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Evolution of memes

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The paper considers the evolution of a population of individuals, where each one initially possesses a certain number of strategies the memory of which does not exceed a depth of 2. All individuals randomly enter into competition in pairs at each stage of evolution. A random pair of individuals conducts a competition between pairs of all their randomly selected strategies when they are interacting. These strategies compete in pairs according to the iterated prisoner's dilemma. In such struggle, strategies earn evolutionary advantage points according to a given payout matrix. The strategy with the most points wins. Two strategies come into this game twice to negate an impact of the first move. The first game starts by one strategy, the second game starts by another one. The winnings are determined by the outcome of both these games. After this competition the winning strategy of one individual replaces the corresponding losing strategy of another individual. Thus, there is an exchange of more "successful" strategies between individuals with the loss of lost strategies. The evolution of the population of such individuals was carried out until the stage of stationary state. There were established patterns of changes in basic properties of strategies of average individual during evolution. It is shown that in the process of evolution the aggression of an individual increases, tenting to the maximum value. The stationary set of strategies of an individual consists of strategies of maximum memory depth and complexity with a certain number of primitive strategies. The complexity and memory depth of an individual's strategies turns out to be evolutionary beneficial. In the stationary state the number of primitive strategies in an individual depends on their initial distribution to individuals. The paper considers two initial distributions, where the first corresponds to the equal probability of any strategy in the distribution by individuals, and the seconds corresponds to equally probable choice in terms of memory depth. The variety of strategies in the process of evolution decreases significantly, making up only a small part of the initial strategies present in the population.

Keywords: *evolution of population, strategies with memory, memory depth, complexity, aggression, prisoner's dilemma, stationary state.*

Еволюція мемів

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В роботі розглянуто еволюцію популяції особин, кожна з яких спочатку має певну кількість стратегій, пам'ять яких не перевищує глибини 2. На кожному етапі еволюції всі особини випадковим чином вступають попарно в конкурентну боротьбу. Випадкова пара особин при взаємодії проводить змагання між парами всіх своїх випадково обраних стратегій. Ці стратегії попарно вступають у змагання відповідно до ітерованої дилеми в'язнів. У такій боротьбі стратегії набирають очки еволюційних переваг відповідно заданої матриці виплат. Виграє стратегія, яка набрала найбільшу кількість очок. Для нівелювання значення першого ходу дві стратегії вступають в таку гру двічі. Спочатку гру починає одна стратегія, в другій грі починає інша стратегія. Виграш визначається за результатом цих двох ігор. Після проведення такого змагання стратегія однієї особини, яка виграла, замінює відповідну стратегію, яка програла, іншої особини. Таким чином, між особинами відбувається обмін більш "процвітаючими" стратегіями з втратою тих, хто програв. Проведена еволюція популяції таких особин до етапу встановлення стаціонарного стану особин популяції. Під час еволюції встановлені закономірності зміни основних властивостей стратегій середньої особини. Показано, що в процесі еволюції агресивність особини зростає, прагнучи до максимального значення. Кількість стратегій особини зменшується, досягаючи деякого стаціонарного значення. Стаціонарний набір стратегій особини складають стратегії максимальної пам'яті та складності з деякою кількістю примітивних стратегій. Складність і глибина пам'яті стратегій особини виявляються еволюційно вигідними. Кількість примітивних стратегій у особини в стаціонарі залежить від їх початкового розподілу по особинам. У роботі розглянуті два початкових розподіли: один відповідає рівноймовірності будь-якої стратегії при розподілі по особинам, другий відповідає рівноймовірному вибору за глибиною пам'яті стратегій. Різноманітність стратегій в процесі еволюції зменшується значно, складаючи лише малу частину присутніх в популяції початкових стратегій.

Ключові слова: *еволюція популяції, стратегії з пам'яттю, глибина пам'яті, складність, агресивність, дилема ув'язнених, стаціонарний стан.*

Эволюция мемов

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В работе рассмотрена эволюция популяции особей, каждая из которых изначально обладает определенным числом стратегий, память которых не превосходит глубины 2. На каждом этапе эволюции все особи случайным образом вступают попарно в конкурентную борьбу. Случайная пара особей при взаимодействии проводит соревнование между парами всех своих случайно выбранных стратегий. Эти стратегии попарно вступают в соревнование в соответствии с итерированной дилеммой заключенных. В такой борьбе стратегии набирают очки эволюционных преимуществ в соответствии с заданной матрицей выплат. Выигрывает стратегия, набравшая больше очков. Для нивелирования значения первого хода две стратегии вступают в такую игру дважды. Сначала игру начинает одна стратегия, во второй игре начинает другая стратегия. Выигрыш определяется по результату этих двух игр. После проведения такого соревнования выигравшая стратегия одной особи заменяет соответствующую проигравшую стратегию другой особи. Таким образом, между особями происходит обмен более "преуспевающими" стратегиями с потерей проигравших. Проведена эволюция популяции таких особей до этапа установления стационарного состояния особей популяции. Во время эволюции установлены закономерности изменения основных свойств стратегий средней особи. Показано, что в процессе эволюции агрессивность особи возрастает, стремясь к максимальному значению. Число стратегий особи уменьшается, достигая некоторого стационарного значения. Стационарный набор стратегий особи составляют стратегии максимальной памяти и сложности с некоторым количеством примитивных стратегий. Сложность и глубина памяти стратегий особи оказываются эволюционно выгодными. Количество примитивных стратегий у особи в стационаре зависит от их первоначального распределения по особям. В работе рассмотрены два начальных распределения: одно соответствует равновероятности любой стратегии при распределении по особям, второе соответствует равновероятному выбору по глубине памяти. Разнообразие стратегий в процессе эволюции уменьшается значительно, составляя только малую часть присутствующих в популяции начальных стратегий.

Ключевые слова: *эволюция популяции, стратегии с памятью, глубина памяти, сложность, агрессивность, дилемма заключенных, стационарное состояние.*

1 Introduction

Nowadays, the evolution of different populations and communities is the subject of interest and active research. Interest in their evolution is closely related to changes in the populations' properties that emerge during the evolution process. The emergence of emergent properties of the population is of particular interest. It should be noted that this method of changing the properties of a population is also useful for artificial populations of various swarm and multi-agent systems. One of the properties that have been attracting attention for a long time is the emergence of cooperative behavior in different populations [1]. Elements of game theory are widely used in modeling the evolution of populations (examples [2,3]). In a result of the conducted researches some mechanisms were found that lead to cooperation in a wide variety of systems (f. e. [4]). Among such mechanisms should be noted: voluntary participation [5], punishment [6], similarity [7], heterogeneous activity [8], social diversity [9, 10], dynamical linking [11], asymmetric interaction and permutation graphs [12], migration [13-15], in-group favoritism [16], interdependent relations [17]. Of course, in a similar way it is possible to find mechanisms of emergence and other properties of populations in the process of evolution.

Memory is one of the most general properties of individuals in a population, so its influence on the evolution of populations is of great interest.The papers [18,19] considers the evolution of a population of strategies limited by memory depth within the scope of generalized prisoner's dilemma. The concept of strategy's complexity is also introduced. Every new generation of strategies loses the most disadvantageous behavior strategies of the previous generation. There is shown that an increase in memory and complexity of population strategies is evolutionary beneficial. Evolutionary selection winners invariably come across strategies with maximum memory and close to maximum complexity.

The paper [20] considers the evolution of a population of strategies with memory in the presence of sources of strategies of different memory depth. There is shown that memory depth and complexity of strategies are evolutionarily advantageous too. In all cases the strategies' aggression decreases, and at the stationary stage all strategies have zero aggression towards each other. This feature can be used as a principle for selecting strategies for possible stationary states. Also, a relation was found between aggression and amount of points received per strategy's move when strategies interact. The higher the aggression, the fever evolutionary advantage points are received by the strategy per move.

The paper [21] considers the alternative evolution of a population of strategies with memory. In such evolution from every generation removes a strategy with the highest amount of evolutionary advantage points. This alternative evolution leads to significant changes in the strategies of the population in comparison to its usual evolution. In a sense, the alternative evolution supports maximum memory depth and complexity even more than regular evolution. However, the major difference lies in the absolute aggression of stationary strategies to each other. The stationary state is formed by the most aggressive strategies.

This paper considers a population of individuals where each has a certain number of strategies with memory. A principle for the exchange of strategies between individuals is proposed, and the evolution of such population is considered. When individuals interact, all their strategies come into competition in pairs according to the iterated prisoner's dilemma. The winning strategy replaces the corresponding losing strategy of the individual. It is shown that in the process of the evolution the aggression of an individual increases tending to the maximum value. The behavior of the distribution of strategies of an individual was studied in details in terms of memory depth and complexity in the process of evolution. The number of individual's strategies decreases reaching a certain stationary value. The stationary set of strategies of an individual consists of strategies of maximum memory and complexity with a certain number of primitive strategies. The variety of strategies in the process of evolution decreases significantly, making up only a small part of the initial strategies present in the population.

2 Population of individuals

In the considering population each individual has a finite set of strategies with memory. To carry out evolution, it is essential to introduce the rules of strategies' meetings, interactions and selection. In the following population at each stage of evolution all individuals are randomly divided into pairs that interact. All strategies of one individual interact with the strategies of another (also by random division into pairs) according to the iterated prisoner's dilemma. To negate the impact of the first move, both strategies are joining the game twice. Initially one strategy starts the game, and another starts the second game. The winnings are determined by the results of these two games. The winning strategy in a pair struggle of strategies replaces the losing strategy of the corresponding individual. In other words, there is an exchange of strategies between individuals with the removal of the loser. After the interactions of all pairs of individuals in accordance with the described rules, the next stage of evolution comes. Again, all individuals are randomly dividing into pairs that interact. In some abstract sense it resembles the evolution of ideas or memes[1] in different societies. The evolution stops when the population reaches a

[1]ThistermwasintroducedbyRichardDawkinsin 1976 tonameaunitofrelevant information within a culture. Amemeisanidea, symbol, manner, ora way of acting, that consciously or unconsciously can be transmitted from one person to another through speech, writing, video, rituals, gestures, etc. In some sense, a meme is similar to a gene as a carrier of cultural information.

stationary state. During the process of evolution, we will monitor the properties of the population's strategies and all individuals included in the population.

Now, let us discuss the choice of the initial distribution of strategies over individuals of the population. The full number of strategies with memory depth 2 is distributed among individuals. The process of distribution can be implemented in different ways. This paper considers two ways of strategies distribution. In the first method, we endow an individual with strategies by choosing them randomly from the set of all strategies with the memory depth not exceeding 2, assuming that strategies are equally probable. This choice does not take into account the properties of strategies, assuming them to be equal. In the second method, we choose strategies for an individual by a uniform distribution over memory. This means all strategies are divided into three sets by memory depth and the choice is equally probable between these sets. Below we will consider the described two methods of forming the initial distribution of strategies among individuals of the population.

Another feature of the considered population of individuals is the emergence of collective variables to describe individuals and not just the population of strategies. Each individual has a distribution of strategies in terms of memory and complexity, and the entire individual can show a definite aggression. Thus, there are properties of an individual, such as distribution of strategies by memory depth and complexity. These distributions define the average memory depth, complexity, aggression of i individual and its average aggression (Formula 2.1).

$$\langle M\rangle_i = \sum_{j=1}^{N_i} \frac{M_{ij}}{N_i}, \langle C\rangle_i = \sum_{j=1}^{N_i} \frac{C_{ij}}{N_i}, \langle A\rangle_i = \sum_{j=1}^{N_i} \frac{a_{ij}}{N_i} \tag{2.1}$$

There are $\langle M\rangle_i$, $\langle C\rangle_i$, $\langle A\rangle_i$ is the average memory, the average complexity, the average aggression respectively of the i individual; N_i is the number of strategies in the i individual; M_{ij} is the memory depth of the j strategy in the i individual; C_{ij} is the complexity of the j strategy in the i individual; a_{ij} is the aggression of the j strategy in the i individual.

These local characteristics can be used to track changes in the properties of individuals in a population during evolution. In addition, global characteristics of the population appear, such as the average values of memory, complexity and aggression of individuals of the whole population (Formula 2.2).

$$\overline{M} = \sum_{i=1}^{S} \frac{\langle M\rangle_i}{S}, \overline{C} = \sum_{i=1}^{S} \frac{\langle C\rangle_i}{S}, \overline{A} = \sum_{i=1}^{S} \frac{\langle A\rangle_i}{S} \tag{2.2}$$

where S is the number of individuals in the population.

These characteristics are calculated by averaging over all individuals in the population and determine the typical characteristics of an individual in the population. One more useful characteristic that determines the variety of strategies in the population is the number of different strategies in the population which will be called as formative strategies. By the formative strategies we imply the number of different strategies present in a population at some stage of evolution. Accordingly, we use Formula 2.3 as characteristics of the formative strategies in the population.

$$\langle M\rangle = \sum_{i=1}^{N} \frac{M_i}{N}, \langle C\rangle = \sum_{i=1}^{N} \frac{C_i}{N}, \langle A\rangle = \sum_{i=1}^{N} \frac{a_i}{N} \tag{2.3}$$

There are N is the number of formative strategies; $\langle M\rangle$, $\langle C\rangle$, and $\langle A\rangle$ are the average memory, complexity, aggression respectively of the formative strategy.

3 Evolution of individuals with a uniform distribution of initial strategies

We are considering the population of 50000 individuals, where each one has 50 strategies. The ratio between these values is chosen in a way all strategies that not exceed memory depth of 2 are present among the individuals of the population. The choice of these strategies is carried out in an equally probable manner from the set of all strategies with the memory depth not exceeding 2. In this case, in the individuals of the population, strategies with memory 2 dominate in accordance with the largest number of such strategies in the initial set of strategies (32640) and a significantly smaller number of strategies with the memory depth of 1 (120 strategies) and a very small number of strategies with the memory depth of 0 (8 strategies). The initial distribution of strategies in the individuals is shown in Fig.3.1 and inherits the distribution properties of all strategies. The overwhelming number of strategies with the memory depth of 2 and maximum complexity is typical for the initial distribution of strategies in an individual (see Fig.3.1), and it is the consequence of the used rule for selecting strategies for an individual. Further, a calculation of the properties of individuals was carried out by averaging over 10 implementations of the initial state and their evolution.

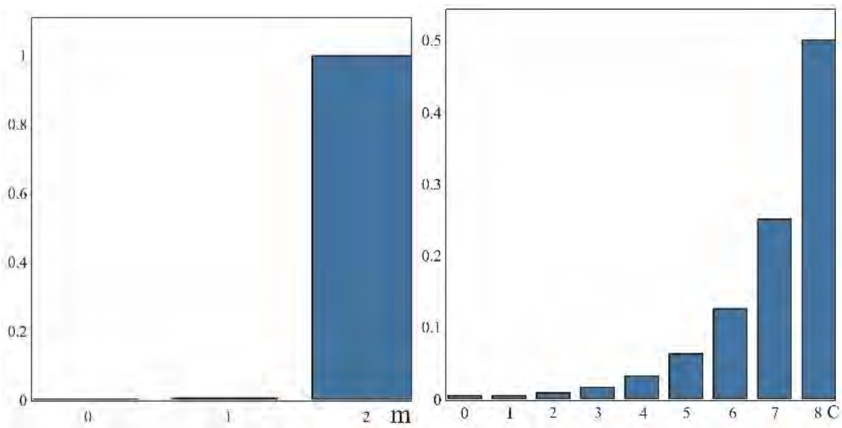

Fig.3.1 In an average individual at the beginning of evolution, the share of strategies of a certain memory depth is to the left, and the share of strategies of varying complexity is to the right. The same histograms determine the probability of detecting a strategy of a certain memory depth (to the left) and complexity (to the right) in an individual at the initial stage of evolution

Let us discuss the initial distribution of aggression which is formed in the individuals of the population by the mentioned initial distribution of strategies. To do this we need to proceed with the first stage of evolution to determine the initial aggression of the individuals in the population. At the first stage of evolution, all individuals are randomly divided into pairs that interact. All strategies of one individual are joining the interaction with the strategies of another, also by dividing strategies randomly into pairs, in accordance with the iterated prisoner's dilemma (the number of moves is 100). They interact twice: in the first meeting the first move starts by the first strategy, and the second meeting starts by the second strategy. This method of competing between two strategies negates the impact of the first move. To identify the winning strategy there applied the payoff matrix [3, 18-20] as shown in the Table 1.

Table 1. Payoff matrix

	Cooperation	Rejection
Cooperation	3,3	0,5
Rejection	5,0	1,1

The winning strategy, or the one scored the highest, replaces the losing strategy of the corresponding individual in a pair struggle of strategies. After the first stage, aggression shown by an individual (or the relative number of refusals to cooperate) is established.

Figure 3.2 shows the distribution of the initial aggression of the individuals in terms of memory and complexity. It is simple to see that average aggression of the individuals in the population is close to 0.5. At the same time, strategies with different memory depth shows approximately the same aggression. The aggression of strategies with zero complexity is maximum, and with the complexity 1 is minimum; strategies of greater complexity have the same aggression (see Fig.3.2). Therefore, the aggression of primitive strategies with the memory depth 0 and the complexity 0 is maximal.

Further evolution makes it possible to establish changes in the basic characteristics of individuals.

First of all, the number of strategies in every individual decreases. Figure 3.3 shows the average decrease in the number of strategies in an individual. In the process of evolution, after the stage of exponential decrease in numbers, it reaches a stationary state.This way individuals develop a stationary set of strategies. With such a ratio of individuals and their strategies the stationary stage is achieved on average at $N_s = 45.5$.

Let us consider how the distribution of strategies by memory depth of an individual changes over time. Figure 3.4 shows the evolution in strategies' distribution by the memory. As a convenient characteristic for this, we use the probability of finding in an individual a strategy with the memory depth $m = 0, 1, 2$ respectively.

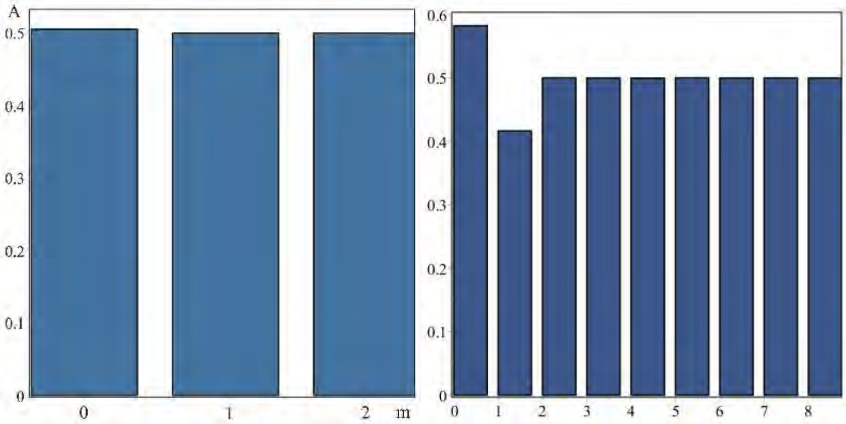

Fig.3.2 The initial distribution of an individual's aggression in the population by memory depth (to the left) and complexity (to the right) calculated at the first step of evolution

The probability of detecting strategies with the memory depth of 0 and 1 in an individual increased up to tenth step, and at the same time the probability of strategies with the memory depth of 2 decreased insignificantly and stabilized at a significant level (Fig.3.4). It is clearly seen that the probability of discovering strategies of maximum memory depth in an individual throughout evolution is overwhelmingly high.

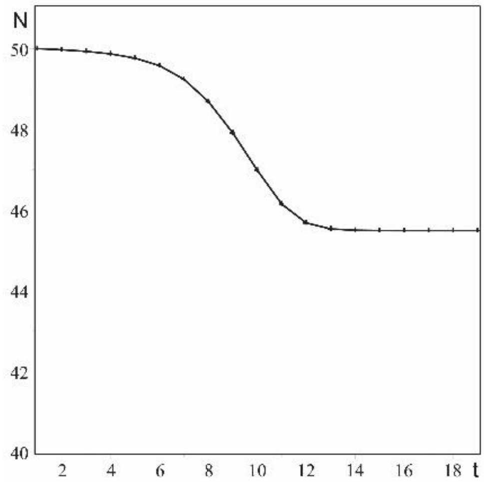

Fig.3.3 The change in the average number of strategies in an individual of the population over time of evolution

Note that the probability of detecting strategies with minimal memory of 0 and 1 did not decrease over time but even increased. So, for the memory 0 by a degree and for the memory 1 by 3 times but still remaining at a low level. The composition of the strategies of an individual also undergoes qualitative changes in terms of complexity. Let us consider the change in the probability of finding a strategy of a certain complexity in an individual in the process of evolution. Figure 3.5 shows the probabilities of finding a strategy of a certain complexity in an individual at the corresponding stages of evolution.

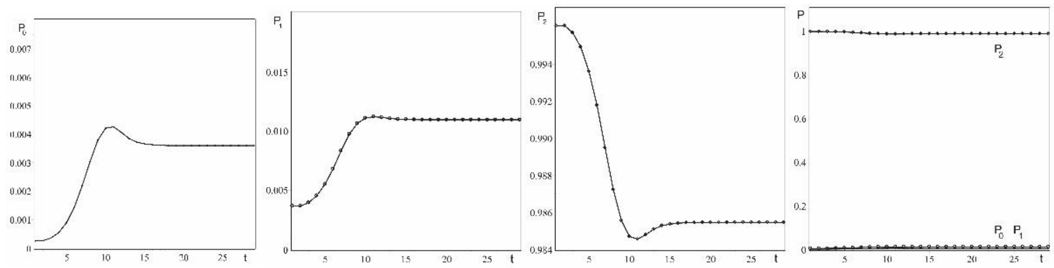

Fig.3.4 Shows the change in the probability of finding in an individual a strategy with memory depth 0 as P_0, with memory depth 1 as P_1, with memory depth 2 as P_2 over time of evolution. To the right these relations are shown in the same scale, so the changes of initial probabilities are relatively minor

Rather diverse behavior of these probabilities is noticeable. Every complexity follows three different types of behavior: decreasing, increasing, oscillating. These probabilities reach certain stationary levels.

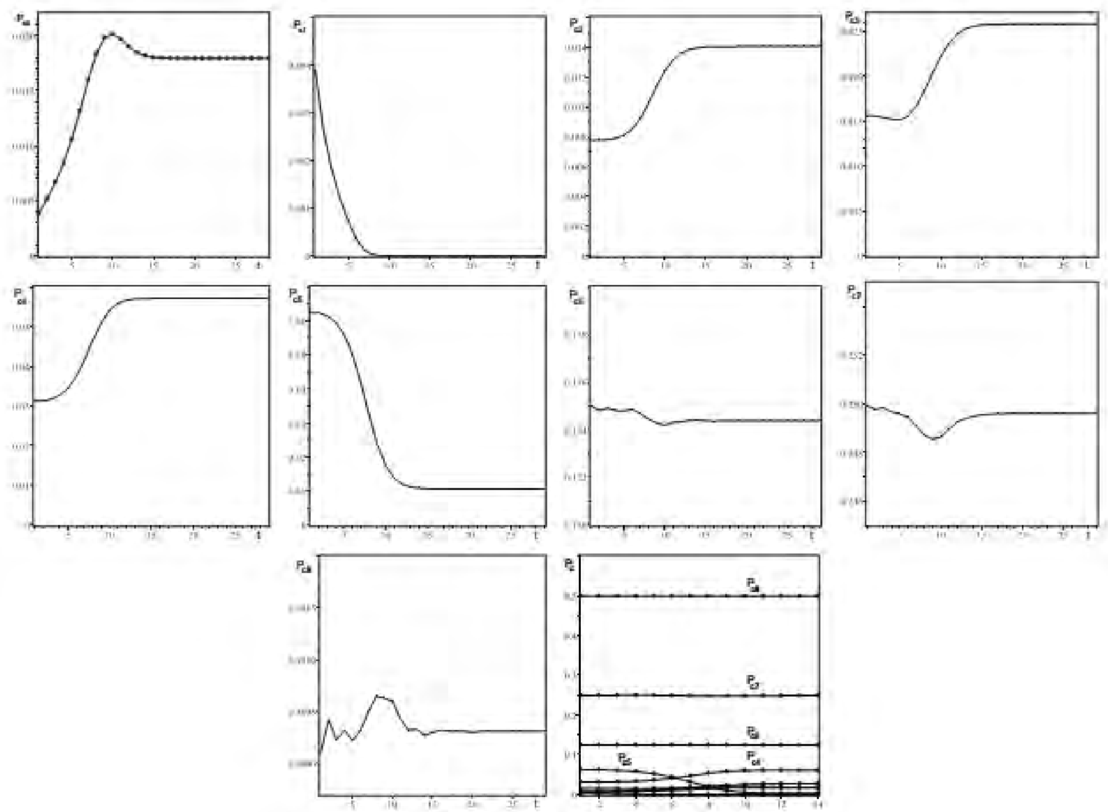

Fig.3.5 Shows the change in the probability of finding a strategy with complexity 0,1,..., 8 in an individual over time of evolution, where strategies of these complexities named as P_{c0}, P_{c1}, ... , P_{c8} respectively. A decrease in the probabilities is observed only for strategies of complexity 1 and 5. The last figure shows these dependencies on the same scale

The most complex strategies of 6, 7, 8 have the highest probability by the end of the evolution ($P_{c6} \approx 0.12$), ($P_{c7} \approx 0.25$), ($P_{c8} \approx 0.5$) and their changes over time are negligible.Strategies of complexity 1 exhibit significantly different behavior, which disappeared from the population after step 8. Because of this, the probability to discover strategies with the memory depth of 0 increases due to an increase in probability of strategies of complexity 0 and 2. It is interesting to note that also the probability of detecting the most primitive strategies with zero memory and complexity increases by $4 \div 5$ times, it remains low at $P_{c0} \approx 0.02$. The increase in strategies with zero memory occurs mainly due to the increase in primitive strategies. The most common behavior is the one of the probabilities to find the most complex strategies P_{c6}, P_{c7}, P_{c8}, which retain their almost initial values. The reason is in their overwhelming quantity and small number of low-complexity strategies, that does not allow them to greatly influence strategies of high complexity. Noticeable changes are undergone in the probabilities of detecting P_{c5}, P_{c4} and strategies of complexity 5 and 4 that compete for presence in an individual, which is also influenced by strategies of lower complexity.

Thus, in an individual in a stationary state, strategies of maximum depth and close to maximum complexity have a dominant probability. Even so, the most primitive strategies did not disappear but increased their own probability of presence in an individual remaining at a low level.

More obvious tendencies are displayed by the changes in the aggression of individuals in the population. The average aggression of individuals in the population tends to the maximum possible value of 1. The aggression distribution by memory depth is shown in Fig.6. Strategies with zero memory demonstrates the highest aggression during all stages of evolution, and strategies with memory depth of 1 demonstrates the lowest aggression. After the tenth stage of evolution the differences in their aggression disappears and coincides with the average aggression (see Fig.3.6). The stratification of aggression by complexity also indicates a typical increase of aggression and striving for maximum value. The only exception is strategies with complexity 1 as the least aggressive strategies that disappear from the population after stage 12. Therefore, the aggression of population's strategies, regardless of memory depth and complexity of strategies, grows with evolution and reaches the maximum value of 1 in the stationary state. The stationary set of individual's strategies consists of strategies with maximum aggression towards strategies of other individuals. The stationary distribution of strategies forms by the strategies of maximum memory depth and complexity. The variety of present strategies in the population decreases significantly. The number of formative strategies in the stationary state decreased by 97.7% in comparison to the initial one, and the number of strategies in individual decreased by only 9%.

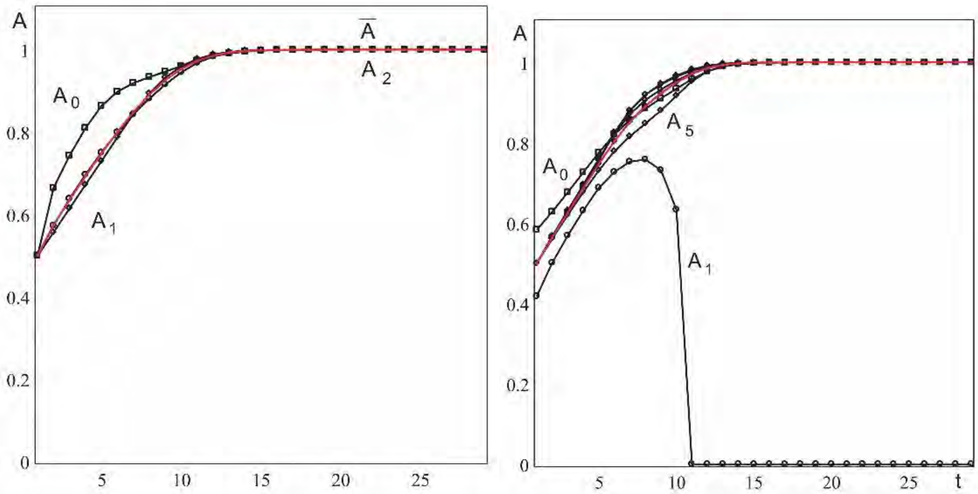

Fig.3.6 The average aggression of an individual of the population over time of evolution. By the memory depth to the left, where $\overline{A}$ is the average aggression of the whole population, and A_0, A_1, A_2 are the average aggressions of strategies with memory depth 0, 1, 2. By complexity to the right, where $\overline{A}$ is the average complexity of the population, and $A_0, A_1, ..., A_8$ are the average aggressions of strategies with complexity from 0 to 8

Note that in the process of evolution no strategy was able to spread to all individuals. The maximum presence of one strategy over individuals averaged 21%, which differs from the initial number by no more than 0.4%. Also, an increase in the initial number of individuals does not affect the typical behavior of their strategies. All tendencies of changes in individuals keeps the same as well as the stationary values. Apparently, further increase in the size of population does not lead to significant changes because of the presence of all formative strategies in the initial population.

4 Evolution of individuals with a uniform distribution of initial strategies by memory

Let us now consider the evolution of a population with another initial distribution of strategies over individuals. The initial distribution of strategies forms by equally probable choice between strategies of different memory depth from formative strategies. This means that all formative strategies are divided into three classes of memory depth and the probability of choice between them is equal. All characteristics are averaged by 10 experiments with the same initial properties. In this section the number of individuals is 50000 too, but the number of strategies for each is 24. With this choice each individual has an equal number of strategies of different memory depth (Fig. 4.1) at the beginning of evolution. So, the main difference from the previous case is the equal share of strategies of the memory depth of 0, 1, and 2 in each individual in initial population. Strategies with greater memory can be more complex and this affects the initial distribution of strategies in complexity in a complex way. In a result, the initial distribution of strategies' complexity in a typical individual has a non-monotonic dependence as shown in Figure 4.1 on the right. Here, the initial number of complex strategies $C = 6, 7, 8$ decreases.

To find the aggression of population's strategies let us proceed with the first stage of evolution. Figure 4.2 shows the simulation's results and the relation of aggression on the memory depth and the complexity of population strategies.

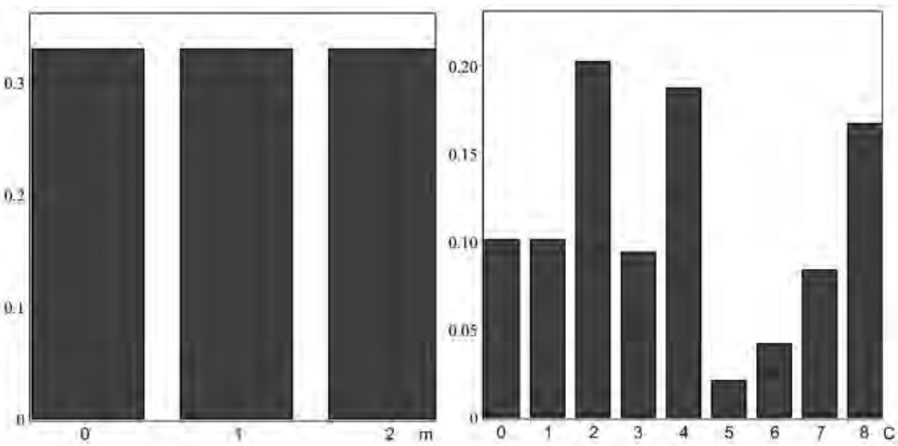

Fig.4.1 The initial distribution of strategies in an average individual in the population by memory depth (to the left) and complexity (to the right)

First of all, note that despite the significantly different distribution of strategies among individuals, the initial distribution of aggression in terms of memory depth and complexity remains the same. But the tendency for an increased average aggression of strategies of complexity 0 and a decreased aggression of strategies of complexity 1 even solidified due to an increase in the number of these strategies.

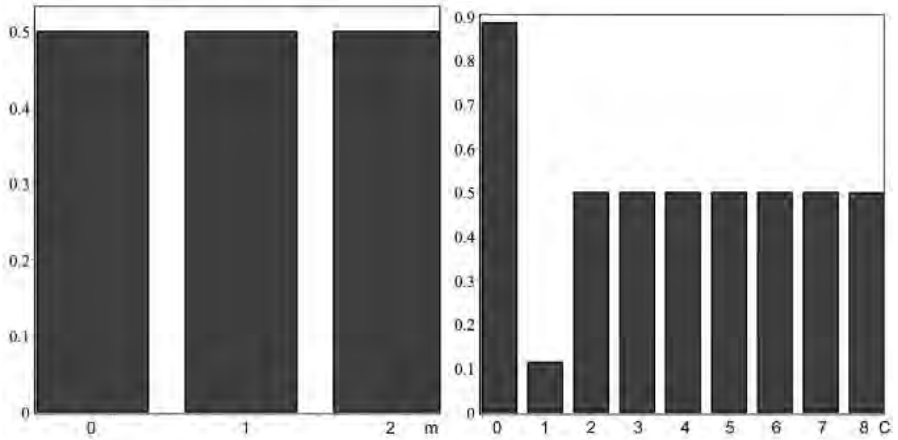

Fig.4.2 The histogram that determines the distribution of strategies' initial aggression in an individual of the population by memory depth (to the left) and complexity (to the right)

Let us now consider how the evolution affects the number of strategies of an individual and their memory depth with such initial population. Figure 4.3 on the left shows the change in the average number of strategies in an individual. The change of the number of strategies in an individual is universal demonstrating a decrease of their numbers in the process of evolution. To control the change in memory depth of an individual we use the probability of finding a strategy of a certain memory depth, which is shown in Figure 4.3 on the right.

At the initial stage these probabilities are equal. During evolution the probability of detecting strategies with zero memory depth falls to the certain stationary level of $P_{s0} \approx 0.15$.

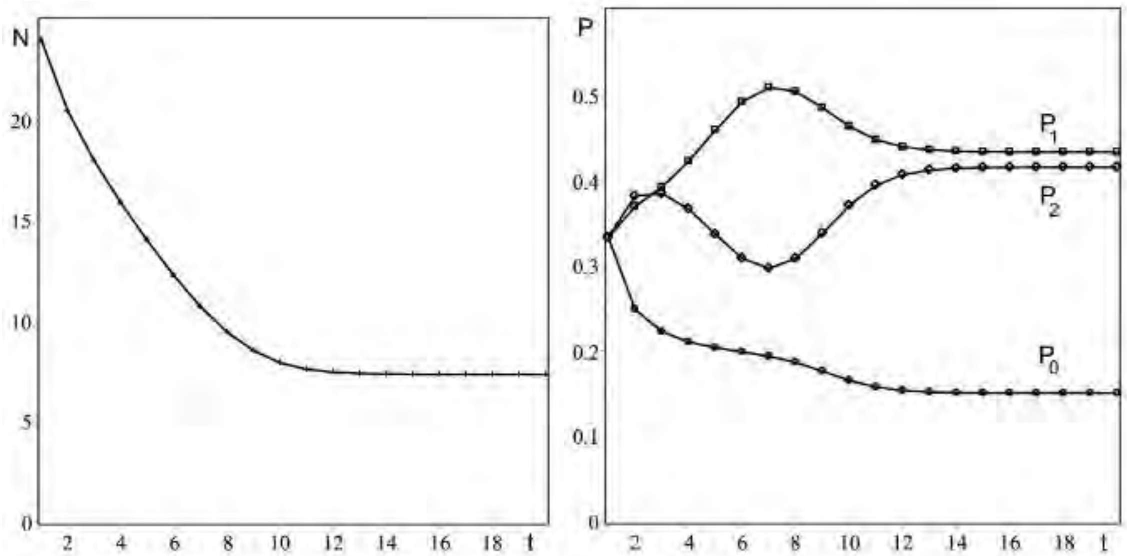

Fig.4.3 The change in the total number of strategies in an average individual (to the left) and the probability of finding in them the presence of a strategy with a certain memory depth (to the right), where P_0 is the probability of finding a strategy with memory depth 0, P_1 for memory depth 1, P_2 for memory depth 2

The probability of detecting strategies with the memory depth of 1 increases to a certain value and then falls to the stationary value of $P_{s1} \approx 0.43$. In contrast to the previous case, strategies with the memory depth of 1 are dominant in the stationary state. Probability of detecting a strategy with the memory depth of 2 changes in a more complex way. There are two stages of increase of this probability divided by the stage of decrease, and in the end they reach the stationary value of $P_{s2} \approx 0.41$. It is interesting to note that the minimum value of the probability P_2 is reached at the stage of the maximum value of the probability P_1. In other words, it is these components that compete with each other for their presence in an individual. Thus, in the stationary state strategies with the memory depth of 1 and 2 are dominant with a significant presence of strategies with a zero memory depth. The stationary values for this distribution are radically different from the corresponding values found in the previous section. The nature of these probabilities has also changed dramatically. Now, in the process of evolution the probability P_0 does not increase, and the probabilities P_1 and P_2 exceed their initial level. This way there is a seizure of individual by the strategies of the memory depth of 1 and 2.

For a more detailed analysis of the behavior of strategies of an individual let us consider how the complexity of an individual and the complexity of strategies with a certain memory evolve. Figure 4.4 shows the average complexity of an individual in the population and the change in the complexity components of strategies with a certain memory depth.

It is simple to spot two regions of increase in complexity of an average individual divided by the stage of complexity decrease. The time period of complexity decrease can be called a period of decline or a primitive period of evolution. Periods of complexity increase can be called periods of growth. The first period of growth lasted 2 stages of evolution (or 9% of the evolution time). The period of decline took 4 stages (or 18% of the evolution time). And the last one lasted until reaching the stationary state after 22 evolution stages (or 72% of the evolution time). These periods correlate with the behavior of strategies with a certain memory depth. Thus, for example, probability of finding a strategy of the memory depth of 2 in an individual correlate with a change in complexity of the individual's strategies (see Fig.4.3). The reason is related to the greater complexity of the strategies contained in the class of strategies of the memory depth of 2. Note that the average complexity of an individual is significantly lower than in the previously considered case. This is due to the large number of strategies with zero and one memory depth in an individual and a low initial complexity of individual's strategies. In the process of evolution, complex strategies dominate and therefore the stationary level of complexity ($\overline{C}_s \approx 4.3$) of strategies of an individual exceeds the initial ($\overline{C}\,|_{t=0} \approx 3.8$).

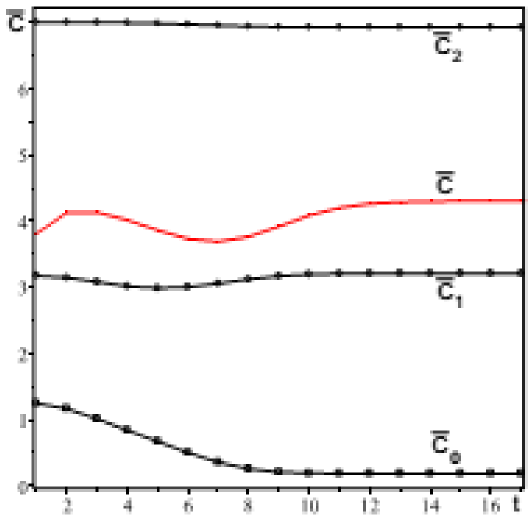

Fig.4.4 The evolution of the average complexity of an individual in the population $\overline{C}$ *and by memory depth (*$\overline{C}_0$ *is the complexity of individual's strategies with memory depth 0,* $\overline{C}_1$ *for memory depth 1,* $\overline{C}_2$ *for memory depth 2*

A more detailed behavior of the average complexity of strategies in an individual can be obtained by plotting the relation of the number or the probability of the presence of a strategy of a certain complexity on the time of evolution. The change in probability of finding a certain strategy in an individual in terms of complexity is quite complicated depending on the value of complexity (Fig. 4.5). So, strategies of complexity 0, 4, 6, 7, 8 increased their presence in an individual in the process of evolution. The most primitive zero-complexity strategies came out on the third place in terms of probability after strategies of complexity 4 (maximum value) and complexity 8. The probabilities of strategies with complexity 1, 2, 3, and 5 decreased their presence in individuals of the population. Strategies of complexity 1, as the most non-aggressive ones, have completely disappeared from the population.

Moving on to the evolution of an individual's aggression in the population, Figure 4.6 shows that strategies with memory depth 0 are more aggressive and strategies with memory depth 1 are less aggressive over the course of evolution. In terms of complexity, strategies with complexity 0 are distinguished as the most aggressive; complexity 1 as the most non-aggressive (disappears from the population after step 12); complexity 8 as the least aggressive of all others after complexity 1. In the previous type of distribution, strategies of complexity 1 also disappeared from the population and all the others tend to the maximum possible value of the aggression, which indicates the universality of these patterns. Also, with the previous distribution, the aggression of the strategies of complexity 0 was the greatest. The average aggression of an individual is determined by the aggression of the strategies of memory depth 2 and complexity 8 as shown on Fig.4.6.

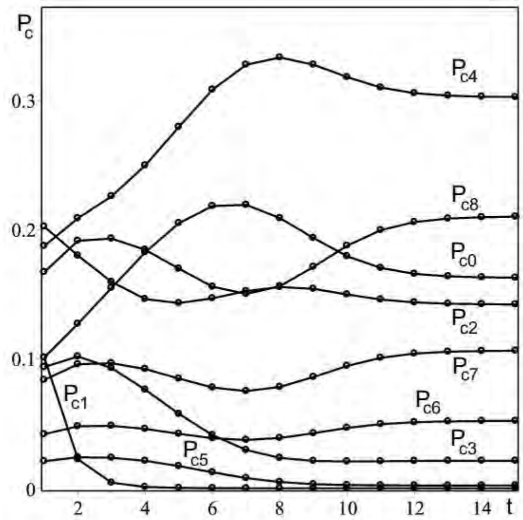

Fig.4.5 The probability of finding a strategy with a certain complexity in an average individual of the population ($P_1 \ldots P_8$ represents complexity from 0 to 8 respectively)

Thus, the stationary stage is also formed in this case by the most aggressive individuals, whose strategies have the greatest complexity and memory depth. The presence of strategies of zero memory depth in the stationary state increased in comparison to the initial one by approximately 2 times. Moreover, it is the primitive strategies of zero complexity that have survived, while the less aggressive strategies of complexity 1 have disappeared from the population. Complex strategies ($C = 6, 7, 8$) have increased their presence in individuals of the population as well as of complexity $C = 4$. Strategies with complexity $C = 1, 2, 3, 5$ have decreased their presence.

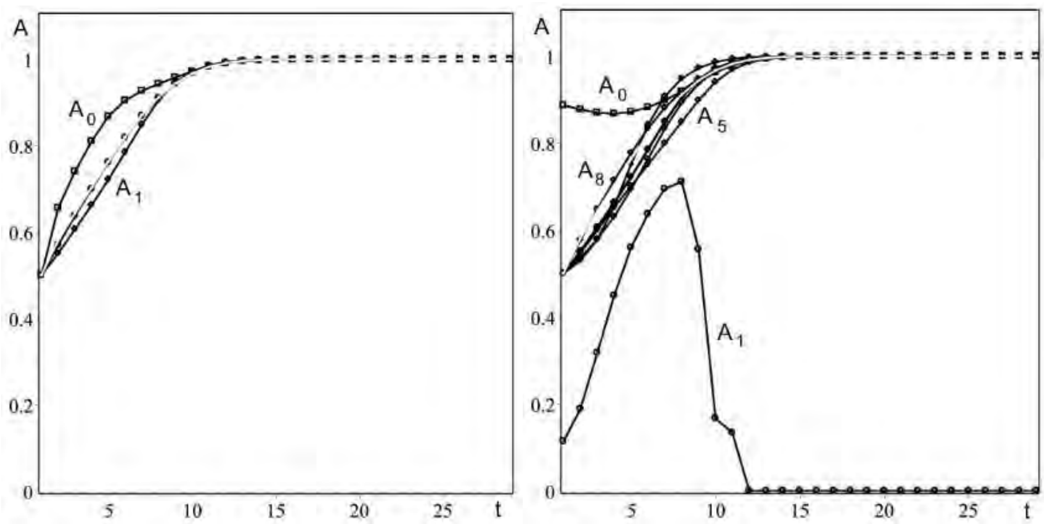

Fig.4.6 Aggression of the population by memory depth (to the left) and by complexity (to the right), where A is the average aggression; A_0, A_1, A_2 are the aggressions for memory depth from 0 to 2; $A_0, A_1, \ldots, A_8$ are the aggressions for complexities from 0 to 8

Let us now discuss the effect of an increase in the number of strategies in an individual while maintaining the initial distribution of strategies with equal probabilities by memory depth. Naturally, this will lead to an increase in the average strategies' complexity of an individual because of the increased number of strategies of maximum complexity. Therefore, the stationary value of the average complexity of the strategies of an individual will also increase. The dependence of its changes over time with the presence of two periods of growth and one period of decline will remain (quite similar to the example shown on Figure 4.4).

The probabilities of finding strategies of a certain memory depth in an individual are subject to more radical changes (see Fig.4.3). For example, when choosing the initial number of strategies in an individual as 50, at all times of evolution the probability of finding a strategy of memory depth 2 in it exceeds the probability of finding a strategy with memory depth 1 in contrast to the one shown on Figure 4.3 for the initial number of strategies in an individual of 24. Thus, the tendency for the dominance of strategies of maximum memory and complexity in an individual only becomes higher with an increase in the number of its strategies.

5 Conclusion

Therefore, evolution with the used method of strategies' exchange supports individuals with maximum aggression, maximum memory depth, and high complexity of strategies. The stationary set of strategies of an individual consists mainly of strategies with some share of the most primitive strategies. The number of strategies of the average individual decreases with the evolution time reaching a certain stationary value, which depends on the initial distribution of strategies. The variety of strategies with evolution decreases more significantly: about 6% of the original formative strategies remains in the stationary state. Strategies of complexity 1, as the least aggressive, even disappear from the population. Complex behavior and periods of growth and decline in complexity appear with significant proportion of strategies of low complexity and low memory, otherwise these strategies do not affect the nature of evolution being suppressed at early stages by more complex strategies with a large memory depth.

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