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Multidimensional generalizations of atomic radial basis functions

Possible approaches to generalizing multidimensional atomic radial basis functions are presented. The functions of mathematical physics are used in solving two-dimensional and three-dimensional boundary value problems with partial derivatives. Depending on the problem, the functions of different dimensions are used, that is, the functions generated by various differential operators. The functional-differential equations that generate those functions are considered and families of finite solutions for those equations generated by the differential operators of Laplace, Helmholtz, etc. have been constructed according to the given scheme. The results are presented in the form of theorems. In order to expand the class of functions and improve their properties, the construction of the atomic radial basis function family of three independent variables on the example of a functional-differential equation of the appropriate type is considered. The solution methods refer to seedless schemes and combine the possibilities of constructing the boundaries of regions by using R-functions. Atomic functions are convenient for implementing computational algorithms for constructing approximate solutions of boundary value problems in 2D and 3D domains by using meshless schemes. The properties of these functions make it possible to use them as basic functions in solving boundary value problems by meshless methods based on collocation methods. For atomic functions, the dependence on the compression ratio is provided, which is specified in the process of constructing the solution of the boundary value problem as necessary to ensure certain properties of the functions. The scheme for constructing solutions of heat conduction problems by using a gridless scheme is provided.
Keywords: atomic radial basic functions, boundary problems of mathematical physics, meshless methods for solving boundary value problems..

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The necessity to construct various types of multidimensional generalizations of atomic radial basic functions is determined by the requirements of their application in the numerical analysis. For example, when constructing an interpolation spline it is necessary to solve systems of algebraic equations. The dimension of the system is determined by the number of interpolation conditions and may be sufficiently huge. The task of interpolation is simplified by using local splines, as to calculate each of the splenic parameters several interpolation conditions are utilized. Atomic functions not only enable

the further simplification of the solution of the interpolation problem, but allow us to consider more complex situations of interpolation of functions which are determined on various geometric manifolds.

An attempt to non-trivial generalization of atomic functions in the case of many variables was provided in the research [1] where a number of necessary conditions for the existence of finite solutions of some functional-differential equations is given without evidence. Multidimensional generalizations of atomic functions appeared as a result of solving one of the lists of relevant tasks of the theory of atomic functions [2] on the construction of finite solutions of functional-differential equations like the following:

$$Lu(x_1,\dots,x_n)=\int_{\partial\Omega}\varphi(\xi_1,\dots,\xi_n)u(ax_1-\xi_1,\dots,ax_n-\xi_n)ds+ku(ax_1,\dots,ax_n)$$

where L is a linear differential operator; $\partial\Omega$ is the boundary of the convex closed area. The cases of existence and unity of the solution of this problem when the operators of Laplace, Helmholtz and Klein Gordon and others are considered as generating operators.

Let us consider the equation

$$\Delta u(x_1,x_2)=\lambda\oint_{\partial\Omega}u[3(x_1-\xi_1),3(x_2-\xi_2)]ds+\mu u(3x_1,3x_2)$$

where $\partial\Omega$ is a circle: $\xi_1^2+\xi_2^2=\rightleftarrows r^2,\Delta=\partial^2/\partial x_1^2+\partial^2/\partial x_2^2$ is a Laplace operator. The consideration of these functional-differential equations follows from the condition that the Laplace operator is invariant relatively to rotations and a finite system of points. In the case when it contains more than one point it is not invariant. Summation in the right part of the equation (similar to one-dimensional case when the equation like [3] $Ly(x)=\lambda\sum_{k=1}^{M}c_k y(ax-b),|a|>1,$ is considered where L is a linear differential operator with constant coefficients) does not provide the finite solution.

However, if instead of the subformation we carry out an integration operation by a circle, it is possible to establish a solution with a compact carrier (that is the finite solution of the output equation). The resulting functions have preserved the name of atomic ones. The finite solution is called $Plop(x_1,x_2)$, it exists and is the only one for the following values of coefficients: $\mu=-4\pi\lambda/3$ and $\lambda=3^5/(4\pi)$.

$$Plop(x_1,\,x_2)=\sum_{r=0}^{\infty}\sum_{q=0}^{\infty}a_{rq}\cos(r\pi x_1)\cos(q\pi x_2)$$

$$a_{oo}=\frac{1}{4};\quad a_{ro}=\frac{1}{2}U(r\pi,0)=\prod_{h=0}^{\infty}\frac{1-J_o(2q\pi/3^{h+1})}{3^{-2h-2}(q\pi)^2};$$

$$a_{oq}=\frac{1}{2}U(0,q\pi)=\prod_{h=0}^{\infty}\frac{1-J_o(2r\pi/3^{h+1})}{3^{-2h-2}(r\pi)^2};$$

$$a_{rq}=U(r\pi,q\pi)=\prod_{h=0}^{\infty}\frac{1-J_o\left(\frac{2}{3^{h+1}}\sqrt{(r\pi)^2+(q\pi)^2}\right)}{3^{-2h-2}[(r\pi)^2+(q\pi)^2]}$$

The graph and the level lines of the atomic function $Plop(x_1,x_2)$ the support of which is a part of the region $[-1,1]\times[-1,1]$ are presented in Fig. 1:

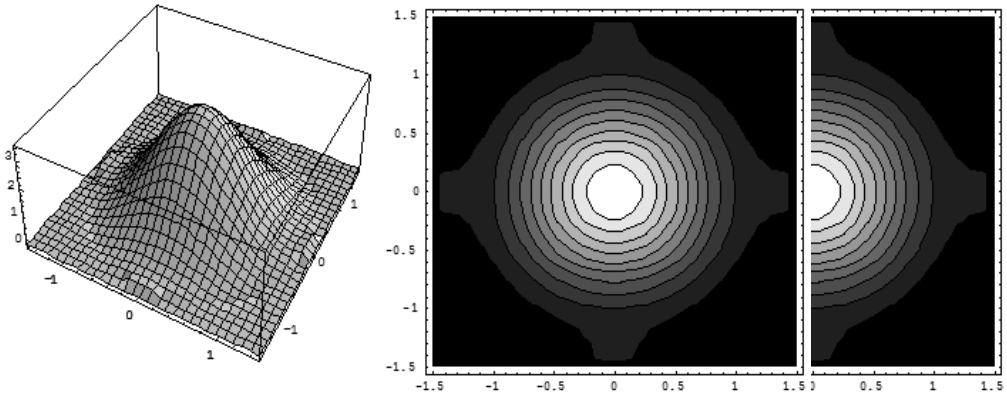

Fig. 1. The graph (a) and the level lines (b) of the atomic function $Plop(x_1, x_2)$.

In the case of three variables the desired function is obtained when solving the equation:

$$\Delta u(x_1,x_2,x_3) = \lambda \int \oint_{\partial\Omega} u[3(x_1-\xi_1),3(x_2-\xi_2),3(x_3-\xi_3)]ds + \mu u(3x_1,3x_2,3x_3)$$

where $\partial\Omega$ is the surface of the ball: $\xi_1^2+\xi_2^2+\xi_3^2=\frac{4}{9}$, $\Delta = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2 + \partial^2/\partial x_3^2$ is the Laplace operator in R^3. That function is called $Corp(x_1, x_2, x_3)$ and support of that function is shown in Fig. 2 and Fig. 3 (a, b)

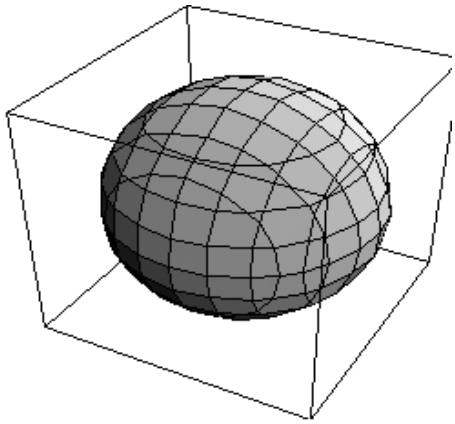

Fig. 2. The support of the function $Corp(x_1, x_2, x_3)$.

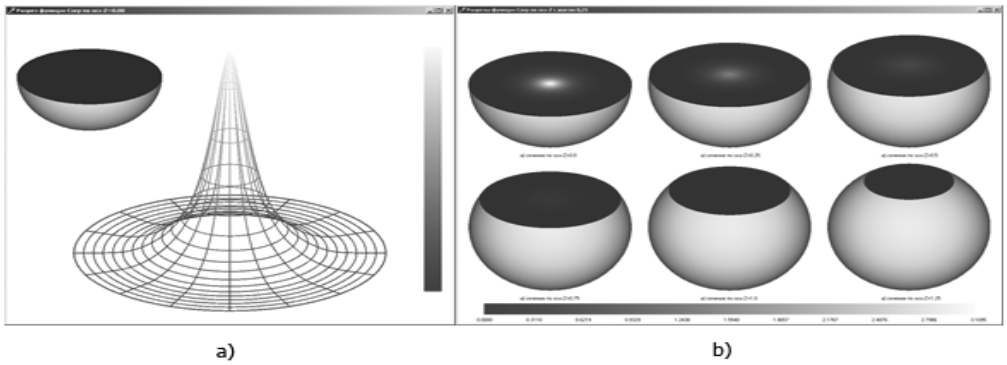

Fig. 3 The graph of the atomic function: a) with different sections of the OZ and b) plane cross section visualization of its distribution density

In numerical schemes for solving boundary value problems, in addition to the above functions, the following functions are also required: $\Delta Plop(x_1, x_2)$, $\Delta Corp(x_1, x_2, x_3)$, $\Delta^s Plop(x_1, x_2)$, $\Delta^s Corp(x_1, x_2, x_3)$, $s > 1$.

To solve the equation of thermal conductivity according to the mesh scheme, it makes sense to use the atomic solution of the following functional-differential equation:

$$\Delta u(x_1, x_2) - \delta^2 u(x_1, x_2) = \lambda \int_{\partial\Omega} u(3(x_1 - \xi_1), 3(x_2 - \xi_2)) d\omega + \mu\, u(3x_1, 3x_2)$$

where $\partial\Omega: \xi_1^2 + \xi_2^2 = 1; \Delta = \partial^2 / \partial x_1^2 + \partial^2 / \partial x_2^2$, with the appropriate choice of parameters λ and μ : $\mu = 2\pi\lambda J_0(ic), \lambda = \dfrac{9c^2}{2\pi\left[J_0(ic) - J(0)\right]}$.

The support of the function $Hlop(x_1, x_2)$ is a circle of radius 1.5 (Fig. 4).

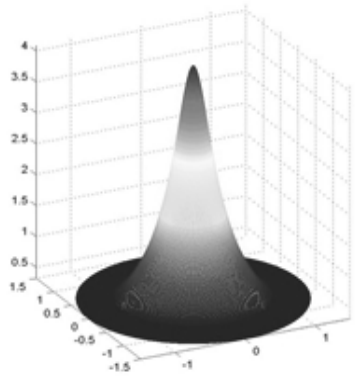

Fig. 4 The graph of atomic function $Hlop(x_1, x_2)$

The information about other atomic functions such as $KGlop(x_1, x_2)$ and $Blop(x_1, x_2)$ functions generated by a biharmonic differential operator can be found in [5, 6, 8].

Obtaining functions of three independent variables $Corp(x_1, x_2, x_3)$ and $Horp(x_1, x_2, x_3)$ is ensured when finding finite solutions of functional differential equations like

$$Lu(x_1, x_2, x_3) = \lambda \iint_{\partial\Omega} u\left[a(x_1 - \xi_1), a(x_2 - \xi_2), a(x_3 - \xi_3)\right] + \mu\, u(ax_1, ax_2, ax_3)$$

where the atomic function $Corp(x_1, x_2, x_3)$ is a solution to the functionally differential equation generated by the Laplace operator: $\Delta = \sum_{i=1}^{3} \dfrac{\partial^2}{\partial x_i^2}$; the atomic function $Horp(x_1, x_2, x_3)$ to the equation generated by the Helmholtz operator: $\Delta \pm \delta^2$, where δ^2 is the parameter of the Helmholtz equation.

The corresponding supports of the functions $Corp(x_1, x_2, x_3)$ and $Horp(x_1, x_2, x_3)$ are a ball of a certain radius $supp\, F(x_1, x_2, x_3) = M; M: x_1^2 + x_2^2 + x_3^2 \le R^2$; $F(x_1, x_2, x_3)$ is one of the functions $Corp(x_1, x_2, x_3)$ or $Horp(x_1, x_2, x_3)$. Each of these functions is normalized by the condition $\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(x_1, x_2, x_3)dx = 1$.

These three-dimensional atomic functions are convenient for implementing computational algorithms when constructing approximate solutions to boundary value problems in 3D domains using meshless schemes. The properties of these functions make it possible to use them as basic ones for solving boundary value problems by meshless methods based on collocation methods.

Atomic functions of three independent variables are solutions of functional differential equations with the Helmholtz operator

$$\Delta u(x_1, x_2, x_3) - \delta^2 u(x_1, x_2, x_3) = \lambda \iint\limits_{\partial\Omega} u\left[3(x_1 - \xi_1), 3(x_2 - \xi_2), 3(x_3 - \xi_3)\right] d\omega + \mu\, u(3x_1, 3x_2, 3x_3), \quad (1)$$

where $\partial\Omega$ is the sphere: $\xi_1^2 + \xi_2^2 + \xi_3^2 = \frac{4}{9}$; $\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}$ is the Laplace operator; λ, μ are the parameters, the values of which are refined in the process of finding a finite solution to this equation; δ^2 is the parameter of the Helmholtz equation.

The functions $Horp(x_1, x_2, x_3)$ form a subclass of atomic functions that are generated by the modified Helmholtz differential operator $\Delta - \delta^2$. Other subclasses of atomic functions which are generated by the Laplace operator, biharmonic and polyharmonic operators, have been investigated in articles [5, 6, 7].

Fig. 5–7 show the visualization of the result of the numerical construction of a three-dimensional function $Horp(x_1, x_2, x_3)$, its first and second derivatives with respect to the arguments x_1, x_2, and the results of the action of the Laplace and Helmholtz operators on the function $Horp(x_1, x_2, x_3)$: $\Delta Horp(x_1, x_2, x_3)$, $\left(\Delta - \delta^2\right) Horp(x_1, x_2, x_3)$ respectively.

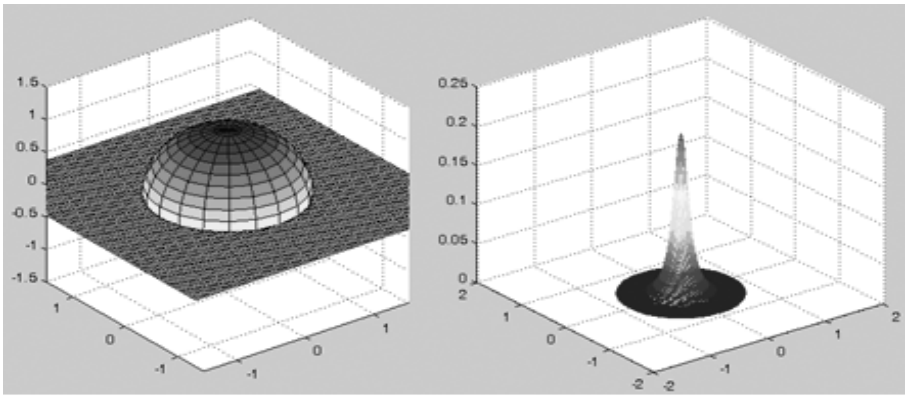

Fig. 5 Visualizing the graph of the $Horp(x_1, x_2, x_3)$ function projection in space $Ox_1x_2Horp(x_1, x_2, const)$: a) determining the support points under the condition $x_3 = const$ (section by the plane $x_3 = const$ of the support – a unit ball centered at the point (0, 0, 0)); b) the graph of the function projection

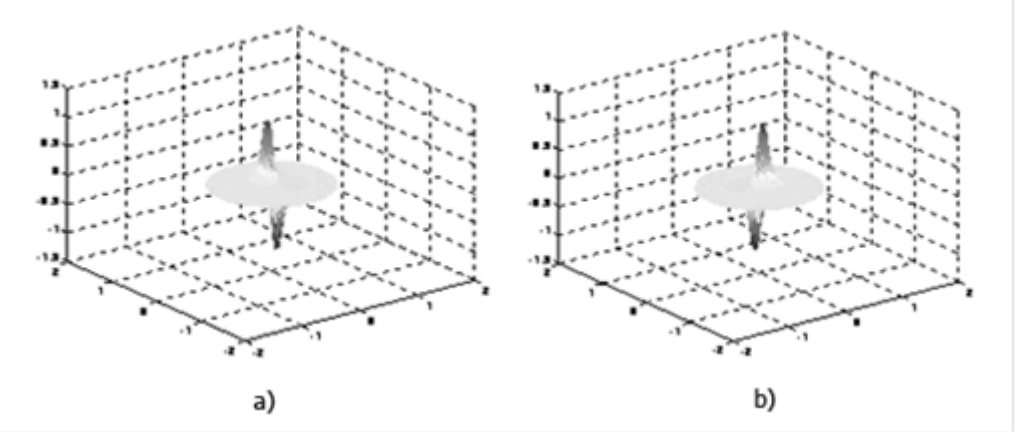

Fig. 6 The graphs of projections of the first derivatives of the function $Horp(x_1, x_2, x_3)$ with respect to variables x_1, x_2, provided $x_3 = const$: a) the graph of $\frac{\partial Horp(x_1, x_2, const)}{\partial x_1}$ in space $Ox_1x_2 \frac{\partial Horp(x_1, x_2, const)}{\partial x_1}$; b) the graph of $\frac{\partial Horp(x_1, x_2, x_3)}{\partial x_2}$ in space $Ox_1x_2 \frac{\partial Horp(x_1, x_2, const)}{\partial x_2}$.

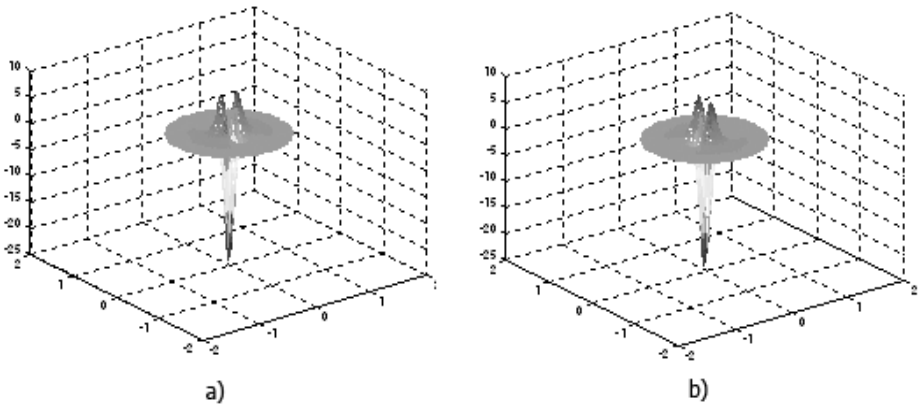

Fig. 7 The graphs of projections of the second derivatives of the function $Horp(x_1,\ x_2,\ x_3)$ *on the variables* $x_1,\ x_2$ *provided:* $x_3 = const$ *a) the graph of* $\dfrac{\partial^2 Horp(x_1,\ x_2,\ const)}{\partial x_1^2}$ *in space* $Ox_1x_2\dfrac{\partial^2 Horp(x_1,\ x_2,\ const)}{\partial x_1^2}$ *; b) the graph of* $\dfrac{\partial^2 Horp(x_1,\ x_2,\ x_3)}{\partial x_2^2}$ *in space* $Ox_1x_2\dfrac{\partial^2 Horp(x_1,\ x_2,\ x_3)}{\partial x_2^2}$

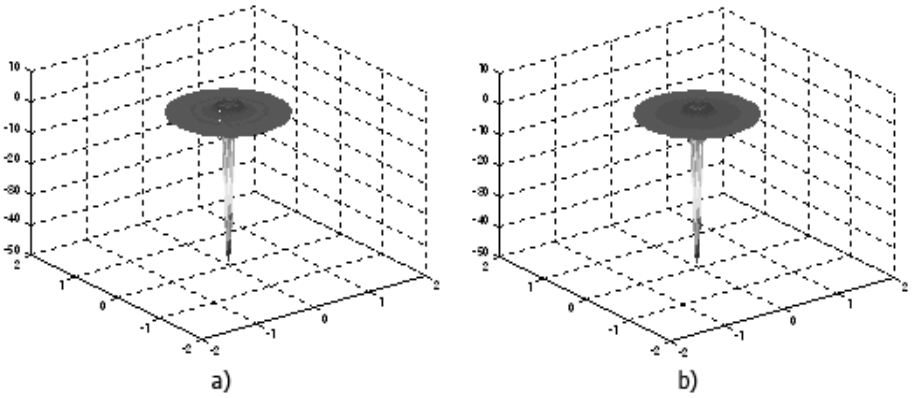

Fig. 8 The results of the action of the Laplace Δ *and Helmholtz* $\left(\Delta - \delta^2\right)$ *differential operators on the function* $Horp(x_1,\ x_2,\ x_3)$ *: a) the projection graph of* $\Delta Horp(x_1,\ x_2,\ const)$ *in space* $Ox_1x_2\Delta Horp(x_1,\ x_2,\ const)$ *; b) the graph of* $\left(\Delta - \delta^2\right)Horp(x_1,\ x_2,\ const)$ *in space* $Ox_1x_2\left(\Delta - \delta^2\right)Horp(x_1,\ x_2,\ const)$

The scheme for constructing a finite solution to a functional differential equation (1) can also be used to find a finite solution to a functional differential equation of the following form:

$$\Delta u(x_1,\ x_2,\ x_3) + \delta^2 u(x_1,\ x_2,\ x_3) = \lambda \iint\limits_{\partial\Omega} u\left[3(x_1 - \xi_1),\ 3(x_2 - \xi_2),\ 3(x_3 - \xi_3)\right] d\omega + \mu u(3x_1,\ 3x_2,\ 3x_3), \quad (2)$$

where all the notations of the equation (1) are preserved.

The atomic function $KGorp(x_1,\ x_2,\ x_3)$ which is a solution to the functional differential equation (2) where $\partial\Omega$ is the sphere $\xi_1^2 + \xi_2^2 + \xi_3^2 = \dfrac{4}{9}$ and the values of the coefficients $\lambda = \dfrac{3^5\delta^3}{8\pi\left[\sin\dfrac{2}{3}\delta - 2\delta\right]}$, $\mu = \dfrac{8\pi}{3\delta}\lambda\sin\dfrac{2}{3}\delta$ will be a bounded infinitely differentiable function supported in the form of a ball of unit radius and normalized by the condition

$\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} KGorp(x_1, x_2, x_3)dx_1dx_2dx_3 = 1$ which is represented in the cube $[-1, 1]\times[-1, 1]\times[-1, 1]$ by the Fourier series

$$KGorp(x_1, x_2, x_3) = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{r=0}^{\infty} a_{pqr}\cos(p\pi x_1)\cos(q\pi x_2)\cos(r\pi x_3)$$

where a_{pqr}, $p, q, r = 1, 2, ...$ are the Fourier coefficients:

$$a_{000} = \frac{1}{8},$$

$$a_{p00} = \frac{1}{4}KG\tilde{o}\tilde{r}\tilde{p}(p\pi,0,0),\ a_{0q0} = \frac{1}{4}KG\tilde{o}\tilde{r}\tilde{p}(0,q\pi,0),\ a_{00r} = \frac{1}{4}KG\tilde{o}\tilde{r}\tilde{p}(0,0,r\pi),$$

$$a_{pq0} = \frac{1}{2}KG\tilde{o}\tilde{r}\tilde{p}(p\pi,q\pi,0),\ a_{0qr} = \frac{1}{2}KG\tilde{o}\tilde{r}\tilde{p}(0,q\pi,r\pi),\ a_{p0r} = \frac{1}{2}KG\tilde{o}\tilde{r}\tilde{p}(p\pi,0,r\pi),$$

$$a_{pqr} = KG\tilde{o}\tilde{r}\tilde{p}(p\pi,q\pi,r\pi).$$

$$KG\tilde{o}\tilde{r}\tilde{p} = \prod_{h=0}^{\infty}\frac{\mu - \frac{16\pi}{3^2}\lambda\frac{\sin\left(\frac{2\sqrt{t_1^2+t_2^2+t_3^2}}{3^{h+1}}\right)}{\frac{2\sqrt{t_1^2+t_2^2+t_3^2}}{3^{h+1}}}}{3^3\left(\frac{t_1^2+t_2^2+t_3^2}{3^{2h}} - \delta^2\right)}.$$

Fourier function transformation $KGorp(x_1, x_2, x_3)$ is a rapidly decreasing function of exponential type for $t_1^2 + t_2^2 + t_3^2 \to \infty$.

The proof of the existence of a solution is similar to the given above.

The considered atomic functions $Horp(x_1, x_2, x_3)$ and $KGorp(x_1, x_2, x_3)$ are the radial basis functions that can be used to construct approximate solutions of boundary value problems according to meshless schemes for differential equations in the formation of which Helmholtz-type operators are used.

In order to expand the class of functions and improve their properties, let us consider the construction of a family of the atomic radial basic functions (ARBFs) of three independent variables by using the example of a functional differential equation in the form

$$\Delta u(x_1,x_2,x_3) - \delta^2 u(x_1,x_2,x_3) = \\ = \lambda\oint_{\partial\Omega} u(k(x_1-\xi_1), k(x_2-\xi_2), k(x_3-\xi_3))d\omega + \mu u(kx_1,kx_2,kx_3), \quad (3)$$

where $\partial\Omega: \sum_{i=1}^{3}\xi_i^2 = r_k^2$ and $r_k = r_k(k)$.

It should be noted that the region $\partial\Omega$ is dependent on the compression ratio and can be refined in the process of constructing a solution to the boundary value problem if it necessary to ensure certain properties of functions.

That function will be denoted $Horp_k(x_1, x_2, x_3)$ and, according to [7], will be considered an atomic function. It should be noted that the index indicates the possibility of extending the subclass of the functions $Horp_k(x_1, x_2, x_3)$ in order to provide the necessary characteristics of the function.

The function $Horp_k(x_1, x_2, x_3)$ is even with respect to its variables and can be expanded in a threefold Fourier series [4]

$$Horp_k(x_1, x_2, x_3) = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{r=0}^{\infty} a_{pqr}\cos(p\pi x_1)\cos(q\pi x_2)\cos(r\pi x_3) \quad (4)$$

in which the Fourier coefficients are calculated by using the following formulas:

$$
\begin{aligned}
a_{000} &= \frac{1}{8};\ a_{p00} = \frac{1}{4} H\tilde{o}\tilde{r}p_k\left(p\pi,\ 0,\ 0\right);\\
a_{pq0} &= \frac{1}{2} H\tilde{o}\tilde{r}p_k\left(p\pi,\ q\pi,\ 0\right);\ a_{0q0} = \frac{1}{4} H\tilde{o}\tilde{r}p_k\left(0,\ q\pi,\ 0\right);\\
a_{0qr} &= \frac{1}{2} H\tilde{o}\tilde{r}p_k\left(0,\ q\pi,\ r\pi\right);\ a_{p0r} = \frac{1}{2} H\tilde{o}\tilde{r}p_k\left(p\pi,\ 0,\ r\pi\right);\\
a_{00r} &= \frac{1}{4} H\tilde{o}\tilde{r}p_k\left(0,\ 0,\ r\pi\right);\ a_{pqr} = H\tilde{o}\tilde{r}p_k\left(p\pi,\ q\pi,\ r\pi\right),
\end{aligned}
\tag{5}
$$

where $p, q, r = 1, 2, ..$

The functions $Horp_k(x_1,\ x_2,\ x_3)$ forms a family of a subclass of atomic functions that are generated by the modified Helmholtz differential operator $\Delta - \delta^2$ (Fig.9).

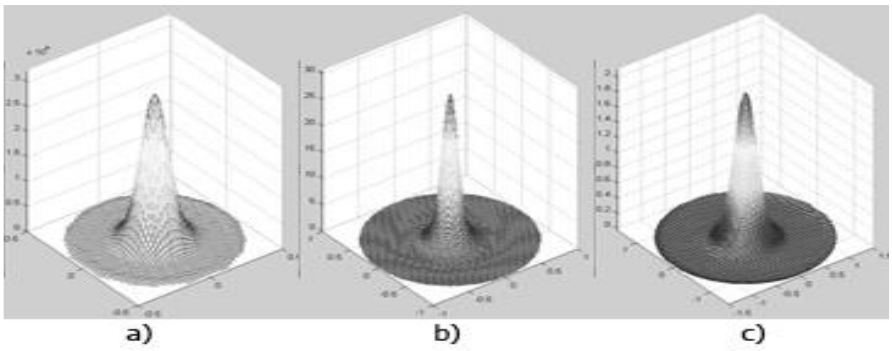

Fig. 9 The $Horp_k(x_1,\ x_2,\ x_3)$ function of ARBF family with variation in parameters r (support radius) and k (compression ratio)

According to the scheme above, it is possible to construct families of finite solutions to the considered functional differential equations which are generated by differential operators of Laplace, Helmholtz, etc.

The proposed families of atomic radial functions can be used as basic ones in the case of solving boundary value problems by using meshless schemes [9, 10]. Expanding the subclass of functions will allow selecting functions from families. That will provide the best approximation of the desired solution to the boundary value problem.

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Багатовимірні узагальнення атомарних радіальних базисних функцій

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Представлені можливі підходи узагальнення багатовимірних атомарних радіальних базисних функцій. Функції математичної фізики використовуються при розв'язанні двовимірних та тривимірних крайових задач з частинними похідними. Відповідно до задач мають місце використання функції згідно до розмірності, тобто функції, що породжені різними диференційними операторами. Розглянуті функціонально-диференційні рівняння, що породжують саме ці функції. Згідно з наведеною схемою, будують сімейства фінітних рішень розглянутих функціонально-диференціальних рівнянь, які породжуються диференціальними операторами Лапласа, Гельмгольця та ін. Підсумки наведені у вигляді теорем. З метою розширення класу функцій та удосконалення їх властивостей розглядається побудова сімейства АРБФ трьох незалежних змінних на прикладі функціонально-диференціального рівняння відповідного виду. Методи розв'язання відносяться до безіткових схем та поєднують можливості побудови границь областей за допомогою R-функцій. Атомарні функції зручні при реалізації обчислювальних алгоритмів побудові наближених рішень крайових завдань у 2D та 3D областях за безсіточними схемами. Властивості цих функцій дозволяють використовувати їх як базисні при вирішенні крайових задач безсіточними методами на основі методів колокації. Для атомарних функцій надана залежність від коефіцієнта стиснення, яка уточнюється в процесі побудови рішення крайової задачі за необхідністю для забезпечення певних властивостей функцій. Надано схему побудови розв'язків задач теплопровідності за безсітковою схемою.

Ключові слова: атомарні радіальні базисні функції, крайові задачі математичної фізики, безсіткові методи розв'язування крайових задач.

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