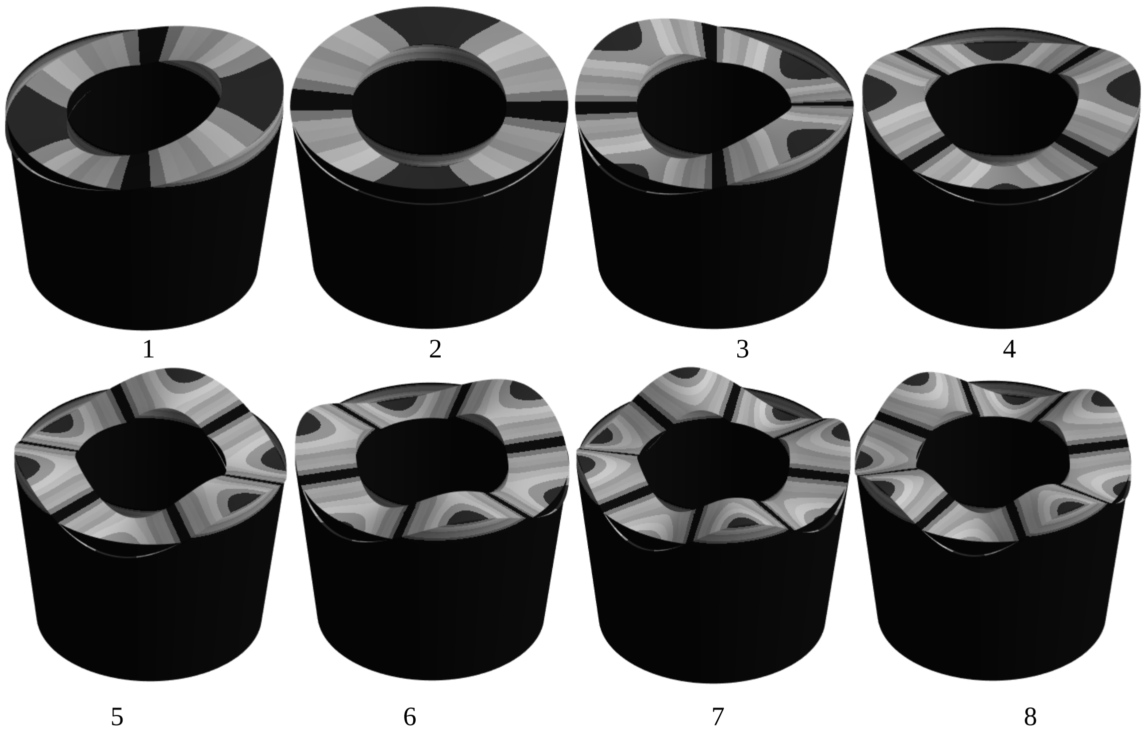


Fig. 6.3. Vibration modes $\varphi_k(r,\theta,H)$ of the free surface in shell structure 1b).

Thus, the spectral problem of determining the frequencies and modes of fluid vibrations in coaxial shell structures has been solved. This makes it possible to study the movement of liquid in fuel tanks and reservoirs under the action of external loads.

Conclusion

The method has been proposed for determining the frequencies and modes of fluid vibrations in coupled shells of revolution. The boundary element method is used to solve the spectral boundary value problem within these coupled shells for the first time. This approach will be pivotal in computer modeling to understand the dynamic behavior of liquid tanks and to investigate the stability of liquid movement in the fuel tanks of complex-shaped launch vehicles. Future research will focus on studying the vibrations of elastic coaxial shells containing liquid, incorporating various composite materials [19].

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Спектральна крайова задача для коаксіальних оболонок обертання

Основною метою цього дослідження є розробка ефективного чисельного підходу з використанням граничних елементів для оцінки власних частот коливань рідини у складених резервуарах. Проаналізовано власні коливання конструкцій оболонок, що включають циліндричні та конічні оболонки, поєднані кільцями. Область між оболонками заповнена ідеальною нестисливою рідиною. У числовому моделюванні використовуються метод суперпозиції у поєднанні з методом граничних елементів. Здійснено числовий розв'язок спектральної граничної задачі щодо коливань рідини в жорстких оболонкових конструкціях. Частоти і форми визначаються шляхом розв'язання систем сингулярних інтегральних рівнянь. Для оболонки обертання, ці системи спрощуються до одновимірних рівнянь, де інтеграли обчислюються вздовж кривих і відрізків прямих. Для обчислення одновимірних інтегралів із логарифмічними особливостями та особливостями типу Коші використовуються ефективні числові процедури. Тестові розрахунки підтверджують високу точність і ефективність запропонованого методу. Важливість і практична значимість методу полягає в можливості досліджувати коливання рідини в реальних складених паливних баках ракет-носіїв за різні умови навантаження.
Ключові слова: *циліндрично-конічні резервуари, системи сингулярних інтегральних рівнянь .метод граничних елементів, плескання рідини.*

Надійшла у першій редакції 22.08.2024, в останній - 14.09.2024.
The first version has been received on 22.08.2024, the final version - on 14.09.2024.