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Evolution of a population of strategies with memory in the presence of a distinct small group

Topicality. Today, the human population is constantly growing. The influence of societies and subcultures on each other and on the population as a whole is also increasing. It is necessary to expand knowledge in the field of social behavior to improve the understanding of interaction within the population.
Key goals. The purpose of the article is to study the influence of selected distinct groups on the evolution of the population. The nature of changes in evolution is studied in the presence of groups of different complexity and size in the population. This work also aims to establish the most evolutionarily advantageous composition and size of a small group and the stage at which a population will be dominated by such a group, if it is possible.
Research methods. The evolution of a population with a full set of behavioral strategies, limited only by memory depth, in the presence of a distinct group is considered. Each strategy carrier interacts with all others, themselves included, according to an iterative model of the prisoner's dilemma. Rewards are determined by payment matrices. Each subsequent generation successively loses the most disadvantageous behavioral strategies of the previous generation. Carriers who are members of a distinct group behave more aggressively towards "strangers" and more cooperative with "their own". Different variants of the initial composition of the group are considered, namely the case when the average complexity of a small group is greater than the average complexity of the population as a whole, less, or comparable to it. Variants with different initial size of the small group have been considered – 5, 10, 15, 20% of the population size.
The results. The paper shows how the presence of small groups of different complexity and size affects population dynamics. An increase in the aggressiveness of the population has been revealed if there is a small group of any composition in it. The size and composition of a small group that makes it possible to dominate the society and the stage of evolution when it takes place have been established.
Conclusion. Based on the results of the article, the main consequences of the influence of distinct small groups on the evolution of the population have been determined.

Key words: *evolution, population, society, strategy, distinct group, complexity, aggressiveness, cooperation.*

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1. Introduction

Today, research on behavior in society, development and evolution of the groups forming it are becoming more relevant [1]. The population of people is growing and the influence of societies on each other is also increasing due to the rapid development of information technologies. People from different continents and countries, different religions and beliefs, with different cultures and traditions now coexist in a single information space, and therefore constantly influence each other. The study of such interaction is a key to understanding the future of humanity not only as a society, but also as a biological species.

Modern game theory [2] is one of the areas used to study the interaction of populations. A clear example of game models used for this purpose are the models based on the so-called "prisoner's dilemma" [3]. These models represent a paradigm which has helped to discover such mechanisms of cooperative behavior as voluntary participation, punishment, heterogeneous activity and social diversity.

The model of the prisoner's dilemma game is that two players can "cooperate" with each other (help each other) or "betray" the opponent (behave aggressively). At the same time, the reward matrix is chosen in such a way that the best payoff for both players can be obtained only when they both choose cooperation, but the highest payoff for one player can be obtained if one of them chooses betrayal and the other chooses cooperation. Thus, the locally optimal solution for both players is aggressive behavior, which is due to the real-life simulation: cooperative behavior usually requires additional resources, unlike selfish behavior.

In [4-7] the evolution of populations with a full set of behavioral strategies (limited only by the depth of memory) within the framework of a generalized prisoner's dilemma has been considered. Increases in average difficulty and average memory depth over time, as well as decreases in average aggressiveness, have been analyzed. Complexity has been recognized as an evolutionarily advantageous characteristic.

In this work, we have analyzed how the course of evolution changes when a distinct group, whose members actively support each other and behave more aggressive towards others, appears in the population. The main points of interest are: to what extent the appearance of such a group will increase the level of aggressive behavior in society, and whether being in such a group is evolutionarily beneficial for the strategy carriers. Additionally, it is interesting to see how a change in the composition of a small group will affect the development of society, what composition and size of the group are sufficient for its complete dominance in society, as well as how soon such dominance will occur.

2. Description of strategies with memory

A behavior strategy is a rule which determines a move depending on the known move of the opponent. Strategies that imply only the reaction to the opponent’s current move are called the strategies without memory, and strategies that allows decisions based on knowledge of the opponent’s several previous moves are the strategies with memory. The memory depth is the number of the opponent’s moves, over the one available at a current stage, which is taken into account when choosing a move. Therefore, the strategies without memory have a memory depth of 0, the strategies that remember an additional 1 move to the existing one have a depth of 1, and so on.

Strategies without memory choose a move based on the opponent's previous move, i.e., the one made during the previous iteration. There are two move options - cooperation (marked as 1) and betrayal (marked as 0). To denote a strategy without memory, it is necessary to specify the carrier’s responses to the opponent's moves 0 or 1. Those strategy responses can be set as follows 0: 0, 1: 0, that is, "if the opponent move was 0, then we choose the move 0, and if 1, then move 0 anyway." We have decided to denote such strategies as sequences of digits {0,1} (i.e., as numbers in binary representation). Then the bit number is the opponent's move, and the value in it is the response to such a move. For example, the strategy "if the other player moved with 0, choose move 1, and if 1, then choose move 0" can be written in the form: 01. It should be noted that in the [4-7], the strategy record is inverted, and the reaction to move is placed on the left. But we have chosen to use the notation described above for greater similarity with the binary notation and for simplifying bitwise operations in the program.

Therefore, all strategies with zero memory depth can be written as a binary two-digit number – 00, 01, 10, 11. It should be mentioned, that the choice of the first move is a unique situation, at this point there is no information about previous opponent’s moves and a special procedure for such a case is required. That is why we should add one auxiliary bit to determine a first agent’s move, increasing the number of strategies from 4 to 8: [0]00, [1]00, [0]01, [1]01, [0]10, [1]10, [0]11, [1]11.

Strategies with memory depth of 1 make a move taking into account the opponent's current and previous move. So, the opponent's moves will be denoted with two bits: 00, 01, 10, 11. Therefore, a reaction to the opponent's moves (i.e., description or the "name" of the strategy) must be 4 characters long. If we convert those combinations of the opponent's moves from the binary system to the decimal system: 00 - 0, 01 – 1, 10 – 2, 11 – 3, we will get the bit numbers in which the answers to the corresponding combinations of moves should be stored. For example, denoting a strategy reaction to an opponent's moves as 00: 1, 01: 0, 10: 1, 11: 0, in binary notation would be 0101 (the least significant 00th bit rightmost). So, if we need 2 bits to describe a strategy with a memory depth of 0, and 4 bits for a memory depth of 1, then we need 2^{k+1} bits to describe a strategy with an arbitrary memory depth k. 2^N strategies can be formed, where N is the number of bits. So, for an arbitrary memory depth k, there are $2^{2^{k+1}}$ unique strategies. Therefore, for memory depth of 1, we have $2^{2^{1+1}}$=2^4=16 unique strategies.

The question now is how the strategy with memory depth 1 is supposed to react if only current enemy's move is known. That is why it is necessary to add additional reaction bits – a response to the opponent's first move. These strategies should be denoted [01]1001, where the 0th bit in the brackets is a response to the first move of 0, and 1st, to the move of 1. Afterwards, there will have been sufficient data to create a 4-digit description of the strategy. Furthermore, the strategies with memory depth of 1 require an additional bit for the first move like memoryless ones. Therefore, the notation of the strategy with a memory depth of 1 is: [0][10]0110.

Similarly, the description of strategies with memory depth of 2 is a bitwise arrangement of the strategy's responses to the opponent's current move and the 2 previous ones, that is, the {0,1} sequence with the length of 3 characters (010, 101, ...). Using the method introduced above – denoting several moves as a number, converting it to decimal (101 - 5, 011 - 3, ...), and writing the response to this sequence of moves in the bit by the received decimal number, we have concluded that 8 bits to denote such a strategy are required. Moreover, the strategies with memory depth of 2 also need extra bits to record situations with insufficient data, when only the current move or the current and one previous move are known. Therefore, the notation of the strategy with memory depth of 2 is: [0][10][0110]11001011. A set of strategies with the same name but different "sub-strategies" forms a family of strategies.

3. Interaction of strategies

Since we have chosen a prisoner's dilemma game model to describe carriers' interaction, the locally optimal solution for both players is aggressive behavior. This is due to the simulation of real life: cooperative behavior usually requires additional resources, unlike selfish behavior. The matrix of rewards represents the "reaction of the world" to the strategy behavior, because, eventually, we use them to determine which behavior helps strategies to survive and which does not. As in [4-7], Axelrod's matrix [8] (Table 1) is used to describe the rewards:

Table 1 Axelrod's reward matrix

A\B	**0**	**1**
0	1	5
1	0	3

After each move, the carrier's strategy is perceived as strategy A, and its move corresponds to the matrix rows, and the opponent's strategy as strategy B, and its response corresponds to the matrix columns. The reward that the strategy receives is located at the intersection. The strategies that are not used by a small group are rewarded according to this matrix.

We can influence group behavior by changing the values of the reward matrix for group members, stimulating them to take certain actions. In this case, we are interested in a classic "sect" behavior: group members should actively cooperate with each other, help each other, but treat "strangers" aggressively. That is why we set two more reward matrices: members of the small group will be rewarded by using the first one, when they interact with "strangers", and by the second one, when they interact with "their own".

M_2 matrix shown in Table 2 is a modification of the M_1 matrix, where aggressive actions are rewarded by more points, and cooperative actions by less points. It is used for "aggressive towards strangers" interactions of the small group.

Table 2 – Reward matrix for "aggressive towards strangers" interactions

A\B	**0**	**1**
0	2	6
1	0	2

For "helpful toward one's own" interactions matrix, the M_3 matrix shown in Table 3 has been chosen, where the rewards for the aggressive behavior are reduced and for the cooperation are increased.

Table 2 – Reward matrix for "helpful toward one's own" interactions

A\B	**0**	**1**
0	0	3
1	2	5

We have used the prisoner's dilemma model iteratively – each strategy carrier interacts with all others (including themself) a given number of times, receiving rewards according to the established reward matrices. After all players in a given generation have played with everyone, we average the scores across all strategy families, then remove the family with the lowest average score from population and start a new generation with the remained strategies.

4. Collective variables

The number of strategies with memory depth of 0 is relatively small, so it is easy to keep track of each individual strategy in a generation. But the situation changes drastically as the memory depth increases, because the number of strategies in this case grows exponentially: $2^{2^{k+1}}$ strategies for the memory depth of k. Therefore, to track the dynamics of the population as a whole, it will be appropriate to introduce some collective variables that will reflect the state of the entire population.

The average complexity of the population is the arithmetic average of the complexities of all strategies in a generation. We introduce the concept of strategy complexity as described in [4-7], so the result can be compared with the usual process of evolution. This approach is based on the principle of comparative complexity of polynomial functions, which states that polynomials of a higher degree are more complex than polynomials of a lower degree.

The sequences of symbols $\{0,1\}$ $x = x_1 x_2 \ldots x_n$ can be perceived as a function that for each integer value i returns the value $x_i \in \{0,1\}$. Then, to calculate the complexity of the function, the difference operator $A{:}M \rightarrow M$ is used:

$$y = Ax, \tag{4.1}$$

where elements of sequence $y = y_1 y_2 \ldots y_n$ can be defined as

$$y_i = x_{i+1} - x_i, \tag{4.2}$$

where i=1, 2,…,n is an element number in a sequence.

If we use the cyclicity of the binary sequence, or assume that $x_{n+1} = x_1$, then it is possible to calculate the elements of the sequence y for any strategy, i.e., find the strategy into which the chosen one is switched by using the specified differential operator. We assume that the strategies into which the initial strategies are switched by using the operator are simpler. Thus, a graph of strategies is formed, with the simplest and most aggressive strategy – 00 (or 0000 or 00000000 for strategies with memory), which by using the operator are switched into itself, at the bottom. Assuming that its complexity is 0, and the complexity for the each subsequent level of such a graph increases by one, then for the strategies without memory we have the following graph:

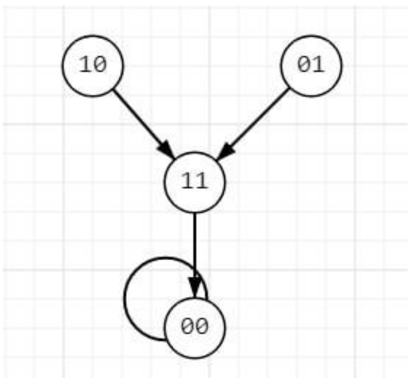

Fig. 4.1 – Complexity graph for memory depth of 0

Using such the approach, it can be determined that the strategy 11 has a complexity of 1, and the strategies 10 and 01 have a complexity of 2. In the same way, we can build a complexity graph for the strategy with any memory depth, for example, for depth 1 we have the graph shown in Fig. 2.

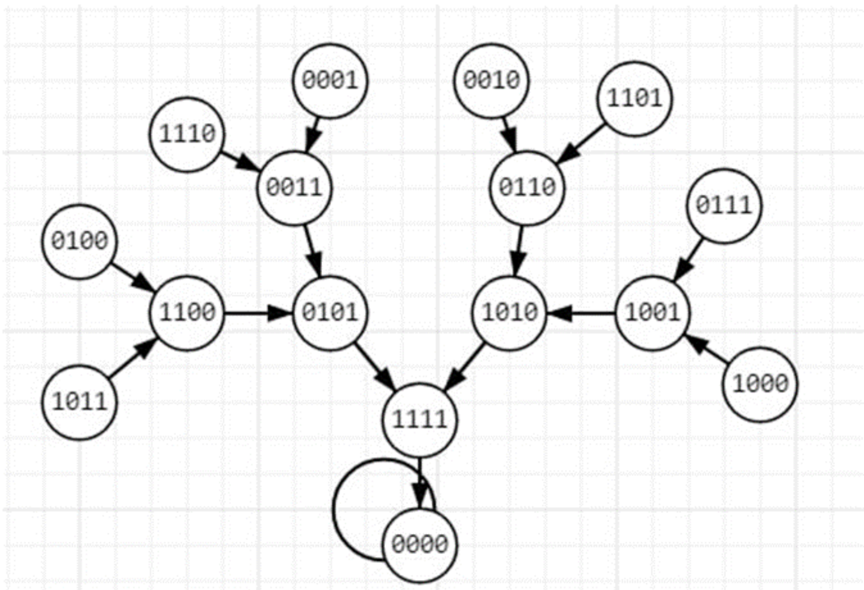

Fig. 4.2 – Complexity graph for memory depth of 1

The level of the graph corresponds to the strategy complexity. By constructing such a graph for strategies of memory depth 2, we obtain the complete complexity distribution for all carriers in the population.

The next collective variable considered is the average memory depth. Even though the strategy complexity directly depends on its memory depth, this variable is still interesting to consider, because it shows how strongly more primitive strategies behave in population of more advanced individuals and how they affect it.

A very important value is the average aggressiveness of the population, i.e., the relative part of aggressive moves that the strategy performs in each generation. Since we have stimulated the aggressive behavior of group members, aggressiveness is expected to exceed the level of the standard evolution.

It is also interesting to consider such a value as the average "earnings" – the average number of points that the strategy receives in one move. Using it, we can determine the reward "profitability" for certain modifications of the population.

Moreover, the size of the group relative to the population should also be considered – a percentage of population size in each generation. What is also interesting is the data on the size and composition of the group at the moment it is able to completely "dominate" the population displacing all other strategies, and how quickly that can happen.

5. Modelling

Since the influence of a small group on the evolution of a population can depend on the composition of a small group directly, several options for dividing strategies into large and small groups should be considered. A convenient measure for comparing groups of strategies is the average complexity. From [4], we know that the average complexity of a population with a memory depth of 2 without modifications at the beginning of the simulation is approximately equal to 7. Therefore, it would be interesting to consider how the nature of evolution can change if strategies of different complexity are adopted by a small group. Let us consider three cases, when the average complexity of the group is greater, smaller, or comparable with the complexity of the population.

Another factor that can strongly influence the nature of evolution is the size of a small group relative to the population. For each of the three variants of group complexity, we have conducted separate studies where the size of group is 5, 10, 15 and 20% of population size.

Strategies that have the same complexity may differ in other collective variables, such as aggressiveness, and therefore may affect the population differently.

For the next step, several groups with a random selection of its participants but with the same average difficulty have been considered. Averaging the results of modeling the influence of such groups on the population allows obtaining more universal dependencies. Such dependencies should not be sensitive to the composition of groups. A series of ten experiments have been conducted with identical inputs of complexity and group size. The results have been averaged to obtain the final values of the collective variables.

5.1. The small group with greater complexity than the population

First, we have considered the case when the average complexity of a small group is greater than that of the population. That is, we have had a separate intellectual group of more "highly developed" strategies. Since the maximum possible complexity of the strategy at memory depth 2 is 8, we have created a group only from representatives of this complexity, and considered the evolution where the group size is 5, 10, 15 and 20% of the population.

At the size of 5%, a group does not make serious changes in the nature of the average complexity dynamics (Fig. 5.1). It can be seen that starting from the 217th generation, the average complexity of a small group drops, this is due to the fact that in some experiments the members of the group disappear in these generations (so its complexity is equal to zero), which lowers complexity, when averaged over 10 experiments. A size of 10% extends the survival period of the group, but still not enough to survive to the end of modelling. But the size of 15% is enough for a group to dominate in the later stages of evolution. At the same time, we can observe an increase in the average complexity of society. The increase in complexity is insignificant, which can be explained by the small difference between the complexity of the group (8) and the population (~7).

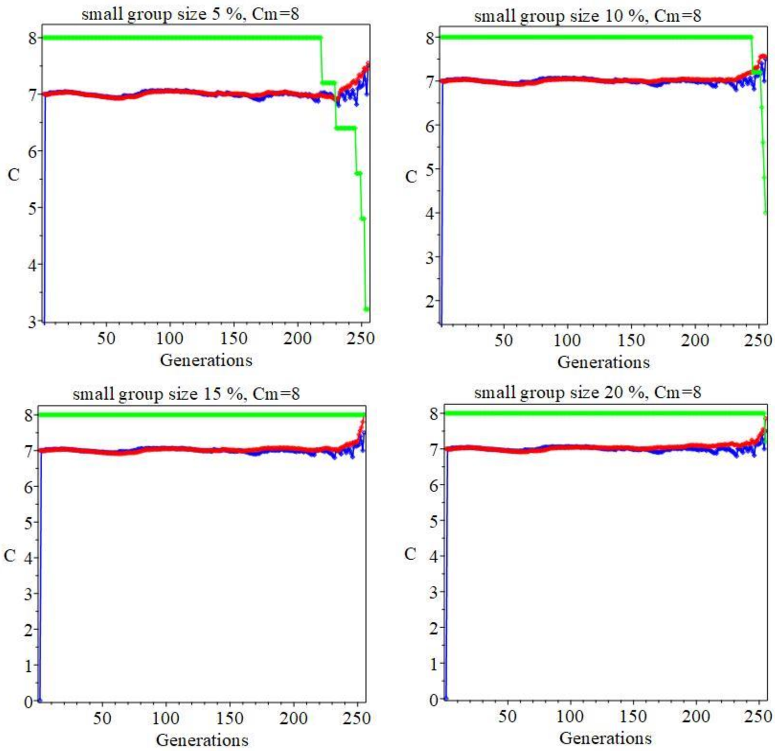

Fig. 5.1 Average complexity dynamics. Red color indicates data for the population with a small group, blue - data for the population without a small group, green - characteristics of the small group itself. The complexity of the small group is equal to 8. There is a noticeable increase in the average complexity of the population even in comparison with the change in the average complexity of the population without the presence of a group. Each picture corresponds to a certain initial size of the small group, which is indicated above it.

The average memory depth of the population slightly decreases at the later stages of evolution (Fig. 5.2) with a big group percentage (15-20%). This is because in the later stages, most of the surviving strategies of memory depth 2 are members of the group, and simpler strategies of smaller memory depths remain in the population.

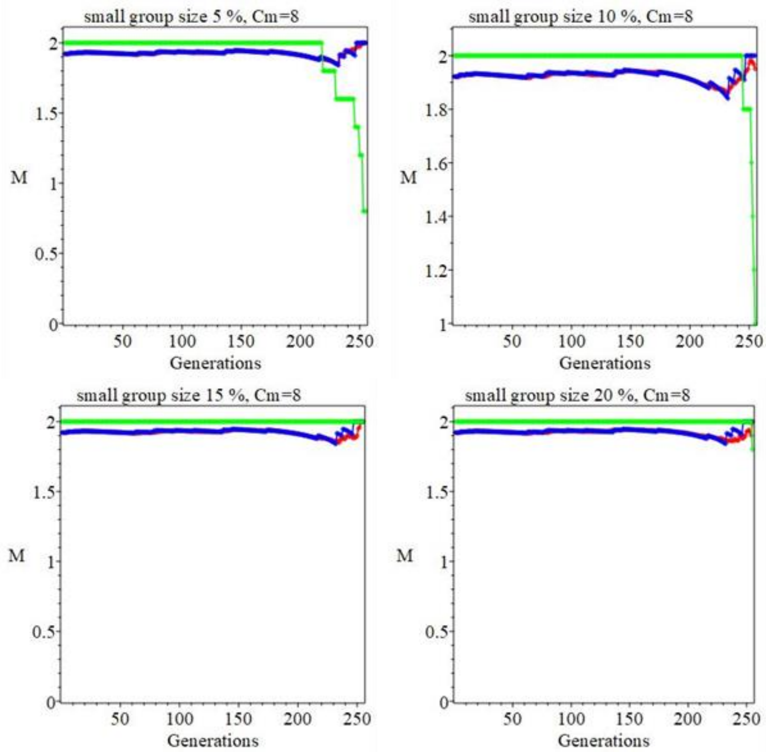

Fig. 5.2 Changes of the average memory depth in the population with a small group (red), without a small group (blue) and in the small group itself (green).

The average aggressiveness of both the population and the small group increases compared to the standard version of the population (see Fig. 5.3). As expected, the small group itself has `very high aggressiveness regardless of its size. Nevertheless, its size strongly affects the aggressiveness of the population - it increases slightly at 5%, becomes increasingly larger at 10 and 15%, and at a size of 20% aggressiveness almost reaches the level of the small group itself.

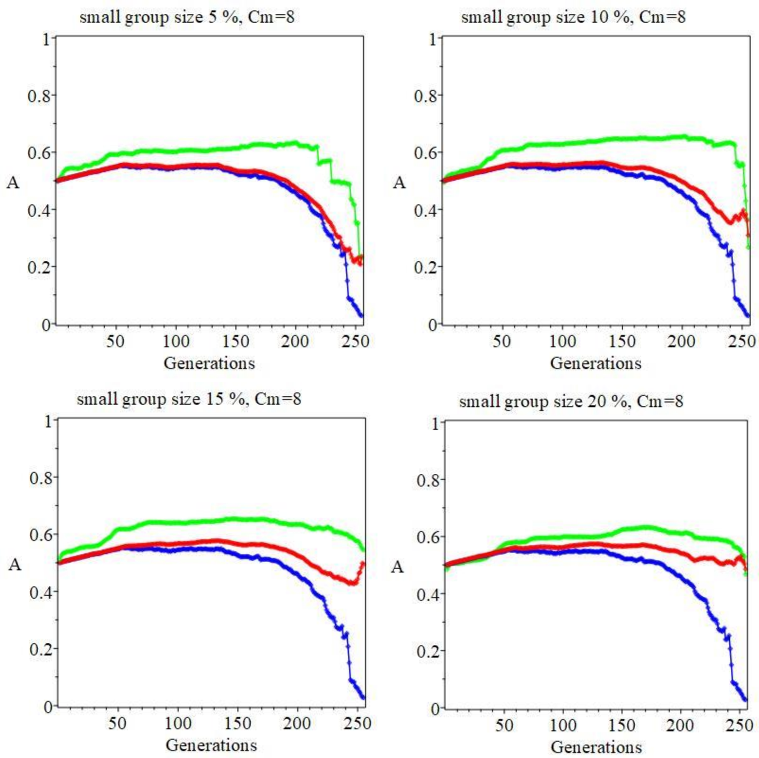

Fig. 5.3 Evolution of the average aggressiveness of the population with a small group (red), without a group (blue) and of the group itself (green).

As shown in [5], the value of the average number of points per move and the aggressiveness of the population are related by the equation:

$$P(t) = P_{\max} - \frac{\left(A(t) - A_{\min}\right)^{\alpha}}{\lambda} \tag{5.1}$$

where $\alpha = 2$ in the absence of a small group, and λ is determined from the requirement of coincidence at the initial point.

However, it turns out that the number of small groups affects the value of α . It decreases in proportion to the initial number in the group. That dependence is in good agreement with the behavior of the average number of points obtained during modeling (see Fig. 5.4). That is, as the average aggressiveness increases, the average earnings per turn decreases, so it becomes lower and lower as the group size increases. As expected, the average earnings of the small group are higher throughout most of the evolution, but on the later stages it declines as the group strategy displaces other strategies. Thus, the number of a small group affects the relationship between aggressiveness and the average number of points per move, significantly reducing them with an increase in the number of a small group.

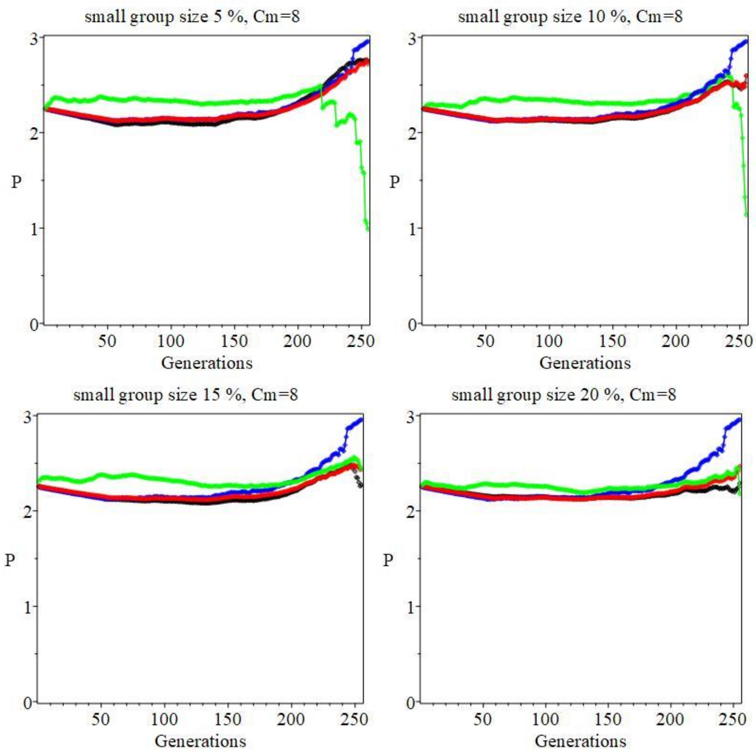

Fig. 5.4 The change in average carriers's points per turn in the population with a small group (red), without a small group (blue), in the small group itself (green). Equation (5.1) is shown in black. α = 1.6 for a small group of 5%, α = 1.2 for 10%, for 15% - α = 0.8, and for 20% = α = 0.4. The good consistency of this dependence with the simulation data (red) is noticeable.

The graphs of the relative group size (group size divided by size of the whole population) best demonstrate population changes with evolution (see Fig. 5.5) – we can see that a size of 5% for a small group does not provide much of survival advantage. A size of 10% of the population gives a slightly better result, but the group is still a minority even on the late stages. Only size of 15-20% allows a small group to dominate the society.

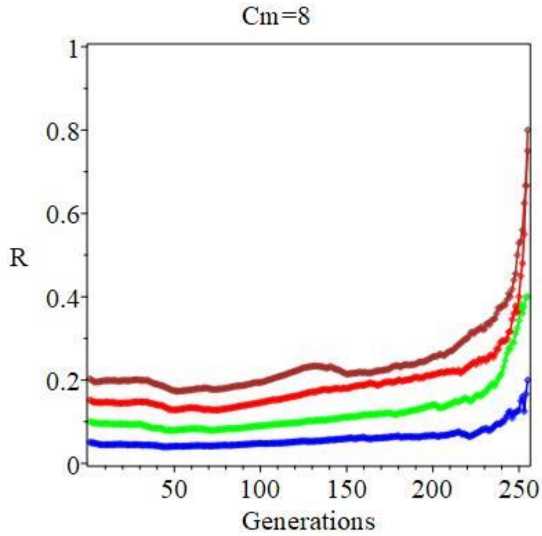

Fig. 5.5. The relative size of groups R changes during evolution for different initial sizes. Brown – initial 20%, red – 15%, green – 10% and blue – 5%. In all cases, the size of small groups is increasing.

5.2. The small group with the same complexity as the population

Let us now consider the modeling of the evolution with a different composition of the small group. This time the average complexity of the "sect" will be compared to the complexity of the population as a whole (~7). The group will randomly include strategies with complexity 6 and 8, but the main part of strategies will have complexity of 7. Again, in order to level the influence of the choice of strategies, the results have been averaged over ten experiments.

An interesting result is that the presence of a selected group, which has a similar "gene pool" to the population, has almost no impact (see Fig. 6.6) on the development of society, the average complexity and average memory depth remain on the same levels, as in the standard version of evolution.

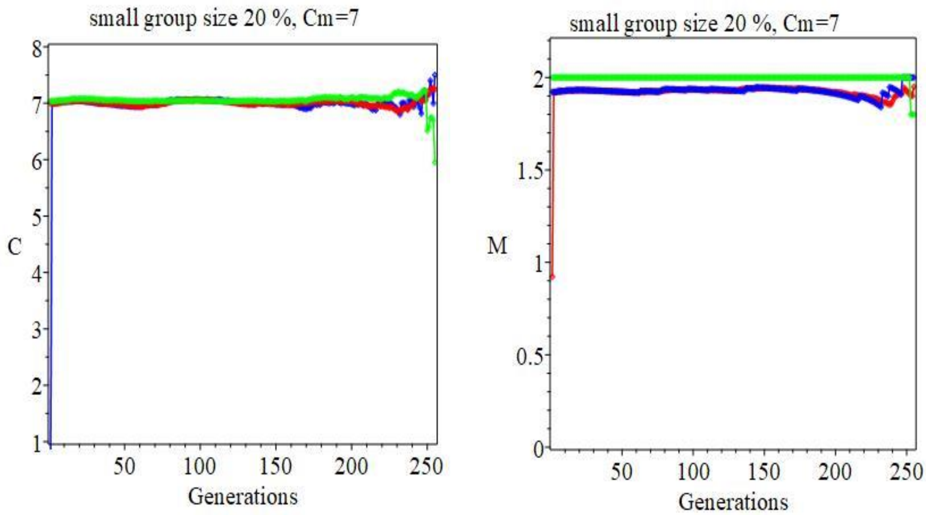

Fig. 5.6. The changes in average complexity (left) and average memory depth (right) of populations during evolution process. They are given for the largest number of the small group (20%) where the differences are most noticeable. Population with a small group marked with red, without a small group with blue, with the small group itself with green.

In such a population, the aggressiveness is the most variable value, which is shown in Fig. 5.7. When small group size is equal to 20%, the aggressiveness of the population as a whole is almost equal to the aggressiveness of a small group.

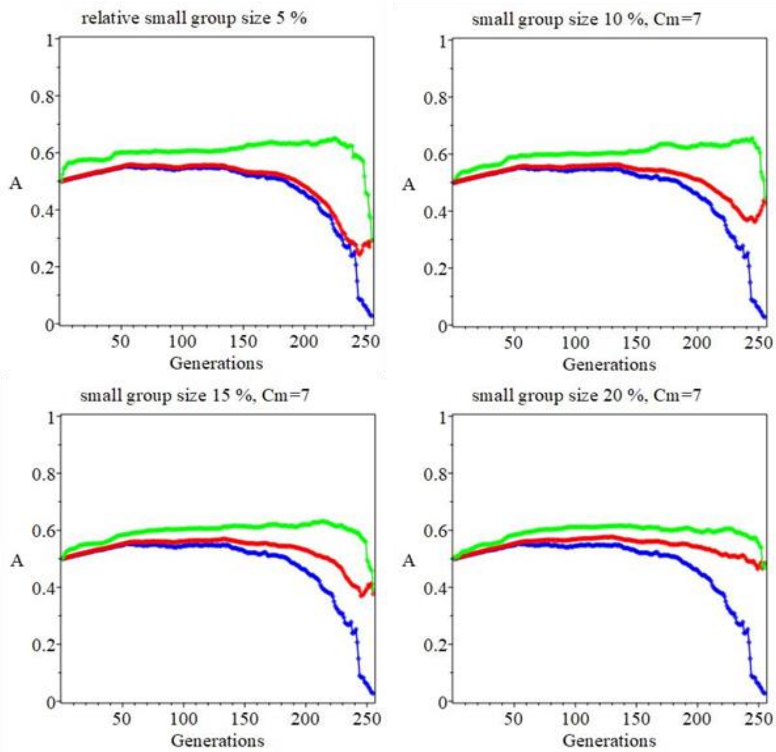

Fig. 5.7. The dynamics of the average aggressiveness of populations and groups over "time". Significant changes in aggressiveness are noticeable in comparison with the aggressiveness of the population without the small group. The difference is especially noticeable at the final stages of evolution, where the aggressiveness of a population with a small group approaches the aggressiveness of a small group itself.

The average number of points per move is roughly the same as in the case of the "more complex" small group. But it can be noticed that the group has a higher survival rate – group members more often survive to the end of the simulation, and in fewer experiments the group disappear completely at the later stages.

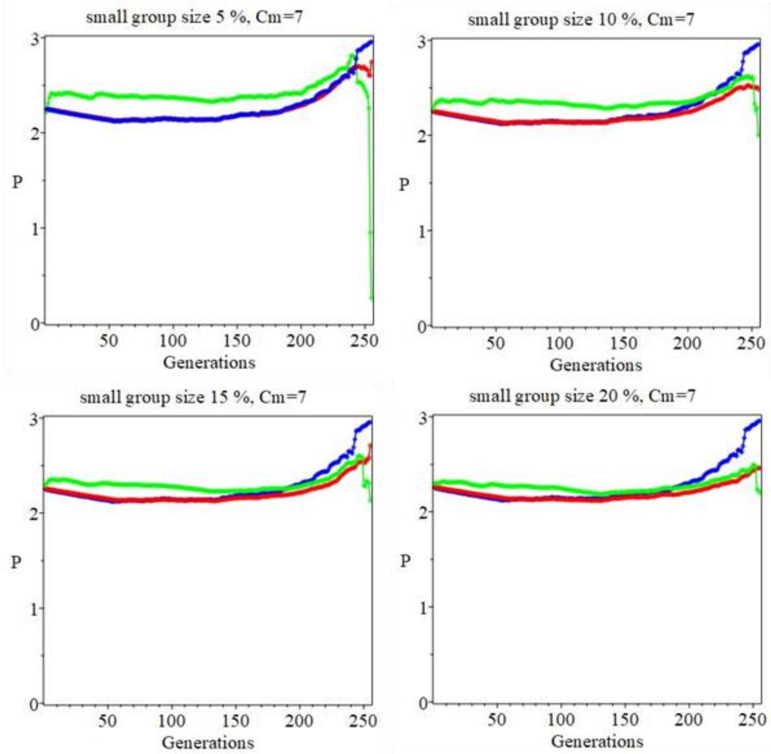

Fig. 5.8. The dynamics of the average number of points per move of populations and groups over "time". The results are better than for the previous group composition (see Fig. 5.4).

The relative size of a small group during the modeling is less (see Fig. 5.9.) than for the “intellectual” group, but then in the later stages of evolution there is a steeper “striking” – so, the group becomes a majority of the finalists by 10% of the population size.

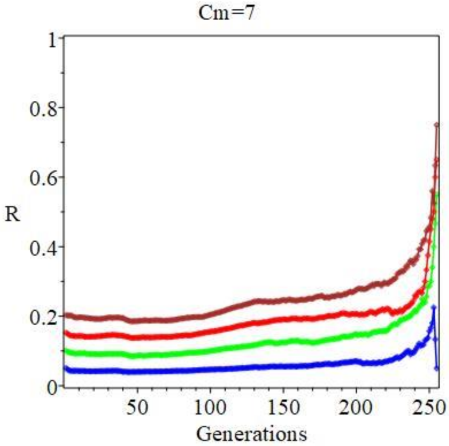

Fig. 5.9. The dependence of changes in the relative size of groups over the "time" for different initial sizes. Brown is initial 20%, red is 15%, green is 10% and blue is 5%.

5.3. The small group with lower complexity than the population

To study the impact of a more primitive group on society, we have added speakers with complexity 4, 5 and 6 to the "sect". The average complexity of the small group at the beginning of the simulation is approximately 5.45.

Observing the change in the average complexity, we can see that even a size of 5% of the population negatively affects the complexity of the whole population on later stages (compared to the standard evolution), but this size is not enough for dominating the group in all conducted experiments (Fig. 5.10). But starting with a size of 15%, the group of strategies with lower complexity succeeds in dominating the majority, which greatly reduces the average complexity of the population as a whole.

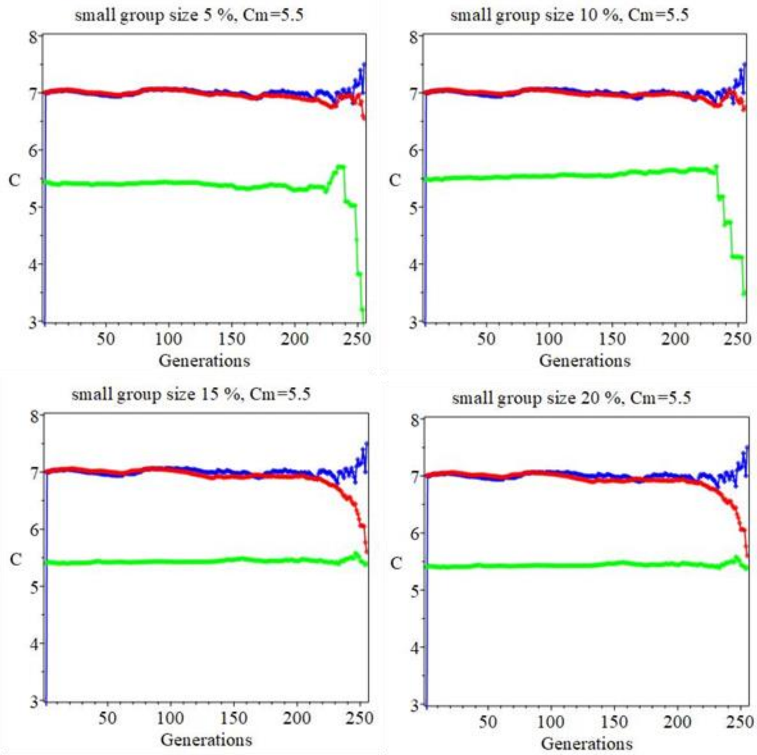

Fig. 5.10. A significant influence of a small group on the nature of the change in population complexity is noticeable. Instead of an increase in complexity, as in a population without a small group, a decrease in the complexity of the population is observed.

The same negative effects are observed with the average memory depth of the population (see Fig. 5.11). At the final stages of evolution, the nature of dynamics fundamentally changes. The difference become especially noticeable with an increase in the initial number of a small group (starting from 15%).

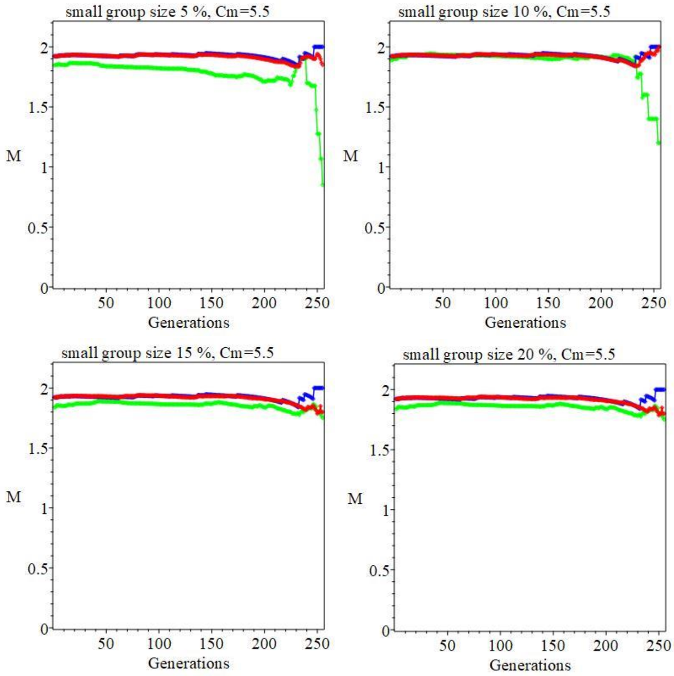

Fig. 5.11. The changes in the average memory of the population with a small group (red), without a small group (blue) and of the small group itself (green) are shown. For 15% and 20%, the average memory of the population approaches the average memory of a small group.

Aggressiveness of the behavior increases significantly. The average number of aggressive moves in the population reaches the highest values compared to previous experiments. At the size of 20% of the population, the average aggressiveness of society is almost equal to the average aggressiveness of a small group.

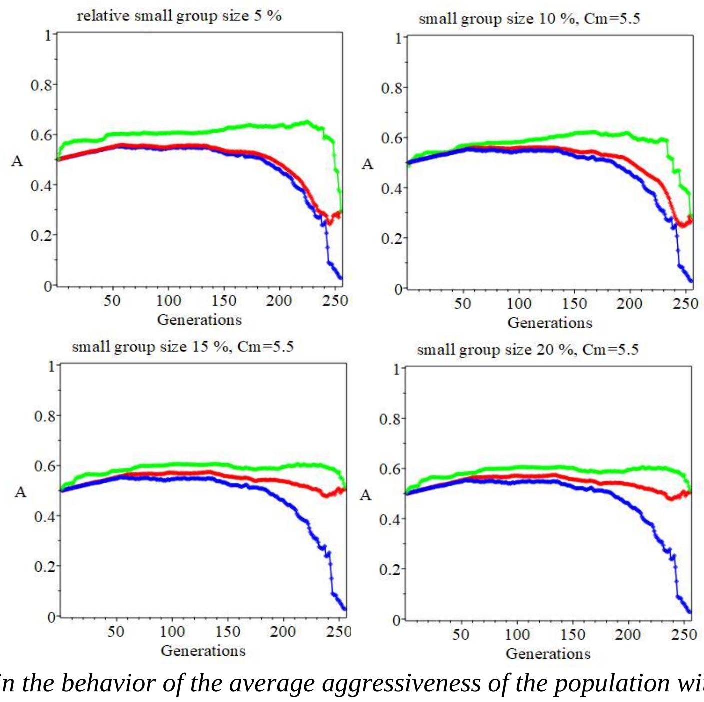

Fig. 5.12 Differences in the behavior of the average aggressiveness of the population with a small group (red), without a small group (blue) and the small group itself (green). Different pictures correspond to different sizes of the small group.

The typical changes in obtaining points per move during the evolution remain. Although the average earnings per move is lower than for the standard evolution, it still brings better results compared to previous experiments.

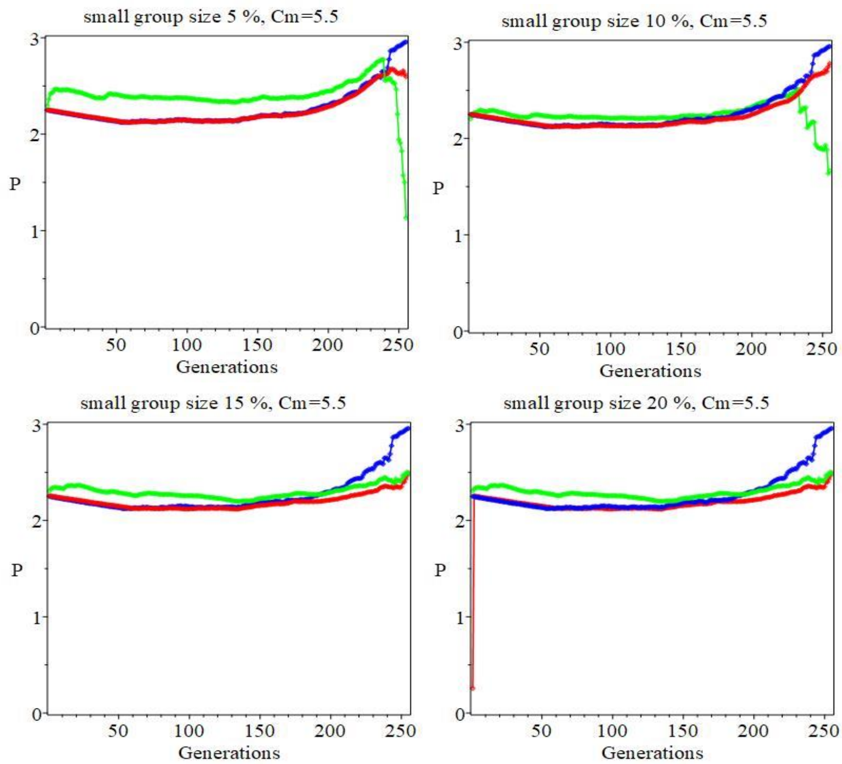

Fig. 5.13 A significant decrease in rewards is noticeable especially for a small group with an insignificant initial number of the group of 5%, 10%. With larger quantities, the rewards of group members are larger than in the population.

We can also observe that this version of the group composition has the highest survival rate among the conducted experiments. This is also confirmed by the charts of the dynamics of small group's relative size – small group has the majority on the later stages of evolution already on 10% of the size. With a size of 20%, the number of the group is record-breaking – a group size reaches the half of the population in the 237 generation.

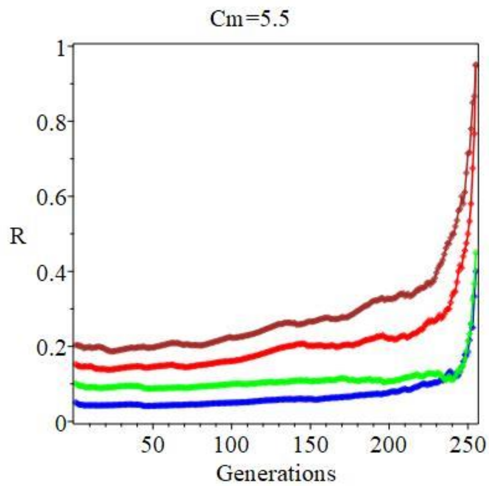

Fig. 5.14. A typical increase in the relative group size in the evolution process. Brown is initial 20%, red is 15%, green is 10% and blue is 5%.

The following chart (Fig. 5.15) shows the relative size of the population for 10 generations before the end of the simulation depending on the initial size of the group, for three variants of the composition of such a group. We can see that such a function has a much larger angle of growth for a primitive "sect", although its initial values are smaller than in more developed versions of society.

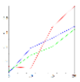

Fig. 5.15. The relative size of the population for 10 generations before the end of the simulation depending on the initial size of the group. Blue color stands for the group with higher complexity, green for the same complexity, and red for the group with lower complexity as in population

6. Conclusions

Comparing the evolution of the population with the distinct group with the evolution of the population without any distinct groups, we can identify typical features of the influence of group presence on the evolution of the population.

As a result, we see that the presence of selected groups of strategies in the population has a significant impact on the nature of evolution. The strategies of the small group interact with each other and the strategies of the population according to different reward matrices. It should be emphasized that more cooperative interaction between group members and more aggressive towards other members of the population, as discussed in the paper, is typical for isolated groups even in real societies. "Intellectual sects" with an increased level of complexity of participants have a positive effect on the "intellectual" development of society, and groups with strategies of lower complexity impacts the population negatively. In all variants of the experiments, the aggressiveness of the population increases significantly with a small group of any composition. If a small group of strategies has a complexity comparable to the complexity of society, aggressiveness is the only indicator that shows any changes. That is, the presence of isolated groups always leads to an increase in the aggressiveness of the population. That results in decreasing points per move of the population strategies and thus the average rewards when the strategies interact. It has been determined that the relationship (5.1) between aggressiveness and points per move is preserved, but the number of a small group leads to a decrease of α dependence indicator in proportion to its number. It should be emphasized that a particularly significant decrease in points is observed for group members. An exception is the case of groups of low complexity with a relatively large initial number (15%, 20%) where the same level of rewards is achieved.

The relative group size increases in the process of evolution. An exception has been observed in the case of the group with complexity 7 with an initial concentration of 5%. An increase in the relative size of the group in the population is typical. A group of low complexity relative to the complexity of the population at an initial number of 20% even dominates the population (i.e., fully displaces all strategies that are not in the group).

An interesting fact is that it is most evolutionarily beneficial for carriers to be a part of “primitive” sect – such a group dominates the majority in society the fastest, “knocking off” the other strategies that do not have an additional “stimulus”.

The issue of higher complexity of a small group (more than 8) remains open for the further research due to the operational complexity of calculations (strategy complexities of more than 8 require calculations at a memory depth of 3). It can be supposed that if the difference between the average complexity of the population and the complexity of the “intellectual” sect will be greater, being in such a group will be more rewarding than in the “primitive” one.

The impact of the reward matrix on the influence of the group on the evolution of the population also remains unexplored. Among the other interesting options, we can mention the study of cooperative behavior towards "one's own" without stimulating aggressive actions towards "strangers". That should not lead to such a rapid increase of aggressiveness as has been observed in this study. The reverse option is also of interest to research.

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Еволюція популяції стратегій з пам'яттю з особливою малою групою

Актуальність. На сьогоднішній день популяція людей постійно зростає. Вплив суспільств та субкультур одне на одного та на популяцію в цілому теж збільшується. Необхідно розширити знання в області поведінки в соціумі для покращення розуміння процесів взаємодії всередині популяції.
Мета. Метою роботи є дослідження впливу виділених особливих груп на еволюцію популяції. Досліджується характер змін в еволюції за умови знаходження в популяції груп різної складності і кількості. Дослідження також має на меті встановити найбільш еволюційно вигідний склад і розмір малої групи і етап, на якому популяція буде захоплена такою групою, якщо це можливо.
Методи дослідження. Розглянуто еволюцію популяції з повним набором стратегій поведінки, обмежених тільки глибиною пам'яті, за наявності в ній виділеної особливої групи. Кожен носій стратегії взаємодіє з кожним, включаючи себе згідно з ітеративною моделлю дилеми ув'язненого. Винагороди визначаються за матрицями виплат. Кожне наступне покоління популяції послідовно втрачає найбільш невигідні стратегії поведінки попереднього покоління. Носії, що є членами особливої групи поводиться більш агресивно до «чужинців», і більше кооперується зі «своїми». Розглянуто різні варіанти початкового складу групи, а саме випадок, коли середня складність малої групи більша за середню складність популяції в цілому, менша, або порівняна з нею. Розглянуто варіанти з різним початковим розміром малої групи – 5, 10, 15, 20% від розміру популяції.
Результати. В роботі показано, як наявність малих груп різної складності та розміру впливає на динаміку популяції. Виявлено збільшення агресивності популяції за умови знаходження в ній малої групи будь-якого складу. Встановлено кількість та склад малої групи, за яких їй вдається захопити більшість в соціумі. Встановлено етап еволюції, на якому це відбувається.
Висновки. За результатами роботи визначено головні наслідки впливу особливих малих груп на еволюцію популяції.

Ключові слова: *еволюція, популяція, суспільство, стратегія, особлива група, складність, агресивність, кооперація.*

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