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Evolution of memes

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The paper considers the evolution of a population of individuals, where each of them initially possesses a certain number of strategies whose memory depth does not exceed 2. All individuals randomly enter into competition in pairs at each stage of evolution. In each random pair of individuals a competition between pairs of all their randomly selected strategies is performed. These strategies compete in pairs according to the iterated prisoner's dilemma. Afterwards, strategies earn evolutionary advantage points according to a given payout matrix. The strategy with the most points wins. To negate an effect of the first move each pair of strategies play this game twice. The first game is started by one strategy and the second game by another. The winnings are determined by the outcome of both games. After this competition the winning strategy of one individual replaces the corresponding losing strategy of another individual. Thus, there is an exchange of the "successful" strategies between individuals with the corresponding elimination of the losing strategies. The evolution of the population is carried out until the stationary state is achieved. The patterns of changes in basic properties of average individual strategies during evolution have been established. It is shown that in the process of evolution the aggression of an individual increases, tending to the maximum value. The stationary set of an individual's strategies consists of strategies of maximum memory depth and complexity along with a certain number of primitive strategies. The complexity and memory depth of an individual's strategies turn out to be evolutionary beneficial. In the stationary state the number of primitive an individual's strategies depends on their initial distribution. The paper considers two initial distributions, where the first corresponds to the equal probability of any strategy in the distribution by individuals, and the seconds corresponds to equally probable choice in terms of memory depth. The variety of strategies in the process of evolution decreases significantly, making up only a small part of the initial strategies present in the population.

Keywords: *evolution of population, strategies with memory, memory depth, complexity, aggression, prisoner's dilemma, stationary state.*

Еволюція мемів

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В роботі розглянуто еволюцію популяції особин, кожна з яких спочатку має певну кількість стратегій, пам'ять яких не перевищує глибини 2. На кожному етапі еволюції всі особини випадковим чином вступають попарно в конкурентну боротьбу. Випадкова пара особин при взаємодії проводить змагання між парами всіх своїх випадково обраних стратегій. Ці стратегії попарно вступають у змагання відповідно до ітерованої дилеми в'язнів. У такій боротьбі стратегії набирають очки еволюційних переваг відповідно заданої матриці виплат. Виграє стратегія, яка набрала найбільшу кількість очок. Для нівелювання значення першого ходу дві стратегії вступають в таку гру двічі. Спочатку гру починає одна стратегія, в другій грі починає інша стратегія. Виграш визначається за результатом цих двох ігор. Після проведення такого змагання стратегія однієї особини, яка виграла, замінює відповідну стратегію, яка програла, іншої особини. Таким чином, між особинами відбувається обмін більш "процвітаючими" стратегіями з втратою тих, хто програв. Проведена еволюція популяції таких особин до етапу встановлення стаціонарного стану особин популяції. Під час еволюції встановлені закономірності зміни основних властивостей стратегій середньої особини. Показано, що в процесі еволюції агресивність особини зростає, прагнучи до максимального значення. Кількість стратегій особини зменшується, досягаючи деякого стаціонарного значення. Стаціонарний набір стратегій особини складають стратегії максимальної пам'яті та складності з деякою кількістю примітивних стратегій. Складність і глибина пам'яті стратегій особини виявляються еволюційно вигідними. Кількість примітивних стратегій у особини в стаціонарі залежить від їх початкового розподілу по особинам. У роботі розглянуті два початкових розподіли: один відповідає рівноймовірності будь-якої стратегії при розподілі по особинам, другий відповідає рівноймовірному вибору за глибиною пам'яті стратегій. Різноманітність стратегій в процесі еволюції зменшується значно, складаючи лише малу частину присутніх в популяції початкових стратегій.

Ключові слова: *еволюція популяції, стратегії з пам'яттю, глибина пам'яті, складність, агресивність, дилема ув'язнених, стаціонарний стан.*

Эволюция мемов

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В работе рассмотрена эволюция популяции особей, каждая из которых изначально обладает определенным числом стратегий, память которых не превосходит глубины 2. На каждом этапе эволюции все особи случайным образом вступают попарно в конкурентную борьбу. Случайная пара особей при взаимодействии проводит соревнование между парами всех своих случайно выбранных стратегий. Эти стратегии попарно вступают в соревнование в соответствии с итерированной дилеммой заключенных. В такой борьбе стратегии набирают очки эволюционных преимуществ в соответствии с заданной матрицей выплат. Выигрывает стратегия, набравшая больше очков. Для нивелирования значения первого хода две стратегии вступают в такую игру дважды. Сначала игру начинает одна стратегия, во второй игре начинает другая стратегия. Выигрыш определяется по результату этих двух игр. После проведения такого соревнования выигравшая стратегия одной особи заменяет соответствующую проигравшую стратегию другой особи. Таким образом, между особями происходит обмен более "преуспевающими" стратегиями с потерей проигравших. Проведена эволюция популяции таких особей до этапа установления стационарного состояния особей популяции. Во время эволюции установлены закономерности изменения основных свойств стратегий средней особи. Показано, что в процессе эволюции агрессивность особи возрастает, стремясь к максимальному значению. Число стратегий особи уменьшается, достигая некоторого стационарного значения. Стационарный набор стратегий особи составляют стратегии максимальной памяти и сложности с некоторым количеством примитивных стратегий. Сложность и глубина памяти стратегий особи оказываются эволюционно выгодными. Количество примитивных стратегий у особи в стационаре зависит от их первоначального распределения по особям. В работе рассмотрены два начальных распределения: одно соответствует равновероятности любой стратегии при распределении по особям, второе соответствует равновероятному выбору по глубине памяти. Разнообразие стратегий в процессе эволюции уменьшается значительно, составляя только малую часть присутствующих в популяции начальных стратегий.

Ключевые слова: *эволюция популяции, стратегии с памятью, глубина памяти, сложность, агрессивность, дилемма заключенных, стационарное состояние.*

1. Introduction

Nowadays, the evolution of different populations and communities is the subject of interest and active research. Such interest is closely related to changes in the populations' properties that emerge during the evolution process and the emergent properties of the population are of particular interest. It should be noted that this method of changing the properties of a population is also useful for artificial populations of various swarm and multi-agent systems. One of the properties that have been attracting attention for a long time is the emergence of cooperative behavior in different populations [1]. Elements of game theory are widely used in modeling the evolution of populations [2,3]. The conducted researches revealed some mechanisms that lead to cooperation in a wide variety of systems [4]. Among such mechanisms the following can be noted: voluntary participation [5], punishment [6], similarity [7], heterogeneous activity [8], social diversity [9, 10], dynamical linking [11], asymmetric interaction and permutation graphs [12], migration [13-15], in-group favoritism [16], and interdependent relations [17]. It is possible to find mechanisms of emergence of other properties of populations in the process of evolution in a similar way.

Memory is one of the most general properties of individuals in a population, so its influence on the evolution of populations is of great interest. The evolution of a population of strategies limited by memory depth within the scope of generalized prisoner's dilemma is considered in [18,19]. The concept of strategy's complexity is also introduced. Every new generation of strategies loses the most disadvantageous behavior strategies of the previous generation. It is shown that an increase in memory and complexity of population strategies is evolutionary beneficial. Evolutionary selection winners invariably come across strategies with maximum memory and close to maximum complexity.

The evolution of a population of strategies with memory in the presence of sources of strategies of different memory depth is considered in [20]. It is shown that memory depth and complexity of strategies are evolutionarily advantageous too. In all cases the strategies' aggression decreases, and at the stationary stage all strategies have zero aggression towards each other. This feature can be used as a principle for selecting strategies for possible stationary states. Also, a relation between aggression and the amount of points received per strategy's move when strategies interact has been found. The higher the aggression, the fever evolutionary advantage points are received by the strategy per move.

The alternative evolution of a population of strategies with memory is considered in [21]. In such evolution a strategy with the highest amount of evolutionary advantage points is removed from every generation. This alternative evolution leads to significant changes in the strategies of the population in comparison to the usual evolution. In a sense, the alternative evolution supports maximum memory depth and complexity even more than the regular evolution. However, the major difference lies in the absolute aggression of stationary strategies. The stationary state is formed by the most aggressive strategies.

This paper considers a population of individuals where each has a certain number of strategies with memory. A principle for exchanging strategies between individuals is proposed, and the evolution of such population is considered. When individuals interact, all their strategies compete in pairs according to the iterated prisoner's dilemma. The winning strategy replaces the corresponding losing strategy of an individual. It is shown that in the process of the evolution the aggression of an individual increases tending to the maximum value. The behavior of the distribution of an individual’s strategies in terms of memory depth and complexity in the process of evolution has been studied in details. The number of individual's strategies decreases reaching a certain stationary value. The stationary set of an individual’s strategies consists of strategies of maximum memory and complexity with a certain number of primitive strategies. The variety of strategies in the process of evolution decreases significantly, making up only a small part of the initial strategies present in the population.

2. Population of individuals

In the considered population each individual has a finite set of strategies with memory. For the evolution it is essential to introduce the rules of strategies' meetings, interactions and selection. In the population at each stage of evolution all individuals are randomly divided into pairs that interact. All strategies of one individual interact with the strategies of another (also randomly divided into pairs) according to the iterated prisoner's dilemma. To negate the impact of the first move, both strategies play the game twice. One strategy starts the first game, and another starts the second game. The winnings are determined by the results of these two games. The winning strategy replaces the losing strategy of the corresponding individual. In other words, there is an exchange of strategies between individuals where a loser gets removed. After the interactions of all pairs of individuals in accordance with the described rules, the next stage of evolution begins. All individuals again are randomly divided into the interacting pairs. It can be compared to the evolution of ideas or memes[1] in different societies. The evolution stops

[1]This term was introduced by Richard Dawkins in 1976 to name a unit of relevant information within a culture. A meme is an idea, symbol, manner, or a way of acting, that consciously or unconsciously can be transmitted from one person to another through speech, writing, video, rituals, gestures, etc. In some sense, a meme is similar to a gene as a carrier of cultural information.

when the population reaches a stationary state. During the process of evolution, we will monitor the properties of the population's strategies and all the individuals included in the population.

Now, let us discuss the choice of the initial distribution of strategies over individuals of the population. The full number of strategies with the memory depth 2 is distributed among individuals. The process of distribution can be implemented in different ways. This paper considers two ways of strategies distribution. In the first method, we assign strategies to an individual by choosing them randomly from the set of all strategies with the memory depth not exceeding 2, assuming that strategies are equally probable. This choice does not take into account the properties of strategies, assuming them to be equal. In the second method, we choose strategies for an individual by a uniform distribution over memory. Therefore all strategies are divided into three sets by memory depth and the choice is equally probable between these sets. We will discuss those two methods of forming the initial distribution of strategies among individuals of the population further.

Another feature of the considered population of individuals is the emergence of collective variables to describe individuals and not just the population of strategies. Each individual has a distribution of strategies in terms of memory and complexity, and the entire individual can show definite aggression. Thus, there are properties of an individual, such as distribution of strategies by memory depth and complexity. These distributions define the average memory depth, complexity, aggression of i individual and its average aggression:

$$\langle M\rangle_i = \sum_{j=1}^{N_i}\frac{M_{ij}}{N_i}, \langle C\rangle_i = \sum_{j=1}^{N_i}\frac{C_{ij}}{N_i}, \langle A\rangle_i = \sum_{j=1}^{N_i}\frac{a_{ij}}{N_i} \quad (2.1)$$

where $\langle M\rangle_i$, $\langle C\rangle_i$, $\langle A\rangle_i$ are the average memory, the average complexity, and the average aggression of the i individual respectively; N_i is the number of strategies of the i individual; M_{ij} is the memory depth of the j strategy of the i individual; C_{ij} is the complexity of the j strategy of the i individual; a_{ij} is the aggression of the j strategy of the i individual.

These local characteristics can be used to track changes in the properties of individuals during evolution. In addition, global characteristics of the population appear, such as the average values of memory, complexity and aggression of the whole population:

$$\overline{M} = \sum_{i=1}^{S}\frac{\langle M\rangle_i}{S}, \overline{C} = \sum_{i=1}^{S}\frac{\langle C\rangle_i}{S}, \overline{A} = \sum_{i=1}^{S}\frac{\langle A\rangle_i}{S} \quad (2.2)$$

where S is the number of individuals in the population.

These characteristics are calculated by averaging over all individuals in the population and determine the typical characteristics of an individual. Another useful characteristic that determines the variety of strategies in the population is the number of different strategies which we call formative strategies. By the formative strategies we imply the number of different strategies present in a population at some stage of evolution. Accordingly, we use the following formula as characteristics of the formative strategies in the population.

$$\langle M\rangle = \sum_{i=1}^{N}\frac{M_i}{N}, \langle C\rangle = \sum_{i=1}^{N}\frac{C_i}{N}, \langle A\rangle = \sum_{i=1}^{N}\frac{a_i}{N} \quad (2.3)$$

where N is the number of formative strategies; $\langle M\rangle$, $\langle C\rangle$, and $\langle A\rangle$ are the average memory, complexity, aggression of the formative strategy respectively.

3. Evolution of individuals with a uniform distribution of initial strategies

The population of 50000 individuals, where each one has 50 strategies has been considered. The ratio between these values is chosen so that all strategies which do not exceed the memory depth of 2 are present among the individuals of the population. The choice of these strategies is carried out in an equally probable manner from the set of all strategies with the memory depth not exceeding 2. In this case, the strategies with the memory depth of 2 dominate due to the largest number of such strategies in the initial set of strategies (32640). The number of strategies with the memory depth of 1 is significantly smaller (120 strategies) and the number of strategies with the memory depth of 0 is the smallest (8 strategies). The initial distribution of strategies is shown in Fig. 3.1 and inherits the distribution properties of all strategies. The overwhelming number of strategies with the memory depth of 2 and maximum complexity is typical for the initial distribution of an individual's strategies (see Fig.3.1) as the consequence of the rule used for selecting strategies for an individual. The calculation of the properties of individuals has been carried out by averaging over 10 implementations of the initial state and their evolution.

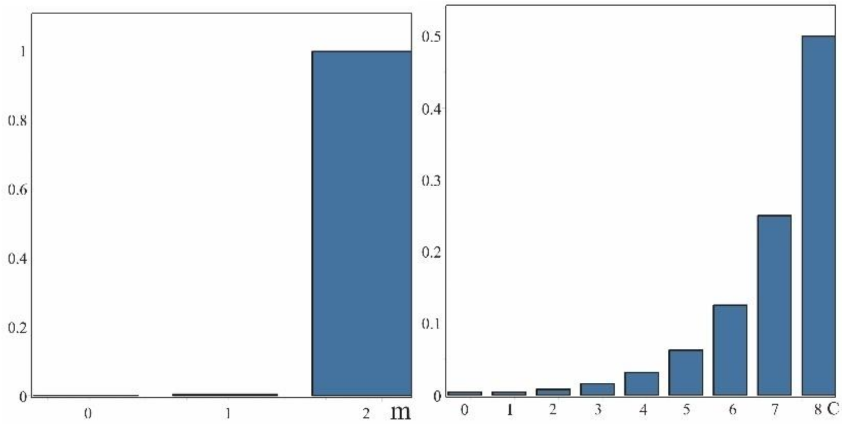

Fig.3.1 For an average individual at the beginning of evolution, the share of strategies of a certain memory depth is shown on the left, and the share of strategies of varying complexity on the right. The same histograms determine the probability of detecting a strategy of a certain memory depth (left) and complexity (right) of an individual at the initial stage of evolution

Let us discuss the initial distribution of aggression which is formed in the individuals by the chosen initial distribution of strategies. To do this we need to proceed with the first stage of evolution. At the first stage of evolution, all individuals are randomly divided into pairs that interact. All strategies of one individual interact with the strategies of another by dividing strategies randomly into pairs in accordance with the iterated prisoner's dilemma (the number of moves is 100). They interact twice: in the first game started by the first strategy, and the second game started by the second strategy. This method of competing between two strategies negates the impact of the first move. To identify the winning strategy the payoff matrix [3, 18-20] have been applied as shown in the Tab. 1.

Table 1. Payoff matrix

	Cooperation	Rejection
Cooperation	3,3	0,5
Rejection	5,0	1,1

The winning strategy (one with the highest score) replaces the losing strategy of the corresponding individual for an each pair of strategies. After the first stage, aggression shown by an individual (or the relative number of refusals to cooperate) is established.

Fig. 3.2 shows the distribution of the initial aggression of the individuals in terms of memory and complexity. It is easy to see that average aggression of the individuals in the population is close to 0.5. At the same time, the strategies with different memory depth show approximately the same aggression. The aggression of the strategies with zero complexity is maximal, and the strategies with the complexity 1 are minimal. The strategies of greater complexity have the same aggression (see Fig. 3.2). Therefore, the aggression of primitive strategies with the memory depth 0 and the complexity 0 is maximal.

Further evolution makes it possible to discern the changes in the basic characteristics of individuals.

First of all, the number of strategies of every individual decreases. Fig. 3.3 shows the average decrease in the number of an individual's strategies. In the process of evolution, after the stage of exponential decrease in numbers, it reaches a stationary state. Therefore individuals develop a stationary set of strategies. With such a ratio of individuals and the strategies the stationary stage is achieved on average at $N_s = 45.5$.

Let us consider how the individual distribution of strategies by memory depth changes over time. Fig. 3.4 shows the evolution in strategies' distribution by the memory. For that we use the probability of finding an individual's strategy with the memory depth $m = 0,1,2$ respectively as a convenient characteristic.

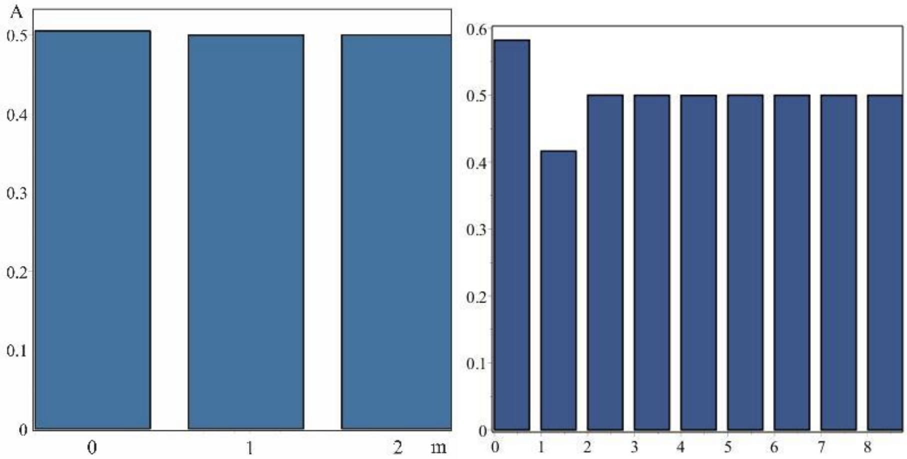

Fig.3.2 The initial distribution of an individual's aggression in the population by memory depth (left) and complexity (right) calculated at the first step of evolution

The probability of detecting an individual's strategies with the memory depth of 0 and 1 increases up to the tenth step, while the probability of strategies with the memory depth of 2 decreases insignificantly and then stabilizes (Fig. 3.4). It is obvious that the probability of finding an individual's strategies of maximum memory depth throughout evolution is overwhelmingly high.

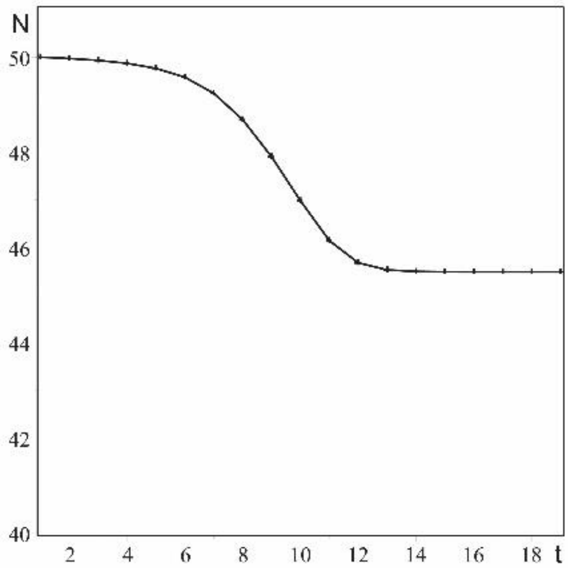

Fig.3.3 The change in the average number of individual's strategies over time of evolution

Note that the probability of detecting strategies with the minimal memory of 0 and 1 increase over time but still remains at a low level. The composition of an individual's strategies also undergoes qualitative changes in terms of complexity. Let us consider the change in the probability of finding an individual's strategy of certain complexity in the process of evolution. Fig. 3.5 shows the probabilities of finding an individual's strategy of a certain complexity at the corresponding stages of evolution.

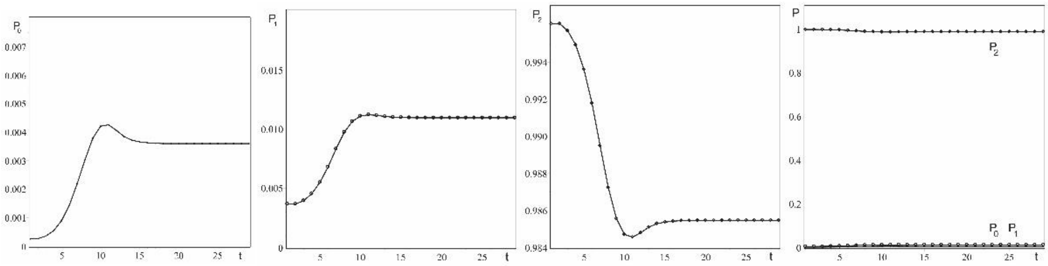

Fig. 3.4 shows the change in the probability of finding an individual's strategy with the memory depth 0 as P_0, with the memory depth 1 as P_1, and with the memory depth 2 as P_2 over time of evolution. These relations are shown in the same scale on the right, so the changes of initial probabilities are relatively minor

Rather diverse behavior of these probabilities is noticeable. Every complexity follows three different types of behavior: decreasing, increasing, oscillating. These probabilities reach certain stationary levels.

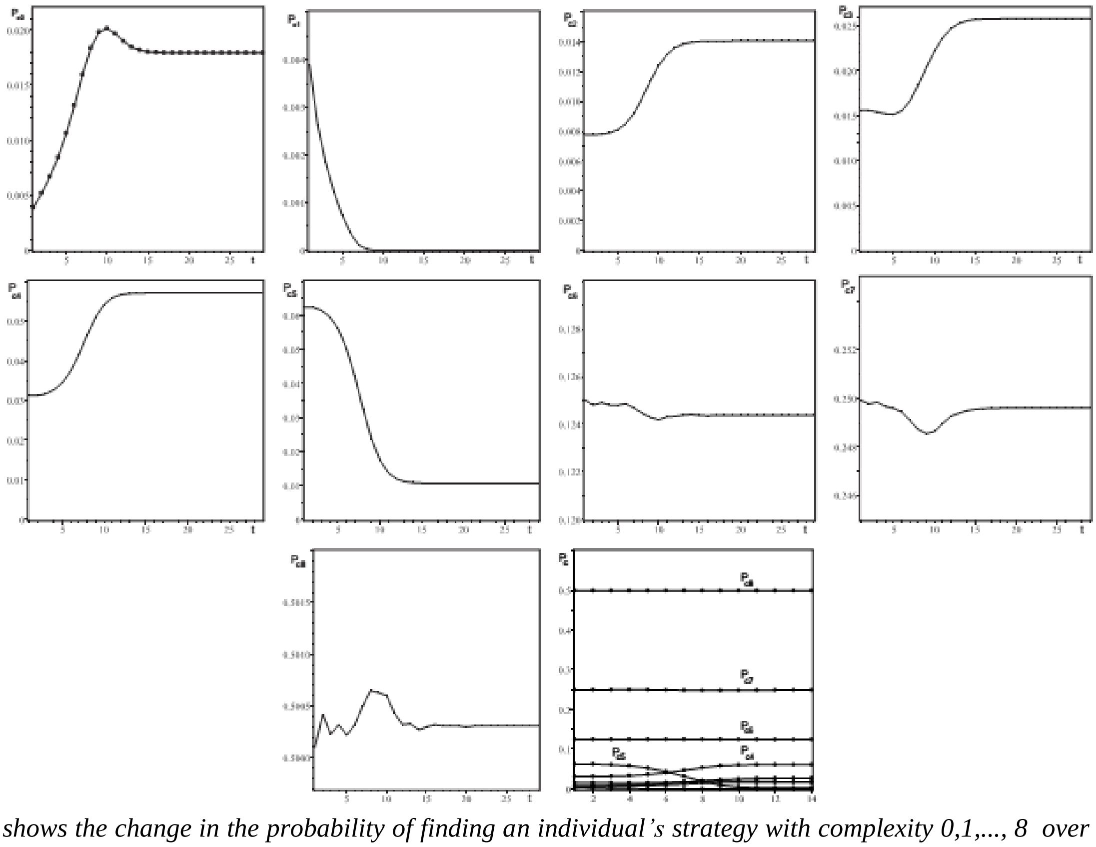

Fig.3.5 shows the change in the probability of finding an individual's strategy with complexity 0,1,..., 8 over time of evolution, where strategies of these complexities denoted as P_{c0}, P_{c1}, ... , P_{c8} respectively. A decrease in the probabilities is observed only for strategies of complexity 1 and 5. The last graph shows these dependencies on the same scale

The most complex strategies of 6, 7, 8 have the highest probability by the end of the evolution ($P_{c6} \approx 0.12$), ($P_{c7} \approx 0.25$), ($P_{c8} \approx 0.5$) and their changes over time are negligible. The strategies of complexity 1 exhibit significantly different behavior and disappear after step 8. Therefore, the probability to discover strategies with the memory depth of 0 increases due to an increase in probability of strategies of complexity 0 and 2. Interestingly, the probability of detecting the most primitive strategies with zero memory and complexity increases by $4 \div 5$ times, while remaining low at $P_{c0} \approx 0.02$. The increase in strategies with zero memory occurs mainly due to the increase in primitive strategies. The probabilities for finding the most complex strategies P_{c6}, P_{c7}, P_{c8}, mostly retain their initial values. This is due to their overwhelming quantity; therefore a small number of low-complexity strategies cannot significantly influence strategies of high complexity. Noticeable changes can be seen in the probabilities of detecting strategies of complexity P_{c5}, P_{c4} that compete directly and are influenced by the strategies of lower complexity as well.

Thus, for an individual in a stationary state, strategies of maximum depth and close to maximum complexity have the highest probability. Even so, the most primitive strategies do not disappear but increase their probability while remaining at a low level.

More obvious tendencies are displayed by the changes in the aggression of individuals. The average aggression of individuals in the population tends to the maximum possible value of 1. The aggression distribution by memory depth is shown in Fig. 6. The strategies with the zero memory depth demonstrate the highest aggression during all stages of evolution, and the strategies with the memory depth of 1 demonstrate the lowest aggression. After the tenth stage of evolution the differences in their aggression disappears and coincides with the average aggression (Fig. 3.6). The stratification of aggression by complexity also indicates a typical increase of aggression and striving for maximum value. The only exception is the strategies with complexity 1 as the least aggressive strategies that disappear from the population after stage 12. Therefore, the aggression of population's strategies, regardless of memory depth and complexity of strategies, grows with evolution and reaches the maximum value of 1 in the stationary state. The stationary set of an individual's strategies consists of strategies with maximum aggression towards strategies of other individuals. The stationary distribution of strategies is formed by the strategies of maximum memory depth and complexity. The variety of present strategies in the population decreases significantly. The number of formative strategies in the stationary state decreases by 97.7% in comparison to the initial one, and the number of strategies in individual decreases just by 9%.

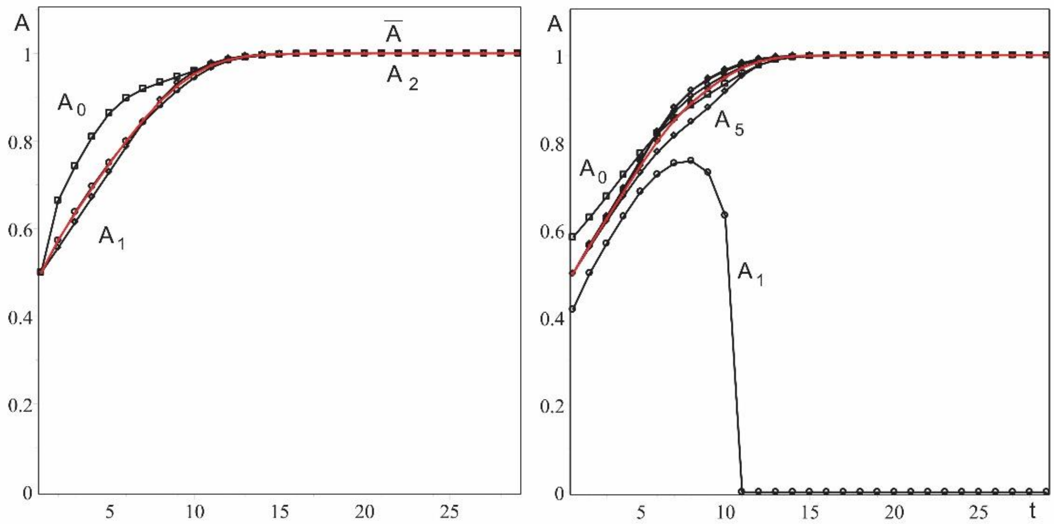

Fig. 3.6 shows average aggression of an individual of the population over time of evolution. Average aggression by memory depth (left), where $\overline{A}$ is the average aggression of the whole population, and A_0, A_1, A_2 are the average aggressions of strategies with memory depth 0, 1, 2. Average aggression by complexity to the right, where $\overline{A}$ is the average complexity of the population, and $A_0, A_1, \ldots, A_8$ are the average aggressions of strategies with complexity from 0 to 8

It should be noted that in the process of evolution no strategy is able to spread to all individuals. The averaged maximum presence of one strategy over individuals is 21%, which differs from the initial number by no more than 0.4%. Also, an increase of the initial number of individuals does not affect the typical behavior of their strategies. All tendencies of an individual’s changes, as well as the stationary values stay the same. Apparently, further increase in the size of population does not lead to significant changes because of the presence of all formative strategies in the initial population.

4. Evolution of individuals with a uniform distribution of initial strategies by memory depth

Let us now consider another initial distribution of strategies for the population evolution. The initial distribution of strategies is formed by an equally probable choice between strategies of different memory depth from formative strategies. This means that all formative strategies are divided into three classes by memory depth and the probability of choice between them is equal. All characteristics are averaged by 10 experiments with the same initial properties. In this experiment the number of individuals is 50000 as well, but the number of strategies for each is 24. With this choice each individual has an equal number of strategies of different memory depth at the beginning of evolution (Fig. 4.1). Therefore, the main difference from the previous case is the equal share of strategies of the memory depth of 0, 1, and 2 for each individual in initial population. Strategies with the greater memory depth can be more complex, which sophisticates the initial distribution of strategies by complexity. Consequently, the initial distribution of strategies by the complexity for a typical individual has a non-monotonic dependence as shown in Fig. 4.1 on the right. It can be seen that the initial number of complex strategies $C = 6, 7, 8$ decreases.

To find the aggression of population's strategies let us proceed with the first stage of evolution. Fig. 4.2 shows the simulation results and the impact of aggression on the memory depth and the complexity of population strategies.

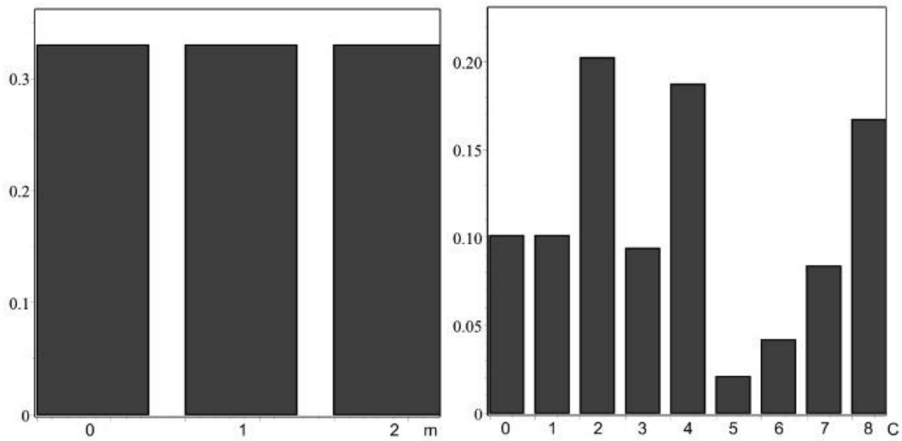

Fig.4.1 The initial distribution of strategies for an average individual in the population by memory depth (left) and complexity (right)

Let us note that despite the significantly different distribution of strategies among individuals, the initial distribution of aggression in terms of memory depth and complexity remains the same. But the tendency for an increased average aggression of strategies of the complexity 0 and a decreased aggression of strategies of the complexity 1 becomes even more prominent due to an increase in the number of these strategies.

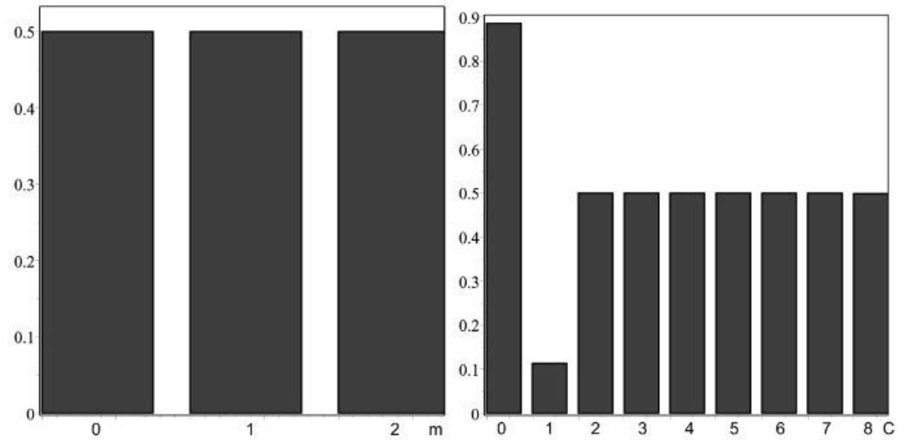

Fig.4.2 The histogram that determines the initial aggression distribution of strategies for an individual of the population by memory depth (left) and complexity (right)

Let us consider how the evolution affects the number of an individual's strategies and their memory depth for such an initial population. The change in the average number of strategies for an individual is shown in Fig. 4.3 on the left. The changes in the number of strategies for an individual are universal and decrease in the process of evolution. To control the change in the memory depth of an individual we use the probability of finding a strategy of a certain memory depth shown in Fig. 4.3 on the right.

At the initial stage these probabilities are equal. During the evolution the probability of detecting strategies with zero memory depth falls to the certain stationary level of $P_{s0} \approx 0.15$.

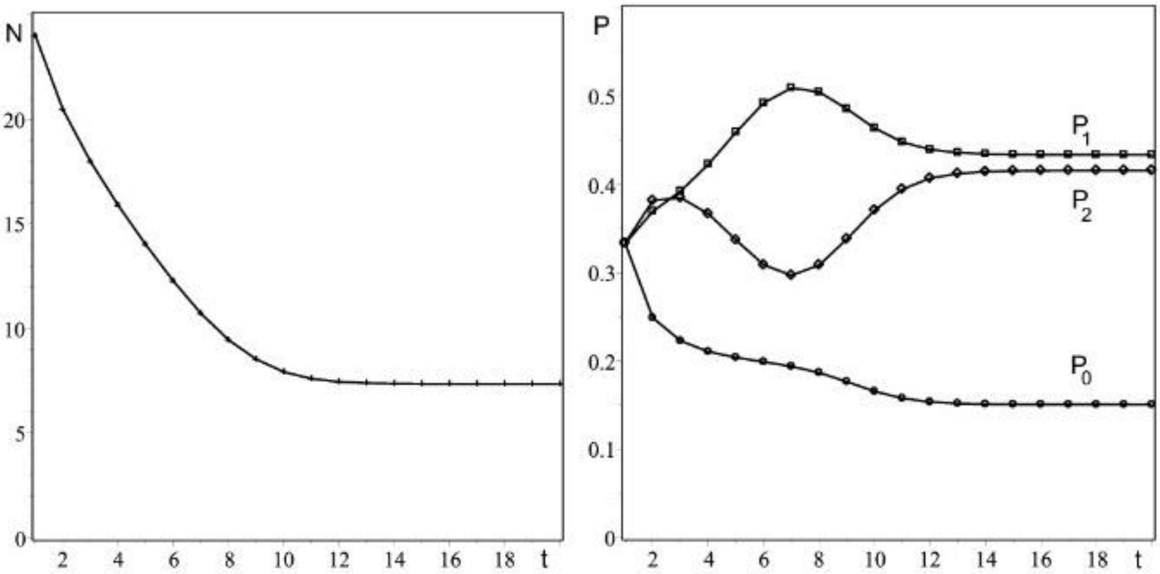

Fig.4.3 The change in the total number of strategies for an average individual (left) and the probability of finding the presence of a strategy with a certain memory depth (right), where P_0 is the probability of finding a strategy with the memory depth 0, P_1 with the memory depth 1, P_2 with the memory depth 2

The probability of detecting strategies with the memory depth of 1 increases to a certain value and then falls to the stationary value of $P_{s1} \approx 0.43$. In contrast to the previous case, strategies with the memory depth of 1 are dominant in a stationary state. Probability of detecting a strategy with the memory depth of 2 changes in a more complex way. There are two phases of increasing this probability divided by the phase of decreasing. Finally, it reaches the stationary value of $P_{s2} \approx 0.41$. It is interesting that the minimum value of the probability P_2 corresponds to the maximum value of the probability P_1. Therefore, these components compete with each other to be available for an individual. Thus, in the stationary state strategies with the memory depth of 1 and 2 are dominant with a significant presence of strategies with a zero memory depth. The stationary values for this distribution are radically different from the corresponding values found in the previous experiment. The nature of these probabilities has also changed dramatically. In that case, in the process of evolution the probability P_0 does not increase and the probabilities P_1 and P_2 exceed their initial level.

For a more detailed analysis of the behavior of an individual's strategies let us consider how the complexity of an individual and the complexity of strategies with a certain memory depth evolve. Fig. 4.4 shows the average complexity of an individual in the population and the change in the complexity components of strategies with a certain memory depth. It could be seen that two phases of increase in complexity of an average individual divided by the phase of decrease. The time period of complexity decrease can be called a period of decline or a primitive period of evolution. Periods of complexity increase can be called periods of growth. The first period of growth lasts 2 stages of evolution or 9% of the evolution time. The period of decline takes 4 stages or 18% of the evolution time. And the second period lasts till the stationary state, which takes 22 evolution stages or 72% of the evolution time. These periods correlate with the behavior of strategies with a certain memory depth. Thus, for example, probability of finding a strategy of the memory depth of 2 for an individual correlates with a change in complexity of the individual's strategies (Fig.4.3). The reason is related to the greater complexity of the strategies with the memory depth of 2. Note that the average complexity of an individual is significantly lower than in the previously considered case. This is due to the large number of strategies with the memory depth of 0 and 1 for an individual and a low initial complexity of individual's strategies. In the process of evolution, complex strategies dominate and therefore the stationary level of complexity ($\overline{C}_s \approx 4.3$) of an individual's strategies exceeds the initial ($\overline{C}|_{t=0} \approx 3.8$).

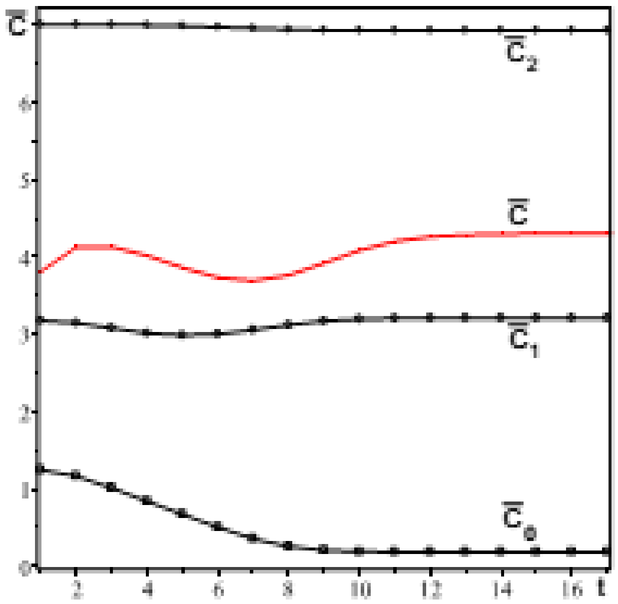

Fig.4.4 The evolution of the average complexity of an individual in the population $\overline{C}$ *and by memory depth.* $\overline{C}_0$ *is the complexity of individual's strategies with the memory depth 0,* $\overline{C}_1$ *for the memory depth 1,* $\overline{C}_2$ *for the memory depth 2*

A more detailed behavior of the average complexity of strategies for an individual can be obtained by plotting the probability of the presence of a strategy with a certain complexity at the certain time of evolution. The change in probability of finding a strategy with a certain complexity is quite complicated depending on the value of complexity (Fig. 4.5). Therefore, the strategies of the complexity 0, 4, 6, 7, 8 increase their presence in the process of evolution. The most primitive zero-complexity strategies take the third place in terms of probability after the strategies of the complexity 4 (maximum value) and the complexity 8. The strategies with the complexity 1, 2, 3, and 5 decrease their presence. The strategies of complexity 1 have completely disappeared from the population being the most non-aggressive ones.

As for the evolution of an individual's aggression in the population, Fig. 4.6 shows that the strategies with the memory depth 0 are more aggressive and the strategies with the memory depth 1 are less aggressive over the course of evolution. In terms of complexity, the strategies with the complexity 0 are most aggressive; the strategies with the complexity 1 are the most non-aggressive and disappear from the population after step 12. For the previous type of distribution, the strategies of the complexity 1 also disappear from the population and others tend to the maximum possible value of the aggression, which indicates the universality of these patterns. The aggression of the strategies of the complexity 0 is the greatest as well. The average aggression of an individual is determined by the aggression of the strategies of the memory depth 2 and the complexity 8 as shown on Fig. 4.6.

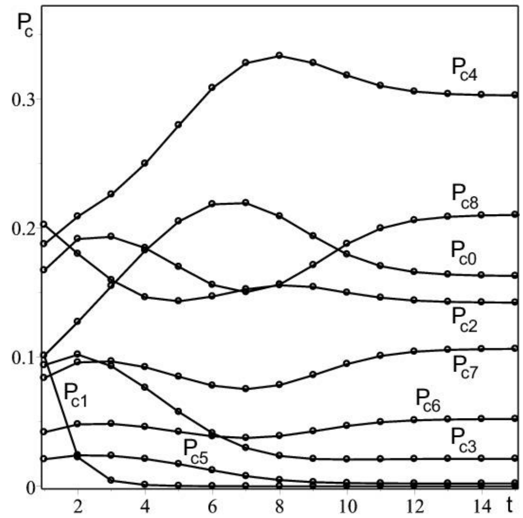

Fig.4.5 The probability of finding a strategy with a certain complexity for an average individual of the population ($P_1 \dots P_8$ *represents complexity from 0 to 8 respectively)*

Thus, in this case the stationary stage is also formed by the most aggressive individuals, whose strategies have the greatest complexity and memory depth. The presence of strategies of zero memory depth in the stationary state increases in comparison to the initial one by approximately 2 times. Moreover, it is the primitive strategies of zero complexity that have survived, while the less aggressive strategies of complexity 1 have disappeared from the population. Complex strategies ($C = 6,7,8$) have increased their presence, as well as those of complexity $C = 4$. Strategies with complexity $C = 1,2,3,5$ have decreased their presence.

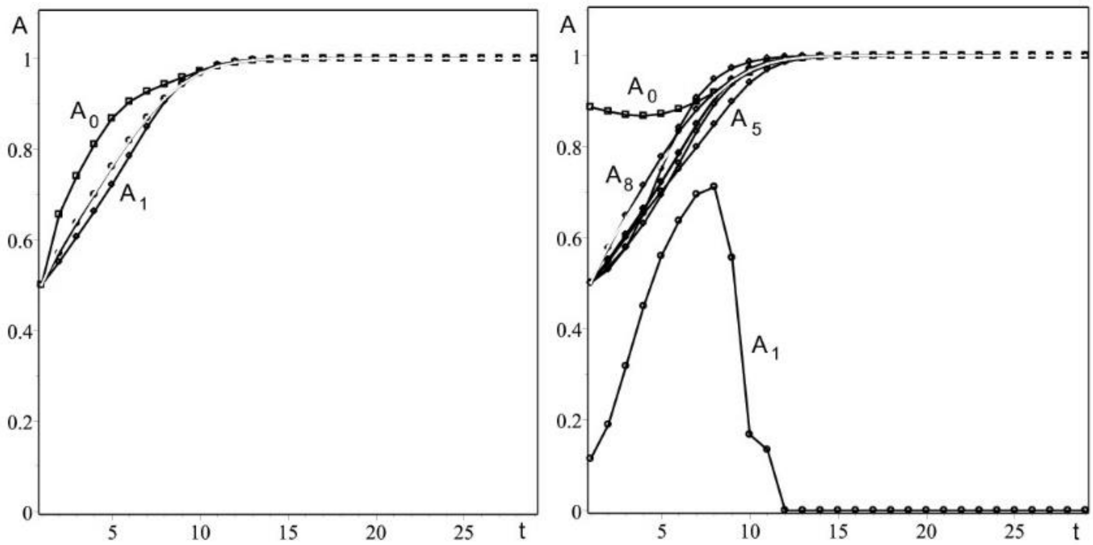

Fig.4.6 Aggression of the population by memory depth (left) and by complexity (right), where A *is the average aggression;* A_0, A_1, A_2 *are the aggressions for memory depth from 0 to 2;* $A_0, A_1, \dots, A_8$ *are the aggressions for complexities from 0 to 8*

Let us now discuss the effect of an increase in the number of strategies for an individual in case of initial distribution of strategies with equal probabilities by memory depth. Naturally, this will lead to an increase in the average strategy complexity of an individual because of the increased number of strategies of maximum complexity. Therefore, the stationary value of the average complexity of an individual's strategies will also increase. The dependence of its changes over time with the presence of two periods of growth and one period of decline will remain (quite similar to the example shown in Fig. 4.4).

The probabilities of finding strategies of a certain memory depth are a subject to more radical changes (see Fig. 4.3). For example, when choosing the initial number of strategies for an individual as 50, at all times of evolution the probability of finding a strategy of the memory depth 2 in it exceeds the probability of finding a strategy with the memory depth 1 in contrast to the case where the initial number of strategies is 24 (Fig. 4.3). Thus, the tendency for the dominance of strategies of maximum memory and complexity becomes ever more prominent with an increase in the number of its strategies.

5 Conclusion

Therefore, the evolution where the method of strategy exchange is used supports individuals with maximum aggression, maximum memory depth, and high complexity of strategies. The stationary set of an individual's strategies consists mainly of such strategies with addition of some most primitive strategies. The number of strategies of the average individual decreases with the evolution time reaching a certain stationary value, which depends on the initial distribution of strategies. The variety of strategies with evolution decreases more significantly: about 6% of the original formative strategies remains in the stationary state. The strategies of the complexity 1, being the least aggressive, disappear from the population. Complex behavior and periods of growth and decline in complexity appear with significant proportion of strategies with low complexity and low memory, otherwise these strategies do not affect the nature of evolution being suppressed at early stages by more complex strategies with a large memory depth.

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