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The calculation of the thermal stressed state of multilayer plates of a non-canonical shape

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The solution of the problem of stationary thermal elasticity of multilayer plates of a non-canonical shape in a plane is proposed. It is based on the immersion method, where a complex non-canonical shape is merged into the canonical shape. Therefore, a plate of a non-canonical shape with arbitrary boundary conditions is "immersed" into a canonical one. To ensure that the specified boundary conditions are met compensating loads distributed along the contour of the initial structure are added to the auxiliary structure. The intensities of compensating loads are determined from a system of integral equations. Deformation of the layers of the plates are described within the framework of the first-order theory, taking into account a transverse shear strain in each layer. The field of temperature loads is obtained by solving the non-stationary problem of thermal conductivity of a multilayer plate. The temperature stresses in a five-layer plate when heated by a film heat source have been investigated.

Keywords: *multilayer plate, complex shape, heat source, temperature, thermal elasticity*

Розрахунок термонапруженого стану багатошарових пластин неканонічної форми

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Однією з актуальних задач в сучасній техніці є достовірне визначення температурних полів і напружень в елементах конструкцій. Огляд моделей і методів розв'язання задач і термопружності показав, що найбільш досліджені однорідні конструкції. Пропонується метод розв'язання задачі стаціонарної термопружності багатошарових пластин неканонічної форми в плані, який базується на прийомі занурення складної області в область канонічної форми. Задана пластина неканонічної форми з довільними граничними умовами «занурюється» в пластину канонічної форми. Найпростіший розв'язок задачі в аналітичному вигляді можна отримати, коли у якості допоміжної застосовується шарнірно оперта пластина прямокутної форми у плані з тією ж композицією шарів. Умови навантаження допоміжної конструкції збігаються з умовами навантаження вихідної конструкції. Для забезпечення виконання заданих граничних умов до допоміжної конструкції додаються додаткові компенсуючі навантаження, що розподілені вздовж контуру вихідної конструкції. Інтенсивності компенсуючих навантажень визначаються з системи інтегральних рівнянь, в основі якої лежать вихідні граничні умови. Розв'язок системи одержано шляхом розвинення шуканих функцій у тригонометричні ряди у допоміжній області і вздовж контуру вихідної конструкції та подальшого розв'язання системи лінійних алгебраїчних рівнянь відносно коефіцієнтів розвинень. Після знаходження значень компенсуючих навантажень визначаються переміщення та напруження у шарах вихідної конструкції. Деформації шарів пластин описуються у рамках теорії першого порядку, що враховує деформації поперечного зсуву в кожному шарі. Рівняння термопружної рівноваги та граничні умови одержані з варіаційного принципу. Поле температурних навантажень отримано з розв'язку нестаціонарної задачі теплопровідності багатошарової пластини. На верхній та нижній поверхнях пластини відбувається конвективний теплообмін, а бічна поверхня вважається ідеально теплоізольованою. Досліджено температурні напруження у п'ятишаровій пластині складної форми при нагріванні плівковим джерелом тепла.

Ключові слова: *багатошарова пластина, складна форма, джерело тепла, температура, термопружність.*

Расчет термонапряженного состояния многослойных пластин неканонической формы

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Предлагается метод решения задачи стационарной термоупругости многослойных пластин неканонической формы в плане, который базируется на приеме погружения сложной области в область канонической формы. Пластина неканонической формы с произвольными граничными условиями «погружается» в пластину канонической формы. Для обеспечения выполнения заданных граничных условий к вспомогательной конструкции добавляются дополнительные компенсирующие нагрузки, распределенные вдоль контура исходной конструкции. Интенсивности компенсирующих нагрузок определяются из системы интегральных уравнений. Деформации слоев пластин описываются в рамках теории первого порядка, учитывает деформации поперечного сдвига в каждом слое. Поле температурных нагрузок получено в результате решения нестационарной задачи теплопроводности многослойной пластины. Исследованы температурные напряжения в пятислойной пластине при нагревании пленочным источником тепла.

Ключевые слова: *многослойная пластина, сложная форма, источник тепла, температура, термоупругость.*

1. Introduction

Widespread usage of multilayer structural elements in various fields of technology is associated with the need to develop the methods for their calculation at different loads, such as thermal effects. The analysis of literature sources shows that the homogeneous structures are the most studied ones [1–5]. The models and methods of solving heat conduction and thermal elasticity problems are reviewed in [6, 7]. The problems of thermal elasticity of multilayer plates and shells are most often solved for objects of canonical shape, and the law of temperature distribution over the thickness and surface of the structure is usually given, but not obtained from solving the problem of thermal conductivity of these structural elements [6–12].

In this work, we propose a method for solving the problem of thermal elasticity of multilayer plates of noncanonical shape in the plane under the influence of temperature fields, which are determined from the solution of the problem of stationary thermal conductivity.

2. Unresolved issues and goals of the work

From the analysis of publications on thermoelastic deformation of multilayer structural elements we can conclude that each of these research methods has its advantages and disadvantages which limit their scope. In addition, the deformation of multilayer plates is often considered by using the classical theory, although the plate may consist of layers whose physical and mechanical characteristics differ significantly. Therefore, the development of new methods and improvement of existing methods for calculating multilayer plates of a complex shape is of great importance.

3. Basic relations

Let us consider a multilayer plate assembled from I layers of constant thickness and assigned to the Cartesian coordinate system, which is connected to the outer surface of the first layer. On the coordinate surface the plate occupies an area G bounded by the contour L. The plate is subjected to a system of force and heat loads.

The behavior of the plate is described within the framework of a refined first-order theory that takes into account the shear strains [5, 13]. It is assumed that the contact between the layers prevents their delamination and mutual slippage. The displacements of the plate points are represented as

$$u^i = u + \sum_{j=1}^{i-1} h_j \psi_x^j + (z - \delta_{i-1})\psi_x^i, \quad v^i = v + \sum_{j=1}^{i-1} h_j \psi_y^j + (z - \delta_{i-1})\psi_y^i,$$
$$w^i = w, \quad \delta_i = \sum_{j=1}^{i} h_j, \quad \delta_{i-1} \le z \le \delta_i, \quad i = \overline{1, I} \tag{1}$$

where $u = u(x, y)$, $v = v(x,y)$, $w = w(x,y)$ are displacements of the coordinate surface point in the direction of the coordinate axes; $\psi_x^i = \psi_x^i(x, y)$, $\psi_y^i = \psi_y^i(x, y)$ are angles of rotation of a normal element in the *i*- th layer around the axes $0x$ and $0y$; h_j is the thickness of the *j*-th layer. The layer strains $\varepsilon_x^i, \varepsilon_y^i, \gamma_{xy}^i, \gamma_{xz}^i, \gamma_{yz}^i$ are determined according to the Cauchy formulas. Stresses and strains in the *i*-th layer are related by Hooke's law

$$\sigma_x^i = \frac{E_i}{1-\nu_i^2}\left(\varepsilon_x^i + \nu_i \varepsilon_y^i\right) - \frac{E_i}{1-\nu_i}\alpha_i^t T_i, \quad \sigma_y^i = \frac{E_i}{1-\nu_i^2}\left(\varepsilon_y^i + \nu_i \varepsilon_x^i\right) - \frac{E_i}{1-\nu_i}\alpha_i^t T_i,$$
$$\tau_{xy}^i = G_i \gamma_{xy}^i, \quad \tau_{xz}^i = G_i \gamma_{xz}^i, \quad \tau_{yz}^i = G_i \gamma_{yz}^i, \quad i = \overline{1, I}, \tag{2}$$

where $G_i = \frac{E_i}{2(1+\nu_i)}$,

E_i is the Young's modulus of the *i*- th layer material; ν_i is the Poisson's ratio; α_i^t is the coefficient of linear thermal expansion of the *i*-th layer material; T_i is the temperature change in relation to the temperature of the unstressed state.

Equilibrium equations for a multilayer plate are obtained on the basis of the Lagrange's variational principle

$$\boldsymbol{C}\ \boldsymbol{U} = \boldsymbol{Q}^t - \boldsymbol{Q}, \quad (x,y) \in G, \tag{3}$$

as well as the corresponding boundary conditions

$$\boldsymbol{\Gamma U} = \boldsymbol{Q}^L. \tag{4}$$

Here **Q** is the vector of force loads, **U** is the vector of the required displacement functions,

$$\boldsymbol{U} = \left\{u,\ v,\ w,\ \psi_x^i,\ \psi_y^i\right\}^T; \ \boldsymbol{Q}^t = \left\{C_{t,x}^I,\ C_{t,y}^I,\ 0,\ D_{t,x}^i,\ D_{t,y}^i\right\}^T; \ \boldsymbol{Q}^L = \left\{C_t^I,\ 0,\ 0,\ D_t^i,\ 0\right\}^T;$$
$$C_t^I = \sum_{i=1}^{I} N_t^i, \quad D_t^i = h_i \sum_{j=i}^{I-1} N_t^{j+1} + M_t^i;$$
$$N_t^i = \frac{E_i \alpha_i^t}{1-\nu_i}\int_{\delta_{i-1}}^{\delta_i} T_i dz, \ M_T^i = \frac{E_i \alpha_i^t}{1-\nu_i}\int_{\delta_{i-1}}^{\delta_i} T_i (z - \delta_{i-1}) dz, \quad i = \overline{1, I}.$$

The elements of the symmetric matrix **C** (3) and the matrix **Γ** (4) are given in [5, 11].

4. Solution method

The solution to the problem of thermal elasticity is based on the immersion method developed previously for solving the problem of unsteady dynamics of multilayer structures [14]. The initial multilayer plate of arbitrary shape in plane is immersed in an auxiliary enveloping multilayer plate with the same composition of layers. The shape of the enveloping plate is chosen in such a way that a simple analytical solution can be obtained. In this work a hinged rectangular multilayer plate is chosen as an auxiliary one. The loading conditions for the auxiliary plate in the area G coincide with the loading conditions for the initial plate.

To ensure the fulfillment of the actual boundary conditions (4), additional compensating forces and moments are applied to the auxiliary plate along the contour L $\boldsymbol{Q}^c = \{q_j^c(x,y)\}$, $(x,y) \in L$, $j = \overline{1,2I+3}$. The functions of compensating loads enter the equations of thermoelastic equilibrium (3) in the form of integral relations:

$$p_j^c(x,y) = \int_0^{s^*} q_j^c(x_L, y_L)\delta(x - x_L, y - y_L)ds, \quad j = \overline{1,2I+3}, \tag{5}$$

where $\delta(x - x_L, y - y_L)$ is the two-dimensional Dirac δ- function; L: $x_L = x(s)$, $y_L = y(s)$, $0 \le s \le s^*$; s is the current arc length; s^* is the plate perimeter.

Boundary conditions (4) taking into account (5) lead to a system of integral equations for determining the intensities of compensating loads

$$\boldsymbol{\Gamma U}[\boldsymbol{Q}^c(x,y)] = \boldsymbol{Q}^L, \ (x,y) \in L. \tag{6}$$

The method for solving system (6) is that the functions of displacements **U**, force **Q**, temperature $\boldsymbol{Q}^t$ and compensating $\boldsymbol{P}^c$ (5) loads are expanded into trigonometric series in functions that satisfy the conditions for the hinge support of the auxiliary rectangular plate

$$u_j(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\varphi_{jmn}B_{jmn}(x,y), \ q_j(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}q_{jmn}B_{jmn}(x,y),$$
$$q_j^t(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}q_{jmn}^t B_{jmn}(x,y), \ p_j^c(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}p_{jmn}^c B_{jmn}(x,y), \tag{7}$$
$$B_{1mn} = cos\frac{m\pi x}{A} sin\frac{n\pi y}{B}, \ B_{2mn} = sin\frac{m\pi x}{A} cos\frac{n\pi y}{B}, \ B_{3mn} = sin\frac{m\pi x}{A} sin\frac{n\pi y}{B},$$
$$B_{3+i,mn} = B_{1mn}, B_{3+I+i,mn} = B_{2mn}, \quad i = \overline{1,I}, j = \overline{1,2I+3};$$

A and B are geometric dimensions of the plate according to the coordinate axes.

At the same time, the functions of compensating loads and functions included in the boundary conditions (4) are expanded in a series along the contour L

$$q_j^c(s) = \sum_{\alpha=1,2}\sum_{\mu=0}^{\infty} f_{j\alpha\mu}d_{\alpha\mu}(s), u_j(s) = \sum_{\alpha=1,2}\sum_{\mu=0}^{\infty} u_{j\alpha\mu}d_{\alpha\mu}(s), \quad j = \overline{1,2I+3}, \tag{8}$$

where $d_{1\mu} = sin[\mu\gamma(s)]$, $d_{2\mu} = cos[\mu\gamma(s)]$, $\gamma(s) = 2\pi\int_0^s d\tilde{s} / \int_0^{s^*} d\tilde{s}$, $0 \le \gamma(s) \le 2\pi$.

As a result of the expansions (7), (8) and further transformations, the system of integral equations (6) is reduced to a system of linear algebraic equations for the coefficients of expansion into a series of functions of compensating loads$f_{j\alpha\mu}$, the solution of which makes it possible to determine the values of compensating loads. Finally, the solution to the problem (3), (4) can be represented as

$$u_j(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{2I+3}\eta_{jk}^{mn}\left[q_{jmn} - q_{jmn}^t + \sum_{\alpha=1,2}\sum_{\mu=0}^{\infty} f_{j\alpha\mu}\theta_{j\alpha\mu}^{mn}\right]B_{jmn}, \quad j = \overline{1,2I+3},$$

where η_{jk}^{mn} are the coefficients of the inverse matrix obtained by expanding the displacement functions from the equilibrium equations (3) into series in functions satisfying the conditions for the hinged support of the auxiliary rectangular plate;

$$\theta_{j\alpha\mu}^{mn} = \frac{4}{AB}\int_0^{s^*} d_{\alpha\mu}(s)B_{jmn}(x_L, y_L)ds \quad j = \overline{1,2I+3}.$$

5. Numerical results

The efficiency and effectiveness of the developed method is illustrated by solving the problem of thermal elasticity of a multilayer plate, the contour of which is made up of K line segments and K arcs of circles conjugated to them ($K = 4$). The sections of the contour s_{2k-1}, which are line segments, are given by the following equations:

$$x_L = x_{2k-1} + \left(S - S_{2(k-1)}\right)cos\,\alpha_{2k-1}, \ y_L = y_{2k-1} + \left(S - S_{2(k-1)}\right)sin\,\alpha_{2k-1}, \ k = \overline{1,K},$$

where the point with coordinates $(x_{2k-1}, \ y_{2k-1})$ is the beginning of $(2k-1)$-th straight line segment.

The sections of the contour s_{2k}, which are circular arcs, are given by the ratios:

$$x_L = x_{2k} + R_k\left[sin\left(\frac{S - S_{2k-1}}{R_k} + \alpha_{2k-1}\right) - sin\,\alpha_{2k-1}\right],$$
$$y_L = y_{2k} - R_k\left[cos\left(\frac{S - S_{2k-1}}{R_k} + \alpha_{2k-1}\right) - cos\,\alpha_{2k-1}\right], \ k = \overline{1,K},$$

where the point $(x_{2k}, \ y_{2k})$ is the end of $(2k-1)$- th straight line segment, α_{2k-1} is the angle between the $(2k-1)$-th straight line segment on the contour and the positive direction of the X-axis, S is the length of the contour section from the origin [point$(x_1,\ y_1)$] to the current point$(x,\ y)$ on this contour section; $S_k = \sum_{i=1}^{k} s_i,\ \ S_0 = 0$.

Fig. 1 shows the design scheme of the plate, $l_1 = 74$ mm, $l_2 = 53$ mm, $l_3 = 77$ mm, $l_4 = 60$ mm, $R_i = 30$ mm, $i = \overline{1,\ 4}$. The layers of the plate are made of materials with the following characteristics: $E_i = 6.8 \cdot 10^4$ MPa, $\nu_i = 0{,}22$, $\alpha_i^t = 9 \cdot 10^{-6}$°C^{-1}, $i = 1,\ 3{,}5$; $E_i = 2{,}2 \cdot 10^2$ MPa, $\nu_i = 0{,}38$, $\alpha_i^t = 8.3 \cdot 10^{-5}$°C^{-1}, $i = 2{,}4$; $h_1 = 5$mm, $h_2 = 3$mm, $h_3 = 19$mm, $h_4 = 2$mm, $h_5 = 12$mm.

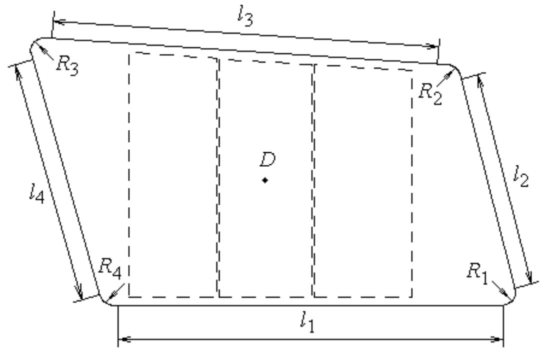

Fig. 1. Design scheme

It is assumed that there are no power loads. The field of temperature loads is obtained from the solution of the non-stationary problem of thermal conductivity of multilayer plates [15-17], taking into account the effect of a film heat source. Thermal insulation of the side surface of the plate is considered to be ideal. The heat conductivity problem has been solved with the following initial data: $k_i = 1.08$ W/(m·°C), $i = 1,\ 3{,}5$; $k_i = 0.22$ W/(m·°C), $i = 2{,}4$ (thermal conductivity coefficients of the i-th layer material); $H_1 = 433$ W/(m·°C), $H_2 = 20$ W/(m·°C) (convective heat transfer coefficients on the upper and lower surfaces of the plate); $T_1 = -30^oC$, $T_2 = 20^oC$ (environment temperature at the upper and lower surfaces boundary). A film heat source with a capacity of $q = 6$ kW/m^2 is located between the first and second layers of the plate. The source location is shown in Fig. 1 by a dashed line.

Fig. 2 shows the distribution of temperature and the principal stress σ_1^i ($i = \overline{1, I}$) over the thickness of the plate at point *D* (see Fig. 1), which located in the middle of the region occupied by the heat source. The stress is obtained for the temperature distribution at the time when the temperature on the surface with the source reaches the highest value. Layer composition is also shown. At the interface between the first and second layers of the plate, a sharp change in temperature and stress caused by the presence of a heat source is observed.

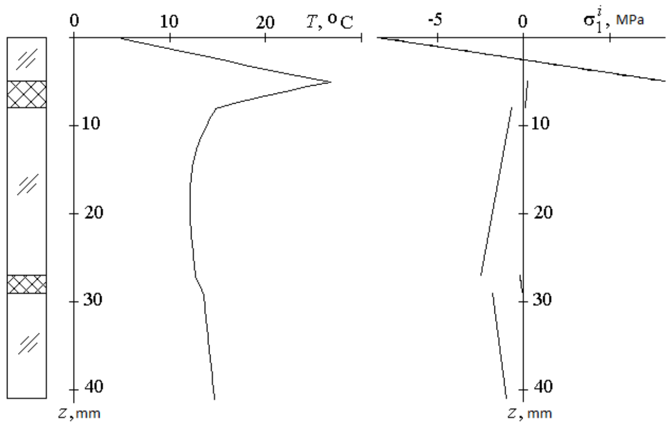

Fig. 2. Distribution of temperature and stress over the thickness of the plate.

6. Conclusions

A method for solving problems of thermal elasticity of multilayer plates of non-canonical shape in plane has been developed, which makes it possible to present the solution of the problem in an analytical form.

Deformation of plates is described within the framework of the linear refined first-order theory which accounts for transverse shear strain in each layer along with the polygonal line hypothesis for a pack. The thermal elastic equilibrium equations and the boundary conditions on the contour are obtained by using Lagrange's variational principle. The method of solving the thermal elasticity problem for multilayer plates with non-canonical shape is based on the immersion method.

The proposed approach can be applied in the design of heating systems for multilayer glazing in various vehicles.

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