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The application of the orthogonal decomposition method for the algebraic solver separator

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The method which is valid for improving the transmission of information and a clean separation of signal and noise is suggested in [1]. The basis of the proposed new way of developing the theory and methods of communication is the rejection of the probabilistic method for evaluating noisy signals according to the maximum likelihood rule. This method contains the mathematical procedure for absolutely accurate separation, as well as proving the absence of any fundamental theoretical limits [2,3] on the effectiveness of communications, including the absence of channel capacity limitations [4]. This approach includes the fundamentally new concept and the technical aspects of implementing telecommunication systems and uses systems of linear algebra equations (SLAE) to filter signals from noise. The SLAE matrix is the linear algebraic matrix (LSM) that separates and extracts the true values of informative signal parameters. Such a SLAE always has the solution, but obtaining this solution sometimes requires a particular method to be used, because its matrix is not always square, but can be rectangular as well. Therefore, the method of the orthogonal decomposition is proposed in this paper. For obtaining the matrices of orthogonal decomposition the Gram–Schmidt process, which is suitable for matrices of any size and composition, can be used. The method of solving a SLAE includes full description of solution and acceptable for matrices of any size. The example of solving the SLAE with a small matrix is presented in the paper. The MathCad Prime has been implemented for a bigger matrix. The implementation includes the functions that can be used in any other programming language. The solution has minimal norm and acceptable for linear algebraic matrices that separate signal and noise.

Key worlds: *separation, systems of linear algebra equations, signal, noise, orthogonal decomposition.*

Застосування методу ортогонального розкладення в алгебраїчній сепарації

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Винахід у роботі [1] пропонує метод, який пов'язано з системами для покращення передачі та чистого відділення шуму та сигналу. Базова ідея, яка підкреслюється у роботі для нового шляху розробки теорії та техніки комунікації, це відхилення методу, який побудовано на імовірності, для оцінки сигналу згідно з правилом найбільшої ймовірності. Це математична процедура для абсолютно чіткого відокремлення сигналу та шуму та доказ відсутності будь-яких фундаментальних теоретичних обмежень [2,3] на ефективність комунікації, включно відсутність обмежень ємності каналу [4]. Такий підхід розглядає нову концепцію та технічні аспекти імплементації телекомунікаційних систем та використовує системи алгебраїчних рівнянь (СЛАР) для того, щоб відфільтрувати сигнал від шуму. Матриця СЛАР – це лінійна алгебраїчна матриця, що сепарує та виділяє правдиві значення інформативних параметрів сигналу. Такі СЛАР завжди мають рішення, але ці рішення потребують інколи особливого методу, щоб бути вирішеними, тому що їх розмір не завжди квадратний, іноді прямокутний. Таким чином у цій роботі пропонується метод ортогонального розкладення. Для отримання матриць з ортогональним розкладенням можна використовувати метод Грама Шмідта для матриць з будь-яким розміром навіть якщо є стовпці або строки, що повторюються у матриці. Метод для вирішення СЛАР містить повний опис рішення та придатний для довільного розміру матриць. У роботі є приклад з вирішення з малим розміром матриці. Також є приклад імплементації з матрицею набагато більшого розміру у середовищі MathCad Prime. Імплементація містить функції, які можна використовувати для інших мов програмування. Отримане рішення має мінімальну норму та придатне для лінійних алгебраїчних матриць, що сепарують сигнал від шуму.

Ключові слова: *сепарація, системи лінійних алгебраїчних рівнянь, сигнал, шум, ортогональне розкладення.*

1. Introduction

The possibility of a new method of improving the transmission of information and a clean separation of signal and noise is suggested by the discovery presented in [1]. This method uses systems of linear algebra equations (SLAE) to filter noise from signals. To correct errors the system is configured to carry out mixture Y(t) at specific time $T_0,\ldots, T_{2M}$ and SLAE can be:

$$A\cdot X = B, \tag{1.1}$$

The algebraic solver separator is designed to separate the signal and noise into two independent processes that occur in time [1]. The QR decomposition method of solving such a SLAE is considered in the paper. This method allows solving a square and rectangular SLAE. The solution has a minimal norm, otherwise:

$$\|X\| = \sqrt{{x_0}^2 + {x_1}^2 + {x_2}^2 + \ldots + {x_{n_1}}^2} \tag{1.2}$$

where coefficients of a matrix A are:

$$A=\underbrace{\left.\begin{pmatrix} A_{0,0} & A_{0,1} & A_{0,2} & \cdots & A_{0,M} & A_{0,M+1} & \cdots & A_{0,2M} \\ A_{1,0} & A_{1,1} & A_{1,2} & \cdots & A_{1,M} & A_{1,M+1} & \cdots & A_{1,2M} \\ A_{2,0} & A_{2,1} & A_{2,2} & \cdots & A_{2,M} & A_{2,M+1} & \cdots & A_{2,2M} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ A_{2M,0} & A_{2M,1} & A_{2M,2} & \cdots & A_{2M,M} & A_{2M,M+1} & \cdots & A_{2M,2M} \end{pmatrix}\right\}}_{2M+1} 2M+1 \tag{1.3}$$

In [1] a matrix A is square $(2M+1)\times(2M+1)$, but it can be possible to use matrices that are not square where: $n_1>n_2$, $n_1<n_2$, $n_1=n_2$. n_1 and n_2 are possible sizes of a matrix A:

$$A=\underbrace{\begin{pmatrix} A_{0,0} & A_{0,1} & A_{0,2} & \dots & A_{0,n_1-1} & A_{0,n_1} & \dots & A_{0,n_2-1} \\ A_{1,0} & A_{1,1} & A_{1,2} & \dots & A_{1,n_1-1} & A_{1,n_1} & \dots & A_{1,n_2-1} \\ A_{2,0} & A_{2,1} & A_{2,2} & \dots & A_{2,n_1-1} & A_{2,n_1} & \dots & A_{2,n_2-1} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ A_{n_1-1,0} & A_{n_1-1,1} & A_{n_1-1,2} & \dots & A_{n_1-1,n_1-1} & A_{n_1-1,n_1} & \dots & A_{n_1-1,n_2-1} \end{pmatrix}}_{n_2} \Bigg\} n_1 . \tag{1.4}$$

And a vector-column of free members can be:

$$B=\begin{vmatrix} Y(T_0) \\ Y(T_1) \\ Y(T_2) \\ Y(T_3) \\ \dots \\ Y(T_{n1}) \end{vmatrix} \begin{matrix} \leftarrow for\ T_0 \\ \leftarrow for\ T_1 \\ \leftarrow for\ T_2 \\ \leftarrow for\ T_3 \\ \dots \\ \leftarrow for\ T_{n_1} \end{matrix} \tag{1.5}$$

2 Introduction of SLAE matrices with the orthogonal decomposition

One of the most important variants of orthogonal decompositions is a so-called QR decomposition:

$$A = QR, \tag{2.1}$$

where A is a matrix from (1.1);
Q is an orthogonal matrix.

The matric R can be obtained by the formula:

$$R = Q^T \cdot A \tag{2.2}$$

The variables $N_1=n_1-1$, $N_2=n_2-1$, $N_3=|N_1-N_2|$ are introduced for convenience. The size of a QR decomposition matrix is: $N_1 \times (N_1+N_2)$ if $n_1>n_2$ or $N_1 \times (2 \cdot N_1+N_3)$ if $n_1<n_2$.

For a rectangular matrix with size $n_1 \times n_2$ where $n_1>n_2$, a QR decomposition matrix can be as shown in (2.3) or (2.4):

$$QR=\begin{pmatrix} Q_{0,0} & Q_{0,1} & Q_{0,2} & \dots & Q_{0,N_1} & R_{0,0} & R_{0,1} & R_{0,2} & \dots & R_{0,N_1} & E_{0,0} & E_{0,1} & E_{0,2} & \dots & E_{0,N_3-1} \\ Q_{1,0} & Q_{1,1} & Q_{1,2} & \dots & Q_{1,N_1} & 0 & R_{1,1} & R_{1,2} & \dots & R_{1,N_1} & E_{1,0} & E_{1,1} & E_{1,2} & \dots & E_{1,N_3-1} \\ Q_{2,0} & Q_{2,1} & Q_{2,2} & \dots & Q_{2,N_1} & 0 & 0 & R_{2,2} & \dots. & R_{2,N_1} & E_{2,0} & E_{2,1} & E_{2,2} & \dots & E_{2,N_3-1} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ Q_{N_1,0} & Q_{N_1,1} & Q_{N_1,2} & \dots & Q_{N_1,N_1} & 0 & 0 & 0 & \dots & R_{N_1,N_1} & E_{N_1,0} & E_{N_1,0} & E_{N_1,0} & \dots & E_{N1,N_3-1} \end{pmatrix} \tag{2.3}$$

$$QR = [Q(N_1 \times N_1) \quad R(N_1 \times N_1) \quad E(N_1 \times (N_1 - N_2 - 1))]. \tag{2.4}$$

If $n_1<n_2$, a QR decomposition matrix can be as shown in (2.5) or (2.6):

$$QR=\begin{pmatrix} Q_{0,0} & Q_{0,1} & Q_{0,2} & \dots & Q_{0,N_1} & R_{0,0} & R_{0,1} & R_{0,2} & \dots & R_{0,N_2} \\ Q_{1,0} & Q_{1,1} & Q_{1,2} & \dots & Q_{1,N_1} & 0 & R_{1,1} & R_{1,2} & \dots & R_{1,N_2} \\ \dots & \dots & \dots & \dots & \dots & 0 & 0 & R_{2,2} & \dots & R_{2,N_2} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & 0 & 0 & 0 & 0 & R_{N_2,N_2} \\ \dots & \dots & \dots & \dots & \dots. & 0 & 0 & 0 & 0 & 0 \\ Q_{N_1,0} & Q_{N_1,1} & Q_{N_1,2} & \dots. & Q_{N_1,N_1} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.5}$$

$$QR = \begin{bmatrix} Q(N_1 \times N_1) & R(N_2 \times N_2) \\ & 0(N_3 \times N_2) \end{bmatrix} \tag{2.6}$$

If $n_1=n_2$, then a QR decomposition matrix is:

$$QR = \begin{pmatrix} Q_{0,0} & Q_{0,1} & Q_{0,2} & ... & Q_{0,N_1} & R_{0,0} & R_{0,1} & R_{0,2} & ... & R_{0,N_1} \\ Q_{1,0} & Q_{1,1} & Q_{1,2} & ... & Q_{1,N_1} & 0 & R_{1,1} & R_{1,2} & ... & R_{1,N_1} \\ Q_{2,0} & Q_{2,1} & Q_{2,2} & ... & Q_{2,N_1} & 0 & 0 & R_{2,2} & ... & R_{2,N_1} \\ ... & ... & ... & ... & ... & ... & ... & ... & ... & ... \\ Q_{N_1,0} & Q_{N_1,1} & Q_{N_1,2} & ... & Q_{N_1,N_1} & 0 & 0 & 0 & ... & R_{N_1,N_1} \end{pmatrix} \tag{2.7}$$

$$QR = \left[Q(N_1 \times N_1) \quad R(N_1 \times N_1) \right] \tag{2.8}$$

Any SLAE can be represented by the following formulas (2.9), (2.10), (2.11) after obtaining a QR-decomposition matrix:

$$Q^T \cdot A = Q^T \cdot B \tag{2.9}$$

$$R = Q^T \cdot B . \tag{2.10}$$

$$R = QTB , \tag{2.11}$$

where $QTB = Q^T \cdot B$.

SLAE for (2.3) and (2.4) can be like (2.12):

$$\begin{aligned} &R_{0,0} \cdot x_0 + R_{0,1} \cdot x_1 + R_{0,2} \cdot x_2 + ... + R_{0,N_1} \cdot x_{N_1} = QTB_0 - E_{0,0} \cdot x_{N_1+1} - E_{0,1} \cdot x_{N_1+2} - E_{0,2} \cdot x_{N_1+3} - ... - E_{0,N_3-1} \cdot x_{N_1+N_3} \\ &R_{1,1} \cdot x_1 + R_{1,2} \cdot x_2 + ... + R_{1,N_1} \cdot x_{N_1} = QTB_1 - E_{1,0} \cdot x_{N_1+1} - E_{1,1} \cdot x_{N_1+2} - E_{1,2} \cdot x_{N_1+3} - ... - E_{1,N_3-1} \cdot x_{N_1+N_3} \\ &R_{2,2} \cdot x_2 + ... + R_{2,N_1} \cdot x_{N_1} = QTB_2 - E_{2,0} \cdot x_{N_1+1} - E_{2,1} \cdot x_{N_1+2} - E_{2,2} \cdot x_{N_1+3} - ... - E_{2,N3} \cdot x_{N_1+N_3} \\ &... \\ &R_{N_1,N_1} \cdot x_{N_1} = QTB_{N_1} - E_{N1,0} \cdot x_{N_1+1} - E_{N1,1} \cdot x_{N_1+2} - E_{N1,2} \cdot x_{N_1+3} - ... - E_{N1,N3} \cdot x_{N_1+N_3} \end{aligned}$$

. (2.12)

The norm of the solution vector looks like:

$$C = x_0^2 + x_1^2 + x_2^2 + ... + x_{N_1}^2 + x_{N_1+1}^2 + x_{N_1+2}^2 + ... + x_{N_1+N_3}^2 . \tag{2.12}$$

The partial derivatives are equated to zero for calculating the minimum of C (2.12):

$$\begin{aligned} &\frac{\partial C}{\partial x_{N_1+1}} = x_0 \cdot \frac{\partial x_0}{\partial x_{N_1+1}} + x_1 \cdot \frac{\partial x_1}{\partial x_{N_1+1}} + x_2 \cdot \frac{\partial x_2}{\partial x_{N_1+1}} + ... + x_{N_1} \cdot \frac{\partial x_{N_1}}{\partial x_{N_1+1}} + x_{N_1+1} = 0 \\ &\frac{\partial C}{\partial x_{N_2+2}} = x_0 \cdot \frac{\partial x_0}{\partial x_{N_2+2}} + x_1 \cdot \frac{\partial x_1}{\partial x_{N_2+2}} + x_2 \cdot \frac{\partial x_2}{\partial x_{N_2+2}} + ... + x_{N_1} \cdot \frac{\partial x_{N_1}}{\partial x_{N_1+2}} + x_{N_2+2} = 0 \\ &... \\ &\frac{\partial C}{\partial x_{N_1+N_3}} = x_0 \cdot \frac{\partial x_0}{\partial x_{N_1+N_3}} + x_1 \cdot \frac{\partial x_1}{\partial x_{N_1+N_3}} + x_2 \cdot \frac{\partial x_2}{\partial x_{N_1+N_3}} + ... + x_{N_1} \cdot \frac{\partial x_{N_1}}{\partial x_{N_1+N_3}} + x_{N_1+N_3} = 0 \end{aligned} \tag{2.13}$$

N_1 variables can be derived from (2.12):

$$x_{N_1}=\frac{QTB_{N_1}}{R_{N_1,N_1}}-\frac{E_{N1,0}\cdot x_{N_1+1}}{R_{N_1,N_1}}-\frac{E_{N1,1}\cdot x_{N_1+2}}{R_{N_1,N_1}}-\frac{E_{N1,2}\cdot x_{N_1+3}}{R_{N_1,N_1}}-...-\frac{E_{N_1,N_3-1}\cdot x_{N_1+N_3}}{R_{N_1,N_1}}$$

...

$$x_2=\frac{QTB_2}{R_{2,2}}-\frac{E_{2,0}\cdot x_{N_1+1}}{R_{2,2}}-\frac{E_{2,1}\cdot x_{N_1+2}}{R_{2,2}}-\frac{E_{2,2}\cdot x_{N_1+3}}{R_{2,2}}-...-\frac{E_{2,N_3-1}\cdot x_{N_1+N_3}}{R_{2,2}}-\frac{R_{2,N_1}x_{N_1}}{R_{2,2}}$$

$$x_1=\frac{QTB_1}{R_{1,1}}-\frac{E_{1,0}\cdot x_{N_1+1}}{R_{1,1}}-\frac{E_{1,1}\cdot x_{N_1+2}}{R_{1,1}}-\frac{E_{1,2}\cdot x_{N_1+3}}{R_{1,1}}-...-\frac{E_{1,N_3-1}\cdot x_{N_1+N_3}}{R_{1,1}}-\frac{R_{1,2}\cdot x_2}{R_{1,1}}-...-\frac{R_{1,N_1}\cdot x_{N_1}}{R_{1,1}}$$

$$x_0=\frac{QTB_0}{R_{0,0}}-\frac{E_{0,0}\cdot x_{N_1+1}}{R_{0,0}}-\frac{E_{0,1}\cdot x_{N_1+2}}{R_{0,0}}-\frac{E_{0,2}\cdot x_{N_1+3}}{R_{0,0}}-...-\frac{E_{0,N_3-1}\cdot x_{N_1+N_3}}{R_{0,0}}-\frac{R_{0,2}\cdot x_2}{R_{0,0}}-\frac{R_{0,1}\cdot x_1}{R_{0,0}}-...-\frac{R_{0,N_1}\cdot x_{N_1}}{R_{0,0}}$$

. (2.14)

Any lines of SLAE (2.14) can be represented like the expressions (2.15):

$$\begin{aligned}
&x_0=e_0+d_0\cdot x_{N_1+1}+c_0\cdot x_{N_1+2}+b_0\cdot x_{N_1+3}+...+a_0\cdot x_{N_1+N_3}\\
&x_1=e_1+d_1\cdot x_{N_1+1}+c_1\cdot x_{N_1+2}+b_1\cdot x_{N_1+3}+...+a_1\cdot x_{N_1+N_3}\\
&x_2=e_2+d_2\cdot x_{N_1+1}+c_2\cdot x_{N_1+2}+b_2\cdot x_{N_1+3}+...+a_2\cdot x_{N_1+N_3}\\
&....\\
&x_{N_1}=e_{N_1}+d_{N_1}\cdot x_{N_1+1}+c_{N_1}\cdot x_{N_1+2}+b_{N_1}\cdot x_{N_1+3}+...+a_{N_1}\cdot x_{N_1+N_3}
\end{aligned} \quad . \qquad (2.15)$$

Then the expressions (2.13) will take the following form:

$$\begin{aligned}
&\frac{\partial C}{\partial x_{N_1+1}}=x_0\cdot d_0+x_1\cdot d_1+x_2\cdot d_2+...+x_{N_1}\cdot d_{N_1}+x_{N_1+1}=0\\
&\frac{\partial C}{\partial x_{N_2+2}}=x_0\cdot c_0+x_1\cdot c_1+x_2\cdot c_2+...+x_{N_1}\cdot c_{N_1}+x_{N_1+2}=0\\
&...\\
&\frac{\partial C}{\partial x_{N_1+N_3}}=x_0\cdot a_0+x_1\cdot a_1+x_2\cdot a_2+...+x_{N_1}\cdot a_{N_1}+x_{N_1+N3}=0
\end{aligned} \quad . \qquad (2.16)$$

The SLAE (2.16) is a system with a square matrix. According to (2.7) and (2.8) such systems are solved by the method of “direct run” because it can be represented in the form of the triangular matrix R. It is possible to obtain N3 variables. N_1+1 variables are derived from (2.12).

3 An example of solving a SLAE by using QR decomposition matrices

For the demonstration of the solution, matrices A and B have been chosen:

$$A=\begin{bmatrix}1 & 20 & 2 & 3\\ 7 & 8 & 56 & 200\end{bmatrix},\ B=\begin{bmatrix}2\\ 24\end{bmatrix}.$$

The parameters of the A matrix $n_1=2$, $n_2=4$, $N_1=2-1=1$, $N_2=4-1=3$, $N3=N_1-N_2=2$. The size of the QR matrix A is $N_1\times(2\cdot N_1+N_3)=2\times(2\cdot 2+2)$ at 2<4:

$$AQR=\begin{pmatrix}0.141 & -0.99 & 7.071 & 10.748 & 55.72 & 198.414\\ 0.99 & 0.141 & 0 & -18.668 & 5.94 & 25.314\end{pmatrix}.$$

The expression (2.3) will be represented in the form:

$$AQR=\begin{bmatrix}Q_{0,0} & Q_{0,1} & R_{0,0} & R_{0,1} & E_{0,0} & E_{0,1}\\ Q_{1,0} & Q_{1,1} & 0 & R_{1,1} & E_{1,0} & E_{1,1}\end{bmatrix}$$

$$QTB = Q^T \cdot B = \begin{bmatrix} QTB_0 \\ QTB_1 \end{bmatrix}.$$

According to (2.14) it is possible to present:

$$R_{0,0} \cdot x_0 + R_{0,1} \cdot x_1 = QTB_0 - E_{0,0} \cdot x_2 - E_{0,1} \cdot x_3 \quad (3.1)$$

$$R_{1,1} \cdot x_1 = QTB_1 - E_{1,0} \cdot x_2 - E_{1,1} \cdot x_3 \quad (3.2)$$

$$x_1 = \frac{QTB_1}{R_{1,1}} - \frac{E_{1,0}}{R_{1,1}} \cdot x_2 - \frac{E_{1,1}}{R_{1,1}} \cdot x_3$$

$$x_0 = \frac{QTB_0}{R_{0,0}} - \frac{E_{0,0}}{R_{0,0}} \cdot x_2 - \frac{E_{0,1}}{R_{0,0}} \cdot x_3 - \frac{R_{0,1}}{R_{0,0}} \cdot x_1$$

$$x_0 = \frac{QTB_0}{R_{0,0}} - \frac{E_{0,0}}{R_{0,0}} \cdot x_2 - \frac{E_{0,1}}{R_{0,0}} \cdot x_3 - \frac{R_{0,1}}{R_{0,0}} \cdot \left(\frac{QTB_1}{R_{1,1}} - \frac{E_{1,0}}{R_{1,1}} \cdot x_2 - \frac{E_{1,1}}{R_{1,1}} \cdot x_3 \right)$$

$$x_0 = \left(\frac{QTB_0}{R_{0,0}} - \frac{R_{0,1}}{R_{0,0}} \cdot \frac{QTB_1}{R_{1,1}} \right) - \left(\frac{E_{0,0}}{R_{0,0}} - \frac{R_{0,1}}{R_{0,0}} \cdot \frac{E_{1,0}}{R_{1,1}} \right) \cdot x_2 - \left(\frac{E_{0,1}}{R_{0,0}} - \frac{R_{0,1}}{R_{0,0}} \cdot \frac{E_{1,1}}{R_{1,1}} \right) \cdot x_3 .$$

The variables x_1 and x_0 for the SLAE are possible to represent as in (2.3) and (2.4):

$$x_0 = c_0 + b_0 \cdot x_2 + a_0 \cdot x_3 , \quad (3.3)$$

where $c_0 = \frac{QTB_0}{R_{0,0}} - \frac{R_{0,1}}{R_{0,0}} \cdot \frac{QTB_1}{R_{1,1}}$, $b_0 = \frac{E_{0,0}}{R_{0,0}} - \frac{R_{0,1}}{R_{0,0}} \cdot \frac{E_{1,0}}{R_{1,1}}$, $a_0 = \frac{E_{0,1}}{R_{0,0}} - \frac{R_{0,1}}{R_{0,0}} \cdot \frac{E_{1,1}}{R_{1,1}}$.

$$x_1 = c_1 + b_1 \cdot x_2 + a_1 \cdot x_3 , \quad (3.4)$$

where $c_1 = \frac{QTB_1}{R_{1,1}}$, $b_1 = \frac{E_{1,0}}{R_{1,1}}$, $a_1 = \frac{E_{1,1}}{R_{1,1}}$.

The norm of the vector:

$$C = x_0^2 + x_1^2 + x_2^2 + x_3^2 . \quad (3.5)$$

The partial derivatives are zero for calculating the minimum of (3.5):

$$\frac{\partial C}{\partial x_2} = x_0 \cdot \frac{\partial x_0}{\partial x_2} + x_1 \cdot \frac{\partial x_1}{\partial x_2} + x_2 = 0 , \quad \frac{\partial C}{\partial x_3} = x_0 \cdot \frac{\partial x_0}{\partial x_3} + x_1 \cdot \frac{\partial x_1}{\partial x_3} + x_3 = 0 .$$

Another way:

$$\frac{\partial C}{\partial x_2} = x_0 \cdot b_0 + x_1 \cdot b_1 + x_2 = 0 , \quad \frac{\partial C}{\partial x_3} = x_0 \cdot a_0 + x_1 \cdot a_1 + x_3 = 0 . \quad (3.6)$$

$$\frac{\partial C}{\partial x_2} = -29.424 + 71.052 \cdot x_2 + 252.354 x_3 = 0 ,$$

$$\frac{\partial C}{\partial x_3} = -105.983 + 252.354 \cdot x_2 + 910.126 \cdot x_3 = 0 .$$

Or:

$$262.166 \cdot x_2 + 944.457 \cdot x_3 = 29.424$$

$3.781 \cdot x_3 = 0.401$, then: $x_2 = 0.107$, $x_3 = 0.034$.

Other variables can be found from the following expressions:

$$\begin{aligned} R_{0,0} \cdot x_0 + R_{0,1} \cdot x_1 &= QTB_0 - E_{0,0} \cdot x_3 - E_{0,1} \cdot x_4 \\ R_{0,1} \cdot x_1 &= QTB_1 - E_{1,0} \cdot x_3 - E_{1,1} \cdot x_4 \end{aligned} .$$

$$\begin{aligned} 7.071 \cdot x_0 + 10.748 \cdot x_1 &= 0.912 \\ -18.668 \cdot x_1 &= -1.497 \end{aligned}$$, then $x_1 = 0.08$, $x_0 = 0.007$.

4. The implementation of MathCad Prime for solving a SLAE by using QR decomposition matrices

The matrix A of the size 190×324 (Fig. 4.1), and the matrix B (Fig. 4.2) with 189 elements have been used for demonstration.

	0 …	181	182	183	184	185	186	187	188	189	… 323
0	⋱										
⋮											
110		−0.852	0.852	0.085	−0.928	0.751	0.252	−0.978	0.628	0.412	
111		−0.654	0.974	−0.252	−0.739	0.94	−0.135	−0.814	0.893	−0.017	
112		−0.396	0.996	−0.56	−0.458	1	−0.502	−0.517	0.999	−0.443	
113		−0.102	0.915	−0.804	−0.119	0.921	−0.794	−0.135	0.928	−0.783	
114		0.202	0.739	−0.956	0.236	0.716	−0.966	0.268	0.692	−0.974	
⋮											⋱
189											

Fig. 4.1 The matrix A for a SLAE

$$B = \begin{array}{r} 0 \\ 1 \\ 2 \\ 3 \\ \vdots \\ 189 \end{array} \begin{bmatrix} 0.783 \\ 1.002 \\ 0.498 \\ 0.068 \\ \vdots \end{bmatrix}$$

Fig. 4.2 The matrix B for a SLAE

(2.14) and (2.15) is presented in the block x (Fig. 4.3), where the matrix RTGR is the matrix R, described in the second part of the article. (2.16) is presented in the block C (Fig. 4.4). The method of “direct run” is presented in Fig. 4.5. The block of “direct run” implementation is shown in Fig. 4.6 and Fig. 4.7. The verification of obtained solution is shown in Fig. 4.8 where the result of multiplication AX_{qr} completely matches the matrix B.

$$
\begin{aligned}
x :=\ & x_{0,0} \leftarrow \frac{QTb_{N1}}{RTRG_{N1,N1}} \\
& \textbf{for } j \in 0..N3-1 \\
& \quad x_{0,j+1} \leftarrow \frac{-E_{N1,j}}{RTRG_{N1,N1}} \\
& \textbf{for } k \in 1..N1 \\
& \quad x_{k,0} \leftarrow \frac{QTb_{N1-k}}{RTRG_{N1-k,N1-k}} - \left(\sum_{h=0}^{k-1} \frac{RTRG_{N1-k,N1-h} \cdot x_{h,0}}{RTRG_{N1-k,N1-k}} \right) \\
& \quad \textbf{for } j \in 0..N3-1 \\
& \quad\quad x_{k,j+1} \leftarrow \frac{-E_{N1-k,j}}{RTRG_{N1-k,N1-k}} - \left(\sum_{h=0}^{k-1} \frac{RTRG_{N1-k,N1-h} \cdot x_{h,j+1}}{RTRG_{N1-k,N1-k}} \right) \\
& x
\end{aligned}
$$

Fig. 4.3 The implementation of x

$$
\begin{aligned}
&C := \text{for } z \in 0..N3-1\\
&\quad \text{for } j \in 0..N3\\
&\quad\quad \text{if } z+1 = j\\
&\quad\quad\quad C_{z,j} \leftarrow \sum_{i=0}^{N1} x_{i,j} \cdot x_{i,1+z} + 1\\
&\quad\quad \text{else}\\
&\quad\quad\quad C_{z,j} \leftarrow \sum_{i=0}^{N1} x_{i,j} \cdot x_{i,1+z}\\
&\quad C
\end{aligned}
$$

Fig. 4.4 The implementation of C.

$$
\begin{aligned}
trg(A,b) := \;& N \leftarrow \text{cols}(A)\\
& x_{N-1} \leftarrow \frac{b_{N-1}}{A_{N-1,N-1}}\\
& \text{for } i \in N-2..0\\
& \quad x_i \leftarrow \frac{1}{A_{i,i}} \cdot \left(b_i - \sum_{j=i+1}^{N-1} A_{i,j} \cdot x_j\right)\\
& x
\end{aligned}
$$

Fig.4.5 The implementation of the “direct run” method

$$
YQR := trg\left(CQR, -QQR^{\mathrm{T}}\ \text{submatrix}(C,0,133,0,0)\right) = \begin{bmatrix} 0.107 \\ -0.069 \\ 0.042 \\ 0.032 \\ \vdots \end{bmatrix}
$$

Fig. 4.6 The implementation of the “direct run” method

$$
Xqr := \text{stack}\left(trg\left(RTRG, QTb - \sum_{i=0}^{\text{rows}(YQR)-1} \text{submatrix}(E,0,N1,i,i) \cdot YQR_i\right), YQR\right) = \begin{bmatrix} 0.076 \\ -0.24 \\ 0.068 \\ -0.42 \\ \vdots \end{bmatrix}
$$

Fig. 4.7 The implementation of the “direct run” method for finding N1+1=190 variables

$$
\mathrm{A} \cdot Xqr = \begin{bmatrix} 0.783 \\ 1.002 \\ 0.498 \\ 0.068 \\ \vdots \end{bmatrix}
$$

Fig. 4.8 The verification of the solution, $AX_{qr}=B$

5. Conclusions

The possibility of a new method of improving the transmission of information and a clean separation of signal and noise is suggested by the discovery presented in [1]. The basis of the proposed new way of developing the theory and methods of communication is the rejection of the probabilistic method for evaluating noisy signals according to the maximum likelihood rule. This method contains the mathematical procedure for absolutely accurate separation, as well as proving the absence of any fundamental theoretical limits on the effectiveness of communications, including the absence of channel capacity limitations. This approach includes the fundamentally new concept and the technical aspects of implementing telecommunication systems and uses systems of linear algebra equations (SLAE) to filter signals from noise. The SLAE matrix is the linear algebraic matrix (LSM) that separates and extracts the true values of informative signal parameters. Such a SLAE always has the solution, but obtaining this solution sometimes requires a particular method to be used, because its matrix is not always square, but can be rectangular as well. Therefore, the method of the orthogonal decomposition is proposed in this paper. For obtaining the matrices of orthogonal decomposition the Gram–Schmidt process, which is suitable for matrices of any size and composition, can be used. The method of solving a SLAE includes full description of solution and acceptable for matrices of any size. The example of solving the SLAE with a small matrix is presented in the paper. The MathCad Prime has been implemented for a bigger matrix. The implementation includes the functions that can be used in any other programming language. The solution has minimal norm and acceptable for linear algebraic matrices that separate signal and noise.

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