DOI: 10.26565/2304-6201-2020-48-08

УДК 593.3

Simulation of liquid movement in cylindrical shells

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The motion of a viscous incompressible fluid with constant coefficients in case of the Hagen-Poiseuille flow is considered in this paper. The equation of motion of the elastic shell in the absence of external perturbations is obtained on the basis of the Ostrogradsky-Hamilton principle. Assuming the Hagen-Poiseuille flow, the expression for the kinetic energy of a moving fluid in a nanotube is obtained, and the equations to determine oscillation frequencies of a tube with a fluid are constructed, that allows investigating the stability of motion. Using the Poiseuille formula it is possible to effectively determine the movement of fluid in tubes. According to the Bernoulli's equation, when the fluid is stationary along a rectilinear horizontal tube of a constant cross-section, the fluid pressure must be the same along the entire length of a tube.

In recent years, a new trend associated with nanotechnology has been developing in hydromechanics. The relevance of modeling fluid flow through micro- and nanotubes is confirmed by the results of many experiments conducted over two decades. A nanotube can be represented as a graphite plane rolled into a cylinder (hollow tube), which is a set of regular hexagons with carbon atoms in the vertices, and having the diameter of several nanometers. The fluid flow through micro and nanotubes is a common phenomenon in various biological and technical devices and systems and therefore is of great importance. Consequently, flows in nanometer-sized channels are being studied intensively. The numerical values of the oscillation frequencies of the elastic cylindrical shell for the cases of absence of fluid and the presence of fluid with different pressure values have been obtained.

Отримані числові значення частот коливань пружної циліндричної оболонки без урахування наявності рідини, а також з рідиною при різних значеннях перепаду тиску.

Keywords: *nanotubes, ideal liquid, boundary conditions, incompressible fluid, Hagen-Poiseuille flow.*

Моделювання руху рідини в циліндричних оболонках

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В роботі розглянуто рух в'язкої нестисливої рідини в пружній циліндричній оболонці. Рівняння руху пружної оболонки за відсутністю зовнішніх збурень отримано на основі принципу Остроградського – Гамільтона. За припущеннями наявності течії Хагена – Пуазейля отримано вираз для кінетичної енергії рухомої рідини в нанотрубці та побудовані рівняння для визначення частот коливань трубки з рідиною, що дає змогу дослідити стійкість руху. Отримані числові значення частот коливань пружної циліндричної оболонки без урахування наявності рідини, а також з рідиною при різних значеннях перепаду тиску.

Ключові слова: *нанотрубки, ідеальна рідина, граничні умови, нестислива рідина, течія Хагена – Пуазейля.*

Моделирование движения жидкости в цилиндрических оболочках

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В последние годы, в гидромеханике получает развитие новое направление, связанное с распространенным применением нанотехнологий. Движение жидкости по трубкам является очень распространенным явлением в природе и технике. В работе рассмотрены движение вязкой несжимаемой жидкости в упругой цилиндрической оболочке. Уравнения движения упругой оболочки при отсутствии внешних возмущений получено на основе принципа Остроградского - Гамильтона. В предположениях, характеризующих течение Хагена – Пуазейля получено выражение для кинетической энергии движущейся жидкости в нанотрубке, построены уравнения для определения частот колебаний трубки с жидкостью, что позволяет исследовать устойчивость движения.

Ключевые слова: *нанотрубки, идеальная жидкость, граничные условия, несжимаема жидкость, течение Хагена - Пуазейля.*

1 Introduction

The movement of fluid is a phenomenon inherent in many technical processes. It is known that a stationary liquid in a vessel transmits external pressure in all directions according to Pascal's law, and conditions at all points of the volume of the liquid medium are constant and identical. However, if the fluid flows through the pipe without friction, the cross-sectional area varies in different areas. The pressure is different along the pipe lengths [1]. There is an effective way to determine the movement of fluid in tubes by using the Poiseuille formula. According to the Bernoulli's equation, if the fluid is stationary along a rectilinear horizontal tube of constant cross-section, the fluid pressure must be the same along the entire length of the tube. But in reality the fluid pressure drops along with the direction of its movement. To use the Poiseuille approach it is necessary to ensure stationarity, namely, to maintain a constant pressure difference at the ends of the tube, balancing the forces of internal friction that occur during the movement of the fluid [2, 3].

In recent years, a new trend associated with nanotechnology has been developing in hydromechanics. The relevance of modeling fluid flow through micro- and nanotubes is confirmed by the results of many experiments conducted over two decades. A nanotube can be represented as a graphite plane rolled into a cylinder (hollow tube), which is a set of regular hexagons with carbon atoms in the vertices, and having the diameter of several nanometers. Nanotubes can consist of a single layer of atoms - single-walled nanotubes (SWNT), or several "nested" single-layer tubes - multi-walled nanotubes (MWNT). It has been found that if the size of volume is about 10 molecular diameters or less, the liquid behaviour changes significantly. The studies show that in a microtube with a diameter of 50 nm the flow is continuous but in a microtube with a diameter of 5 nm it is not. The high value of the elasticity modulus of carbon nanotubes makes it possible to create composite materials that could be extremely resistant to ultra-high elastic deformations.

2 The Poiseuille flow simulation.

Stating that molecules in a microtube form a solid wall and atoms in the wall are subjected to chaotic motion at a finite wall temperature, there are the following peculiarities of the Poiseuille flow: in the intermediate range of the Knudsen numbers, liquid particles slide into the walls. Three possible cases are described in [4]:

1) the liquid is stationary (no slippage);
2) there is a flow with slippage;
3) there is no friction.

For the classical hydrodynamics, Navier (C.L.M.H.Navier, 1823) proposed a boundary condition that took into account the slippage of a liquid along the surface a solid body. O. Girard used this model to describe his experimental data [5].

According to the Navier boundary condition, the value of the liquid slip rate is proportional to the velocity gradient on the wall:

$$\vartheta\big|_{z=R} = L_s \left.\frac{d\vartheta}{dz}\right|_{z=R} \qquad (1)$$

Scientists from the University of Wisconsin-Madison (USA) have been able to prove that the laws of friction for nanostructures do not differ from the classical laws [6].

The inner surface of the nanotube is considered to be smooth even on the atomic level. But the macroscopic surfaces of porous materials are rough on a much smaller level. Therefore, the gas that propagates through the inner cavity of the nanotube meets significantly less surface resistance. Moreover, not only gaseous but liquid substances can pass through the nanotubes as well. The experiments have proved that the liquid carrying capacity of nanotubes is more than three orders of magnitude higher than the corresponding value obtained on the basis of the classical the Hagen-Poiseuille formula. This effect could be explained by the complicated nature of the fluid interaction with the inner walls of the membrane. The membrane is a part of a filter element, which is a nanotube. A liquid slides along the surface of a nanotube; therefore, the traditional boundary conditions (according to which the speed of movement on the wall is zero) are no longer satisfied. On a manometer scale, the liquid exhibits atypical properties, such as a sharp increase in viscosity and density near the walls of nanocapillaries, the changes in thermodynamic parameters, and the atypical chemical activity at the boundary between solid and liquid phases. Experiments have shown a significant increase in effective viscosity in such volumes compared to macroscopic values. It is determined that the effective viscosity of a liquid in a nanotube depends on its diameter.

In classical hydrodynamics, the atomic (molecular) structure of a liquid is not taken into account, and the liquid motion in nanochannels with a diameter of 10 nm and smaller is not described. One of the main problems is that the laws of classical physics are not applicable in the nanotechnology.

At present, the main methods of studying the flow of fluid in nanotubes are the molecular dynamics and continuum mechanics. The method of modeling molecular dynamics is an effective method, but it requires a great amount of computational time, especially for large systems. Therefore, it is expedient to model large systems by the method of continuum mechanics [7], [8], [9].

During the physical adsorption of the substance on the inner and outer surfaces of the tube, the density of a layer formed can be similar to the density of a condensed state of the substance. Thus, on the one hand, the tube could be as a container for storing aggressive substances without using common thick-walled vessels. On the other hand, the properties of the tubes themselves will be modified by the absorbed particles, and that allows creating various heterostructures. [10].

The fluid flow through micro and nanotubes is a common phenomenon in various biological and technical devices and systems and therefore is of great importance. Consequently, flows in nanometer-sized channels are being studied intensively. At the first stage it is necessary to investigate the adequacy of the classical models of shells.

3 Problem formulation and method of solution

The equations of hydrodynamics are nonlinear, even at constant coefficients. Due to the fact that the inertial terms of the equation include the square velocity, relatively few exact solutions are known. Therefore, it is necessary to find such approximate solutions that would be a set of basic currents present in technological processes.

As the first approximation, we will consider the Hagen-Poiseuille flow in an elastic cylindrical tube. Let us present the value G, which characterizes the volumetric flow rate of the liquid

$$G = \int_0^R U(r) 2\pi r dr = \frac{\pi R^4 \Delta p}{8 \mu L} \qquad (2)$$

where υ is a speed, R is a tube radius, L is a tube length, μ is a dynamic viscosity of the liquid, Δp is a pressure drop.

Using the Hagen-Poiseuille formula (2), we can find the average cross-sectional velocity in the pipe

$$U_{cep} = \frac{G}{\pi R^2} \quad (3)$$

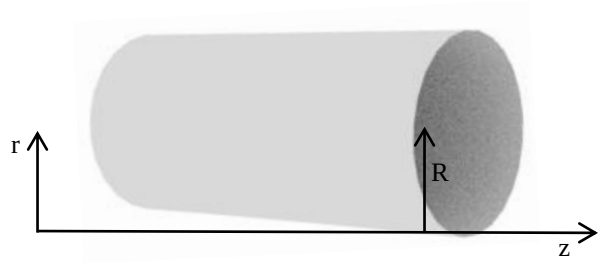

Figure 1. Coordinate system for the Hagen - Poiseuille flow

Here is a layered flow with a single non-zero z-velocity component, namely:

$$U_z = U \neq 0 \quad (4)$$

A characteristic feature of layered flow is the presence of the only one non-zero velocity component. This means that the trajectories of all fluid particles are co-aligned with one of the coordinate lines [11].

The equation of the elastic shell motion in the absence of external perturbations can be obtained on the basis of the Ostrogradsky-Hamilton principle:

$$\int_{t_1}^{t_2} \left(\delta\Pi - \delta T\right) dt = 0 \quad (5)$$

where Π, T- potential and kinetic energy.

Let $U = (Ur, Uz, U_\theta)$ be a shell displacement vector in the cylindrical coordinate system r, z, θ. Let us present the vector of displacement in the form an expansion into Fourier series by a circumferential coordinate

$$\boldsymbol{U}(r,z,\theta) = \sum_{\alpha=0}^{\infty} \boldsymbol{u}_\alpha(r,z)\boldsymbol{C}_\alpha(\theta) \quad (6)$$

where

$$\boldsymbol{C}_0 = \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ \boldsymbol{C}_\alpha = \begin{bmatrix} \cos\alpha\theta & 0 & 0 \\ 0 & \cos\alpha\theta & 0 \\ 0 & 0 & \sin\alpha\theta \end{bmatrix}, \ \alpha = 1, 2, ...\infty$$

The potential strain energy can be represented as follows:

$$\Pi = \frac{1}{2}\iint_S \sigma_{ij}\varepsilon_{ij} dS_1 dS_2, \quad (7)$$

where σ_{ij}, ε_{ij} are components of tension and strain tensors.

The expressions:

$$dS_1 = A_1\, d\alpha_1, \ dS_2 = A_2\, d\alpha_2 = r d\theta \quad (8)$$

are the differentials of the arcs of the median surface along the directions of the coordinate lines α_1 and α_2.

For the kinetic energy of a shell that does not contain a liquid, we have the expression:

$$T_s = \frac{1}{2}\iint_S \rho_s \left(\dot{U}_r^2 + \dot{U}_z^2 + \dot{U}_\theta^2\right) dS_1 dS_2. \quad (9)$$

In the presence of the Hagen-Poiseuille flow, the expression for the kinetic energy of a moving fluid in a shell which experiences small oscillations has the form:

$$T_f = \frac{1}{2}\iint_S \rho_f \left(\dot{U}_r^2 + \left(\dot{U}_z + U_{cep}\frac{\partial U_z}{\partial z} \right)^2 + \dot{U}_\theta^2 \right) dS_1 dS_2 \quad (10)$$

where the quantities ρ_s , ρ_f are the densities of the shell material and the liquid, respectively, [12].

The overall kinetic energy of the system is equal to

$$T = T_s + T_f . \quad (11)$$

The equation of the shell motion has the form

$$\mathbf{LU} + \mathbf{M}\ddot{\mathbf{U}} + \mathbf{D}\dot{\mathbf{U}} = \mathbf{Q} \quad (12)$$

where **L** is an elastic force operator, **M** is a mass operator, **D** is a damping operator, **Q** is a vector describing external influences.

We will look for a solution of the system of equations (12) in the form

$$\mathbf{U} = \sum_{k=1}^{\infty} c_k(t)\mathbf{u}_k , \qquad \mathbf{u}_k = (u_r^k, u_\theta^k, u_z^k), \quad (13)$$

where $c_k(t)$ are unknown coefficients that depend only on time, $\boldsymbol{U_k}$ are modes of oscillations of an elastic unfilled shell. $c_k(t)$ can be considered as generalized coordinates.

Substituting the series (13) into equation (12) and taking into account the expressions obtained by using the Hamilton-Ostrogradsky principle and the formula (11), we obtain

$$\sum_{k=1}^{\infty} c_k(t)\mathbf{L}(\mathbf{u}_k) + \sum_{k=1}^{\infty} \ddot{c}_k(t)\mathbf{M}(\mathbf{u}_k) + \sum_{k=1}^{\infty} \dot{c}_k(t)\mathbf{D}(\mathbf{u}_k) = \mathbf{Q}. \quad (14)$$

Let us perform the scalar product of equation (14) on $\boldsymbol{U_j}$, j=1,2,…

Let us use the relations (13)

$$\mathbf{L}(\mathbf{u}_k) = \Omega_k^2 \mathbf{M}(\mathbf{u}_k), \quad (15)$$

in which Ω_k is an oscillation frequency of the unfilled shell.

From (15) we obtain

$$(\mathbf{L}(\mathbf{u}_k), \mathbf{u}_j) = \Omega_k^2 (\mathbf{M}(\mathbf{u}_k), \mathbf{u}_j) \quad (16)$$

Taking into account the formulas (9), (10) for the kinetic energy of the fluid and the expression (13) the expression ($\mathbf{D}(u_k)$, u_j) takes the following form

$$(\mathbf{D}(\mathbf{u}_k), \mathbf{u}_j) = \frac{R}{2}\rho_f U_{cep}\left(u_z^j, \frac{\partial u_z^k}{\partial z} \right).$$

Taking into account the above relations, the equation (14) takes the form

$$\ddot{c}_k(t)\left(\rho_s h + \rho_l \frac{R}{2} \right)\delta_{kj} + \frac{R}{2}\rho_s U_{cep}\sum_{k=1}^{\infty} \dot{c}_k(t)\left(u_z^j, \frac{\partial u_z^k}{\partial z} \right) + \Omega_k^2 \rho_s h \delta_{kj} c_k(t) = (\mathbf{Q}, \mathbf{u}_j), \quad (17)$$

where h is shell thickness.

To solve the problem of eigenvalues (determination the natural frequencies of oscillations of the shell with a liquid) we assume that

$$Q = 0, \quad c_k(t) = C_k \exp(\omega t), \quad C_k = const .$$

Therefore, we obtain the equation for determining the oscillation frequencies ω and unknown coefficients C_k, which determine the modes of oscillations, in the form of

$$\omega^2\left(1 + \frac{\rho_f R}{2\rho_s h} \right)\delta_{kj} + \Omega_k^2 \delta_{kj} C_k + \omega U_{cep}\sum_{k=1}^{\infty} D_{kj} C_k = 0 , \; D_{kj} = \frac{\rho_f R}{2\rho_s h}\left(u_z^j, \frac{\partial u_z^k}{\partial z} \right). \quad (18)$$

We reduce (18) to the standard problem of eigenvalues. Let us limit to a finite number of *n* members of the series in (13).

We introduce the vector **X** in the form of

$$\mathbf{X}= (\mathbf{C}, \omega\mathbf{C}) \tag{19}$$

(18) could be rewritten in a matrix form

$$\omega^2 \mathbf{BC}+\mathbf{\Omega C} + \omega U_{cep} \mathbf{DC} =0, \tag{20}$$

where the following order matrices are used (n×n).

$$\mathbf{B}=\left(1+\frac{\rho_f R}{2\rho_s h}\right)\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ \dots & \dots & \dots & \dots\\ 0 & 0 & 0 & 1\end{pmatrix},\ \mathbf{\Omega}=\begin{pmatrix}\Omega_1^2 & 0 & 0 & 0\\ 0 & \Omega_2^2 & 0 & 0\\ \dots & \dots & \dots & \dots\\ 0 & 0 & 0 & \Omega_n^2\end{pmatrix}$$

The expression for the matrix **D** is given in the formula (18).

From the equation (20) we obtain

$$\omega^2 \mathbf{BC}= - \ (\mathbf{\Omega} + \omega U_{cep} \mathbf{D})\mathbf{C},$$

$$\omega^2 \mathbf{C}= - (\mathbf{B}^{-1} \mathbf{\Omega} + \omega U_{cep} \mathbf{B}^{-1} \mathbf{D})\mathbf{C}. \tag{21}$$

We introduce a dimensional matrix *2n×2n*

$$\mathbf{A}=\begin{pmatrix}0 & \mathbf{E}\\ -\mathbf{B}^{-1}\mathbf{\Omega} & -U_{cep}\mathbf{B}^{-1}\mathbf{D}\end{pmatrix}.$$

From (19), (20) we have the equality

$$\begin{pmatrix}0 & \mathbf{E}\\ -\mathbf{B}^{-1}\mathbf{\Omega} & -U_{cep}\mathbf{B}^{-1}\mathbf{D}\end{pmatrix}\begin{pmatrix}\mathbf{C}\\ \omega\mathbf{C}\end{pmatrix}=\begin{pmatrix}\omega\mathbf{C}\\ \omega^2\mathbf{C}\end{pmatrix}=\omega\begin{pmatrix}\mathbf{C}\\ \omega\mathbf{C}\end{pmatrix}.$$

Therefore, the standard problem of natural oscillations is obtained

$$\mathbf{AX}= \omega\mathbf{X} . \tag{22}$$

In contrast to the equations used in [13, 14], the characteristic equation in (22) can have complex natural frequencies. If the real parts of such roots are positive, there is a stability loss. The described approach can be used to study the stability of the elastic tubes with a fluid.

4 Analysis of the numerical results

The elastic cylindrical shell with the following geometric and physical parameters is considered: *R=0.05m, L=3m, μ=1.004·10^{-3}Pa·s, Δp=0.5Pa, h=0.005m*, the Young's module *E=20000MPa*, the Poisson's ratio *υ=0.3*, p_f *=1000kg/m^3* , p_s*=7800kg/m^3*. We suppose that the shell is rigidly fixed at *z =0*.

Table 4.1 shows the values ω of the first five oscillation frequencies for the empty elastic shell, for the shell filled with the stationary liquid (Δp=0), and for the case of the shell with moving fluid (Δp=0.5Pa).

Table 4.1. Shell oscillation frequencies

ρ_f, *kg/m3*	Δp,Pa	*n*=1	*n*=2	*n*=3	*n*=4	*n*=5
0	0	261.699	422.243	785.098	1266.42	1308.50
1000	0	181.964	293.579	545.895	880.595	909.827
1000	0.5	(0,181.964*i*)	(-1.8580, 293.579*i*)	(0,545.895*i*)	(-1.3854, 880.595*i*)	(0,909.827*i*)

From the data we can see that the complex frequencies appear in the case of moving fluid. The real parts of these frequencies are negative, i.e. the oscillations are damped. The presence of fluid leads to a decrease in the frequency of the shell oscillations.

5. Conclusions

The motion of the viscous incompressible fluid in the elastic cylindrical shell has been considered in the paper. The expressions for kinematic and potential energy have been derived. The equations for determining the frequencies of the fluid oscillation according to the assumptions of the Hagen-Poiseuille flow have been obtained. The frequencies of the shell oscillation have been determined taking into account the presence of moving fluid. The approach to studying the elastic shell stability in case of fluid motion has been developed.

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Надійшла у першій редакції 07.10.2020, в останній - 12.11.2020.