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Pryimak Oleksii *Candidate of Engineering Sciences*
e-mail: pryimak@mail.com
https://orcid.org/0000-0003-2633-7456

Yanovsky Volodymyr *Doctor of Physical and Mathematical Sciences, Professor*
V. N. Karazin Kharkiv National University,
6 Svobody Sq., Kharkiv, 61022, Ukraine;
Institute for Single Crystals, NAS Ukraine,
60 Nauky Ave., Kharkiv, 61001, Ukraine
e-mail: yanovsky@isc.kharkov.ua
https://orcid.org/0000-0003-0461-749X

Head-on collision of flocks

Relevance. The study of the collective behavior of flocks of intelligent agents by using the mathematical and numerical methods is related to the multi-agent systems and artificial intelligence, which are actively researched nowadays.
Goal. To reveal the patterns of flocks dispersion during their collision and to obtain the analytical ratios of flocks kinematics.
Research methods. The work is based on the methods of mathematical and numerical modeling of multi-agent systems.
The results. The main parameter that determines the flocks behavior during their interaction is the acceleration. When the impact parameter is increased, the changes in the characteristics of the flocks become less noticeable. The obtained dependence of the acceleration on the value of the aiming parameter resembles the dependence that is typical for phase transitions.
Conclusions. The main regularities of flocks dispersion are determined, as well as the analytical ratios of flocks kinematics, which are in good conformity with the simulation data.
Keywords: *collective behavior, multi-agent system, flock, intelligent agent, force vector, kinematics, acceleration, impact parameter.*

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1 Introduction

Currently the collective behavior of self-driven or motile elements is of great interest (see [1] for example). Such systems include flight of flocks of birds [2-4], movement of schools of fish [5-7], insect migrations [8, 9] and herds of animals [10]. The patterns of their collective motion and the nature of the appearance of coordinated motions are intensively studied [11]. However, that interest is not limited to natural objects. Similar problems arise with the emergence of artificial multi-agent systems capable of manifesting themselves in collective behavior [12-14]. Even the influence of artificial objects — robots on the behavior of natural systems is also considered [15, 16]. At the same time, on the one hand, such systems can be considered as simple models of natural objects, on the other hand, as the emergence of swarm intelligence in artificial intelligent systems. A new direction in physics has even appeared — active matter [17, 18].

In nature, the behavior of systems consisting of many interacting moving individuals is extremely diverse. Despite the widespread occurrence of such phenomena, there have been no systematic studies of self-propelled interacting objects till recent times. One of the first works [19] is devoted to modeling the movement of schools of fish using the rules: (a) approach movement to allow aggregation, (b) parallel orientation movement and (c) cohesion. The speed and direction of individuals were considered stochastic, and the direction of movement of individuals was determined by the position and direction of movement of their neighbors. It has been noted that a collective movement can arise even in the absence of a leader. Much more famous was the later work of Reynolds [20] in which he simulated the movement of bird-like objects — boyds. A more detailed discussion of the various patterns of movement of interacting individuals can be found in the review [1].

In this work, the behavior of flocks during their head-on collision is considered. Two flocks have been simulated by using the flocking algorithm and their behavior after the collision has been studied. Actually, this type of collision in physics is the beginning of studying the scattering of bodies in a collision. The features of flocks collisions and the differences between them and the regularities of scattering of physical particles are discussed. The main patterns of collisions of such flocks have been determined.

2 Model

The collective behavior of flocks of agents has been modeled by using the flocking algorithm proposed by Reynolds [20]. In its basic form, flocking describes the collective movement of agents according to three simple rules: separation, alignment, cohesion. The process of moving is iterative. At each iteration, the change in the velocity of each agent is calculated as the sum of the changes in speed in accordance with the three rules of the flock. The change in the position of the i-th agent is determined by a simple iterative equation

$$\vec{x}_i(t+1) = \vec{x}_i(t) + \vec{v}_i(t+1)\cdot\Delta t. \tag{2.1}$$

The change in the velocity of the i-th agent depends on the neighboring agents marked with the index j, which falls into the sphere of radius R centered on the i-th particle in accordance with the equation

$$\vec{v}_i(t+1) = \vec{v}_i(t) + \Delta\vec{v}_i(t), \tag{2.2}$$

where $\vec{v}_i(t)$ is the velocity vector of the i-th agent at discrete time t. For flocking, the change in speed is determined by three factors, as noted above

$$\Delta\vec{v}(t)_i = 5\cdot\Delta\vec{v}_{separation,i}(t) + 3\cdot\Delta\vec{v}_{alignment,i}(t) + \Delta\vec{v}_{cohesion,i}(t).$$

Numeric multipliers determine the weights or significance of the respective contributions to the velocity change and may vary. To describe the model, we define these terms [21]. So let us start with the separation

$$\Delta\vec{v}_{sep\,i}(t) = \left(v_0 - v_i(t)\right)\frac{\left\langle x_i(t) - x\ (t)\right\rangle_R}{\left|\left\langle x_i(t) - x\ (t)\right\rangle_R\right|},$$

where v_0 is the magnitude of velocity vector of the agents and $v_i(t) = |\vec{v}_i(t)|$ is the magnitude of velocity vector of the i-th agent. The factor $\left(v_0 - v_i(t)\right)$ is present in all contributions and it is needed to return the velocity value to the value of v_0. We will also use the notation

$$\left\langle \vec{x}_i(t) - \vec{x}\ (t)\right\rangle_R = \frac{1}{n}\sum_j^n \frac{\vec{x}_i(t) - \vec{x}_j(t)}{\left|\vec{x}_i(t) - \vec{x}_j(t)\right| d_{ij}(t)}.$$

Here $\langle\ldots\rangle_R$ denotes averaging (or summation) of the velocities within a circle of radius R surrounding agent i. For each rule, the radii of the neighborhood can be chosen separately. Now we define the value of $\Delta\vec{v}_{separation,i}(t)$ as

$$\Delta\vec{v}_{separation,i}(t) = \frac{\Delta\vec{v}_{sep}(t)}{\left|\Delta\vec{v}_{sep}(t)\right|}\cdot\sigma \quad ; \quad \sigma = \begin{cases} 1 \ \ if \left|\Delta\vec{v}_{sep}(t)\right| \le 0.03 \\ 0.03 \ \ if \left|\Delta\vec{v}_{sep}(t)\right| > 0.03 \end{cases}.$$

The σ factor is called steering force. It is introduced to smooth non-physical sharp turns. The value $\sigma = 0.03$ is chosen for modeling and may change.

Let us move on to alignment and define

$$\Delta\vec{v}_{alig\,i}(t) = \left(v_0 - v_i(t)\right)\frac{\left\langle \vec{v}(t)\right\rangle_R}{\left|\left\langle \vec{v}(t)\right\rangle_R\right|}.$$

Here we again use the natural notation

$$\left\langle \vec{v}(t)\right\rangle_R = \frac{1}{n}\sum_j^n \vec{v}_j(t),$$

where $\langle\ldots\rangle_R$ - means averaging over the velocities of the agents inside the sphere of radius R surrounding the i-th agent. It is obvious that $\dfrac{\langle\vec{v}_j\rangle_R}{|\langle\vec{v}_j\rangle_R|}$ is a unit vector in the direction of the mean motion of neighboring agents. This provides alignment. Let us define $\Delta v_{alignment}$ as

$$\Delta\vec{v}_{alignment,i}(t)=\frac{\Delta\vec{v}_{alig,i}(t)}{\left|\Delta\vec{v}_{alig,i}(t)\right|}\cdot\sigma \quad ; \quad \sigma=\begin{cases}1 \;\; if \;\left|\Delta\vec{v}_{alig,i}(t)\right|\le 0.03\\ 0.03 \;\; if\;\left|\Delta\vec{v}_{alig,i}(t)\right|>0.03\end{cases}.$$

Let us proceed to the description of the last contribution of the person responsible for <<cohesion>> and define

$$\Delta\vec{v}_{coh\,i}\left(t\right)=\left(v_0-v_i\left(t\right)\right)\frac{\left\langle\Delta\vec{x}_i\left(t\right)\right\rangle}{\left|\left\langle\Delta\vec{x}_i\left(t\right)\right\rangle\right|},$$

where

$$\left\langle\Delta\vec{x}_i\right\rangle_R=\frac{1}{n}\sum_j^n\vec{x}_j\left(t\right)-\vec{x}_i\left(t\right)$$

deviation of the position of the i-th agent from the average position of the neighbors in the sphere of radius R. Finally

$$\Delta\vec{v}_{cohesion,i}(t)=\frac{\Delta\vec{v}_{coh,i}(t)}{\left|\Delta\vec{v}_{coh,i}(t)\right|}\cdot\sigma \quad ; \quad \sigma=\begin{cases}1 \;\; if \;\left|\Delta\vec{v}_{coh,i}(t)\right|\le 0.03\\ 0.03 \;\; if\;\left|\Delta\vec{v}_{coh,i}(t)\right|>0.03\end{cases}.$$

When modeling, two flocks of agents are considered to move in a two-dimensional space without obstacles. When determining the change in the corresponding velocities of the agent, the different radii of the spheres, over which the averaging is performed, are chosen:

$$R_{separation}=500\;m,$$

$$R_{alignment}=1000\;m,$$

$$R_{cohesion}=2000\;m.$$

The velocity of the agent v in space per iteration is limited by the expression $v_{min}\le v\le v_{max}$, and the rotation angle is limited by the expression $\Delta A\le\Delta A_{max}$. The agent's velocity tends to v_0, which is taken into account in the three flocking rules. The magnitude of the velocity of each agent at the initial moment of time is equal to v_0. The listed characteristics are assigned the following values:

$$v_{min}=30\;m/1,$$

$$v_0=50\;m/1,$$

$$v_{max}=70\;m/1,$$

$$\Delta A_{max}=\frac{\pi}{30}.$$

At the initial moment of time, the agents of each flock are located in space in a "chessboard" order according to the following rules. In each column, the agents have the same position along the X axis, and the distance between the neighboring agents along the Y axis is 500 meters. The distance between adjacent columns along the X axis is 500 meters. There is one less agent in the even columns than in the odd ones, and the agents are located with a shift along the Y axis by 250 meters (Fig. 1). The cases with the following number of agents in a flock are considered: 3, 14, 33, 60, 77, 95.

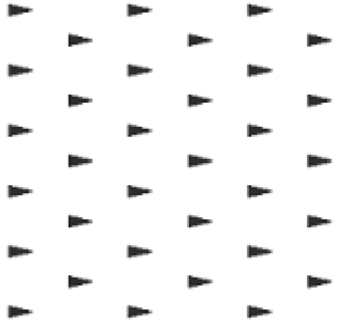

Fig.1 An example of the location of a flock of 33 agents at the initial time

In a numerical experiment, two flocks with the same number of agents collide. The centers of two colliding flocks at the initial moment of time are located at a distance of 10^5 meters from each other along the X axis. The center of a flock is determined as the arithmetic average of the positions of the agents of flocks separately for each dimension.

The direction of movement at the initial moment of time for the agents of the first flock is equal to 0 radians, and for the agents of the second flock to π radians.

The flocks move towards each other. The collection of statistics on the movement of flocks begins with an iteration when the distance between any two agents of two flocks does not exceed $R_{cohesion}$, i.e. agents of different flocks begin to interact with each other.

Flocks collide and fly past each other, after which the agents of different flocks stop interacting with each other. The interaction of agents of different flocks lasts for t_{int}, and the collection of flock movement statistics starts from the moment the interaction of agents of different flocks begins on the interval $4 \cdot t_{int}$ for flocks of sizes 3, 14, 33 agents and $8 \cdot t_{int}$ for flocks of 60, 77, 95 agents.

The average speed of the flock is equal to the arithmetic mean of the speed of the agents of the flock, and the average direction of movement of the flock is equal to the arithmetic mean of the direction of the agents of the flock.

The obtained simulation data are dimensionless. The characteristic scale for nondimensionalization is $S = R_1 + R_2 + R_{cohesion}$, where the flock radius R is equal to the maximum distance among flock agents from the center of the flock.

3 Collision of flocks

Let us consider a head-on collision between two flocks. The geometry of their scattering is shown in Fig. 2. This is the simplest and in some sense the canonical case of scattering. Initially, the agents of the respective flocks occupy certain areas. When modeling, they have been distributed over areas close to a circle. The initial distance between flocks across the direction of movement corresponds to the impact parameter ρ. During the simulation, the impact parameter changes from 0 to l_{int}. For large values of ρ, there is no interaction between flocks and, accordingly, their scattering.

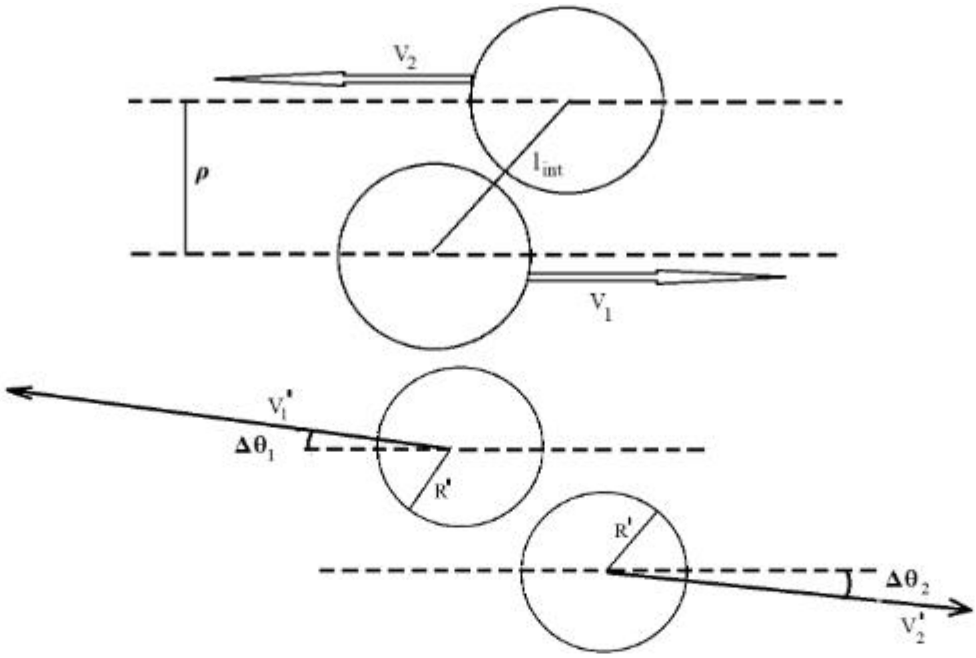

Fig.2 The top image shows the directions of movement of flocks, their velocity and sizes at the moment of the beginning of the interaction of agents of flocks. It is convenient to skip the free movement stage. Possible a priori changes in flock parameters and their movement after interaction are shown below. So, the directions of their movement can change, which are characterized by the scattering angles $\Delta\theta$, the indices indicate the flocks, the velocity of their movements and the characteristic sizes and even the shape of the flocks. The circles show the areas occupied by the agents of the respective flocks

There are specific features of the interaction of flocks. First, their interaction occurs at a finite distance not exceeding l_{int}, and, accordingly, takes a finite time interval from 0 to t_{int}. The flock characteristics may change during interaction, and even after interaction, some changes may persist. These features distinguish flock scattering from such a well-studied process as the scattering of interacting particles. In this sense, the stage of asymptotic return to free motion with new parameters may not occur. Therefore, it makes sense for flocks to determine scattering data starting from the end of scattering, and to track their change in the future at the moments of relaxation to some steady state, if it occurs.

Another a priori difference may lie in the dependence of the scattering pattern on the shape of the colliding flocks. For different forms with the same exposure parameters, different proportions of agents can participate in the interaction and, accordingly, affect their flocks in different ways.

The nature of the interaction between agents, based on the information they receive, complicates the analytical construction of the theory of flock dispersion. Therefore, in the future, we will use the simulation results to establish the main patterns of flock dispersion.

4 Flocks collision simulation results

Let us start with an analysis of the change in the distance between the centers of the flocks during interaction. Typical examples of distance changes obtained as a result of flock collision simulation are shown in Fig. 3. The behavior of distances is universal and most stable, in the sense of the absence of noticeable fluctuations.

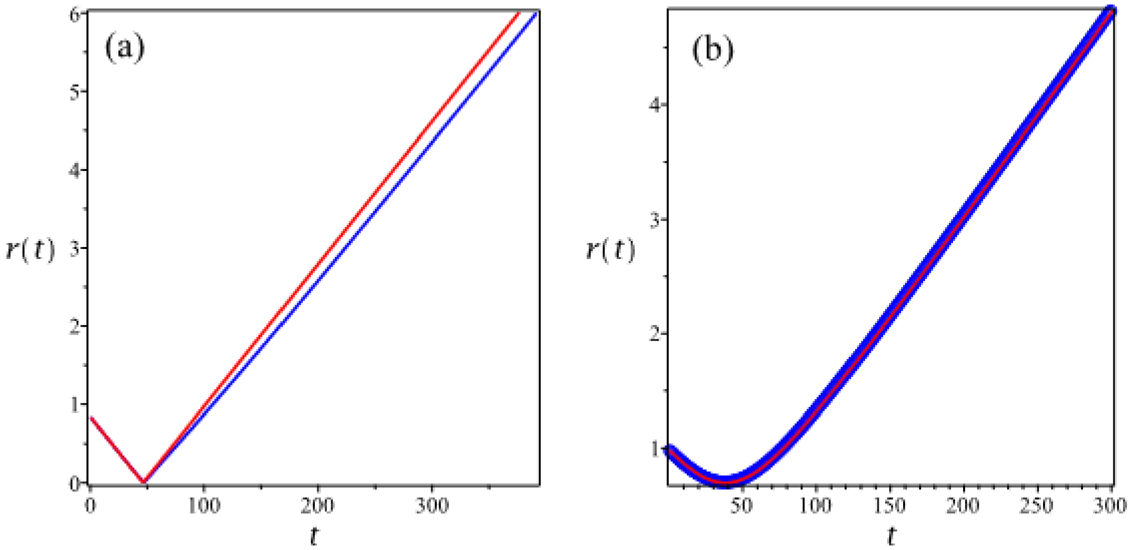

Fig.3 Typical changes in the distance between flocks over time, obtained as a result of numerical simulation. Red line is the distance between non-interacting (phantom) flocks, blue line is the distance between interacting flocks. (a) — impact parameter $\rho = 0.014$*, characteristic interaction scale* $l_{int} = 0.82608$*, and minimum position* $t_{min} = 47$*. (b) — impact parameter* $\rho = 0.0693$*. With the same number of agents in the flock* $N_1 = N_2 = 33$

We can note an important consequence of comparing the change in distances between interacting and non-interacting flocks. For all the initial parameters and the number of agents in the flocks, there is a close location of the minima and a good conformity between the minimum distance and the impact parameter. Noticeable differences can be observed in the asymptotic change of distances with time at times $t >> t_{int}$ or at distances $r >> l_{int}$ (see Fig. 3). The difference in the slope angles of the asymptotics means the difference in the velocities of these flocks. In other words, when flocks interact, it reduces their movement speed. In a certain sense, the differences between these dependences determine the degree of influence of the interaction of flocks on the impact parameter. So, for large values of the parameter, only a small part of the agents of both flocks interact, and this does not have a noticeable effect on the behavior of the flocks as a whole. Therefore, the interaction of agents does not always lead to the interaction of flocks.

Another important observation is related to the value of the minimum distance between flocks. For example, Fig. 4 shows the change in the minimum distance of approach of flocks from the impact parameter. The number of flocks is the same and equal to 95 individuals. In fact, it can be seen that the minimum distance of approach of interacting flocks coincides with the value of the impact parameter $r_m = \rho$. This dependence is retained for all considered numbers of flocks.

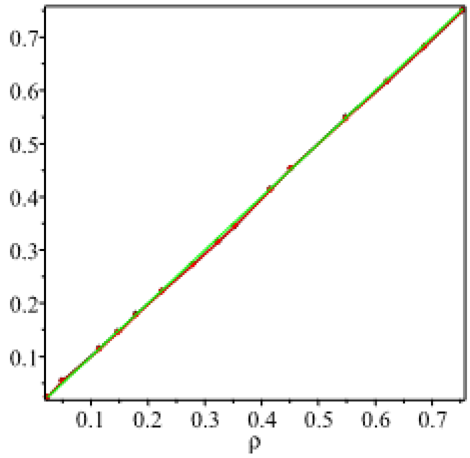

Fig.4 The dependence of the minimum distance of approach of flocks on the impact parameter is shown. Black line is for interacting flocks, red line is for phantom flocks, and green line is for $r_m = \rho$ *. One can see the complete coincidence of these dependencies for flocks* $N_1 = N_2 = 95$*. The same coincidence is observed for flocks of different numbers*

The coincidence of the minimum distance with the impact parameter means the absence of transverse displacements as a result of interaction. Therefore, a small scattering angle for any values of the impact parameter can be observed. Fig. 5 shows flock scattering angles $N_1 = N_2 = 60$ obtained by modeling flock collisions. Similar dependences are also observed for flocks with a different number of agents in flocks. It can be assumed that during a head-on collision, flocks retain their direction of movement after the collision due to symmetry considerations.

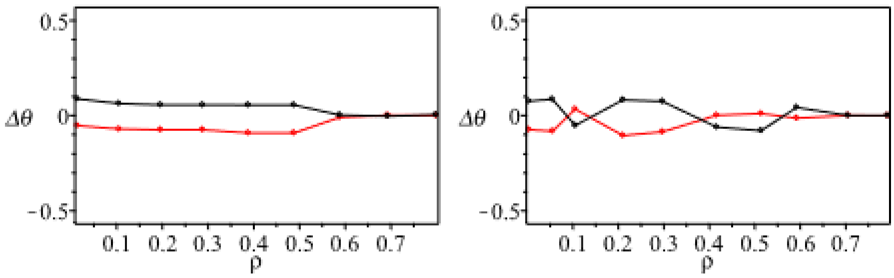

Fig.5 The dependence of the scattering angles of the first flock is shown in red, the second flock in black. Data about flocks with $N_1 = N_2 = 33$ *agents are presented on the left, and flocks with* $N_1 = N_2 = 60$ *on the right. In fact, the deviation of the directions of movement is small and does not have a systematic character*

It is important to remember that flocks interact at a finite distance l_{int} , after which the interaction stops and flocks continue to move freely with new parameters. In fact, if the direction of movement is preserved, then one of the important parameters is the speed of movement of flocks after interaction.

Let us now discuss in more detail how the speed of interacting flocks changes. The nature of the interaction depends on the impact parameter. Thus, at small values, the nature of flock speed changes is clearly expressed and has a typical form, shown in Fig. 6 with noticeable sharp decrease in the speed of the flock and a slower return to the initial speed of the flock. The characteristic velocity relaxation time for these parameters is $t_{rel} \approx 647$, and the interaction time is $t_{int} \approx 143$. Thus, the minimum is reached at the stage of interaction, and the increase in speed continues after the interaction of flocks. The ratio of times for this case is easy to establish. So for the first flock, the beginning of the decrease in speed is $t_{Sv1} \approx 10$, the time to reach the minimum is $t_{v1min} \approx 123$ and $t_{Fv1} \approx 657$ ($t_{rel} = t_{Fv} - t_{Sv}$), flock interaction starts at $t = 0$ and lasts $t_{int} \approx 143$. For the second flock, similar values $t_{Sv2} \approx 10$, $t_{v2min} \approx 125$, and $t_{Fv2} \approx 641$ are realized. Thus, the relaxation times are much longer than the interaction time.

For large impact parameters, when a small part of agents from different flocks interact, the behavior of the velocities is significantly different (see Fig. 6). In fact, there are no noticeable changes in the speed of the flock. In such cases, the change in distances between interacting flocks and phantom flocks is indistinguishable.

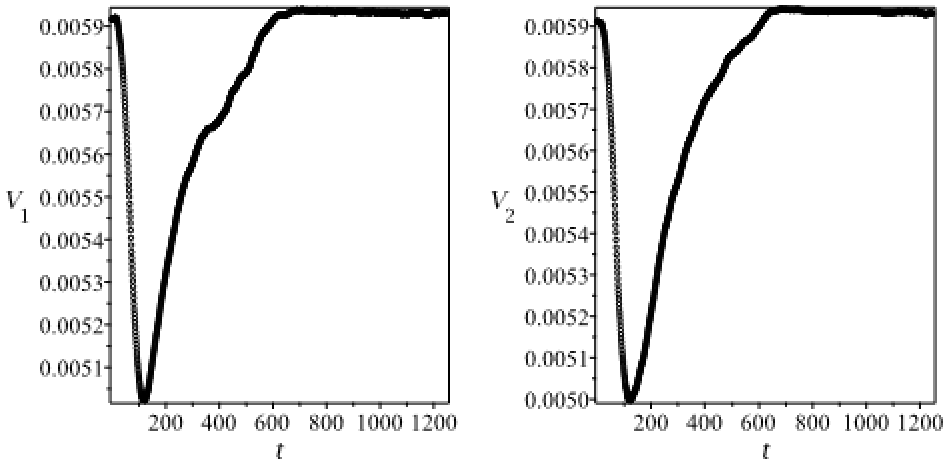

Fig.6 The change in the velocity of the first flock is shown on the left, the second on the right. The number of agents is $N_1 = N_2 = 95$, $\rho = 0.050$

This behavior is observed with a different number of agents in flocks. Fig. 7 shows the dependence of the jump in velocity and relaxation time on the impact parameter. One can see the disappearance of the jump in speed even at the stage of interaction of a certain proportion of the agents of the flocks. Similar dependences are observed for other numbers of flocks.

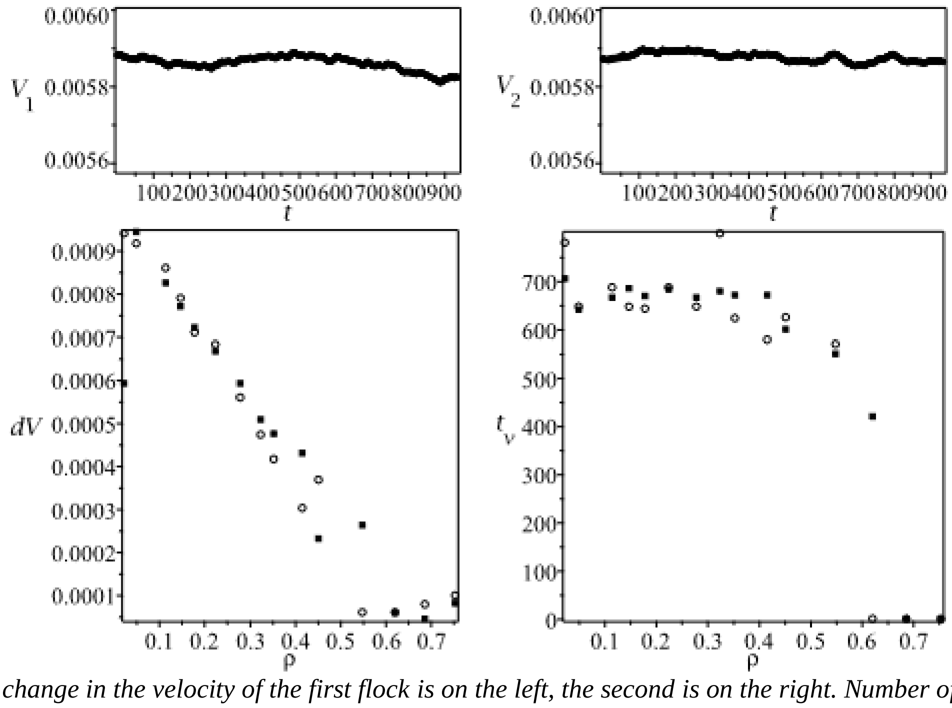

Fig.7 The change in the velocity of the first flock is on the left, the second is on the right. Number of agents is $N_1 = N_2 = 95$, $\rho = 0.687$ *.The magnitude of the jump in velocity is shown below on the left and the relaxation time on the right, depending on the impact parameter. The circles are for the first flock, the squares for the second one.*

Some important changes are also observed in the forms of flocks during interaction. The average radius of the flock has been chosen as a parameter characterizing the shape of the flock. The characteristic differences have been observed again depending on the values of the impact parameter. Fig. 8 shows characteristic changes in the average radii of interacting flocks for different values of the impact parameter. A sharp change in the average radius is noticeable at small values of the impact parameter (see the bottom row of Fig. 8). The change starts at the stage of interaction and continues even in the absence of interaction. At times of the order of the velocity relaxation time, the stage of a slow decrease of the average radius begins. At large values of the impact parameter (see the top row of Fig. 8), a slow monotonous increase in the average radius is observed.

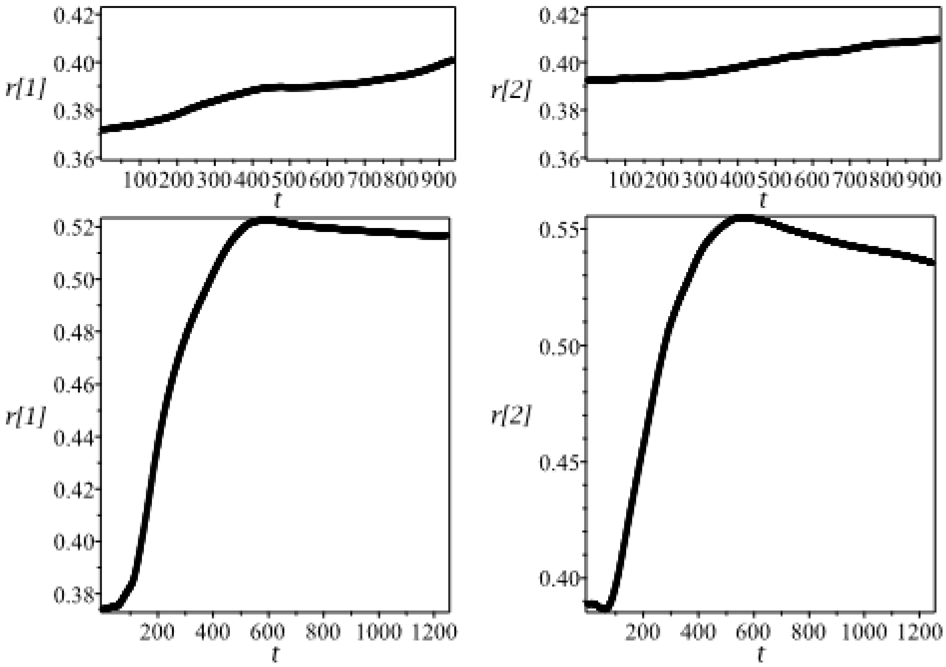

Fig.8 The changes in the average radius of the first flock is on the left, the second is on the right. The number of agents in flocks is $N_1 = N_2 = 95$. *For the top row* $\rho = 0.687$, *for the bottom row* $\rho = 0.050$

It is interesting to consider the change in flock density over time for similar flocks and parameters. Fig. 9 shows the corresponding dependencies. It is easy to see the correlation with the behavior of the mean radius. In addition, an increase in the density of flocks during the interaction and a subsequent sharp decrease in density after the end of the interaction is noticeable.

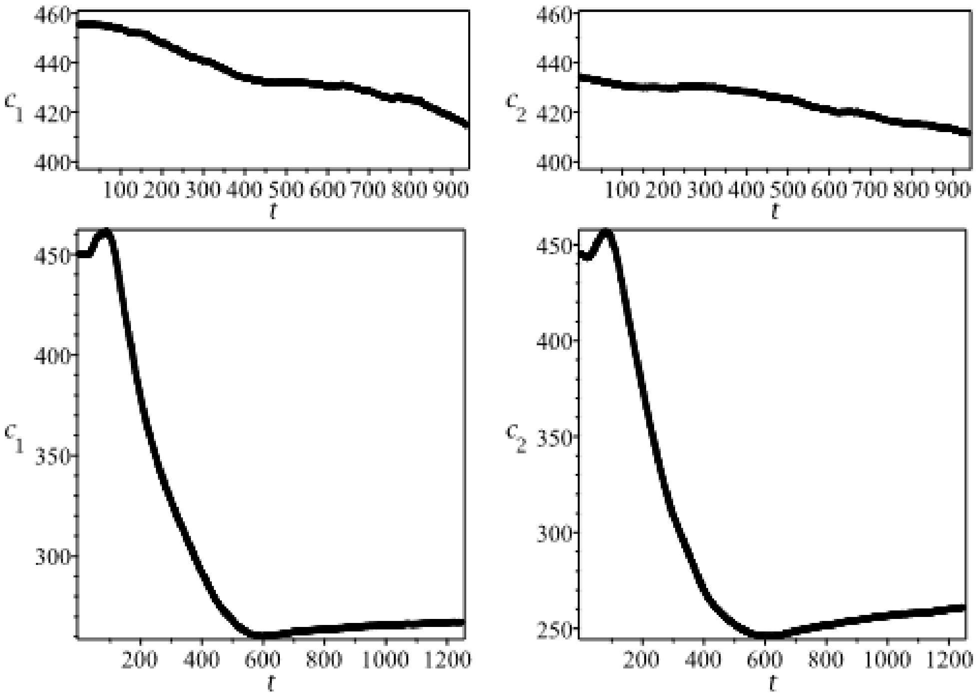

Fig.9 The change in the average density of the first flock is on the left, the second is on the right. The number of agents in flocks is $N_1 = N_2 = 95$. *For the top row* $\rho = 0.687$, *for the bottom row* $\rho = 0.050$

We can say that there is a jump in the Δc density. After times of the order of velocity relaxation, a slow stage of density increase begins. The presence or absence of that jump, as well as the magnitude of the jump, determines the degree of influence of the interaction of flocks on their movement and behavior.

5 Flocks kinematics

Let us consider the kinematics of the flocks' movement. In this case, we consider two cases of flock movement with the same initial data. This is the movement of flocks without interaction and in the presence of interaction between flocks. Flocks without interaction are phantom flocks. Agents of this flock cannot see agents of another flock and pass through them unharmed. Let us start with this case. Let the centers of the flocks move without interaction, then the positions of the flocks change with time as

$$x_1 = x_{01} + v_1 t; \quad y_1 = y_{01};$$

$$x_2 = x_{02} - v_2 t; \quad y_2 = y_{02}.$$

Here, the number of the flock is marked by the index, and the speeds of the flock movement are oppositely directed along the X axis. Let us assume that the beginning of the flocks' movement starts with the distance between them coinciding with the characteristic scale of interaction l_{int}. In other words, $x_{02} = x_{01} + \sqrt{l_{int}^2 - \rho^2}$. Then the distance between the centers of the flocks changes according to

$$r_s(t) = \sqrt{\left(x_1 - x_2\right)^2 + \left(y_1 - y_2\right)^2} = \sqrt{\left(\left(v_1 + v_2\right)t - \sqrt{l_{int}^2 - \rho^2}\right)^2 + \rho^2}, \tag{5.1}$$

where $r_s(t)$ is the distance between freely moving flocks without interaction. The minimum $r_s(t)$ is reached at time $t = t_m = \dfrac{\sqrt{l_{int}^2 - \rho^2}}{\Delta v}$ where the approach velocity is $\Delta v = v_1 + v_2$ and is equal in magnitude to $r_s(t_m) = \rho$. When comparing with experimental data, it is convenient to use an equivalent form, but depending on other parameters

$$r_s(t) = \sqrt{\left(\left(\frac{t}{t_m} - 1\right)\sqrt{l_{int}^2 - \rho^2}\right)^2 + \rho^2}. \tag{5.2}$$

In this relation, the time of the closest approach t_m appears as the main parameter. We also give the initial speed of approach

$$\frac{dr_s(t)}{dt}\Big|_{t=0} = -\frac{l_{int}^2 - \rho^2}{\sqrt{l_{int}^2}\, t_m}. \tag{5.3}$$

Comparing this expression with a similar form obtained from the relation (5.1), we recieve the initial approach velocity through the main parameters

$$v_1 + v_2 = \frac{\sqrt{l_{int}^2 - \rho^2}}{t_m}. \tag{5.4}$$

This relation can be used for an a priori estimate of t_m from the initial data.

Now let us calculate how the distance between the interacting flocks will change. Suppose that the interaction leads to the acceleration of the movement of the flock. We will assume that the acceleration at the moments of interaction is constant and oppositely directed for the first and second flocks. In fact, this assumption is related to the hypothesis of the fulfillment of Newton's third law (the force of action is equal to the force of reaction) and is based on simulation data. Then

$$x_1 = x_{01} + v_1 t - a_{x1}\frac{t^2}{2}; \quad y_1 = y_{01} + a_{y1}\frac{t^2}{2};$$

$$x_2 = x_{02} - v_2 t + a_{x2}\frac{t^2}{2}; \quad y_2 = y_{02} - a_{y2}\frac{t^2}{2}.$$

It remains to calculate the distance between the interacting flocks

$$r_{int}(t) = \sqrt{\left((v_1+v_2)t - \Delta a_x \frac{t^2}{2} - \sqrt{l_{int}^2 - \rho^2}\right)^2 + \rho^2}; \quad 0 \le t \le t_{int}, \tag{5.5}$$

where $\Delta a_x = a_{x1} + a_{x2}$ and for reasons of symmetry and simulation data, we neglect the lateral acceleration $a_y = 0$. This relation is satisfied when the flocks $t \le t_{int}$ interact, where $r_{int} \le l_{int}$. This dependence of the distance between flocks contains natural parameters. Minimum convergence is achieved at times that are easy to find

$$t_{1m} = \frac{v_1 + v_2 - \sqrt{(v_1+v_2)^2 - 2\Delta a_x \sqrt{l_{int}^2 - \rho^2}}}{\Delta a_x},$$

$$t_{2m} = \frac{v_1 + v_2 + \sqrt{(v_1+v_2)^2 - 2\Delta a_x \sqrt{l_{int}^2 - \rho^2}}}{\Delta a_x}.$$

It is easy to see that

$$t_{1m} \le t_{2m} \quad \text{if} \quad a_x > 0,$$

$$t_{1m} < 0, \quad t_{2m} > 0 \quad \text{if} \quad a_x < 0.$$

Therefore, t_{1m} is important for us if $\Delta a_x > 0$, and t_{2m} if $\Delta a_x < 0$. Further, we will be interested in the case $\Delta a_x > 0$. The choice of this case is justified by the data obtained in the simulation of collisions. The condition that these times are real gives an a priori constraint on the acceleration

$$0 \le \Delta a_x \le \frac{(v_1+v_2)^2}{2\sqrt{l_{int}^2 - \rho^2}}.$$

Assuming the acceleration being small, we can obtain a more convenient expression for the time to reach the minimum distance between flocks

$$t_{1m} \approx \frac{\sqrt{l_{int}^2 - \rho^2}}{v_1 + v_2} + \Delta a_x \frac{l_{int}^2 - \rho^2}{2(v_1+v_2)^3}. \tag{5.6}$$

Actually, this ratio explains the closeness of the minimum distances of approach for the phantom and interacting flocks (see the ratio (5.4)). Using the equation (5.6) it is easy to set the acceleration value from the simulation data. Similarly, we can estimate the interaction time $r_{int}(t_{int}) = l_{int}$

$$t_{int} = \frac{(v_1+v_2) - \sqrt{(v_1+v_2)^2 - 4\Delta a_x \sqrt{l_{int}^2 - \rho^2}}}{\Delta a_x}. \tag{5.7}$$

Assuming the acceleration being small, we also obtain an approximate value

$$t_{int} \approx 2\frac{\sqrt{l_{int}^2 - \rho^2}}{v_1 + v_2} + 2\Delta a_x \frac{l_{int}^2 - \rho^2}{(v_1+v_2)^3}. \tag{5.8}$$

The minimum approach distance is still $r_{int}(t_{1m}) = \rho$, as well as the initial approach velocity

$$\frac{dr_{int}(t)}{dt}\Big|_{t=0} = -\frac{2v_1\sqrt{l_{int}^2 - \rho^2}}{\sqrt{l_{int}^2}}.$$

Thus, the dependence (5.5) agrees well with the experimental data. It includes a single unknown parameter Δa_x.

Actually, in a certain sense, it is the parameter which determines the <<force>> of the interaction of flocks. Naturally, we use simulation data to determine it. So, for example, the acceleration of flocks approach Δa_x can be determined by comparing the t_m time to reach the minimum distance with the value t_e obtained in the simulation

$$\Delta a_x = \frac{2((v_1+v_2)t_e - \sqrt{l_{int}^2-\rho^2})}{t_e^2}. \tag{5.9}$$

Using this ratio, it is easy to set the acceleration values from the simulation data.

Finally, after the interaction, the distance changes, as in non-interacting flocks, but with different velocities. These new speeds are the same as $v_1' + v_2' = v_1 + v_2 - \frac{\Delta a_x}{2} t_{int}$. Thus, the change in distance with time $t_{int} \le t$ occurs as

$$r(t) = \sqrt{((v_1'+v_2')t-(l_{int}^2-\rho^2))^2+\rho^2} + \sqrt{((v_1+v_2)t_{int} - \frac{\Delta a_x}{2}t_{int}^2 - (l_{int}^2-\rho^2))^2+\rho^2} + l_{int}. \tag{5.10}$$

This equation is used below for comparison with the simulation data.

6 Results and conclusions

Let us start by comparing the simulation data on the change in the distance between flocks with the obtained dependencies. Fig. 10 shows the simulation data and the corresponding dependencies for the set value of approach acceleration Δa_x. For the chosen initial conditions $\Delta a_x = 0.134 \cdot 10^{-4}$ according to the relation (5.9). There is a noticeable good conformity between the dependence (5.5) and the simulation data during the interaction. The conformity of the minimum distance with the impact parameter and the achievement of l_{int} at time $t = t_{int}$ has been shown as well. At times $t \ge t_{int}$, the dependency (5.10) is shown in green against the simulation data (Fig. 10). The differences between these dependencies are hard to notice. Thus, the hypothesis of uniformly accelerated deceleration of the flocks at the moments of interaction conforms with the simulation data. Such a good match is observed for a different number of agents in flocks and impact parameters. In this sense, acceleration is actually the only parameter that determines the interaction of flocks in a head-on collision. It is natural to assume that it is the acceleration that determines the <<force>> of the interaction of flocks. Therefore, it is of interest to establish its dependence on the impact parameter and on the number of agents.

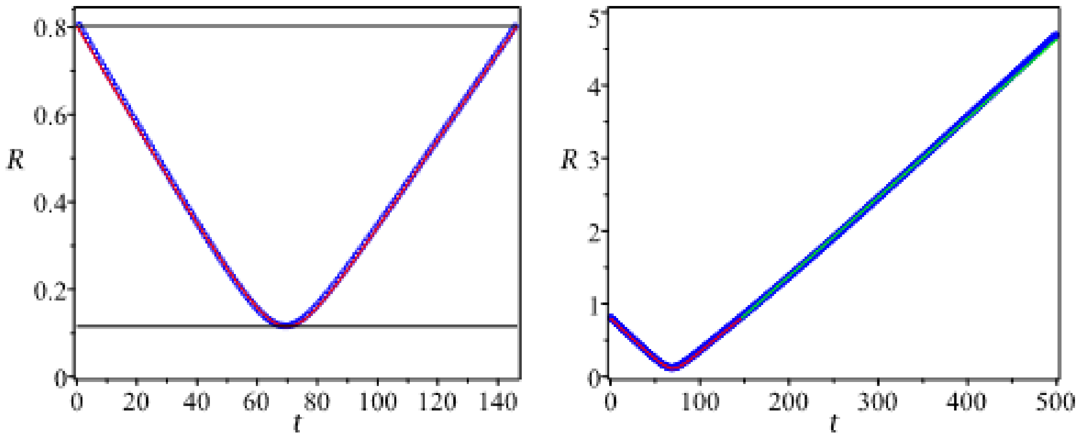

Fig.10 Changing the distance between interacting flocks. The blue color shows the simulation data for flocks $N_1 = N_2 = 95$ and $\rho = 0.115$. The red line corresponds to the dependency (5.5). On the left, at times of flock interaction $0 \le t \le t_{int}$, on the right, at times $t \ge t_{int}$. The two horizontal lines on the left side correspond to l_{int} and ρ. On the right, the dependence (5.10) of the distance change after the end of the interaction of flocks is shown in green

The characteristic change in the approach acceleration Δa_x as a function of the impact parameter is shown in Fig. 11.

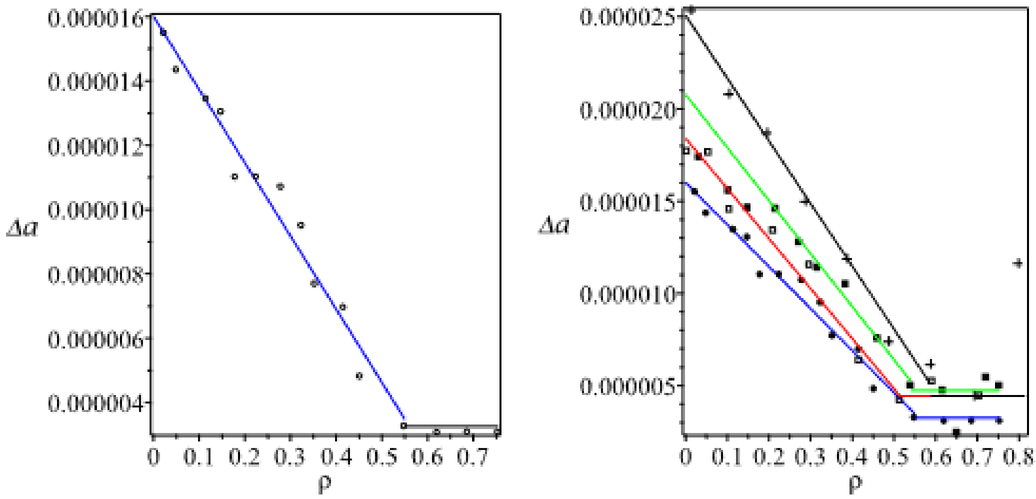

Fig.11 On the left the dependence of acceleration on impact parameter ρ for $N_1 = N_2 = 95$ is shown. Circles are simulation data, blue line is root-mean-square approximation in the region of decline, black line is a constant value corresponding to the absence of acceleration. On the right the dependence of acceleration on the impact parameter for different numbers of flocks is shown. Blue line – $N_1 = N_2 = 95$, black line – $N_1 = N_2 = 33$, green line – $N_1 = N_2 = 77$, and red line – $N_1 = N_2 = 60$

This behavior is typical of the order parameter in phase transitions. This means the disappearance of the interaction of flocks at a certain value of the impact parameter, despite the presence of a certain proportion of interacting agents in the flocks. If we use a linear approximation of the given data in Fig. 11, then the value of the impact parameter at which $\Delta a_x = 0$ for all numbers of flocks is $\rho_c \approx 0.7$. This value is less than l_{int} and, accordingly, part of the agents continues to interact with this value of the impact parameter. In other words, the interaction of agents does not mean the interaction of flocks.

Thus, in a head-on collision at the stage flock interactions, the main parameter acceleration Δa_x is determined by the relation (5.9) and determines the main characteristics of their behavior.

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Приймак О.В. *Кандидат технічних наук*

Яновський В.В. *Доктор фізико-математичних наук, професор*
Харківський національний університет ім. В.Н. Каразіна,
Майдан Свободи 6, м. Харків, 61022;
Інститут монокристалів НАН України,
проспект Науки 60, м. Харків, 61001

Лобове зіткнення зграй

Актуальність. Дослідження колективної поведінки зграй інтелектуальних агентів з використанням методів математичного та чисельного моделювання пов’язане з вирішенням задач у активно досліджуваних у наш час сферах багатоагентних систем та штучного інтелекту.

Мета. Виявити закономірності розсіяння зграй інтелектуальних агентів при їхньому зіткненні та отримати аналітичні співвідношення кінематики зграй.

Методи дослідження. В основі роботи лежать принципи та методи математичного та чисельного моделювання багатоагентних систем. Колективна поведінка агентів змодельована з використанням алгоритму флокування, що належить до методів моделювання руху, заснованих на векторах сил. Використано аналітичні методи опису кінематики поведінки стай, які добре узгоджуються з результатами моделювання.

Результати. Отримано дані, які визначають головні зміни, що відбуваються зі зграями в наслідок їх зіткнення. Визначено особливості, які відбуваються при взаємодії зграй при зміні кількості агентів у зграях. Визначено, що при збільшенні прицільного параметру зміни в характеристиках зграй стають менш помітними, незважаючи на існуючу взаємодію агентів двох зграй. Визначено, що головний параметр, який визначає поведінку зграй, є величина прискорення на стадії їх взаємодії. Отримана залежність прискорення від значення прицільного параметру нагадує залежність, яка є типовою для фазових переходів.

Висновки. В роботі отримано нові наукові результати, що полягають у виявленні головних закономірностей розсіяння зграй інтелектуальних агентів при їхньому зіткненні та отриманні аналітичних співвідношень кінематики зграй, які добре узгоджуються з даними моделювання.

Ключові слова: *колективна поведінка, багатоагентна система, зграя, інтелектуальний агент, флокування, вектор сили, кінематика, прискорення, прицільний параметр.*

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