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Modeling and Analysis of a Dynamic Network of Telephone Subscribers Considering the Degree of Connectivity by Means of Contact Lists

Abstract. Modeling complex dynamic networks, whose components interact and evolve, is essential for understanding and predicting their behaviour. It helps to optimize performance, improve resilience, and effectively manage resources in technological, information, social, and biological networks. **Purpose.** The purpose of the work is to model a dynamic network of telephone subscribers, identify and evaluate its main properties. The focus is on experiments with the resulting model and determining the dependencies of network properties based on simulation data. **Research methods.** The methods of constructing computer models, methods of analyzing network parameters, the method of least squares and the Monte Carlo method of stochastic dynamics of discrete states by using time steps of equal length have been used in the work. The computer model has been developed in Python by using the Pandas, Numpy and NetworkX libraries. **Results.** A model of a dynamic network of telephone subscribers is proposed with imitation of contact lists, which usually include family members, colleagues, and friends. Experiments have been conducted with the model and the dependences of network properties on the number of subscribers and the fraction of contacts within contact lists have been investigated. The values of the model parameters at which the network exhibits the properties of a small-world network has been determined. **Conclusions.** The proposed model of a dynamic network of telephone subscribers with imitation of contact lists has allowed to identify the dependences of the network properties on the number of subscribers and the fraction of contacts within the contact lists. It was revealed that the node degree distribution corresponds to the lognormal law. The number of links in the call graph depends on the number of subscribers linearly, and the higher the fraction of contacts, the fewer links are created when a new subscriber appears. An increase in the number of subscribers affects the network density reducing it according to a hyperbolic law. As the fraction of contacts increases, the network density decreases, since an increasing number of connections are created among a limited number of subscribers. The clustering coefficient changes according to a hyperbolic law as well. The average value of the shortest path length for certain network parameters is well approximated by a logarithmic function when the fraction of contacts is more than 0.80 within contact lists. Finally, the qualities of a small-world network can be recognised in the dynamic network of telephone subscribers when the fraction of contacts in the contact list falls between 0.80 and 0.90, as determined by the coefficient ω (4).

Keywords: *dynamic complex network, mobile call graph, telephone network, lognormal distribution, degree distribution, network density, clustering coefficient, average shortest path length, small-world network.*

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1. Introduction

Various world phenomena – social, physical, biological to name a few – can be represented as complex dynamic networks. In [1], the authors identified four categories of complex networks: technological, informational, social and biological. Representing complex networks as structures that change over time allows identifying structural and temporal patterns. Thus, a dynamic network can be identified as one in which nodes and edges appear and/or disappear over time [2].

Numerous studies have been conducted on the modeling and analysis of complex dynamic networks. In [2], the authors present a classification of dynamic networks and the basics of their modeling using graph neural networks. In [3], the authors propose criteria for choosing a dynamic and static model of a social network to analyze their properties. The social network models predominantly presented in the physics-oriented literature on the complex networks have been classified and compared in [4].

The data obtained from actual networks of phone subscribers have been analyzed in several papers. For instance, in [5], the authors have presented studies on the interaction patterns of millions of mobile phone users. In [6], a network of telephone subscribers of more than 1 million users is studied based on call data presented by the mobile network. The authors evaluate such indicators as the distribution of the number of telephone calls, conversation time, and the number of unique connections per subscriber. In [7], the authors have considered the data on telephone calls from four different geographical regions, have generated call graphs and analyzed their static properties. The dynamic network of telephone subscribers and the influence of the degree of persistence of connections on the structure of the network have been examined in [8]. Other types of dynamic networks have been analyzed as well. The overview of financial networks – definitions of various types of networks and discussion on the dynamic aspect of the functioning and distribution of resources in such networks has been presented in [9]. In [10], the authors describe biological networks, as well as the principles and algorithms of graph neural networks, which can be used to predict the emergence of new connections and diseases.

The papers [5–8] focus on the analysis of existing networks of telephone subscribers. However, only a limited number of works are devoted to the simulation modeling of dynamic networks of telephone subscribers. For example, in [11], the authors proposed a simulation model of a call-center, which allows doing a “what-if” analysis to determine the number of operators required to satisfy a certain requirement for the system. In [12], the authors have developed a telephone network model for an electricity distribution company with the aim of modeling and further optimizing the technical infrastructure of the telephone network. Modeling a dynamic telephone network as a social communications network with different types of subscribers has not been covered in the literature.

The paper proposes a model of a dynamic network of telephone subscribers, in which the number of subscribers is static, and the connections between them change at discrete time intervals According to the terminology presented in [2], such a network can be classified as a discrete node-static temporal network. Furthermore, following the classification of dynamic communication networks proposed in [13], the telephone subscriber network is synchronous with one-to-one connections, which means that connections exist only between two subscribers simultaneously.

2. Modeling dynamic network of telephone subscribers

A network of telephone subscribers is formed by a certain number of nodes (subscribers) and the connections between them. In such a network, a connection is established when two subscribers are engaged in a call. For instance, the average person makes five to seven calls per day, while sale managers may make two or three hundred calls daily. Consequently, at any given moment, a certain number of pairwise connections are established. During the experiment with the model, calls and connections between subscribers have been saved in a log. From the recorded set of paired connections, a dynamic network of telephone subscribers is formed over a specific period. The study aims to analyze the properties of the obtained dynamic network.

The telephone network model proposed in this work differs from the model presented in [14]. The distinction is that each subscriber has a contact list in our model. The fraction of calls between subscribers from the contact list is the specified by the model parameter, and in other cases, the contacts are selected randomly. The lognormal distribution law is used to assign the quantity of calls per day to subscribers

[15]. New calls cannot be placed or received by subscribers while they are engaged in a call. The probability of a call ending at a certain time is used to model the call duration.

The input parameters of the model include the number of subscribers, parameters of the lognormal distribution law for modeling the number of calls per day, the duration of the experiment, the average duration of conversations, the fraction of calls to subscribers from the contact list, and the number of subscribers in the contact list.

With the help of a developed model, such properties of a dynamic network of telephone subscribers as the degree distribution law, the number of created links, network density, clustering coefficient, and the average shortest path length have been investigated.

3. Description of the experiment

During the experiments conducted over a specified period, connections between subscribers are simulated, resulting in a dynamic network. The number of subscribers is predetermined and remains constant. In this model, one unit of time is equivalent to one minute. The duration of the experiment, the parameters of the lognormal distribution law for the daily number of calls, the size of the contact list, and the fraction of calls to subscribers from the contact list are specified. The uniform distribution law is used to select a subscriber for communication, both within the contact list and among all other subscribers. The duration of a call is determined by the probability of its termination at any unit of time, which is set at 0.99 for all experiments. Once a connection is established, subscribers are isolated, which means that they cannot make or receive additional calls. The progress of the experiments is recorded in Table 1:

Table 1. Experiment data sample

idx	caller_id	callee_id	start_time	finish_time
0	12	62	0	1
1	23	4	4	7
2	72	64	8	9

where idx is the record number, caller_id is the caller identifier, callee_id is the callee identifier, start_time is the call starting time, finish_time is the call finishing time, duration is the call duration.

Input data for the experiment: number of subscribers is 1000, duration is 10080 of time units (minutes), the parameters of the lognormal distribution correspond to normal subscribers ($\mu = 1.1, \sigma = 1.0$) [15].

4. Study of the obtained data

During the experiments, data on 34834 subscriber contacts have been obtained. Using this data, the distribution of the number of subscriber calls during the day has been constructed. This distribution can be compared to the distribution obtained in [15], where it has been found that the number of calls follows a lognormal distribution. Therefore, the consistency of the model with the experimental data could be evaluated.

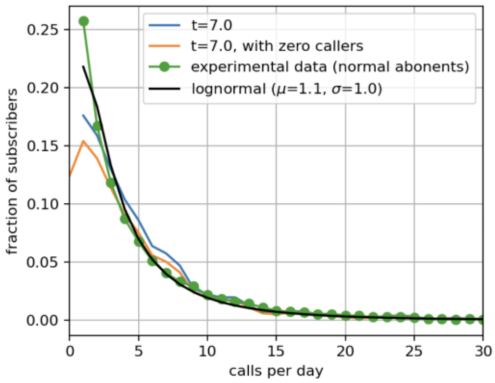


Fig.1 Distribution of the number of subscribers by the number of calls per day. In green – experimental data of a real network, simulation data: yellow – all subscribers are included, blue – only those subscribers who made at least one call, black curve – the lognormal law with μ = 1.1 and σ = 1.0.

The green line corresponds to experimental data from the real telephone network of subscribers [15]. The results obtained from the telephone network simulation are depicted by the orange and blue lines.

The blue line represents the distribution of calls determined without considering subscribers who did not make a single call throughout the entire experiment. The orange line represents the distribution including all subscribers. The black curve corresponds to the data obtained by approximating experimental observations by using a lognormal distribution law with parameters $\mu = 1.1$ and $\sigma = 1.0$.

In the experimental data from a real telephone subscriber network, all subscribers made at least one call. In contrast, the simulated network included subscribers who did not make a single call, as indicated by the distribution depicted by the orange line. That resulted in different data distributions.

The distribution of subscribers who made at least one call during the model simulation is represented by the blue curve in Fig. 1, which shows good agreement with the experimental data, represented by the green curve. Over time, subscribers create a network of connections. We have used the information obtained from experiments with the constructed computer model to conduct a more thorough analysis of the characteristics of the telephone subscriber network.

5. Analysis of dynamic network properties

Numerical experiments have been conducted with varying numbers of subscribers at consistent time intervals to investigate the properties of a dynamic network of telephone subscribers. We have assessed the relationship between these properties and the fraction of calls made within the contact list, as well as the total number of subscribers in the network. Among the properties analyzed, we have examined the number of edges in the network, clustering coefficients, network density, average shortest path length, and degree distribution. Additionally, we have determined the type of network resulting from those model experiments.

A total of 580 experiments have been conducted. The number of subscribers ranged from 10 in the first experiment to 1000 in the last one, with the fraction of calls within the contact list varying from 0.0 to 1.0. The interval was 0.1 from 0.0 to 0.8, and 0.01 from 0.81 to 1.0. In each experiment, the number of subscribers increased by 50. Fig. 2 presents graphs of various networks formed with different fractions of calls but with a consistent number of 100 subscribers. The graphs clearly show the formation of subscriber groups within contact lists and the connections between these groups when calls are made to other subscribers. Those groups or cliques are especially visible in networks with a high fraction of calls within their contact lists.

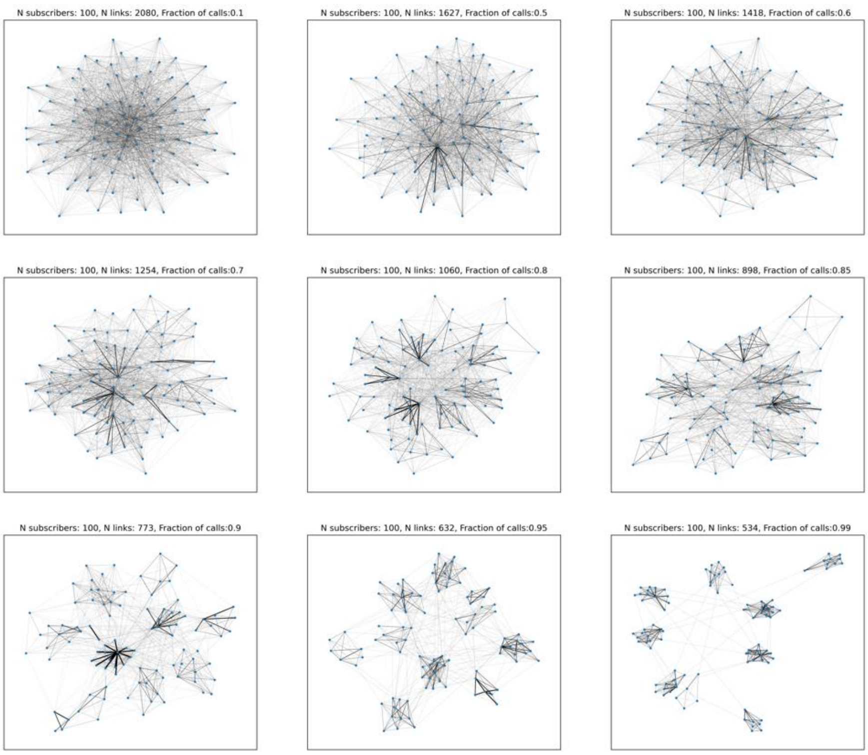
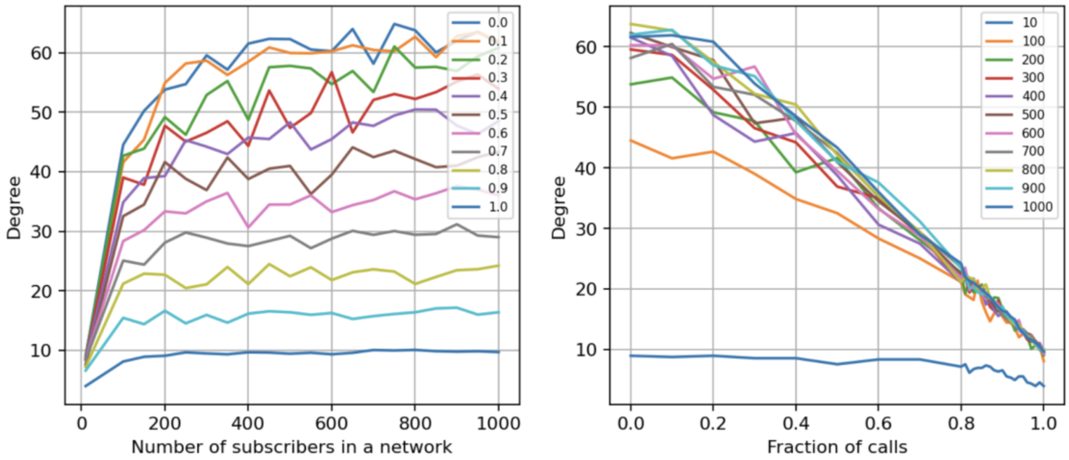


Fig.2 Simulated telephone subscriber networks. N subscribers – number of subscribers, N links – number of links, Fraction of calls – fraction of calls made within subscriber contact lists

The properties of the generated networks have been examined. The form of each network can be explained by using the distribution of node degrees, which represents the number of connections a subscriber established with other subscribers during the experiment. The obtained dependencies are shown in Fig. 3.

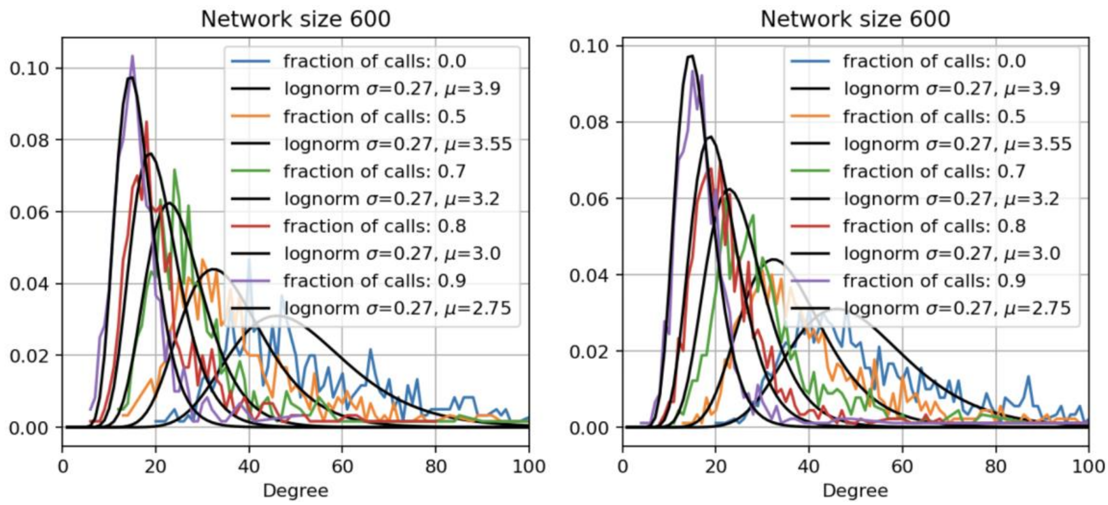


a) *b)*

Fig.3 Average node degrees dependence on a) number of subscribers; and b) fraction of calls within contact lists.

From the graphs presented in Fig. 3, it could be seen that the degrees of the nodes/subscribers weakly depend on the number of subscribers in the network and have a linear dependence on the fraction of calls. For networks with a small number of subscribers where, for example, 10 subscribers have time to form all possible connections, the degree of nodes is equal to the number of subscribers in the network minus 1. Consequently, the higher the fraction of calls within contact lists, the lower the node degrees.

As stated previously, the number of calls per day is distributed according to a lognormal law. The degree of nodes and the number of calls are closely related because connections and degrees in a dynamic network of telephone subscribers are formed through calls between subscribers.



a) *b)*

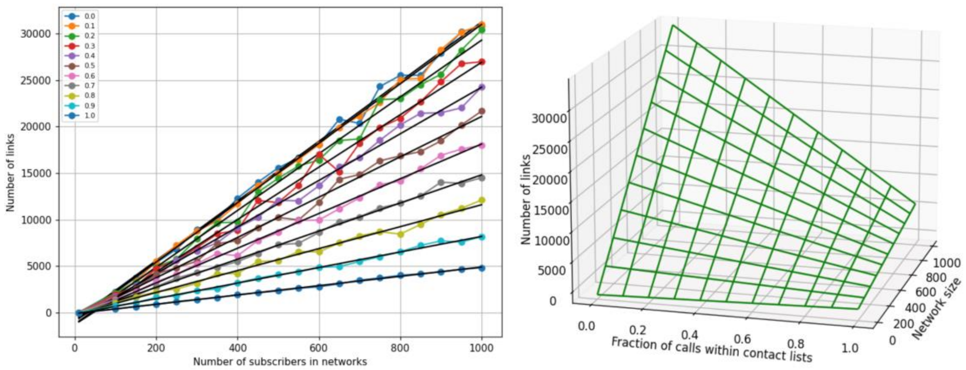
Fig.4 Distribution of node degrees and approximating lognormal distribution curves:
a) network with the 600 subscribers; b) network with the 900 subscribers.

The lognormal distribution curves that correspond to the node degree distributions are shown in Fig. 4. For networks with 600 and 900 subscribers, we have examined whether the node distribution agreed with the lognormal law. The data indicates a decent degree of agreement, with μ depending on the fraction of calls in contact lists (the higher the fraction, the smaller μ); meanwhile, σ remains unchanged.

Next, we have examined the relationship between the number of links and the number of subscribers in the network as well as the fraction of calls. An edge in the call graph between two subscribers is considered to be a link. The dependencies discovered through model experiments are displayed in Fig. 5, along with straight lines representing them. The results show that the number of links made during the experiment is directly correlated with the total number of subscribers in the network. Additionally, the fraction of calls made within contact lists influences the angle of inclination of the straight line. The higher the fraction is, the smaller the angle is and, consequently, the fewer links made over a given amount of time. The relationship between those data, when approximated linearly by using the least squares method, is as follows:

$$L = -30.15NF + 1212.57F + 35.57N - 1184.21 \quad (1)$$

where L is the number of links, N is the number of subscribers in the network, F is the fraction of calls within the contact list.



a) *b)*

Fig.5 Dependence of the number of links on a) the number of subscribers; b) the number of subscribers and the fraction of calls within contact lists.

From that dependence it follows that each new subscriber on average creates 5 new links during the experiment (10800 discrete intervals) when the fraction of calls is equal to 1. For instance, with a fraction of 0.7, the subscriber creates on average about 15 new links etc. Fig. 5b shows the dependence of the number of links on the number of subscribers and the fraction of calls in contact lists.

Next, we present simulation data which shows how the network density changes with an increase in the number of subscribers over the same time interval for different values of the fraction of contacts. Network density is the ratio of an actual number of links to the maximum number of links that can be created with the same number of nodes [1]. The obtained relationships, as well as their approximations, are shown in Fig. 6.

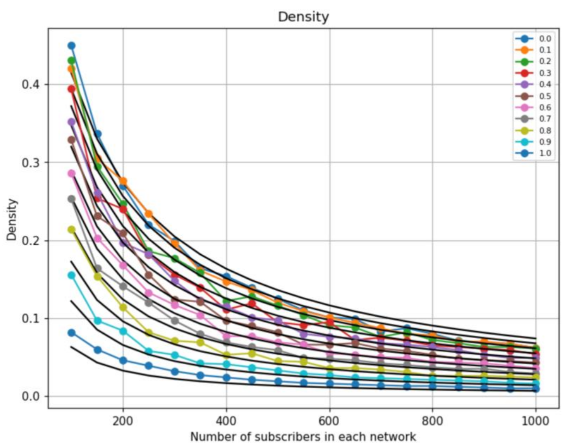


Fig.6 Dependence of network density on the number of subscribers in the network and the fraction of calls

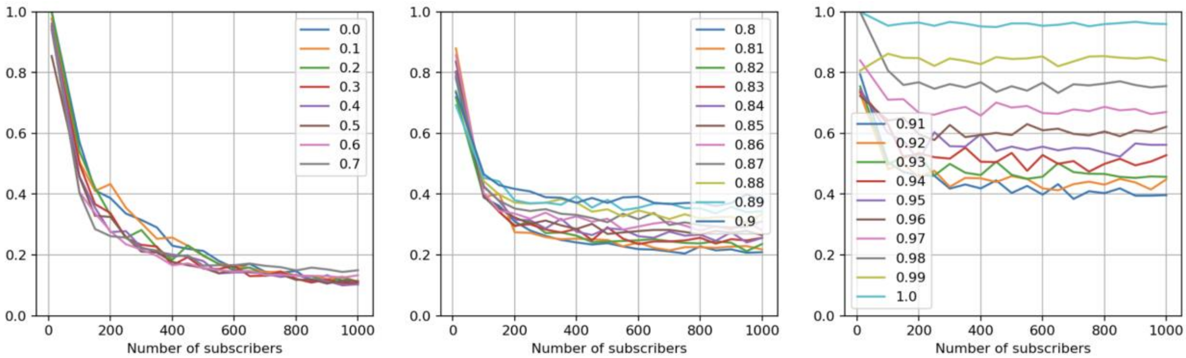
The simulation data is shown in Fig. 6 by the lines with dots, and the approximation of the dependencies is shown in black. An examination of the data has revealed a hyperbolic dependence of density on the number of subscribers in the network and the fraction of calls:

$$\rho = \frac{0.85}{1+\frac{N}{95.30-87.29F}} \tag{2}$$

where ρ is the network density.

Good agreement with the simulation data is noticeable in Fig. 6. As the number of subscribers increases, the network density decreases. Additionally, as the fraction of calls increases, the network density decreases further. That is due to the fact that a higher fraction of calls leads to fewer new links being created among the possible ones, and vice versa. The network density value determined by relation (2) also depends on the duration of the experiment, necessitating further research.

The next network metric analyzed is the average clustering coefficient, which measures the degree of interconnection for subscriber's contacts. A higher clustering coefficient indicates greater connectivity within the graph. Fig. 7 shows how the average clustering coefficient varies with the number of subscribers and the fraction of calls.



a) b) c)

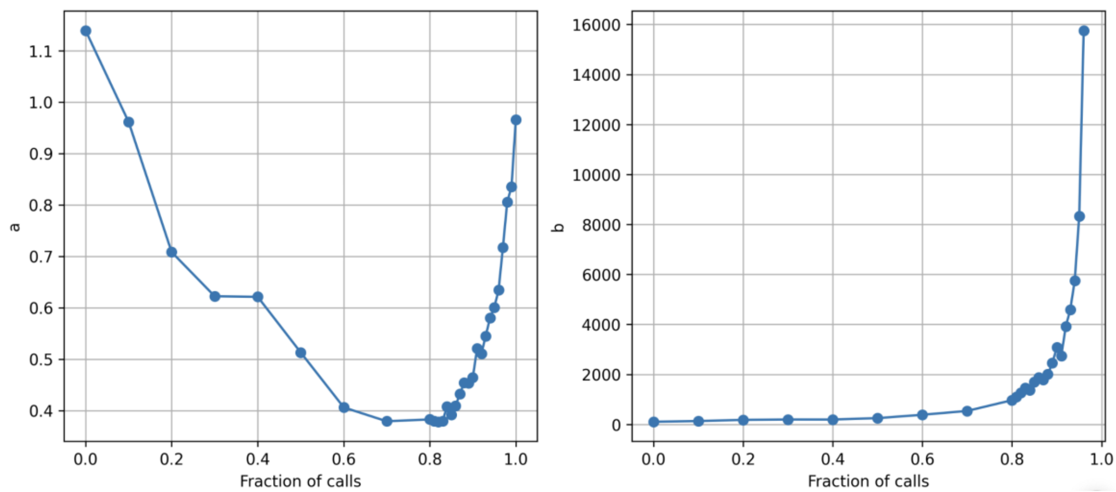
Fig.7 Dependence of the clustering coefficient on the number of subscribers in networks with different fractions of calls

In Fig. 7, the hyperbolic dependence of the clustering coefficient on the number of subscribers in the network with the fraction of calls in the interval [0.0, 0.7] can be observed.

$$C = \frac{a}{1+\frac{N}{b}} \tag{3}$$

where C is the clustering coefficient, a and b are the parameters of the hyperbolic dependence.

The parameters a and b depend on the fraction of calls. The corresponding dependencies are presented in Fig. 8.

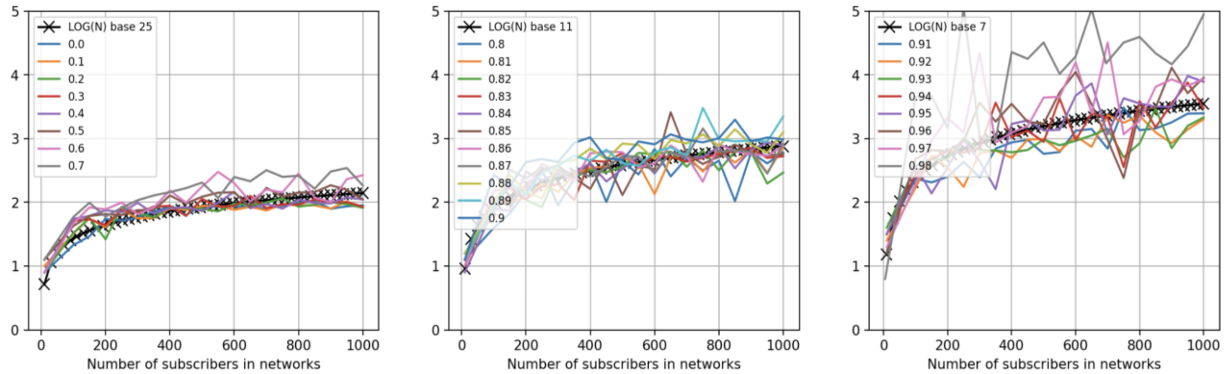


a) b)

Fig.8 Dependence of parameters "a" and "b" on the fraction of calls

It should be noted that the experiments have been carried out with network sizes from 10 to 1000 subscribers and for the same time interval. This explains the transformation of the hyperbolic curve, which depends on the fraction of calls, and the values of the obtained parameters are valid over the time interval of the experiment.

Next, we have considered the changes in the average shortest path length obtained as a result of the simulation. The lengths of all links between adjacent nodes are considered to be equal to one. The average internode distance is the average path over all pairs of nodes between which there is at least one path connecting them. In the categories of the telephone network and connections between people, this indicator characterizes the average number of contacts required to connect any two subscribers.



a) *b)* *c)*

Fig.9 Dependence of the average shortest path length between two subscribers on the number of subscribers in the network and the proportion of contacts within contact lists

In small-world networks, the average shortest path length between two nodes grows in proportion to the logarithm of the number of nodes [16] or subscribers based on the average number of contacts in the case of the telephone network. In the model, the dependence of the average length of the shortest path on the number of subscribers in the network is well approximated by the logarithm to base 11 and 7 for networks with the fraction of contacts in the intervals [0.80, 0.90] and [0.91, 0.98], which is shown in Fig. 9b and 9c, respectively. For networks with fractions of up to 0.7, the logarithmic curve in Fig. 9a corresponds to the logarithm of 25, which is too many regular contacts for normal subscribers.

Let us check if any of the networks obtained as a result of the experiments are small-world networks. Small-world networks are characterized by a high clustering coefficient and a short path length [16, 17]. The distribution of node degrees in small-world networks can be similar to the distributions in random networks [18], so they can be Poisson or normal distributions. The degree to which a network is a small-world network can be measured using the coefficient ω [17], which considers the clustering coefficient and the average shortest path length and compares them with the equivalent regular lattice and random network, respectively:

$$\omega = \frac{L_{rand}}{L} - \frac{C}{C_{latt}} \tag{4}$$

where ω is the small-world measurement

The values of ω are in the range from -1 to 1. Moreover, if the value of ω tends to 1, then it means that the network has the characteristics of a random graph, and if it approaches -1, then the characteristics of a lattice. Values ω around 0 indicate that the network is a small-world network. An alternative small-world measurement metric is the σ proposed in [19]. That metric has several disadvantages, described in [17], so the ω metric has been used to analyze the simulated network.

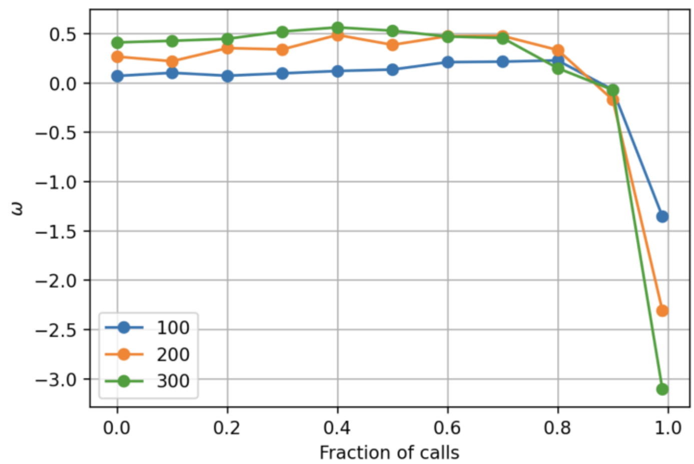


Fig.10 Dependence of ω on the number of subscribers in the network and the fraction of contacts within contact lists

Fig. 10 shows the dependence of ω on the share of contacts and the number of subscribers in the network. Let us reiterate that the proportion of contacts determines the degree of call and, accordingly, at low values ω the network resembles a random graph. In our case, for networks with a fraction of contacts up to 0.80, the values of the coefficient ω are in the interval [0.05, 0.55], which characterizes such networks as random graphs. In a dynamic network of telephone subscribers with fraction values in the range [0.80, 0.90], the coefficient ω crosses the value 0; as a result, those networks show small-world network characteristics. At higher values of the fraction of calls within the contact list, the network has the properties of a regular lattice.

6. Conclusions

In this work, a dynamic network of telephone subscribers has been modeled and its features have been analyzed in relation to the number of subscribers and the fraction of calls made within subscribers' contact lists. The presence of subscribers' contact list, which usually includes family members, colleagues, friends, and the closeness of their connections determines the topology of the network and the dynamics of its properties. Subscribers connected by contact lists and the connections between them are clearly visualized with a high fraction of contacts between them. When the fraction of calls within contact lists is low, the network is transformed into a random graph. As a result of the analysis of the network properties, it has been revealed that the distribution of node degrees corresponds to the lognormal law with parameters μ and σ. The distribution parameters do not depend on the number of subscribers in the network but depend on the fraction of contacts – the higher the fraction, the smaller the parameter μ, and, accordingly, the smaller the value of degrees. The parameter σ is equal to 0.27 and is independent of both the subscriber number and the call fraction. The number of links depends linearly on the number of subscribers – the higher the fraction, the fewer links are created with the appearance of additional subscribers (1). Thus, with a fraction of 0.9, a new subscriber creates on average 8.3 links with other subscribers, and with a share of 0.7, 15 links are created. An increase in the number of subscribers affects the network density, reducing it according to the hyperbolic law (2). As the fraction of contacts increases, the density decreases, since an increasing proportion of links are created among a limited number of subscribers. The clustering coefficient, just like the density, changes according to a hyperbolic law. The parameters of the hyperbolic law have a complex dependence on the fraction of contacts presented graphically in Fig. 8. The average value of the shortest path length is well approximated by a logarithmic function to base 11 and 7 for networks with a fraction of calls within contact list more than 0.80, which indicates the properties of small-world network. Additionally, the presence of small-world properties has been assessed by using the coefficient ω (4) and it has been revealed that a dynamic network of telephone subscribers demonstrates such properties when the fraction of contacts within the contact list is in the interval [0.80, 0.90].

It should be noted that experiments for all networks from 10 to 1000 subscribers have been carried out over the same period. This limitation affects the number of links created between subscribers and, accordingly, in networks with a small number of subscribers, all possible links can be established over a given period, and with a large number, only a small fraction of links can be formed. This model could be used for the further research on the effects of abnormal users (spammers) on telephone subscriber network properties, information propagation speed, and its relationship to contact list sizes and call share.

REFERENCES

1. M. Newman, Networks, vol. 1. Oxford University Press, 2018. DOI: https://doi.org/10.1093/oso/9780198805090.001.0001 (Last accessed: 20.09.2023).
2. Skarding J, Gabrys B, Musial K (2021) Foundations and modelling of dynamic networks using Dynamic Graph Neural Networks: A survey. IEEE Access 9:79143–79168. DOI: https://doi.org/10.1109/ACCESS.2021.3082932 (Last accessed: 20.09.2023).
3. Farine DR (2018) When to choose dynamic vs. static social network analysis. Journal of Animal Ecology 87:128–138. DOI: https://doi.org/10.1111/1365-2656.12764 (Last accessed: 20.09.2023).
4. Toivonen R, Kovanen L, Kivelä M, et al (2008) A comparative study of social network models: network evolution models and nodal attribute models. DOI: https://doi.org/10.1016/j.socnet.2009.06.004 (Last accessed: 20.09.2023).
5. Onnela J-P, Saramäki J, Hyvönen J, et al (2007) Structure and tie strengths in mobile communication networks. Proc Natl Acad Sci USA 104:7332–7336. DOI: https://doi.org/10.1073/pnas.0610245104
6. Seshadri M, Machiraju S, Sridharan A, et al (2008) Mobile call graphs: beyond power-law and lognormal distributions. In: Proceedings of the 14th ACM SIGKDD international conference on Knowledge discovery and data mining. ACM, Las Vegas Nevada USA, pp 596–604. DOI: https://doi.org/10.1145/1401890.1401963 (Last accessed: 20.09.2023).
7. Nanavati AA, Gurumurthy S, Das G, et al (2006) On the Structural Properties of Massive Telecom Call Graphs: Findings and Implications. In: Proceedings of the 15th ACM International Conference on Information and Knowledge Management. Association for Computing Machinery, New York, NY, USA, pp 435–444. DOI: https://doi.org/10.1145/1183614.1183678 (Last accessed: 20.09.2023).
8. Hidalgo CA, Rodriguez-Sickert C (2008) The dynamics of a mobile phone network. Physica A: Statistical Mechanics and its Applications 387:3017–3024. DOI: https://doi.org/10.1016/j.physa.2008.01.073 (Last accessed: 20.09.2023).
9. Bardoscia M, Barucca P, Battiston S, et al (2021) The Physics of Financial Networks. Nat Rev Phys 3:490–507. DOI: https://doi.org/10.1038/s42254-021-00322-5 (Last accessed: 20.09.2023).
10. Muzio G, O'Bray L, Borgwardt K (2021) Biological network analysis with deep learning. Briefings in Bioinformatics 22:1515–1530. DOI: https://doi.org/10.1093/bib/bbaa257 (Last accessed: 20.09.2023).
11. Pichitlamken J, Deslauriers A, L'Ecuyer P, Avramidis AN (2003) Modelling and simulation of a telephone call center. In: Proceedings of the 2003 International Conference on Machine Learning and Cybernetics (IEEE Cat. No.03EX693). IEEE, New Orleans, LA, USA, pp 1805–1812. DOI: 10.1109/WSC.2003.1261636 (Last accessed: 20.09.2023).
12. Niaki S, B. Rad Z (2004) Designing a communication network using simulation. Scientia Iranica 11:165–180. URL: https://www.researchgate.net/publication/236107639_Designing_a_communication_network_using_simulation (Last accessed: 20.09.2023).
13. Lehmann S (2019) Fundamental Structures in Dynamic Communication Networks. DOI: https://doi.org/10.1007/978-3-030-23495-9_2 (Last accessed: 20.09.2023).
14. Danilevskyi M, Yanovsky V (2023) Modeling and Analyzing the Simplest Network of Telephone Subscribers. Bulletin of VN Karazin Kharkiv National University, series «Mathematical modeling Information technology Automated control systems» 59:6–15. DOI: https://doi.org/10.26565/2304-6201-2023-59-01 (Last accessed: 20.09.2023).
15. Danilevskiy V, Yanovsky V (2020) Statistical properties of telephone communication network. arXiv preprint arXiv:200403172. URL: https://arxiv.org/pdf/2004.03172.pdf (Last accessed: 20.09.2023).
16. Watts D, Strogatz S (1998) Collective dynamics of 'small-world' networks. Nature 393:440–442. DOI: https://doi.org/10.1038/30918 (Last accessed: 20.09.2023).

17. Telesford QK, Joyce KE, Hayasaka S, et al (2011) The Ubiquity of Small-World Networks. Brain Connectivity 1:367–375. DOI: https://doi.org/10.1089/brain.2011.0038 (Last accessed: 20.09.2023).
18. Simone A, Ridolfi L, Berardi L, et al Complex Network Theory for Water Distribution Networks Analysis. pp 1971–1962 URL: https://www.researchgate.net/publication/329323388_Complex_Network_Theory_for_Water_Distribution_Networks_analysis (Last accessed: 20.09.2023).
19. Humphries MD, Gurney K, Prescott TJ (2006) The brainstem reticular formation is a small-world, not scale-free, network. Proc R Soc B 273:503–511. DOI: https://doi.org/10.1098/rspb.2005.3354 (Last accessed: 20.09.2023).

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Моделювання та аналіз динамічної мережі телефонних абонентів з урахуванням ступеня зв'язаності засобами контакт-листів

Актуальність. Моделювання складних динамічних мереж, компоненти яких взаємодіють і розвиваються з часом, має важливе значення, оскільки дозволяє розуміти та прогнозувати їхню поведінку. Це допомагає оптимізувати продуктивність, підвищувати стійкість та ефективно управляти ресурсами у технологічних, інформаційних, соціальних та біологічних мережах.

Мета. Метою роботи є моделювання динамічної мережі телефонних абонентів та виявлення основних властивостей мережі. Головна увага зосереджена на експериментах з розробленою моделлю та визначення залежностей властивостей мережі за даними моделювання.

Методи дослідження. В роботі використовувалися методи побудови комп'ютерних моделей, методи аналізу параметрів мереж, метод найменших квадратів і метод Монте-Карло стохастичної динаміки дискретних станів з використанням тимчасових кроків однакової довжини. Комп'ютерна модель розроблена мовою Python із використанням бібліотек Pandas, Numpy та NetworkX.

Результати. Запропоновано модель динамічної мережі телефонних абонентів з імітацією контакт-листів, до яких зазвичай входять члени сім'ї, колеги, друзі. Проведено експерименти з моделлю та досліджено залежності властивостей мережі від кількості абонентів та частки контактів у рамках контакт-листів. Визначено при яких значеннях параметрів моделі мережа, що отримується в результаті моделювання, має властивості мережі тісного світу.

Висновки. Запропонована модель динамічної мережі телефонних абонентів з імітацією контакт-листів дозволила виявити залежності властивостей мережі від кількості абонентів та частки контактів у рамках контакт-листів. Виявлено, що розподіл ступенів вершин відповідає логнормальному закону. Кількість зв'язків лінійно залежить від кількості абонентів, причому чим вища частка контактів, тим менше зв'язків створюється з появою нового абонента. Збільшення кількості абонентів впливає на щільність мережі та знижує її за гіперболічним законом. У разі підвищення частки контактів щільність знижується, оскільки дедалі більша частина зв'язків створюється серед обмеженої кількості абонентів. Коефіцієнт кластеризації так само, як і щільність змінюється за гіперболічним законом. Середнє значення найкоротшого шляху за певних параметрів мережі добре апроксимується логарифмічною функцією за частки контактів більше 0.80. Коефіцієнт ω показує, що при частці контактів в межах контакт-листів в інтервалі [0.80, 0.90] змодельована мережа телефонних абонентів має властивості мережі тісного світу.

Ключові слова: *складна динамічна мережа, граф мобільних викликів, телефонна мережа, логнормальний розподіл, розподіл ступенів, щільність мережі, коефіцієнт кластеризації, середня довжина найкоротшого шляху, мережа тісного світу.*

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