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Mathematical models and approaches in simulation of RRS-based laser sensor system

The article is devoted to the research of mathematical models and approaches in simulation of laser sensor systems based on retroreflective sheeting (RRS) usage. The subject of the study is the mathematical model of the research object - the processes of propagation of a light in the medium and its transformation on retroreflectors with further analysis of the reflected radiation, which, being modulated by interaction with objects in the system, carries information about the state of the system. Overview of existing studies has been made. The analysis of existing approaches to simulation methods has been carried out, as well as the analysis of existing models. Common elements of models applicable in different fields and contexts of such systems have been determined. The scientific methods of research are outlined, such as comparative analysis while studying similar systems, conducting physical experiments in laboratory conditions with a prototype of a computerized detection system for observing phenomena in the system, their systematization and description, synthesis of a mathematical model based on the obtained data, creation of a computer model and, after its verification, and conducting computer experiments. The typical research schema is proposed, suggesting such main stages in simulation modeling of the laser sensing systems based on RRS as generation of the laser emission light field structure, interaction of the light field with environment, transformation of the APD inside the RRS, finding the APD in the reception zone, converting the resulting APD into a photocurrent, finding dependencies between parameters, evaluating detection efficiency, system optimization. The analysis of strong points of different approaches has been made. Some major aspects to be considered when choosing the appropriate method have been pointed out. The guiding principles for the general modeling of such systems illustrated with original concrete examples of such models verified in practice are presented.

Keywords: laser, sensor, retroreflective sheeting, specklometry, simulation modeling.

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1. Introduction

Laser detectors have become widespread due to the properties of the emitters included in their composition. Coherent radiation generated by lasers makes it possible to use the features of the diffraction phenomenon, namely, observing the formed speckle pattern, which allows expanding the scope of application of relatively simple optical detectors significantly. As optical detectors, they have the advantage of low signal response delay, because the speed of light propagation is extremely high. Among other advantages, one cannot fail to mention the fact that laser detectors will allow observing phenomena from a great distance, as this can be used in many fields of technology. In many cases, the condition is also fulfilled that the effect of detector radiation on the investigated system is so small that it can be neglected, which means that the system can be observed with their help without interfering with it, for example, mechanically, which is a great advantage. These devices allow solving a wide range of tasks - vibration control, laser ranging, velocimetry, turbulence control, as well as registration of acoustic oscillations and others [1-8].

One of the significant limitations when using the speckle measurement method is a wide angular spectrum of the laser light scattered on the rough object surface. This leads to a rapid decrease in power of the received radiation with an increase of measurement range. By covering the object surface with retro-reflecting sheeting (RRS) a significant increase of spatial concentration of the scattered radiation can be achieved. These coatings are currently widely used, for example, to enhance the visibility of the road signs, clothing, vehicles, etc. They consist of a large number of glass microspheres or microprisms [9] with characteristic dimensions ranging from tens to hundreds of micrometers and disposed on a substrate in random manner (microspheres) or relatively regular (microprisms). When the laser radiation falls on such surface, the light field is transformed in accordance with the laws of beam pass in each element of the RRS. The transformed radiation propagates in the direction of the laser source within a small angle of a few degrees. That process creates the speckle image within the receiving plane as a result of interference of waves re-emitted by each element. Use of RRS significantly increases the laser radiation capacity observed in a receiver plane. That allows us to increase observation distance and precision of measurements.

The study of the features of RRS-based laser sensor systems is of great interest. As part of the research, it is beneficial to develop simulation models of the system to assist in analyzing the influence of the variation of the system parameters on the quality of detection.

2. Overview of existing studies, analysis and goal definition

The design features of the retroreflectors provide for some useful properties. For example, as in case of the roughsurfaced objects, a speckle pattern is formed in the far zone when they are irradiated with laser radiation, which can be used for laser ranging. It is also possible to vary the microstructure, which opens up an opportunity to influence the transformation characteristics of the light field and intensity distribution of the reflected laser radiation in the far zone (receiving plane). The latter of these features is of particular interest and is the subject of the research [8-11], since it makes it possible to optimize the RRS used to improve the properties of a target, being a part of the specific optical subsystems.

The paper [12] describes the specklometric method of detecting acoustic vibrations, while reflective surfaces based on corner reflectors are used. The article [13] describes an approach to observing and measuring the acoustic dynamics of a gas after an ionizing pulse.

An extremely detailed study of using glass microbeads in road marking has been carried out at the University of Pretoria [14]. The work focuses on the creation of a thermoplastic road marking standard. It serves as a valuable source of both background information and information on different approaches to technology. The model that simulates the reflection of the car headlights from the marking back into the cabin is mentioned in the work. Statistical data on road marking wear over time are presented, an analysis of the dependence of light-reflective properties on the radius of the microspheres used in correlation with wear over time is carried out. The parameters of the structure of the reflective surface and the issues related to the choice of the substrate material are discussed.

Among other promising areas of use for RRS, the creation of reflective screens for image projection could be noted. For example, the article [11] describes a simulation model for studying the properties of such screens. Another similar direction of technology development is the creation of surfaces with a controlled degree of light reflection. The articles [15] and [16] describe such surfaces being developed in the University of Cincinnati. The authors propose a combined approach to the creation of reflective surfaces - prismatic microreflectors are suggested to be filled with an oily solution and light reflection could be controlled by changing the wettability (and thus the spatial shape of the structure) by changing the applied voltage. In [16], the mathematical model is described in detail and the results of physical modeling are presented. Such a serious study of prospective areas of retroreflector application shows us great relevance and broad prospects of using the RRS sensors.

Thus, the article [17] describes the use of RRS in scanning systems in conjunction with lidars. The article [18] describes the principle of creating a linear velocimeter based on the use of RRS and Doppler effect, the article [19] extends this idea to the case of rotating cylindrical objects. The article [20] describes the use of a retroreflective surface for optical tracking of object vibration, providing a mathematical model of laser radiation diffraction on a vibrating surface covered with microprism-based RRS.

The analysis of the literature shows that laser sensor systems with retroreflectors are widely used in industry. Active research of RRS-based laser sensor systems, as well as prospective areas of their application, reflects the interest of the scientific community in their further improvement and development of new areas of their application.

Our goal is to carry out an analysis of existing approaches simulation methods, find common elements of models applicable in different fields and contexts of such systems, as well as analyze strong points of different approaches in order to help in selecting an appropriate method, taking into account coherent laser radiation properties, and provide original examples of such models verified in practice.

3. Typical research schema

From the overview and analysis of the existing models it can be concluded that there is a general underlying schema of research, which we are going to summarize.

The purpose of the research on the modeling of RRS-based laser sensor systems is developing mathematical models of such systems for their further practical optimization and efficiency improvement.

The subject of the study is the mathematical model of the research object - the processes of propagation of a light wave (in the case of lasers - partially coherent) in the medium and its transformation on retroreflectors (microbeads- or microprism-based) with further analysis of the reflected radiation, which, being modulated by interaction with objects in the system, carries information about the state of the system.

There are some widespread scientific research methods, such as comparative analysis, conducting physical experiments in laboratory conditions with a prototype of a computerized detection system for observing phenomena in the system, their systematization and description, synthesis of a mathematical model based on the obtained data, creation of a computer model and, after its verification, conducting computer experiments.

The result of such research is a developed mathematical and computer models describing the system consisting of laser emitters and RRS, which allow improving the efficiency of such systems by providing recommendations on the selection of system parameters. Depending on the type of sensor under consideration and the application field such parameters as the detection efficiency, which is expressed as the ratio of positive activations to all activations in notification systems, or the accuracy of detection of environmental fluctuations, and the speed of movement of the object of observation, etc., can be selected.

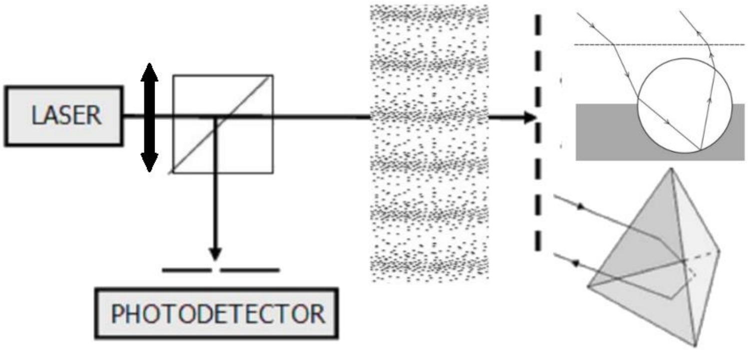

Fig.3.1 Typical setup for the research

A typical [11, 12, 15, 18, 19, 21, 22] setup of laser sensor with RRS, regardless of the application field shown in Fig. 3.1. It includes the main following elements: a source of laser radiation with an external focusing device, some kind of environment that laser beam transverses (might be turbulent atmosphere, regular acoustic oscillations, liquid, etc.) a reflecting surface with a reflective coating applied to it (surface might be moving – simulating vibrations or spatial movements of the target), optical splitter, a radiation collector, a diaphragm, and a photodetector.

4. The main stages in simulation modeling of RRS-based laser sensor systems

4.1. Generation of the laser emission light field structure

In some studies, only the intensity of the emitter is taken into account and diffraction does not play a role in detection, the speckle pattern is not important, and only the change in the amplitude of the detected photocurrent is of interest [11, 14, 15, 17, 21]. This greatly simplifies modeling and is also justified in cases where not only laser but the emitter with a wide spectrum can be used.

In the case when it is necessary to take into account the coherence of laser radiation, one should use the representation of the emitter in the form of the Gaussian beam (in the general case, it is multi-mode, most often it is enough to study the single-mode case). Next, we will consider the multimode laser approach, because multimode lasers are required for the sensors to be applicable in practice. In this case to describe the light field at a point, one should use the multimode Gaussian beams [23]:

$$\begin{gathered} E_{l,m}(x,y,z) = E_0 \frac{w_0}{w(z)} H_l\left(\frac{\sqrt{2}x}{w(z)}\right) H_m\left(\frac{\sqrt{2}y}{w(z)}\right) \times \\ \exp\left(-\frac{x^2+y^2}{w^2(z)}\right) \exp\left(-i\frac{k(x^2+y^2)}{2R(z)}\right) \exp(i\psi(z)) \exp(-ikz) \end{gathered} \tag{4.1}$$

where l, m - a longitudinal mode and a transverse mode respectively, H - the Hermite polynomial.

The cross section of the intensity distribution across the beam propagation direction *l* is shown in the Fig. 4.1 (a), with TEM LM designating modes: L designating the longitudinal mode and M the transverse mode.

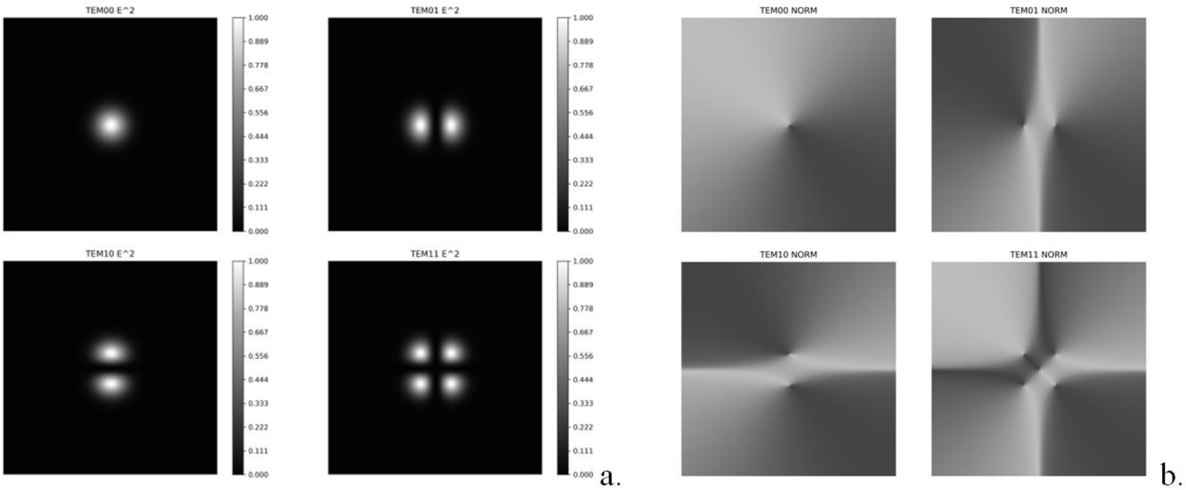

Fig.4.1 Intensity distribution (a) and normal map (b) of the multimode Gaussian beam cross section

For the further transition to the geometric optics, which is used in the model at the stage of calculating the field transformation inside of the RSS, it is necessary to restore the directional vector of the field at a point, as well as the polarization vector perpendicular to it and lying in a given polarization plane. To do this, it is enough to find the value of the gradient of the function at the desired point. One can use the numerical methods (e.g., finite differences methods) [24] for finding the derivative of the equation 4.1, considering the extreme cumbersomeness of the analytical calculation of the derivative function of the multimode Gaussian beam. The attractiveness of the numerical approach is largely explained by the presence of simple dependencies, with the help of which the derivatives at given points can be approximated by several values of the function at those points and points close to them. The construction of approximate differentiation equations based on a fact that the function on the given segment is replaced by the corresponding approximating function, and then it is assumed that the derivatives of the functions coincide. At the same time, the approximating function is most often given in the form of a polynomial [25].

The Fig. 4.1 (b) shows an example of normal maps for the cross sections of the fields of the Gaussian beams of different combinations of the orders of longitudinal and transverse modes. It seems obvious that for a multimode emitter it is possible to carry out an experiment for each of the chosen order of the modes and then combine the obtained intensities in the far zone.

4.2. Interaction of the light field with environment

The environment that laser beam transverses might vary - turbulent atmosphere, regular acoustic oscillations, liquid, etc. Most often it is just regular atmosphere and distances are so small, that it could be ignored altogether. In some cases, atmosphere effects are to be taken into account, Rayleigh scattering and Beer–Lambert law, for example [26].

In case of turbulent atmosphere usage of models with phase screens [27] and ray tracing in context of the refraction are appropriate. The refraction approach can also be used to model liquid environment. When dealing with acoustic wavefront, if width of the laser beam is much smaller than acoustic wavelength, a simple refraction approach is appropriate and described below.

It is assumed that the laser beam width is smaller than the acoustic wavelength in air, therefore, the sound wave can be represented as an acousto-optic refractive deflector - the mechanical propagation of a sound wave is accompanied by a change in the air density, which creates a gradient of optical density and causes the beam to deviate from the axis of its initial propagation.

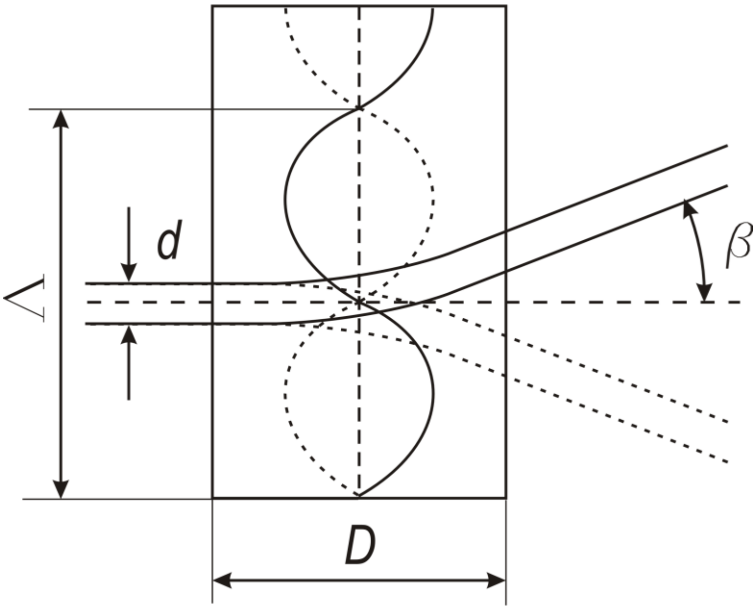

Fig.4.2 Acousto-optic refractive deflector

Then the following equation is applicable for the deflection angle [28]:

$$\beta = \frac{2\pi \Delta n D}{n \Lambda} \sin(2\pi(ft + \varphi)) \tag{4.2}$$

where Δn - a modulation amplitude of the refractive index of the medium, D - acoustic wave thickness, n - a refractive index, Λ - an acoustic wavelength, f - an acoustic frequency, t - time, φ - a phase shift.

For the model, the acoustic wave is assumed to be a plane with the wave propagation axis indicated on it. At a zero point of the axis, the phase is assumed to be zero. For the case of a multimodal laser, one should take into account that dielectric medium for propagation of radiation has a dispersion that can be described by the Sellmeier equation [26].

4.3. Transformation of the APD inside the RRS

Diffraction phenomena being neglected, and only the intensity is of interest, it is enough to calculate the initial aperture, limiting angles of reflection and the reflection coefficient, based on geometrical optics, both analytically as a whole, and by ray tracing, as in work [17], for example.

In the case of the scalar theory of diffraction and taking into account coherent radiation, two approaches are possible. If the RRS grid is strictly periodic (for example, obtained by stamping), analytical transformations according to the laws of diffraction grids are sufficient, as in the work [20], for example.

When the retroreflectors are disordered, as in the case of glass beads, it is necessary to find the transformation of amplitude-phase distribution (APD) on RRS by tracing rays (thus turning to geometric optics), as it is done in the works [11, 29]. We emphasize the fact that the transition to geometric optics is possible only if the retroreflective elements are at least an order of magnitude larger than the wavelength. At the same time, the decomposition of the polarization vector, transmission losses, and the calculation of the phase shift should be taken into account. An example of calculating the transformation of AFD on the retroreflector based on glass beads is given below.

Knowing the field amplitude, phase, directing vector and electric field vector in space at the given point, we can proceed to the second stage - the transformation of the field inside the RRS. To construct the ray trajectory, the directing vector $\overrightarrow{k_1}$ at a point P_1, selected on the RRS plane, transforms into vector $\overrightarrow{k_2}$ according to the Snell's law (Fig. 4.3 (a)). Next, the bead with which the intersection occurs and the point P_2 of the intersection of the ray $\left\{P_1, \overrightarrow{k_2}\right\}$ with the sphere representing the bead is determined. At the found point, a normal to the sphere is constructed, then similar actions are carried out.

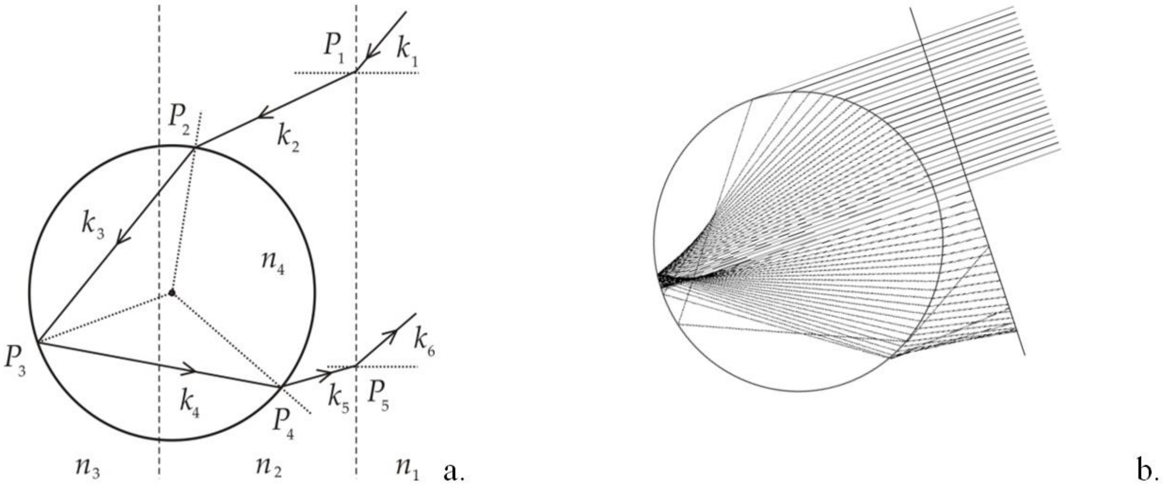

Fig.4.3 Ray trajectory inside glass beads RRS: calculations (a) and simulation example (b)

According to the model, the points P_2 and P_4 should lie on the border between the bead and the covering layer, and the point P_3 should lie on the border between the bead and the substrate; re-reflections inside the bead or between neighboring beads are not considered. The point P_5 is sampled and is the point of the output APD. In general case, the vectors $\overrightarrow{k_1}$ and $\overrightarrow{k_2}$ lie in one plane, and the vectors $\overrightarrow{k_2}$, $\overrightarrow{k_3}$, $\overrightarrow{k_4}$, $\overrightarrow{k_5}$, $\overrightarrow{k_6}$ lie in another. Given each of the segments $\overrightarrow{l_i = P_{i-1}P_i}$ that make up the trajectory of the light beam, the total phase shift φ is calculated:

$$\varphi = \frac{2\pi}{\lambda}\sum_{i=1}^{6} n_i l_i \tag{4.3}$$

where λ - a wavelength in vacuum, n_i - an absolute index of refraction of the medium for λ.

The phase shift when reflected at the point P_3 is considered separately, since the shift *i* for the parallel and perpendicular components *n* the general case is different. Also, for each transition from one medium to another, the amplitude transmission and reflection coefficients for both field components are calculated by using the Fresnel equations [26].

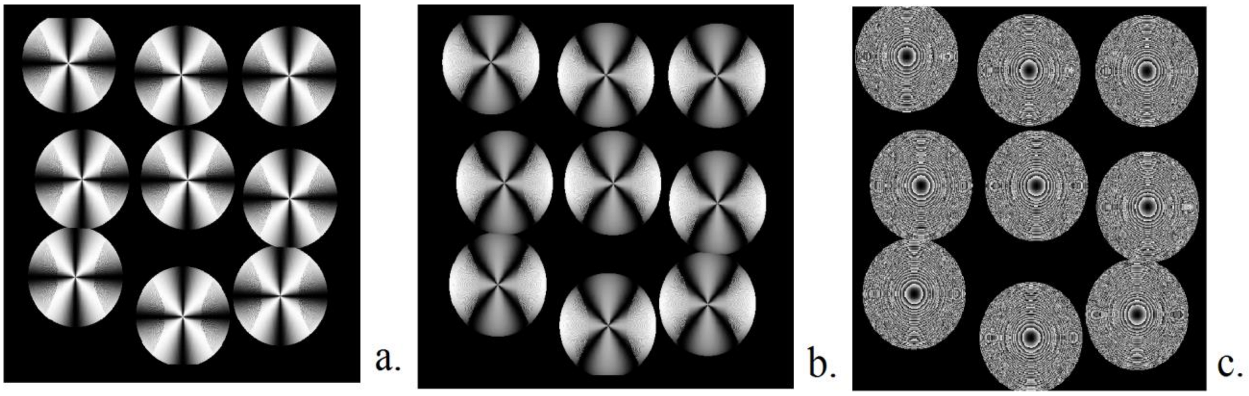

Fig.4.4 Example of the APD after the transformation on the glass beads RRS - parallel amplitude (a), perpendicular amplitude (b) and phase (c)

4.4. Finding the APD in the reception zone (far zone)

In the case of incoherent radiation, only intensity and aperture [11, 14, 15, 17, 21] are of interest. In this case, it is enough to account for the interaction with the environment, as in the modeling step above.

When the radiation is coherent and the RRS plays the role of a diffraction grating, it is necessary to take into account diffraction phenomena. In this case, it is possible to see the structure of the picture or individual speckles to use this information for detection. Then it is necessary to use the scalar theory of diffraction. We will present the general principle of finding speckle pattern in the far zone below.

After the APD is found on the surface of the reflector, one can start searching for the image formed in the far zone. APD on the plane can be considered as a collection of point sources of radiation, the coherence of which is given by the nature of the laser illuminating the surface. The expression for a monochromatic wave has the form [30]:

$$u\left(x,y,t\right)=A(x,y)\cos(2\pi\upsilon t+\varphi(x,y)) \tag{4.4}$$

or, alternatively

$$\begin{aligned} u(x,y,t) &= \mathrm{Re}\left[U(x,y)\exp(-j2\pi\upsilon t)\right] \\ U(x,y) &= A(x,y)\exp(-j\varphi(x,y)) \end{aligned} \tag{4.5}$$

The function $U(x,y)$ from (4.5) also called a phasor and describes the APD. Due to the linearity of the wave equation, as well as the coherence of radiation, the created diffraction pattern can be represented as the integral of a superposition:

$$U\left(x_0,y_0\right)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(x_0,y_0;x_1,y_1)U\left(x_1,y_1\right)dx_1dy_1 \tag{4.6}$$

In equation (4.6), the integrand function of transition has the form:

$$h\left(x_0,y_0;x_1,y_1\right)=\frac{1}{j\lambda}\frac{\exp\left(jkr_{01}\right)}{r_{01}}\cos\left(\vec{n},\overrightarrow{r_{01}}\right) \tag{4.7}$$

The limits of integration in (4.6) are written as infinite due to the fact that outside the source region the function $U\left(x_1,y_1\right)$ turns to zero, which satisfies the Kirchhoff boundary conditions.

If z is sufficiently large, it is possible to represent the integral of the superposition as the Fourier transform by using the Fraunhofer approximation:

$$U\left(x_0,y_0\right)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}U\left(x_1,y_1\right)\exp\left(-j\frac{2\pi}{\lambda z}\left(x_0x_1+y_0y_1\right)\right)dx_1dy_1 \tag{4.8}$$

The resulting Fourier image is a diffraction pattern in the far zone [31]. It should be noted that in practice instead of the amplitude distribution of the received image, the squared one is being used allowing us to obtain a value proportional to the intensity. This is due to the fact that quadratic detectors are widely used, and therefore it is more convenient to evaluate the intensity. Regarding the model, it should be noted that two Fourier transforms are calculated - for parallel and perpendicular components relative to the common base, and then the listed intensities are added. This is due to the fact, that polarization plane shifts during refraction, because the parallel and perpendicular components being affected differently, so to stay within scalar diffraction theory one should consider components separately.

Of course, computer model is able to work only with a limited amount of data, therefore, in practice, only a discrete signal, that can be decomposed into finite Fourier series, can be calculated.

To calculate a two-dimensional DFT, it is enough to calculate one-dimensional complex DFTs of all rows of the image, and then calculate one-dimensional complex DFTs of all columns in the resulting image.

Direct computation of the discrete Fourier transform of the sequence $x(n)$ requires n^2 products (or additions) of complex numbers. Thus, for sufficiently large n, the direct calculation of the DFT requires an excessive number of computational operations. Thus, in practice the fast discrete Fourier transform (FDFT) algorithms are used for more efficient calculations [32].

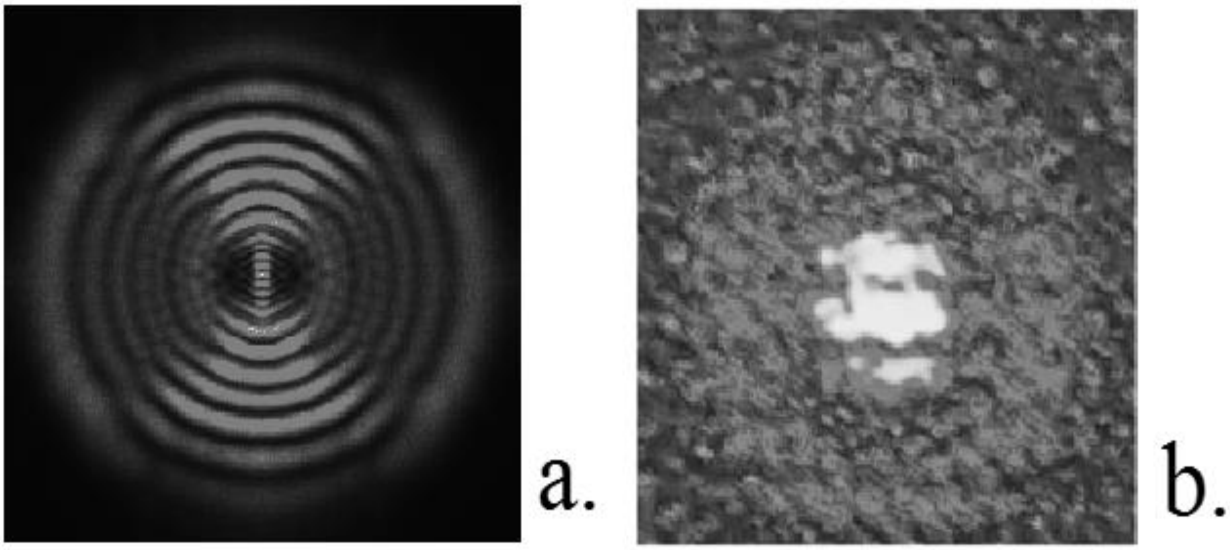

Fig.4.5 Intensity of the APD in far plane after the FDFT (a), speckle structure closeup (b)

4.5. Converting the resulting APD into a photocurrent

Regardless of the way APD is obtained in the far zone, most of the detectors end up converting observed intensity into the photocurrent for further analysis of the fluctuations. We will proceed with explanation on how this part of modeling is approached.

On the previous step we have obtained far-field APD for the perpendicular and parallel polarization components. To convert into a photocurrent, a few simple steps are enough. To begin with, let us translate the amplitude into intensity by squaring. The resulting intensities for the components can be added. At this stage, one can simulate a different receiving aperture by imposing a bit mask on the resulting distribution - multiplying the distribution with an array of zeros and ones of the same size - this way we can cut out the central aperture, make it circular, etc. Further, it is enough to sum up all intensities for each of the polarizations - this is equivalent to the operation of integration over the area, which, in fact, is produced by the lens - the result will be the power. The obtained instantaneous energy characteristic can be represented as directly proportional to the photocurrent - the characteristic of the photodetector can be considered linear and the sensor is infinitely sensitive because this does not apply to detection and is solved in applied cases by typical methods.

Further, one can trace the dynamics of the photocurrent in time by simulating the behavior of the system at successive discrete moments in time. Thus, we receive a photocurrent. Now we need to get the spectral characteristics of this sampled signal. To do this, first we find and subtract the average - this is how we get rid of the constant component. Next, we apply the Fourier transform and obtain the spectrum of the photocurrent - it carries information about the detected acoustic vibrations, since the optical signal that formed it was modulated by a sound wave.

Below, shown on the Fig. 4.6, is an example of the use of FDFT for the analysis of the photocurrent spectrum when detected in the environment of an acoustic column with a frequency of 1100 Hz.

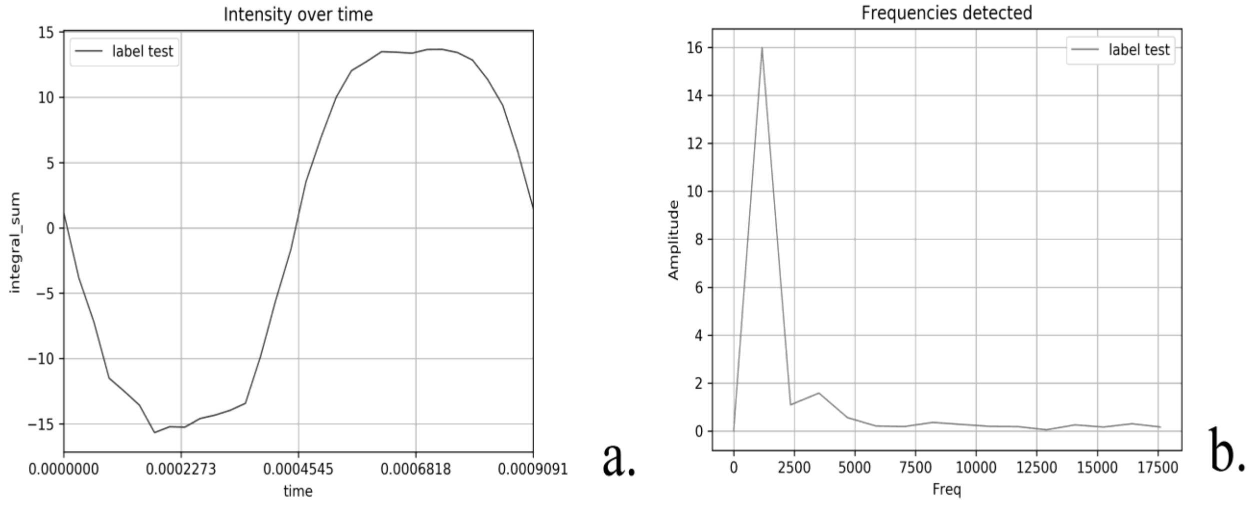

Fig.4.6 Example of the obtained via simulation photocurrent (a), and its spectral image (b)

It worth the mention, that in case you are aiming after modeling velocimetery application, then bumps on the higher frequencies will correspond to the speed of movement of the RRS-coated object, being result of the Doppler effect [18, 19].

4.6. An iterative pass with variation of parameter values of the system

Since for the components of real systems some deviations in the distributions of physical properties are present, it is necessary to conduct computer experiments for various implementations. It is simply impossible to assess whether the simulation model is correct or translate individual results into practice, because this is not a completely analytical mathematical modeling. In fact, the Monte Carlo approach [33] can be used to solve this problem. During iterative runs of experiments with different implementations of the system parts, slightly different results can be obtained. Averaging the results allows us to see which trajectory the system gravitates towards certain parameters, whether it is stable within given limits of deviations.

4.7. Finding dependencies between parameters, evaluating detection efficiency, system optimization

The imitation model ultimately allows, on the one hand, to better understand the processes in detection systems, and on the other hand, to investigate the relationships of parameters in the system without conducting a huge number of physical experiments - which are expensive to implement and take a lot of time. Indeed, the approach to finding relationships between parameters can vary depending on the context of the system. Usually, they become clear quickly enough, since when observing the boundary conditions and constrains, they fall under the theories found in classical works on optics, being a simple combination of known laws. However, in cases where there are a lot of parameters in the system and they are interconnected in such a way that it is not possible to fix all other parameters and look at the influence of one of them on the result, it can be difficult to navigate. In this case, it is advisable to conduct a number of experiments, find correlation between parameters and use a regression model to identify the type of relations [34].

Moreover, based on the results of the experiments with the verified model, it is possible to make recommendations regarding the specific values of the system parameters for the optimal operating mode. In this case, the output parameter is selected, which must be maximized under the given input restrictions. To optimize the target function of efficiency it is necessary to use the method of mathematical programming [35]. The selection of the target metric depends entirely on the application of the model. An example of such a target metric can be, in the case of laser fire detectors, the notification efficiency, defined as the ratio of positive activations to all activations in the condition of external disturbances.

5. Conclusions

The subject of the study is the simulation models of the processes of light wave propagation (in the case of lasers - partially coherent) in the medium and its transformation on retroreflectors (microbeads- or microprism-based) with further analysis of the reflected radiation, which, being modulated by interaction with objects in the system, carries information about the state of the system. Overview of existing studies has been made. Analysis of existing approaches to simulation methods and overview of the existing models has been carried out.

As a result, common elements of models applicable in different fields and contexts of such systems have been found. Scientific methods of research have been outlined and typical research schema is proposed, suggesting the main stages in simulation modeling of RRS-based laser sensor systems. The following topics are covered: generation of the laser emission light field structure, interaction of the light field with environment, transformation of the APD inside the RRS, finding the APD in the reception zone, converting the resulting APD into a photocurrent, finding dependencies between parameters, evaluating detection efficiency, system optimization.

The analysis of strong points of different approaches has been made, the aspects important for choosing the appropriate method have been underlined. The guidance principles for the general modeling of such systems have been given, and original concrete examples of such models verified in practice is provided.

REFERENCES

1. Measures R. M. Laser Remote Sensing: Fundamentals and Applications. Malabar, FL: Krieger Publishing, 1992.
URL: http://www.krieger-publishing.com/Titles/LthruO/STACKmno/stackmno_33.html
2. Tetsuo Fukuchi, Tatsuo Shiina. Industrial Applications of Laser Remote Sensing. Bentham Science Publishers, 2012.
URL: https://doi.org/10.2174/97816080534071120101
3. Korpel, A. Acousto-Optics, Second Edition; Technology & Engineering, CRC Press, 1996.
URL: https://www.academia.edu/8400584/Acousto-optics
4. Magdich, L.N. Acoustooptic Devices and Their Applications; Technology & Engineering, CRC Press, 1989.
URL:https://books.google.com.ua/books/about/Acoustooptic_Devices_and_Their_Applicati.html?id=DPCNu9hQuA4C&redir_esc=y
5. Sirohi, R.S., Ed. Speckle Metrology; Technology & Engineering, CRC Press, 1993.
URL: https://www.routledge.com/Speckle-Metrology/Sirohi/p/book/9780824789329
6. M. Kowalczyk, Pluta M. , Jabczynski J. K., and Szyjer M. Laser speckle velocimetry // Optical Velocimetry, Proc. SPIE 2729, 1996. Pp. 139–145. URL: https://spie.org/Publications/Proceedings/Paper/10.1117/12.233000
7. Raffel M., Willert C., Kompenhans J. Particle Image Velocimetry. A Practical Guide. Springer, 1998.
URL: https://link.springer.com/book/10.1007/978-3-662-03637-2
8. Dolya G. M., Lytvynova O. S. Modeling of fluctuations of laser radiation scattered on the reflector array in a turbulent atmosphere // Laser and Fiber-Optical Networks Modeling (LFNM), 2011. Pp. 108-111.
URL: https://doi.org/10.1109/LFNM.2011.6145015
9. Lloyd J. A brief history of retroreflective sign face sheet materials. The principles of retroreflection. REMA publications, 2008.
URL: http://www.rema.org.uk/pub/pdf/history-retroreflective-materials.pdf
10. Migletz J., Fish J. K., Grahm J. L. Roadway Delineation Practices Handbook. US Department of transportation, Federal highway administration, 1994.

URL: https://safety.fhwa.dot.gov/ped_bike/docs/rdwydelin.pdf

11. Héricz D., Sarkadi T., Erdei G., Lazuech T., Lenk S., Koppa P. Simulation of small- and wide-angle scattering properties of glass-bead retroreflectors // Applied Optics Vol. 56, Issue 14, 2017. Pp. 3969-3976.
URL: http://dx.doi.org/10.1364/AO.56.003969
12. Dolya G. M., Lytvynova O. S Modeling of speckle metrology technique of detecting the medium acoustic oscillations // Bulletin of V.N. Karazin Kharkiv National University, series “Mathematical modelling. Information technology. Automated control systems”, 31(1), 2016. 38-46 p. [in Russian]
URL: https://periodicals.karazin.ua/mia/article/view/6807/6299
13. Wahlstrand J. K., Jhajj N., Rosenthal E. W., Zahedpour S., Milchberg H. M. Direct imaging of the acoustic waves generated by femtosecond filaments in air // Opt. Lett. 39, 2014. 1290-1293 c.
URL: https://doi.org/10.1364/OL.39.001290
14. Naidoo S. The development of thermoplastic road marking material stadart. University of Pretoria, 2016. 302 c.
URL: https://repository.up.ac.za/handle/2263/61319
15. Kilaru M., Yang J., Heikenfeld J. Advanced characterization of electrowetting retroreflectors // OSA, OPTICS EXPRESS, Vol. 17, No. 20, 2009. 17563-17569 c.
URL: https://doi.org/10.1364/OE.17.017563
16. Murali Krishna Kilaru. Electrowetting Switchable Retroreflectors: a dissertation for the degree of doctor of philosophy (Ph.D.), University of Cincinnati, 2009. 148 c.
URL: https://etd.ohiolink.edu/apexprod/rws_etd/send_file/send?accession=ucin1282054545&disposition=inline
17. Anderson J., Massaro R., Lewis L., Moyers R., Wilkins J. Lidar-activated Phosphors and Infrared Retro-Reflectors: Emerging Target Materials for Calibration and Control // Photogrammetric Engineering & Remote Sensing, 2010. 875-879 c.
URL: https://www.researchgate.net/profile/Richard-Massaro/publication/228747484_Lidar-activated_Phosphors_and_Infrared_Retro-Reflectors_Emerging_Target_Materials_for_Calibration_and_Control/links/00b7d5208e69f0a972000000/Lidar-activated-Phosphors-and-Infrared-Retro-Reflectors-Emerging-Target-Materials-for-Calibration-and-Control.pdf
18. Dolya G. M., Lytvynova O. S. Modeling the Method of Laser Doppler Speckle-Velocimetry for Flat Objects with Retroreflective Surface // Asian Engineering Review Vol.4, No. 2, 2017. 7-13 c.
URL: https://doi.org/10.20448/journal.508.2017.42.7.13
19. Dolya G., Bondarenko K. Laser Doppler Velocimetry of Rotating Cylinder with Retroreflecting Surface // Asian Engineering Review, 5(1), 2018. 1-7 pp.
URL: https://doi.org/10.20448/journal.508.2018.51.1.7
20. Dolya G.N., Chudovskaya E.S., Kochin A.V. Mathematical modeling of diffraction of laser radiation on a vibrating surface with a reflective coating // Eastern European journal of advanced technologies 1/6 (37), 2009. 59-62 c. [in Russian]
URL: https://doi.org/10.15587/1729-4061.2009.3174
21. Ishikawa K., Asada K., Tamayama K., Ueda M., Optical Vibration Monitoring System by Means of CCD Camera and Retro-Reflector // The Review of Laser Engineering Vol.30, No.2, 2002 91-93.
URL: https://www.jstage.jst.go.jp/article/lsj/30/2/30_2_91/_pdf
22. Dolya G., Bondarenko K., Laser detector of acoustic oscillations utilizing a retroreflective surface with glass beads // EESA, Vol. 3, No. 11(63), 2020. 7-16 p.
URL: https://archive.eesa-journal.com/index.php/eesa/article/view/30
23. Siegman, Anthony E. Lasers. Revised ed. edition. University Science Books, 1986. 1283 p.
URL: https://uscibooks.aip.org/books/lasers/
24. Kryzhanivska T.V., Boytsova I.A. Synopsis of lectures on the discipline "Numerical methods". Odesa, 2013. – 152 c. [in Ukrainian]
URL:http://eprints.library.odeku.edu.ua/id/eprint/711/1/KryzhanivskaTV_BoitsovaIA_Chyselni_Metody_KL_2013.pdf
25. Feldman L.P. Numerical methods in computer science. - Textbook. К., 2006. – 480с.[in Ukrainian]
URL: http://library.kre.dp.ua/Books/2-

4%20kurs/%D0%90%D0%BB%D0%B3%D0%BE%D1%80%D0%B8%D1%82%D0%BC%D0%B8%20%D1%96%20%D0%BC%D0%B5%D1%82%D0%BE%D0%B4%D0%B8%20%D0%BE%D0%B1%D1%87%D0%B8%D1%81%D0%BB%D0%B5%D0%BD%D1%8C/%D0%A4%D0%B5%D0%BB%D1%8C%D0%B4%D0%BC%D0%B0%D0%BD_%D0%A7%D0%B8%D1%81%D0%B5%D0%BB%D1%8C%D0%BD%D1%96_%D0%BC%D0%B5%D1%82%D0%BE%D0%B4%D0%B8_%D0%B2_%D1%96%D0%BD%D1%84%D0%BE%D1%80%D0%BC%D0%B0%D1%82%D0%B8%D1%86%D1%96_2007.pdf

26. Born M., Wolf E. Principles of optics: electromagnetic theory of propagation, interference and diffraction of light (7th expanded ed.). Cambridge: Cambridge University Press. 1999.
    URL: https://www.worldcat.org/title/1151058062
27. M. Charnotskii. Comparison of four techniques for turbulent phase screens simulation. J. Opt. Soc. Am. A, 37(5):738–747, May 2020.
    URL: https://doi.org/10.1364/JOSAA.385754
28. Giarola A.J., Billeter T.R., Electroacoustic deflection of a coherent light beam // Proc. IEEE 51, #8, 1150, 1963.
    URL: https://doi.org/10.1109/PROC.1963.2463
29. Dolya G., Bondarenko K. Model of laser radiation transformation upon retroreflection from glass beads based surface // East European Scientific Journal, #5(45), 2019. 10-20 p.
    URL: https://eesa-journal.com/wp-content/uploads/EESA_may1.pdf
30. Goodmam J.W. Introduction to Fourier Optics., New York: W. H. Freeman, 2017. 546 p.
    URL: https://www.worldcat.org/title/introduction-to-fourier-optics/oclc/958780856
31. H.L. Kononchuk, V.M. Prokopets, V.V. Stukalenko Introduction to Fourier Optics: A Study Guide. - Kyiv: Publishing and printing center "Kyiv University", 2009. – 320 p.[in Ukrainian]
    URL:http://exp.phys.univ.kiev.ua/ua/Study/Lib/Navch_posibnyky/NP%20-%20Fure%20optyka%20-%20Kononchuk.pdf
32. Fedorov E.E., Nechyporenko O.V., Utkina T.Yu., Korpan Ya.V. Models and methods of computer systems for visual image recognition: monograph. – Cherkasy: ChDTU, 2021. – 482 p. [in Ukrainian]
    URL: https://er.chdtu.edu.ua/bitstream/ChSTU/3502/1/mono_FEE.pdf
33. Kroese, D. P.; Taimre, T.; Botev, Z.I. Handbook of Monte Carlo Methods. New York: John Wiley & Sons. 2011 p. 772.
    URL: https://www.wiley.com/en-au/Handbook+of+Monte+Carlo+Methods-p-9780470177938
34. Rouaud M. Probability, Statistics and Estimation. 2013, p 191.
    URL: http://www.incertitudes.fr/book.pdf
35. Bilogurova G.V., Samoilenko M.I. Mathematical programming: Synopsis of lectures. – Kharkiv: KhNAMG, 2009. – 72 p. [in Ukrainian]
    URL: https://core.ac.uk/download/pdf/11322845.pdf

ЛІТЕРАТУРА

1. Measures R. M. Laser Remote Sensing: Fundamentals and Applications. Malabar, FL : Krieger Publishing, 1992.
   URL: http://www.krieger-publishing.com/Titles/LthruO/STACKmno/stackmno_33.html
2. Tetsuo Fukuchi, Tatsuo Shiina. Industrial Applications of Laser Remote Sensing. Bentham Science Publishers, 2012.
   URL: https://doi.org/10.2174/97816080534071120101
3. Korpel, A. Acousto-Optics, Second Edition; Technology & Engineering, CRC Press, 1996.
   URL: https://www.academia.edu/8400584/Acousto-optics
4. Magdich, L.N. Acoustooptic Devices and Their Applications; Technology & Engineering, CRC Press, 1989.
   URL:https://books.google.com.ua/books/about/Acoustooptic_Devices_and_Their_Applicati.html?id=DPCNu9hQuA4C&redir_esc=y
5. Sirohi, R.S., Ed. Speckle Metrology; Technology & Engineering, CRC Press, 1993.
   URL: https://www.routledge.com/Speckle-Metrology/Sirohi/p/book/9780824789329
6. M. Kowalczyk, Pluta M. , Jabczynski J. K., and Szyjer M. Laser speckle velocimetry // Optical Velocimetry, Proc. SPIE 2729, 1996. Pp. 139–145.

URL: https://spie.org/Publications/Proceedings/Paper/10.1117/12.233000
7. Raffel M., Willert C., Kompenhans J. Particle Image Velocimetry. A Practical Guide. Springer, 1998.
URL: https://link.springer.com/book/10.1007/978-3-662-03637-2
8. Dolya G. M., Lytvynova O. S. Modeling of fluctuations of laser radiation scattered on the reflector array in a turbulent atmosphere // Laser and Fiber-Optical Networks Modeling (LFNM), 2011. Pp. 108-111.
URL: https://doi.org/10.1109/LFNM.2011.6145015
9. Lloyd J. A brief history of retroreflective sign face sheet materials. The principles of retroreflection. REMA publications, 2008.
URL: http://www.rema.org.uk/pub/pdf/history-retroreflective-materials.pdf
10. Migletz J., Fish J. K., Grahm J. L. Roadway Delineation Practices Handbook. US Department of transportation, Federal highway administration, 1994.
URL: https://safety.fhwa.dot.gov/ped_bike/docs/rdwydelin.pdf
11. Héricz D., Sarkadi T., Erdei G., Lazuech T., Lenk S., Koppa P. Simulation of small- and wide-angle scattering properties of glass-bead retroreflectors // Applied Optics Vol. 56, Issue 14, 2017. Pp. 3969-3976.
URL: http://dx.doi.org/10.1364/AO.56.003969
12. Dolya G. M., Lytvynova O. S Modeling of speckle metrology technique of detecting the medium acoustic oscillations // Вісник Харківського національного університету імені В.Н. Каразіна, серія «Математичне моделювання. Інформаційні технології. Автоматизовані системи управління», 31(1), 2016. 38-46 с.
URL: https://periodicals.karazin.ua/mia/article/view/6807/6299
13. Wahlstrand J. K., Jhajj N., Rosenthal E. W., Zahedpour S., Milchberg H. M. Direct imaging of the acoustic waves generated by femtosecond filaments in air // Opt. Lett. 39, 2014. 1290-1293 с.
URL: https://doi.org/10.1364/OL.39.001290
14. Naidoo S. The development of thermoplastic road marking material stadart. University of Pretoria, 2016. 302 с.
URL: https://repository.up.ac.za/handle/2263/61319
15. Kilaru M., Yang J., Heikenfeld J. Advanced characterization of electrowetting retroreflectors // OSA, OPTICS EXPRESS, Vol. 17, No. 20, 2009. 17563-17569 с.
URL: https://doi.org/10.1364/OE.17.017563
16. Murali Krishna Kilaru. Electrowetting Switchable Retroreflectors: a dissertation for the degree of doctor of philosophy (Ph.D.), University of Cincinnati, 2009. 148 с.
URL: https://etd.ohiolink.edu/apexprod/rws_etd/send_file/send?accession=ucin1282054545&disposition=inline
17. Anderson J., Massaro R., Lewis L., Moyers R., Wilkins J. Lidar-activated Phosphors and Infrared Retro-Reflectors: Emerging Target Materials for Calibration and Control // Photogrammetric Engineering & Remote Sensing, 2010. 875-879 с.
URL: https://www.researchgate.net/profile/Richard-Massaro/publication/228747484_Lidar-activated_Phosphors_and_Infrared_Retro-Reflectors_Emerging_Target_Materials_for_Calibration_and_Control/links/00b7d5208e69f0a972000000/Lidar-activated-Phosphors-and-Infrared-Retro-Reflectors-Emerging-Target-Materials-for-Calibration-and-Control.pdf
18. Dolya G. M., Lytvynova O. S. Modeling the Method of Laser Doppler Speckle-Velocimetry for Flat Objects with Retroreflective Surface // Asian Engineering Review Vol.4, No. 2, 2017. 7-13 с.
URL: https://doi.org/10.20448/journal.508.2017.42.7.13
19. Dolya G., Bondarenko K. Laser Doppler Velocimetry of Rotating Cylinder with Retroreflecting Surface // Asian Engineering Review, 5(1), 2018. 1-7 pp.
URL: https://doi.org/10.20448/journal.508.2018.51.1.7
20. Доля Г. Н., Чудовская Е. С., Кочин А. В. Математическое моделирование дифракции лазерного излучения на вибрирующей поверхности со световозвращающим покрытием // Восточно-Европейский журнал передовых технологий 1/6 (37), 2009. 59-62 с.
URL: https://doi.org/10.15587/1729-4061.2009.3174

21. Ishikawa K., Asada K., Tamayama K., Ueda M., Optical Vibration Monitoring System by Means of CCD Camera and Retro-Reflector // The Review of Laser Engineering Vol.30, No.2, 2002 91-93.
URL: https://www.jstage.jst.go.jp/article/lsj/30/2/30_2_91/_pdf
22. Dolya G., Bondarenko K., Laser detector of acoustic oscillations utilizing a retroreflective surface with glass beads // EESA, Vol. 3, No. 11(63), 2020. 7-16 p.
URL: https://archive.eesa-journal.com/index.php/eesa/article/view/30
23. Siegman, Anthony E. Lasers. Revised ed. edition. University Science Books, 1986. 1283 p.
URL: https://uscibooks.aip.org/books/lasers/
24. Крижанівська Т.В., Бойцова І.А. Конспект лекцій з дисципліни „Чисельні методи". Одеса, 2013. – 152 с.
URL:http://eprints.library.odeku.edu.ua/id/eprint/711/1/KryzhanivskaTV_BoitsovaIA_Chyselni_Metody_KL_2013.pdf
25. Фельдман Л.П. Чисельні методи в інформатиці. – Підручник. К., 2006. – 480с.
URL: http://library.kre.dp.ua/Books/2-4%20kurs/%D0%90%D0%BB%D0%B3%D0%BE%D1%80%D0%B8%D1%82%D0%BC%D0%B8%20%D1%96%20%D0%BC%D0%B5%D1%82%D0%BE%D0%B4%D0%B8%20%D0%BE%D0%B1%D1%87%D0%B8%D1%81%D0%BB%D0%B5%D0%BD%D1%8C/%D0%A4%D0%B5%D0%BB%D1%8C%D0%B4%D0%BC%D0%B0%D0%BD_%D0%A7%D0%B8%D1%81%D0%B5%D0%BB%D1%8C%D0%BD%D1%96_%D0%BC%D0%B5%D1%82%D0%BE%D0%B4%D0%B8_%D0%B2_%D1%96%D0%BD%D1%84%D0%BE%D1%80%D0%BC%D0%B0%D1%82%D0%B8%D1%86%D1%96_2007.pdf
26. Born M., Wolf E. Principles of optics: electromagnetic theory of propagation, interference and diffraction of light (7th expanded ed.). Cambridge: Cambridge University Press. 1999.
URL: https://www.worldcat.org/title/1151058062
27. M. Charnotskii. Comparison of four techniques for turbulent phase screens simulation. J. Opt. Soc. Am. A, 37(5):738–747, May 2020.
URL: https://doi.org/10.1364/JOSAA.385754
28. Giarola A.J., Billeter T.R., Electroacoustic deflection of a coherent light beam // Proc. IEEE 51, #8, 1150, 1963.
URL: https://doi.org/10.1109/PROC.1963.2463
29. Dolya G., Bondarenko K. Model of laser radiation transformation upon retroreflection from glass beads based surface // East European Scientific Journal, #5(45), 2019. 10-20 с
URL: https://eesa-journal.com/wp-content/uploads/EESA_may1.pdf
30. Goodmam J.W. Introduction to Fourier Optics., New York: W. H. Freeman, 2017. 546 p.
URL: https://www.worldcat.org/title/introduction-to-fourier-optics/oclc/958780856
31. Г.Л. Кончук, В. М. Прокопець, В.В. Стукаленко. Вступ до фур'є-оптики: навчальний посібник. –К.: Видавничополіграфічний центр «Київський університет», 2009. – 320 с.
URL:http://exp.phys.univ.kiev.ua/ua/Study/Lib/Navch_posibnyky/NP%20-%20Fure%20optyka%20-%20Kononchuk.pdf
32. Федоров Є. Є., Нечипоренко О. В., Уткіна Т. Ю., Корпань Я. В. Моделі та методи комп'ютерних систем розпізнавания зорових образів : монографія. – Черкаси : ЧДТУ, 2021. – 482 с.
URL: https://er.chdtu.edu.ua/bitstream/ChSTU/3502/1/mono_FEE.pdf
33. Kroese, D. P.; Taimre, T.; Botev, Z.I. Handbook of Monte Carlo Methods. New York: John Wiley & Sons. 2011 p. 772.
URL: https://www.wiley.com/en-au/Handbook+of+Monte+Carlo+Methods-p-9780470177938
34. Rouaud M. Probability, Statistics and Estimation. 2013, p 191.
URL: http://www.incertitudes.fr/book.pdf
35. Білогурова Г.В., Самойленко М.І. Математичне програмування: Конспект лекцій. – Харків: ХНАМГ, 2009. – 72 с.
URL: https://core.ac.uk/download/pdf/11322845.pdf

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Математичні моделі та підходи до імітаційного моделювання лазерних датчиків на основі СПП

Стаття присвячена дослідженню математичних моделей і підходів до моделювання лазерних датчиків, які базуються на використанні світлоповертаючих поверхонь. Предметом дослідження є математична модель об'єкта дослідження – процесів поширення світла в середовищі та його трансформації на ретрорефлекторах з подальшим аналізом відбитого випромінювання, яке, модулюючись взаємодією з об'єктами системи, несе інформацію про стан системи. Зроблено огляд існуючих досліджень. Проведено аналіз існуючих підходів до методів моделювання, та аналіз існуючих моделей. Знайдено спільні елементи моделей що можуть бути застосовані в різних прикладних областях та контекстах таких систем. Наведено такі наукові методи дослідження, як порівняльний аналіз при дослідженні подібних систем, проведення фізичних експериментів у лабораторних умовах з прототипом комп'ютеризованої системи детектування для спостереження явищ у системі, їх систематизація та опис, синтез математичної моделі на основі отриманих даних, створення імітаційної комп'ютерної моделі та проведення комп'ютерних експериментів після її верфікації. Запропоновано типову схему дослідження, яка передбачає основні етапи імітаційного моделювання систем лазерного детектування на основі СПП. Розглядаються такі теми, як: генерація структури світлового поля лазерного випромінювання, взаємодія світлового поля з навколишнім середовищем, трансформація АФР всередині СПП, знаходження АФР в зоні прийому, перетворення отриманого АФР у фотострум, знаходження залежностей між параметрами, оцінка ефективності виявлення, оптимізація системи. Проведено аналіз сильних сторін різних підходів, вказано деталі, які необхідно враховувати при виборі відповідного методу з урахуванням властивостей когерентного лазерного випромінювання. Наведено керівні принципи загального моделювання таких систем, що проілюстровані оригінальними конкретними прикладами таких моделей, перевірених на практиці.

Ключові слова: *лазер, датчик, світлоповертаюче покриття, спеклометрія, імітаційне моделювання.*

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