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Damping of liquid sloshing in the tanks subjected to vertical acceleration by using the boundary element method

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The paper presents numerical simulations of liquid sloshing in the partially filled fuel tanks subjected to vertical acceleration. The tanks are considered to be shells of revolution, and a liquid inside the tank is supposed to be incompressible with viscosity effects being accounted for. The liquid motion is irrotational, and a velocity potential can be introduced. The boundary value problem is formulated for the Laplace's equation to obtain the velocity potential and the free surface level. Non-penetration boundary conditions are used at the wetted surface of a shell. The kinematic and dynamic boundary conditions are given on the free liquid surface. Effects of a surface tension are included into the Bernoulli equation as an additional pressure that is proportional to the mean curvature of the free surface. It allows considering coupled effects of both gravitational and capillarity waves. The boundary value problem is solved by using boundary element method. The system of the Mathieu equations is obtained and modified according to the damping effects. These effects are estimated, and stability regions on Ince-Strutt diagram are specified.

***Key words**: mathematical modeling, sloshing, boundary element method, free surface, gravitational waves, capillarity waves*.

Демпфування коливань рідини у резервуарах під дією вертикальної збурювальної сили з використанням методу граничних елементів

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У статті подано чисельне моделювання вимушених вертикальних коливань рідини у частково заповнених паливних баках. Метою даного дослідження є виконання числового аналізу рівня підйому рідини при врахуванні впливу демпфуючих факторів. Резервуари розглядаються як оболонки обертання, що заповнені нестисливою рідиною з урахуванням впливу в'язкості. Для оболонок обертання, заповнених рідиною, розглянуто зв'язану задачу. Рух рідини є безвихровим, і можна ввести потенціал швидкості. Сформульовано крайову задачу для рівняння Лапласа для отримання потенціалу швидкості та рівня вільної поверхні. На змоченій поверхні оболонки використовуються граничні умови непротікання, а також наведені кінематична та динамічна граничні умови на вільній поверхні рідини. Спочатку проведено аналіз впливу демпфування на різні рідини. Припускаємо, що поверхневий натяг не враховується. Отримано систему рівнянь Матьє з урахуванням демпфуючих ефектів. Дано оцінки цих ефектів, і вказані області стійкості на діаграмі Айнса-Стретта. Вивчення впливу поверхневого натягу виконуємо, використовуючи скінченні значення числа Бонда. Поверхневий натяг включаються в рівняння Бернуллі як додатковий тиск, пропорційний середній кривизні вільної поверхні. Це дозволяє враховувати як гравітаційні, так і капілярні явища при русі вільної поверхні рідини. Крайова задача розв'язується методом граничних елементів. Для дослідження явища плескань враховано різні значення числа Бонда для різних рідин. Оцінюються ефекти в'язкості та уточнюється їх вплив на стабільність руху. Було продемонстровано, що в зонах нестійкості діаграми Айнса-Стретта при врахуванні в'язкості з'являються невеликі зони стабільності руху та зони стійкості збільшуються зі збільшенням коефіцієнта демпфірування. Отримані числові дані вказують на те, що збільшення поверхневого натягу приводить до збільшення частоти коливань та зменшення амплітуди коливань.

Ключові слова: *математичне моделювання, плескання, метод граничних елементів, вільна поверхня, гравітаційні хвилі, капілярні хвилі.*

Демпфирование колебаний жидкости в резервуарах под действием вертикальной возмущающей силы с использованием метода граничных элементов

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В статье приведены результаты численного моделирования вынужденных вертикальных колебаний жидкости в частично заполненных топливных баках. Резервуары рассматриваются как оболочки вращения, заполненные несжимаемой жидкостью с учетом влияния вязкости. Движение жидкости является безвихревым, и можно ввести потенциал скорости. Сформулирована краевая задача для уравнения Лапласа для определения потенциала скорости и уровня свободной поверхности. На смоченной поверхности оболочки выполняются граничные условия непротекания, а также приведены кинематическое и динамическое граничные условия на свободной поверхности жидкости. Эффекты поверхностного натяжения включаются в уравнение Бернулли как дополнительное давление, пропорциональное средней кривизне свободной поверхности. Это позволяет учитывать как гравитационные, так и капиллярные явления при движении свободной поверхности жидкости. Краевая задача решается методом граничных элементов. Получена система уравнений Матье с учетом демпфирующих эффектов. Даны оценки этих эффектов и указаны области устойчивости на диаграмме Айнса-Стретта.

Ключевые слова: *математическое моделирование, плескания, метод граничных элементов, свободная поверхность, гравитационные волны, капиллярные волны*.

1 Introduction

The boundary elements methods (BEM) have been widely used in engineering in last decades, especially for solving the fluid-structure interaction (FSI) problems. The main advantage of using BEM is reducing dimensions of the problem without losing accuracy. So, for three-dimensional FSI problems we only have to discretize the boundary of the fluid volume. Now BEM is successfully applied to the multi-medium problems considering fluid-solid, fluid-structure-soil or air-fluid interactions. The special attention here should be paid to the boundary conditions on the interface surfaces. It is topical in aero-space applications, especially considering low-gravity conditions. There is a lot of research devoted to liquid sloshing in ground gravity conditions. The first simulations relied upon mechanical analogies of pendulums or springs to simulate the resultant dynamic pressure on reservoirs during sloshing are presented in [1, 2]. Comprehensive reviews of the sloshing phenomenon, including analytical and experimental research were done by Abramson [3] and Ibrahim [4]. The boundary conditions with the surface tension effect were obtained by Ibrahim in [4] and Myshkis et al in [5] by including the pressure changing across the free liquid surface described by the Laplace – Young equation. The sloshing motion that occurs in a low gravity differs drastically from the sloshing in the ground gravity [6]. It should be noted that in [4] Ibrahim concludes that exact solutions for the linear liquid sloshing are limited to tanks with straight walls (rectangular and upright-cylindrical containers). However, it is specified in [7], that it is difficult to directly apply computational fluid dynamics models to designing spacecraft attitude control devices because the analysis is very cumbersome and time-consuming. So, it is important to elaborate effective methods based on fast numerical procedures [8, 9]. Recently, a series of sloshing model tests for various viscous liquids have been elaborated, that allows investigating the effects of liquid viscosity on sloshing [10, 11]. The experimental results show that the liquid viscosity has an important effect on sloshing pressure [12, 13].

In this paper the boundary element method is applied to simulation of the liquid vibrations in rigid tanks, the effects of viscosity and capillarity being considered. The stability regions on the Ince- Strutt diagram are specified.

2 Problem formulation

Let us consider a shell of revolution partially filled with liquid (Fig. 2.1). Here S_1 is a wetted part of the shell, and S_0 is the liquid free surface. It should be noted that free surface S_0 represents the interface

surface between two mediums: water and air. It is considered as a thin membrane if effects of the surface tension are sufficient.

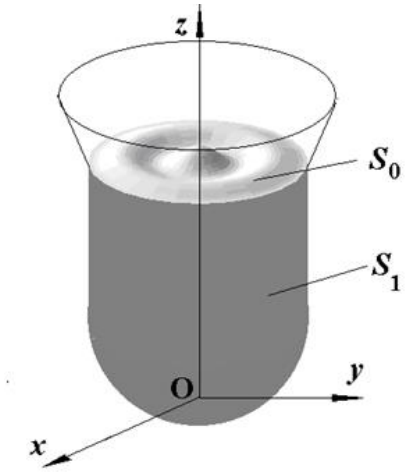

Fig.2.1 Fluid-filled shell of revolution

The fundamental equations of the liquid motion are following:

$$\frac{\partial \rho}{\partial t} = -\operatorname{div}(\rho \boldsymbol{V}_f) + Q, \quad \rho \frac{d\boldsymbol{V}_f}{dt} = \rho \boldsymbol{b} + \operatorname{div}(\boldsymbol{\sigma}_f). \tag{2.1}$$

Here ρ and $\boldsymbol{V}_f$ are the liquid density and velocity, Q is the mass sources, $\boldsymbol{b}$ is the vector of volume forces, and $\boldsymbol{\sigma}_f$ is the stress tensor. So

$$\sigma_f = -p\boldsymbol{I} + \boldsymbol{T}; \quad \boldsymbol{T} = 2\mu\left(\dot{\boldsymbol{S}} - \frac{1}{3}(\operatorname{div}\boldsymbol{V}_f)\right),$$

where p is the liquid pressure, $\boldsymbol{I}$ is the unit tensor, and $\dot{\boldsymbol{S}}$ is the stress deviator. Supposing the fluid flow is irrotational, we can set the velocity potential as $\boldsymbol{V}_f = \operatorname{grad}\varphi$.

As in [8], let us consider the following formulation for determining the velocity potential without including viscosity and capillarity effects

$$\Delta\phi = 0; \quad \left.\frac{\partial^2\phi}{\partial t^2}\right|_{S_0} + g\left.\frac{\partial\phi}{\partial n}\right|_{S_0} = 0; \quad \left.\frac{\partial\phi}{\partial n}\right|_{S_1} = 0; \quad \frac{\partial\zeta}{\partial t} = \left.\frac{\partial\phi}{\partial n}\right|_{S_0}$$

with the following additional condition

$$\iint_{S_0} \frac{\partial\phi}{\partial \boldsymbol{n}} dS_0 = 0.$$

Let σ be a surface tension. It may be included in the Bernoulli equation by using the Laplace-Young equation [14]

$$p_s = \sigma\kappa,$$

where κ is the surface curvature. The expression for κ can be linearized as follows [4]:

$$\kappa = -\Delta_s\zeta.$$

Here Δ_s is the surface Laplace operator, the function ζ describes the shape and location of the free liquid surface. So, the dynamical boundary condition on the free surface can be represented as

$$\frac{\partial\phi}{\partial t} + g\zeta - \frac{\sigma}{\rho_l}\Delta_s\zeta\Big|_{S_0} = 0. \tag{2.2}$$

The above formulation allows us to define the velocity potential and the free surface level without including the effects of viscosity [15].

2.1 Estimation of viscosity effects

The following presentations are applied for the velocity potential and the free surface level functions [14]

$$\varphi = \sum_{m=0}^{N} \cos m\,\theta \sum_{k=1}^{M} d_{mk}(t)\phi_{mk}(r,z),$$
$$\zeta = \frac{1}{g}\sum_{m=0}^{N} \cos m\theta \sum_{k=1}^{M} \chi_{mk}^2 c_{mk}(t)\phi_{mk}(r,0).$$

Here

$$d_{mk}(t) = \dot{c}_{mk}(t), \quad m = 0,1,\dots M, \quad k = 1,2,\dots N.$$

The functions $\phi_{mk}(r,z)$ are obtained by using BEM described in [16]. These functions are the solutions of the following boundary value problems (supposing for simplicity $\phi_{mk}(r,z) = \psi(r,z)$

$$\nabla^2\psi = 0,\ \frac{\partial\psi}{\partial n}\Big|_{S_1} = 0,\ \frac{\partial\psi}{\partial n} = \frac{\chi^2}{g}\psi\Big|_{S_0},\ \iint_{S_0}\frac{\partial\psi}{\partial n}dS_0 = 0. \tag{2.3}$$

The equations (2.3) are reduced to the system of integral equations in the following form:

$$\begin{cases} 2\pi\psi_1 + \iint_{S_1}\psi_1\frac{\partial}{\partial n}\left(\frac{1}{|P-P_0|}\right)dS_1 - \frac{\chi^2}{g}\iint_{S_0}\psi_0\frac{1}{|P-P_0|}dS_0 + \iint_{S_0}\psi_0\frac{\partial}{\partial z}\left(\frac{1}{|P-P_0|}\right)dS_0 = 0, \\ -\iint_{S_1}\psi_1\frac{\partial}{\partial n}\left(\frac{1}{|P-P_0|}\right)dS_1 - 2\pi\psi_0 + \frac{\chi^2}{g}\iint_{S_0}\psi_0\frac{1}{|P-P_0|}dS_0 = 0, \end{cases} \tag{2.4}$$

where $|P-P_0|$ is the Cartesian distance between points P and P_0. In the equations (2.4) we denote by ψ_0 the values of potential ψ in the nodes of the free surface S_0, and by ψ_1 its values in the nodes of the wetted surfaces S_1 of the shell. For the numerical implementation of the equations (2.4) the methods have been developed in [17, 18].

After obtaining functions $\psi_{mk}(r,z)$ and the frequencies χ_{mk} from [14] the next differential equations are formed for evaluating the time-dependent coefficients $c_{mk}(t)$

$$\ddot{c}_{mk}(t) + \chi_{mk}^2\left(1 + \frac{a_z(t)}{g}\right)c_{mk}(t) = 0,\quad m = 0,1,\dots M,\ k = 1,2,\dots N. \tag{2.5}$$

Here g is the gravity acceleration, χ_{mk} are sloshing frequencies.

It should be noted that the equations (2.5) are the Mathieu's equations. Therefore, we suppose that

$$\mathrm{g}_z(t) = a_z\cos\omega_z t\,. \tag{2.6}$$

According to [19], the equations (2.5) are modified to calculate the viscosity and gravity effects as

$$\ddot{c}_{mk}(t) + \gamma\dot{c}_{mk}(t) + \omega_{mk}^2\left(1 + \frac{a_z(t)}{\mathrm{g}_0}\right)c_{mk}(t) = 0,\quad m = 0,1,\dots M,\ k = 1,2,\dots N. \tag{2.7}$$

Here

$$\gamma = 4{,}47\sqrt{\frac{\nu}{\omega_{11}{R_0}^2}},\ {\omega_{mk}}^2 = \mathrm{g}_0\chi_{mk}^2 + \frac{\sigma\chi_{mk}^3}{\rho}. \tag{2.8}$$

It should be noted that the equations (2.7) are so called the damped Mathieu equations [20]. For their numerical implementation the Runge-Kutta 4 order method is used.

3 Numerical simulation

Let us consider a cylindrical tank with the following geometrics: the filling level H=1m, the shell radius R=1m. Taking into account different values of the Bond number, i.e., studying the effect of damping on a liquid at different levels of gravity. First, consider the case when $B_0 = \infty$, so we neglect the surface tension forces. In the equation (2.8) we have the following expression for the frequency: ${\omega_0}^2 = \mathrm{g}_0\chi_{11}^2$. Let us consider the vibrations corresponding to the first harmonic. The values of the characteristic wave numbers are given in Table 1.

Table 1. Characteristic wave numbers for the 1st harmonic

n	Characteristic values χ_{11}
1	1.841183781
2	5.331442774
3	8.536316366
4	11.70600490
5	14.86358863

We consider various variable components of the residual acceleration $\mathrm{g}_z(t)$, but we believe that this acceleration is harmonious.

In Table 2 the values of kinematic viscosity for some typical liquids, such as water, nitric acid, kerosene, hexane, and nitrogen tetraoxid are shown.

Table 2. Kinematic viscosity value

№	Fluid	Kinematic viscosity, ν $10^{-6}\times m^2/sec.$	Temperature, C°
1	Water	1,13	15,6
		0,55	54,4
2	Nitric acid 25 %	0,6957	20,0
	Nitric acid 100 %	0,4934	20,0
3	Kerosene	1,2-4,5 (2,71)	20,0
4	Hexane	0,683	-17,8
		0,401	37,8
5	Nitrogen tetraoxide	0,2917	20,0

The surface tension values for typical liquids at various Bond numbers $B_0 = 1, B_0 = 2, B_0 = 3,\ B_0 = 4, B_0 = 5$ are presented in Table 3.

Table 3. Surface tension values

№	Fluid	Density, kg/ m3	Surface tension, H/m				
			Bond number, B_0				
			1	2	3	4	5
1	Water	1000	100,00	50,00	33,33	25,00	20,00
		986	98,60	49,30	32,87	24,65	19,72
2	Nitric acid 25 %	1150	115,00	57,50	38,33	28,75	23,00
3	Nitric acid 100 %	1520	152,00	76,00	50,67	38,00	30,40
4	Kerosene	850	85,00	42,50	28,33	21,25	17,00
5	Hexane	684	68,40	34,20	22,80	17,10	13,68
		643	64,30	32,15	21,43	16,08	12,86
6	Nitrogen tetraoxide	1440	144,00	72,00	48,00	36,00	28,80

We are interested in the damping effect during vibrations with a frequency ω_z, which approaches to ω_{11} or a double value $2\omega_{11}$.

First, the effect of damping on various fluids is analyzed, the surface tension not being taken into account. The frequency $\omega_z = 3$Hz has been chosen for forced vibrations, and it is not close to either ω_{11} or the double value $2\omega_{11}$.

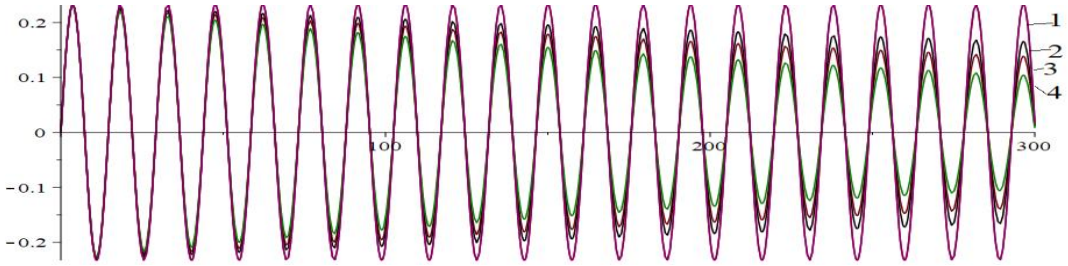

Fig.3.1 Influence of damping on vibration amplitudes for various liquids

Figure 3.1 shows the amplitudes of fluid vibrations under the following conditions. It is assumed that a_z in the formula (2.6) is equal to a_z =0.01, $\omega_z = 3$Hz, $g_0 = 0.1$g.

Number 1 indicates the graph of the function in the absence of damping, namely, at γ = 0, figures 2-4 refer to nitric acid (100%), water and kerosene respectively (the value of kinematic viscosity 2.71 $10^{-6}\times m^2/sec$ is taken).

Further, taking into account the kinematic viscosity, we will consider kerosene, since the damping effect is the most prominent in that case,.

Let us consider forced vibrations with the frequency $\omega_z = 1.8412\text{Hz}$, at $g_0 = 0.1g$ and at different a_z. As could be seen from Table 1, this frequency is practically equal to the fundamental frequency of fluid vibrations.

Figure 3.2 shows the time dependences for the coefficient $c_{11}(t)$ in the expansion for the function ζ.

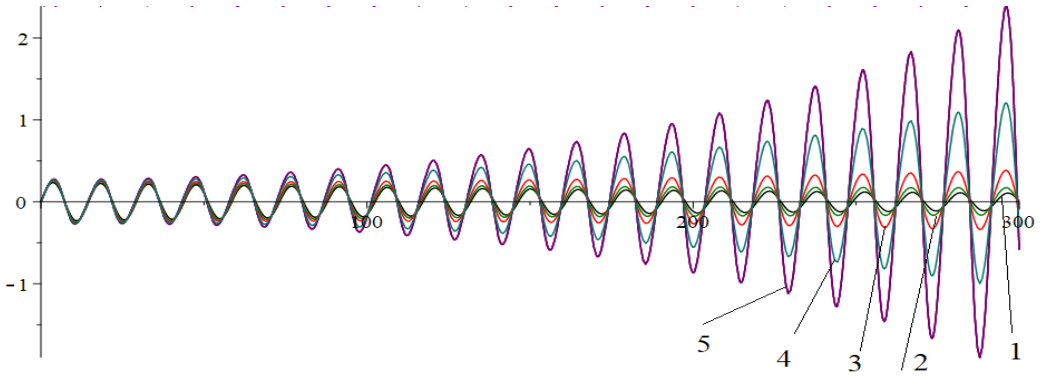

Fig.3.2 Time dependence of $c_{11}(t)$ for different a_z at $\omega_z = 1.8412Hz$

Here numbers 1-5 correspond to the values $a_z = 0.01, 0.05, 0.07, 0.09, 0.1$. In contrast to the Mathieu equation without damping, there is a zone of stability in the first unstable region (Fig. 3.3).

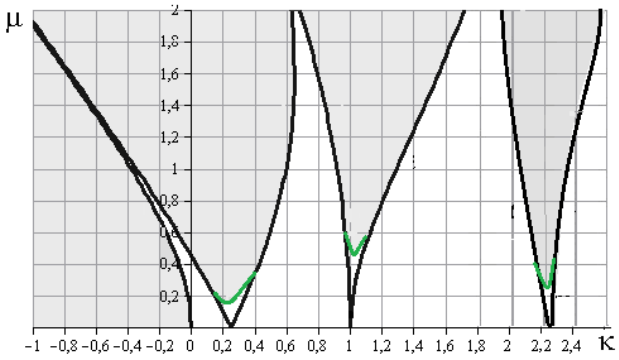

Fig.3.3 Ince-Strutt diagram

It should be noted that in zones of instability of the Ince-Strutt diagram small zones of motion stability appear, when viscosity is taken into account. According to [18], the stability zones increase with the increasing damping coefficient.

A similar pattern is observed with forced vibrations of a fluid with a frequency equal to twice the fundamental. The calculation data are shown in Fig. 3.4.

Here the numbers 1-4 correspond to the values $a_z = 0.01,\ 0.0075,\ 0.005,\ 0.004$. In the case of parametric resonance, only the motions at $a_z \leq 0.004$ will be stable in the second zone (Fig. 3.3).

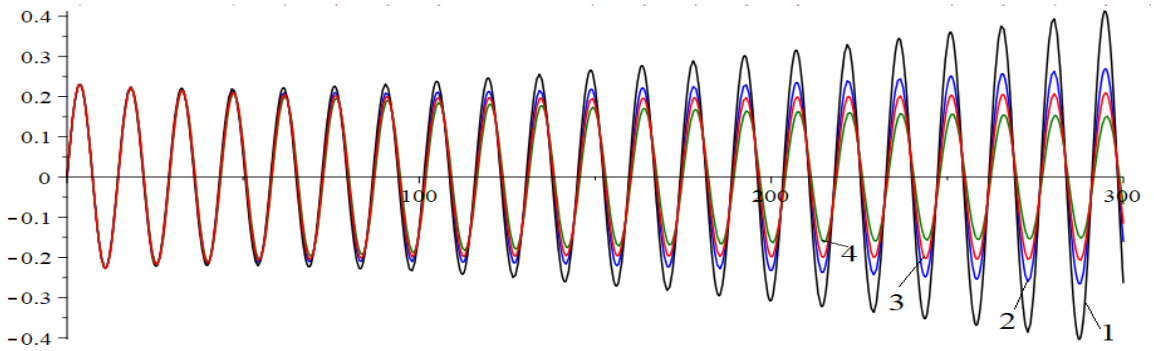

Fig. 3.4 Time dependence of $c_{11}(t)$ for different a_z at $\omega_z = 3.6824Hz$

Next, we study the effect of surface tension, namely, we move to the finite values of the Bond number. In this case, the frequency values are determined by the formula

$$\omega_{mn}^{*2} = \frac{\omega_{mn}^2}{\mathrm{g/R_2}} = k_{mn}\left(1 + \frac{k_{mn}^2}{B_0}\right)\tan\left(k_{mn}\frac{h}{\mathrm{R_2}}\right) \qquad (3.1)$$

Figure 3.5 shows the values of the free surface amplitudes for kerosene at various Bond numbers, namely, $B_0 = 1, B_0 = 2, B_0 = 3, B_0 = 4, B_0 = 5,\ B_0 = \infty$. We investigate the movements caused by the external load with a frequency of $\omega_z = 3\mathrm{Hz}$.

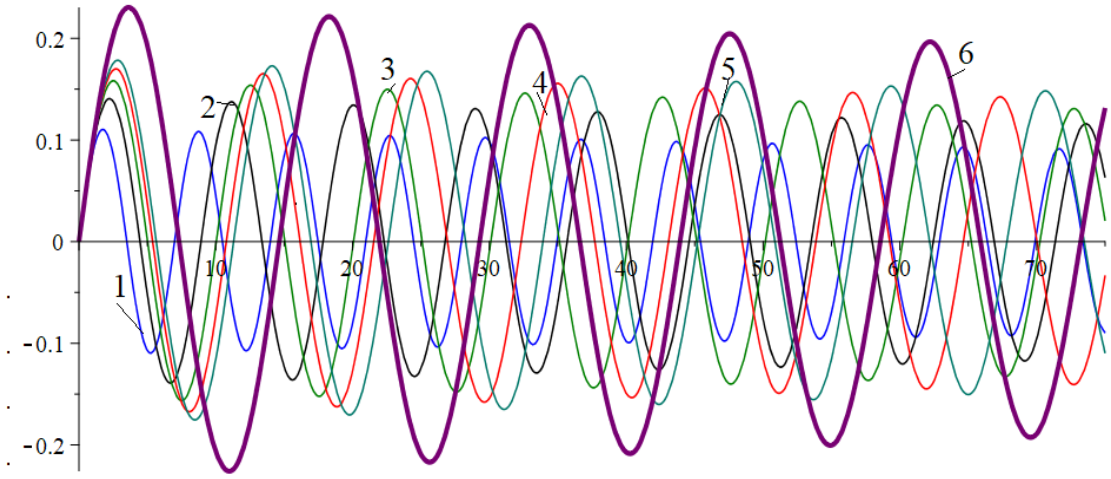

Fig. 3.5 Time dependence of $c_{11}(t)$ for various Bond numbers at $\omega_z = 3.0\ Hz$

Here the numbers 1-6 correspond to the following values of the Bond number: $B_0 = 1, B_0 = 2, B_0 = 3, B_0 = 4, B_0 = 5,\ B_0 = \infty$. From the numerical data it can be concluded that with the increasing surface tension, the vibration frequencies increase and the vibration amplitudes decrease.

4 Conclusions

The boundary element method is applied to numerical estimations of the surface tension and viscosity effects on liquid vibrations in rigid shells of revolution partially filled with the liquid. For the velocity potential the boundary value problem is formulated with modified boundary conditions on the free surface, taking into account the surface tension. The basic functions corresponding to the problem of free surface vibrations have been obtained without considering the surface tension. Then the modified boundary condition on the free surface has been used for obtaining the unknown coefficients. Various Bond's numbers for liquid sloshing of different liquids have been considered. The viscosity effects have been estimated, and their influence on stability of motion has been specified. It has been demonstrated that in the zones of instability of the Ince-Strutt diagram small zones of motion stability appear, when viscosity is taken into account, and the stability zones increase with increasing damping coefficient.

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