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Mathematical model of condition-based preventive maintenance of a complex technical system

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A new approach to mathematical modeling of complex technical systems according to their condition is being developed. Connections between subsystems of a complex system are considered to be arbitrary in terms of reliability. Due to wear, failures of subsystems can happen at random moments of time. Failures of some subsystems can lead to the entire system failure. The purpose of the simulation is to maintain the level of reliability and operability of a complex technical system at an optimal level for an unlimited time interval by means of regular preventive maintenance and repair. Technical instructions and specifications, as well as statistical data, are used in modeling a priori characteristics of subsystems. That information is used to determine the reliability of a complex system and its condition. The mathematical model is built in terms of the Markov decision-making process. The chosen optimization method allows obtaining the best policy for choosing acceptable preventive maintenance policy and repairs at the planned time of inspections and moments of failures.

Key words: *mathematical model, complex system, technical system wear, system operability, system state, control strategy*.

Математична модель профілактики складної технічної системи за станом

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Розвивається новий підхід в математичному моделюванні складних технічних систем з урахуванням їх станів. Розглядаються складні системи, що складаються з кінцевого числа, взагалі кажучи, різних простих підсистем. Допускається, що в сенсі надійності підсистеми з'єднані між собою довільно. Усі підсистеми в процесі роботи зношуються, ймовірність їх відмови збільшується. Відмова однієї або декількох підсистем не обов'язково призводить до відмови усієї системи. Якщо відмова складної системи відбувається, то це призводить до великих втрат із-за її простою і витрат на відновлення. У статті розглянуто випадок роботи системи на необмеженому інтервалі часу з регулярними профілактичними обслуговуваннями і ремонтами в моменти відмов. Допускається безліч різних видів обслуговувань і ремонтів. Метою моделювання системи є знаходження оптимального правила вибору виду обслуговування в планові моменти контролю системи та виду ремонту в моменти відмов, з урахуванням спостережуваних станів, на необмеженому інтервалі часу. Технічні описи та статистичні дані визначають оцінку функції надійності кожної підсистеми. Функції надійності підсистем і інформація про їх з'єднання між собою в сенсі надійності визначають оцінку функції надійності складної системи. Стан системи в момент контролю визначається значеннями параметрів підсистем і функцією надійності системи. Таке визначення стану забезпечує оцінку ймовірності відмови і продуктивності в кожен момент контролю для конкретної розглянутої системи. Використання стану в моделі профілактики є актуальним, оскільки враховуються особливості, ступінь зносу конкретної системи. У існуючих моделях профілактики системи, побудованих на основі "напрацювання на відмову", для прийняття рішень з підтримки оптимального рівня надійності використовуються характеристики, отримані їх усередненням по цілому ансамблю однотипних систем. Зрозуміло, що при цьому особливості конкретної системи не враховуються. Вартість відновлення системи після відмови часто залежить від набору підсистем, які привели її до відмови. У пропонованій статті розглянуто можливість знаходження і обліку таких наборів. Модель побудована в термінах марківського процесу прийняття рішень. Пропонується використати відомий метод оптимізації, заснований на принципі стислих відображень. Цей метод забезпечує знаходження правила вибору профілактик і ремонтів для необхідного рівня ефективності роботи системи. Деякі ключові моменти побудови моделі проілюстровані на конкретному прикладі складної системи.

Ключові слова: *математична модель, складна система, знос технічної системи, працездатність системи, стан системи, стратегія управління.*

Математическая модель профилактики сложной технической системы по состоянию

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Развивается новый подход в математическом моделировании сложных технических систем по состоянию. Подсистемы, рассматриваемой сложной системы, соединены между собой в смысле надежности произвольно. В результате износа допускаются отказы подсистем в случайные моменты времени. Отказы некоторых подсистем могут приводить к отказу всей системы. Целью моделирования является поддержание уровня надежности и работоспособности сложной технической системы на оптимальном уровне на неограниченном интервале времени с помощью регулярных профилактических обслуживаний и ремонтов. Технические инструкции и описания, а также статистические данные, использованы при моделировании априорных характеристик подсистем. Эта информация используется в модели при определении надежности сложной системы и ее состояния. Математическая модель построена в терминах марковского процесса принятия решений. Выбранный метод оптимизации позволяет получить наилучшую стратегию выбора допустимых профилактических обслуживаний и ремонтов в плановые моменты контроля и в моменты отказов. Некоторые ключевые и громоздкие элементы построения модели проиллюстрированы на примере выбранной сложной технической системы.

Ключевые слова: *математическая модель, сложная система, износ технической системы, работоспособность системы, состояние системы, стратегия управления*.

1 Introduction

The method for constructing a mathematical model of a complex technical system in order to optimize the level of its reliability and performance by means of preventive maintenance and repair for an unlimited time interval is discussed in the article.

The class of the systems under consideration is limited by the following conditions. The system consists of a finite number of subsystems. All subsystems in a complex system are used to achieve the same goal. All subsystems wear out during operation, their level of functional quality decreases, and the failure probability increases. Subsystem failure leads to its shutdown and following recovery costs. We assume that in terms of reliability the subsystems are interconnected arbitrarily and function independently of each other. Therefore, the failure of one or more of them does not necessarily lead to the complex system failure. But system failure leads to long downtime and could be very expensive. It follows that the substantiation of the optimal policy for choosing preventive maintenance at the planned inspection time and at the moments of system failure is an important and urgent task. A solution to this problem is proposed in this article.

The existing methods of optimizing the policy of preventive maintenance and repair of technical systems are based on such concepts as “mean time between failures”, “failure rate”, “availability factor”. The solutions based on these characteristics are presented in such well-known works as [1,2,3,4]. Estimations of these characteristics are obtained by processing statistical information from the observation of a wide range of similar systems. It is clear that such estimations can characterize only a generic system and cannot take into account the specifics of a current condition of a specific system. In addition, it is problematic to obtain a representative statistical sample of a sufficiently complex system.

There are some definitions of condition of a complex system which have two values and do not take into account the structure [5]. Other definitions can, for example, introduce a logical variable for identifying the subsystem condition or divide the system operating time into three stages: “burn-in”, “normal work”, “wear-out” and assign the condition to each stage. But there is a difficulty in identifying the condition of a working system. In any case, only generic solutions are available to maintain reliability at a given level, which, obviously, could not be high enough.

A possible option of a system condition is proposed in [6, 7]. The condition in the model is introduced as the value of a random process, the realizations of which are functionally related to the failure probability. The dependence of the failure probability on the condition is estimated according to statistical data. For the convenience of obtaining this dependence, a random process with monotonically increasing trajectories is chosen. All these actions are important for constructing not a generic but specific model. The disadvantage of this approach is that it is impossible to properly define the system condition at the moment of inspection and therefore to make an effective decision.

The model proposed below is a condition-based one. When determining the condition, the following requirements have been accounted for. The assessment of specific system condition in accordance with the observed values of the parameters as well as the available a priori information for each subsystem should be available at any inspection time. It should be noted that the construction of a condition-based mathematical model of system reliability are described as a promising one in [1]. The proposed condition-based model of a system is more suitable for solving the problem of preventive maintenance of a complex system than the ones mentioned above in a brief review of existing approaches, since it takes into account the specific features of a system under consideration. But this method requires larger amount of statistical information.

The system condition forms the basis for the mathematical model of a complex system. The system condition should include the degree of wear of each subsystem, the failure probability of each subsystem, and the interconnection of subsystems in terms of reliability. System failure is allowed. Therefore, reducing the probability of system failure is one of the main tasks for improving the efficiency of the system. It is assumed that at the moment of system failure one of the possible actions is applied. The chosen action ensures that all failed subsystems are restored. Additionally, the chosen action determines one of the subsystems to be updated reducing the likelihood of its failure. We assume that the duration of an emergency shutdown depends on the set of failed subsystems that has caused the failure of a complex system. If there is no failure, the system is stopped for maintenance after operating for a time interval τ specified by the regulations. The duration of the stop is specified by the regulations as well. One of possible actions is applied to ensure the restoration of all failed subsystems to an operating condition, and one of the subsystems determined by the selected action is updated.

When constructing a model, a procedure for determining the basic sets of subsystems the failure of which causes the complex system failure is proposed. We assume that the cost of restoring a system depends on the specified set and the costs of restoring its subsystems, as well as the costs of actions and downtimes are known.

The model is supposed to use certain statistics. All of them refer to the operation of subsystems on time intervals of duration τ. Statistical requirements and their usage will be considered during the model construction process. Real statistical information may be incomplete; therefore it is allowed to use expert assessments for the model construction. The system operation being observed, the missing information can be supplemented and the model refined.

The constructed model allows us to determine the optimal policy for choosing actions at planned and unplanned moments of inspection (action policy). This means that at each moment of inspection all failed subsystems will be restored, and the subsystem for updating will be selected. These actions provide an optimal level of reliability and system performance for an unlimited time interval under the specified conditions and constraints.

The proposed work considers a complex system consisted of simple subsystems. The analyst determines whether the system is simple or complex taking into account the purpose of modeling, the chosen modeling methods, available information about the system, and the required degree of adequacy. To a large extent, the adequacy of the model of a complex system depends not only on the available information, but on the analyst's skill as well.

2 Mathematical model

The purpose of constructing a mathematical model is to optimize the reliability and performance of a complex technical system operating for an unlimited time interval. The model will be based on the system condition. The condition can be assessed at every moment of inspection. It characterizes the performance of the system and the failure probability. System failure as a result of wear of subsystems during operation is allowed as well as its restoration to working condition. Preventive maintenance and repair are supposed to be performed at the specified time intervals or in case of failure. The paper proposes a method for optimizing the choice of preventive maintenance and repair, which considers the system condition.

In this work the model of a complex system built in [8] is modified, the statistical requirements is changed, the assessment of the current system condition is simplified and refined, the adequacy of the model is increased, and the class of modeled systems is expanded.

The model is constructed in terms of the Markov decision-making process [9,10]. The method based on the Banach contraction principle is used for the optimization [9].

Let a complex technical system S be composed of a finite number n of simple subsystems and be examined on an unlimited time interval. It is required that at any moment of time it is possible to unambiguously determine whether the subsystem is in working or non-working (failure) condition. It is convenient to assume that each simple subsystem has one “input” and one “output”. Then the working condition of a simple subsystem can be associated with the presence of an input-output connection, and the non-working condition with the absence of such connection. The model considers and analyzes the sets of several subsystems. Let each set have one input and one output. That will unambiguously identify the working or failure condition of the set in question at any given time.

Simple subsystems are arbitrarily interconnected in terms of reliability. Isolated subsystems are excluded. An example of such complex system is presented in the Appendix.

2.1 System reliability function

It is obvious that the reliability of the system S should depend on the reliability of its constituent subsystems. Let us consider some properties of separate subsystems. Let us adopt the following definition of the reliability function. Let ξ_i be the random time before the failure of the i-th subsystem, $i=1,...,n$. Then $P^{(i)}(t)=P(\xi_i \geq t)$ is a reliability function of the i-th subsystem [1].

Each subsystem, as a rule, has a set of monitored parameters containing information about its reliability and performance. Let the integral indicator of reliability determined by these parameters be denoted by $\theta^{(i)}$, $i=1,...,n$. Let the parameter $\theta^{(i)}$ increase monotonically with increasing wear of the subsystem on the interval $\Theta^{(i)}=\left[\underline{\theta}^{(i)},\overline{\theta}^{(i)}\right]$ [11].

We assume that the system S is inspected at regular time intervals τ if there is no system failure and at the moment of failure if it occurs. At each moment of inspection, one of the possible actions aimed at restoring the failed subsystems and preventive maintenance (update) of one of the subsystems is applied. Let us denote a random variable $\varsigma=\min\{\xi,\tau\}$, where ξ the random time before the system failure is, τ is the time interval specified by the regulations as the interval of inspection for the system S. The beginning of any interval is bound to a $t=0$ moment.

Let us suppose that there is the information about the parameter changes of each of the subsystems at some inspection intervals ς. We assume that it can be limited only to the values at the time of inspection and at the time of subsystem failure. Let us choose the intervals where the information about the trajectory of the parameter of the i-th subsystem is available. Fig. 1 shows an example of three observed trajectories placed on the same interval $[0,\tau)$. At the beginning of the interval at $t=0$ the parameter values are equal to $\theta^{(i)}(0)$. Using interpolation and extrapolation methods, it is possible to estimate the dependence of the parameter of the new subsystem on time [12]. Let us denote it as $\theta^{(i)}(t)$, $t\in[0,T]$, $\theta^{(i)}(0)=\underline{\theta}^{(i)}$, where T is the maximum duration of the time interval provided by statistical data, $i=1,...,n$. In Fig. 1 it is continuous over the time interval $[0,T]$.

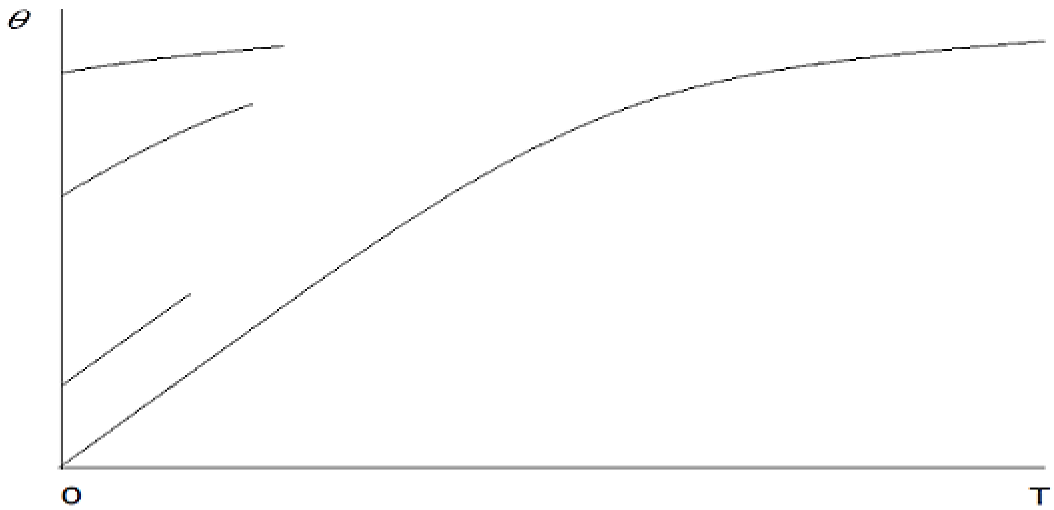

Fig. 1 The estimation of the dependence of the subsystem parameter on time

The adequacy of the model depends on the accuracy of estimating the function $\theta^{(i)}(t)$. Therefore, when changing the operating conditions of the system or obtaining new statistical data on the evolution of a parameter, it is recommended to refine the assessment of its $\theta^{(i)}(t)$ functions.

Let us denote the function inverse to $\theta^{(i)}(t)$, as $t=\psi^{(i)}(\theta)$, $i=1,...,n$. It should be noted that $\psi^{(i)}\left(\underline{\theta}^{(i)}\right)=0$.

Discretization of the set of parameter values. To construct the model it is necessary to discretize the set of parameter values for each subsystem of the system S. For simplicity's sake, we assume that the number of parameter values of any subsystem is equal to m. Let us denote $\delta=\dfrac{\overline{\theta}^{(i)}-\underline{\theta}^{(i)}}{m}$. Let us assign the discrete parameter value $\theta_k^{(i)}=\underline{\theta}^{(i)}+(k-1)\delta$, $k=1,...,m-1$ to the half-interval $\left[\underline{\theta}^{(i)}+(k-1)\delta,\underline{\theta}^{(i)}+k\delta\right)$ and the discrete parameter value $\theta_m^{(i)}=\underline{\theta}^{(i)}+(m-1)\delta$ to the closed interval $\left[\underline{\theta}^{(i)}+(m-1)\delta,\overline{\theta}^{(i)}\right]$. The discrete set of values of the parameter of the i-th subsystem will be denoted by $\tilde{\Theta}^{(i)}$, and a separate value will be denoted by $\tilde{\theta}^{(i)}$ or by $\tilde{\theta}_k^{(i)}$, $k=1,...,m$, $i=1,...,n$.

Let the i-th subsystem have the parameter value $\theta^{(i)}\in\Theta^{(i)}$ at the moment of inspection. It corresponds to a discrete parameter $\tilde{\theta}_j^{(i)}\in\tilde{\Theta}^{(i)}$, the value of which can be calculated by the formula

$$\tilde{\theta}_j^{(i)}=\sum_{k=1}^{m-1}h^+\left(\theta^{(i)}-\left(\underline{\theta}^{(i)}+(k-1)\delta\right)\right)h^-\left(\underline{\theta}^{(i)}+k\delta-\theta^{(i)}\right)\left(\underline{\theta}^{(i)}+(k-1)\delta\right)+$$
$$+h^+\left(\theta^{(i)}-\left(\underline{\theta}^{(i)}+(m-1)\delta\right)\right)h^+\left(\underline{\theta}^{(i)}+m\delta-\theta^{(i)}\right)\left(\underline{\theta}^{(i)}+(m-1)\delta\right),$$

where $h^+(t)=\begin{cases}1, & t\ge 0\\ 0, & t<0\end{cases}$, $h^-(t)=\begin{cases}1, & t>0\\ 0, & t\le 0\end{cases}$ – the variants of the Heaviside function.

Actions. Let a set of actions $Y=Y_1\cup Y_2$ be given. Any action in the set $Y_1=\left\{y^{(1)},...,y^{(n)}\right\}$ can be applied at the scheduled time of inspection. Any action from the set $Y_2=\left\{\hat{y}^{(1)},...,\hat{y}^{(n)}\right\}$ can be applied at the moment of system failure. The cost of actions depends on the subsystem selected for updating.

Let us consider the effect of action $y^{(i)} \in Y_1$. At the scheduled time of inspection, the application of action $y^{(i)}$ leads to the restoration of all failed subsystems and to preventive maintenance of the i-th subsystem. Assume that the following values of the subsystem parameters are recorded after the restoration of the failed subsystems: $\tilde{\theta}^{(j)} \in \tilde{\Theta}^{(j)}$, $j=1,...,n$. Application of action $y^{(i)}$ changes the value of the parameter of the i-th subsystem to the value $\tilde{\theta}_k^{(i)} \in \tilde{\Theta}^{(i)}$ with probability $\rho\left(\tilde{\theta}_k^{(i)} \middle| \tilde{\theta}^{(i)}, y^{(i)}\right)$, $k=1,...,m$. The estimation of the probability distribution ρ can be obtained on the basis of statistical data or by using expert estimations. The distribution ρ for actions from Y_2 can be the same as for actions from Y_1 or it can be estimated separately.

Subsystem reliability function. Numerical information about the probability of subsystem failure is determined by its reliability function. The estimation of the subsystem reliability function can be obtained after processing the available information about the parameter change over time, as well as subsystem failures. Let us consider the case when a failure of any subsystem is detected at the moment of the failure.

Let L be the number of intervals that contain available information. At the beginning of the l-th interval, the action having been applied, the value of the parameter $\theta^{(i)}(l) \in \Theta^{(i)}$ of the i-th subsystem, $i=1,...,n$ is recorded.

If the i-th subsystem does not fail on the l-th interval, then at the time of inspection τ the subsystem is in working condition and the parameter value $\theta^{(i)}(l,\tau) \in \Theta^{(i)}$ is recorded. If the i-th subsystem on the l-th interval fails at a $t^{(i)}(l,\xi)$ moment, then the value of the parameter $\theta^{(i)}(l,\xi) \in \Theta^{(i)}$ is recorded.

We convert the available statistical information on the i-th subsystem for the new i-th subsystem in the following way. Let us set the l-th interval. If at the scheduled time of inspection τ the parameter value equals $\theta^{(i)}(l,\tau) \in \Theta^{(i)}$, then for a new subsystem this value is reached at the time $\tilde{t}^{(i)}(l,\tau) = \psi^{(i)}\left(\theta^{(i)}(l,\tau)\right)$. If at the moment of failure the parameter value equals $\theta^{(i)}(l,\xi) \in \Theta^{(i)}$, then in a new subsystem the failure would occur at the moment of time $\tilde{t}^{(i)}(l,\xi) = \psi^{(i)}\left(\theta^{(i)}(l,\xi)\right)$. Note that if at the moment of failure the parameter value is unknown, but the time of failure $t^{(i)}(l,\xi)$ is known, then $\tilde{t}^{(i)}(l,\xi) = t^{(i)}(l,\xi) + \psi^{(i)}\left(\theta^{(i)}(l)\right)$, where $\theta^{(i)}(l)$ is the parameter value at the beginning of the l-th interval. Let the shifted time moments $\tilde{t}^{(i)}(l,\tau)$, $\tilde{t}^{(i)}(l,\xi)$ be found for all intervals $l=1,...,L$ and for all subsystems $i=1,...,n$. We denote by $T^{(i)}$ the maximum value of all $\tilde{t}^{(i)}(l,\tau)$ and $\tilde{t}^{(i)}(l,\xi)$ for each i.

The empirical reliability function of the i-th subsystem can be defined as follows.

Let us denote by $L^{(i)}(\xi)$ the set of intervals in which the failure of the i-th subsystem occurs.

We denote by $H^{(i)}(t) = \frac{1}{L} \sum\limits_{l \in L^{(i)}(\xi)} h^{-}\left(t - \tilde{t}^{(i)}(l,\xi)\right)$ if $L^{(i)}(\xi) \neq \varnothing$ and assume $H^{(i)}(t) = 0$ if $L^{(i)}(\xi) = \varnothing$. Then for time until the failure of the new i- th subsystem the empirical distribution function has the following form

$$F_0^{(i)}(t) = H^{(i)}(t) + \left(1 - H^{(i)}(t)\right) h^{-}\left(t - T^{(i)}\right),$$

and the empirical reliability function has the form

$$P_0^{(i)}(t) = 1 - F_0^{(i)}(t). \tag{2.1}$$

Afterwards, it is more convenient to use leveled versions of the functions $F_0^{(i)}(t)$ and $P_0^{(i)}(t)$, where $F_0^{(i)}(0) = 0$ and at $t \to \infty$ $F_0^{(i)}(t)$ monotonously tends to 1.

The reliability functions of the new subsystems $P_0^{(i)}(t)$ define the reliability function $P_0(t)$ of the new system S. The reliability function for the model of the system is given in Appendix Item 1.

The prediction of the reliability function. The prediction of the reliability function of the new i-th subsystem at the moment of time δ can be estimated by the function

$$P_\delta^{(i)}(t) = \frac{P_0^{(i)}(t+\delta)}{P_0^{(i)}(\delta)}, \; t > 0, \; i = 1,...,n. \tag{2.2}$$

Here $P_0^{(i)}(t)$, $i = 1,...,n$, the reliability functions of new subsystems. They define the reliability function $P_\delta(t)$ of the new system S at a point in time δ.

Assume that all failed subsystems having been restored, the parameters of the subsystems take on the values $\theta^{(i)}$, $i = 1,...,n$ at the time of inspection t_c. For each i-th subsystem, we will find the shift of values $\hat{t}^{(i)} = \psi^{(i)}\left(\theta^{(i)}\right)$.

The estimation of the reliability function of the i-th subsystem at the time of inspection t_c has the form

$$P_c^{(i)}(t) = \frac{P_0^{(i)}\left(t + \hat{t}^{(i)}\right)}{P_0^{(i)}\left(\hat{t}^{(i)}\right)}, \; t > 0, \; i = 1,...,n. \tag{2.3}$$

Then the system reliability function at the time of inspection t_c can be determined with the functions $P_c^{(i)}(t)$, $i = 1,...,n$. We denote it by $P_c(t)$.

The reliability function of the i-th subsystem in a time interval δ, after evaluating the parameters at a time moment t_c, can be represented in the form

$$P_\delta^{(i)}(t) = \frac{P_0^{(i)}\left(t + \hat{t}^{(i)} + \delta\right)}{P_0^{(i)}\left(\hat{t}^{(i)} + \delta\right)}, \; t > 0, \quad i = 1,...,n. \tag{2.4}$$

Reliability functions $P_\delta^{(i)}(t)$, $i = 1,...,n$, determine the system S reliability function $P_\delta(t)$ in the time interval δ after the moment of inspection.

2.2 System condition

Let the condition x of the new system S at the initial moment of time be equal to 0.

We can calculate the average operating time $\overline{t}$ of the new system S over a time interval τ using the Stieltjes integral

$$\overline{t} = \int_0^\tau t d\left(1 - P_0(t)\right) + \tau \cdot P_0(\tau). \tag{2.5}$$

Let $P_c(t)$ be the reliability function of the system S at the time of inspection t_c. Then the average operating time $\overline{t_c}$ of the system over a time interval $(t_c, t_c+\tau)$ is

$$\overline{t_c} = \int_0^{\tau} td\left(1-P_c(t)\right) + \tau \cdot P_c(\tau). \quad (2.6)$$

Let us assume that at the moment of inspection t_c the condition of the system S is equal to $x_c = \dfrac{\overline{t} - \overline{t_c}}{\overline{t}}$.

The prediction of the reliability function $P_\delta(t)$ of the system S in the time interval δ after the moment of inspection t_c allows us to calculate

$$\overline{t_\delta} = \int_0^{\tau} td\left(1-P_\delta(t)\right) + \tau \cdot P_\delta(\tau). \quad (2.7)$$

Let us assume that the condition of the system S at time $t_c+\delta$ is equal to $x_\delta = \dfrac{\overline{t} - \overline{t_\delta}}{\overline{t}}$.

As a result, the condition of the new system S at the initial moment of time is $x=0$, and the condition of a completely worn out system is close to 1. The set $E=[0,1]$ contains all possible conditions.

In addition, it has been found that the set of parameters of the subsystems $\left\{\theta^{(i)}, i=1,...,n\right\}$, recorded at the moment of inspection t_c after applying the action, makes it possible to estimate the condition of the system S at any moment t, $t_c \le t \le \tau$. It is assumed that estimations of the reliability function of all subsystems have been obtained earlier.

Discretization of the set of conditions. Let us suppose that the discrete set of conditions of the system S consists of N elements and denote it by $\tilde{E}$. Let us assign the discrete condition x_k of the system S, $k=0,...,N-1$ to the half-interval $\left[k\dfrac{1}{N}, (k+1)\dfrac{1}{N}\right)$. Let us assume that the condition x_k has the value $\dfrac{k}{N}$.

Let the condition $x \in E$ be observed at the moment of inspection of the system S. It corresponds to the discrete condition $\tilde{x}_v$, the value of which can be found by the formula

$$\tilde{x}_v = \Lambda(x) = \sum_{k=0}^{N-2} h^-\left(\frac{k+1}{N} - x\right) h^+\left(x - \frac{k}{N}\right)\frac{k}{N} + h^+(1-x)\, h^+\left(x - \frac{N-1}{N}\right)\frac{N-1}{N}. \quad (2.8)$$

Here Λ is a mapping of a set E to a discrete set of conditions $\hat{E} = (x_0,...,x_{N-1})$.

2.3 The process of changing the system condition over time

Subsystems wear out during operation, and some of them may fail at random times. Information about possible failures of subsystems is accounted in the corresponding reliability functions. In addition, the presence of unaccounted factors determines the evolution of the system condition over time as a random process.

If at the beginning of the interval all subsystems are new, then the reliability functions of new subsystems at the moment of time δ are presented in (2.2). Let us consider these functions, assuming δ to be a variable. The corresponding reliability function of the new system S will depend on the variable δ, which we will further denote by z. The time-dependence function of the new system condition will be $x(z)=\dfrac{\bar{t}-\bar{t}(z)}{\bar{t}}\in E$, where $\bar{t}(z)=\int_0^{\tau} td\left(1-P_z(t)\right)+\tau P_z(\tau)$. An explicit form of the dependence and its graph for a specific system can be obtained in the MAPLE system (see Appendix Item 2.).

Let us assume that the dependence of the trajectory of the condition $x(t), t\ge 0$, on time has the form

$$x(t)=\hat{x}(0)+\left(1-\hat{x}(0)\right)\frac{\gamma\cdot t}{\gamma\cdot t+\beta},\quad t\ge 0,\ x(t)\in E,$$

where $\hat{x}(0)$ is the initial condition.

The estimations of γ and β parameters can be obtained from observations of a single trajectory (for example, a new system for which $\hat{x}(0)=0$). If necessary, the shape of the trajectory can be corrected by observing the trajectories of a specific system.

Suppose that at the scheduled time of inspection $t_c=0$, after the restoration of all failed subsystems, a condition $\hat{x}(0)\in\hat{E}$ is observed and the values of the parameters $\left\{\theta^{(j)}, j=1,...,n\right\}$ is recorded. The application of action $y^{(i)}\in Y$ will determine the sets of parameter values $A_k=\left\{\theta^{(1)},...,\theta_k^{(i)},...,\theta^{(n)}\right\}$, $k=1,...,m$. Each set A_k determines the system condition $x(0,k)\in E$ at the beginning of the interval. Let us denote $\hat{x}(0,k)=\Lambda\left(x(0,k)\right)\in\hat{E}_1$. For each j, $j=0,...,N-1$, we define the set $K_j=\left\{k:\Lambda\left(x(0,k)\right)=\hat{x}_j(0)\right\}$, where $\hat{x}_j(0)$ is the condition with the number j from the set $\hat{E}$ at the moment of time $t_c=0$. Let us assume that the desired probability of the discrete condition $\hat{x}_j(0)$ after applying the action $y^{(i)}\in Y$ is

$$P\left(\hat{x}_j(0)\middle|\hat{x}(0),y^{(i)}\right)=\sum_{k\in K_j}\rho\left(\theta_k^{(i)}\middle|\theta^{(i)},y^{(i)}\right),\ j=0,...,N-1. \tag{2.9}$$

Transition probability. For further model construction, it is convenient to expand the discrete set of conditions $\hat{E}$. If a system fails in the condition $\hat{x}_j\in\hat{E}$, then we will formally consider it a different condition and denote it by $\tilde{x}_j$. In this case, the set of conditions will be doubled. We denote the resulting discrete set of conditions by $\tilde{E}=\hat{E}\cup\tilde{E}'=\left\{\hat{x}_0,...,\hat{x}_{N-1},\tilde{x}_0,...,\tilde{x}_{N-1}\right\}$. We assume that in the conditions of the set $\hat{E}$, the actions from the set Y_1, and in the conditions of the set $\tilde{E}'$, the actions from the set Y_2 can be applied. The sets Y_1 and Y_2 do not intersect and $Y=Y_1\cup Y_2$

Let us calculate the probability of each condition from $\tilde{E}$ which the system can assume in a time interval z after applying the action $y^{(i)}\in Y$ in a given condition from the set $\tilde{E}$.

If the action $y^{(i)}\in Y_1$ is applied in the condition $\hat{x}(0)\in\hat{E}$, then the probability of the condition $\hat{x}_j(z)$ that the system will have in the time interval z is denoted by $p\left(\hat{x}_j(z)\middle|\hat{x}(0),y^{(i)}\right)$. It should be noted that after applying the action $y^{(i)}\in Y_1$ in the condition $\hat{x}(0)\in\hat{E}$, the system transitions to the condition $\hat{x}_j(0)$ with probability $P\left(\hat{x}_j(0)\middle|\hat{x}(0),y^{(i)}\right)$, $j=0,...,N-1$.

Let us denote $K_j(z)=\left\{k:\Lambda\left(\hat{x}_k(0)+\left(1-\hat{x}_k(0)\right)\dfrac{\gamma z}{\gamma z+\beta}\right)=\hat{x}_j\right\}$.

Then

$$P\left(\hat{x}_j(z)\middle|\hat{x}(0),y^{(i)}\right)=\sum_{k\in K_j(z)} p\left(\hat{x}_k(0)\middle|\hat{x}(0),y^{(i)}\right),\quad z\in[0,\tau]. \tag{2.10}$$

If the time of inspection $t_c=0$ is unplanned, then the form of formula (2.10) remains correct, provided that $\hat{x}(0)\in\hat{E}$ is replaced by $\tilde{x}(0)\in\tilde{E}'$ and $y^{(i)}\in Y_1$ by $y^{(i)}\in Y_2$.

Let the trajectory of the system condition after the application of the action start from $\hat{x}_j(0)$. Then $\hat{x}_j(0)=\dfrac{\overline{t}-\hat{t}_{md}(\chi_j)}{\overline{t}}$, where $\overline{t}$ is defined in (2.5), $\hat{t}_{md}(\chi_j)=\int_0^\tau t\cdot d\left(1-\dfrac{P_0(t+\chi_j)}{P_0(\chi_j)}\right)+\tau\dfrac{P_0(\tau+\chi_j)}{P_0(\chi_j)}$. Since $\hat{t}_{md}=\overline{t}\left(1-\hat{x}_j(0)\right)$, the value of the shift χ_j of the reliability function of the new system for the initial condition $\hat{x}_j(0)=\dfrac{j}{N}$ is found from the equation $\hat{t}_{md}(\chi_j)=\overline{t}\left(\dfrac{N-j}{N}\right)$, $j=0,...,N-1$. The shifts χ_j, $j=0,...,N-1$ can be calculated in the MAPLE system for all conditions $\hat{E}$ during the model construction before its optimization. These values can be used to construct transition and direct income functions.

2.4 Decision function. Transition matrix

The transition function $Q\left(x_v\middle|x_u,y^{(i)},t\right)$, $x_v\in\hat{E}$, determines the probability distribution on the set $\tilde{E}$ at the moment of the next inspection (planned at $t=\tau$ or unplanned at the moment of system failure), provided that $x_u\in\tilde{E}$ is the condition of the system in which action $y^{(i)}\in Y$ is applied at the beginning of the interval.

Let the mapping $f:\tilde{E}\to Y$ be given, and $f(\hat{x}_j)\in Y_1$, $f(\tilde{x}_j)\in Y_2$, $j=0,...,N-1$. The mapping f is called a decision function. The sequence of decision functions $\pi=\{f_0,f_1,...\}$ is called a action policy. The policy $\pi=f^{(\infty)}=\{f,f,...\}$ is called stationary [9].

For each decision function f there is a corresponding matrix of transition probabilities $Q(f)$ of dimension $2N\times 2N$.

$$Q(f)=\begin{pmatrix} Q_{11} & Q_{12}\\ Q_{21} & Q_{22}\end{pmatrix},$$

We will assume that the matrix $Q(f)$ is regular for any decision function f [9]. In our case, this limitation is not essential.

Let us introduce the notation for the elements of the submatrices Q_{ij}:

$$Q_{11}=\begin{pmatrix} q\left(\hat{x}_0\middle|\hat{x}_0,f(\hat{x}_0)\right) & ... & q\left(\hat{x}_{N-1}\middle|\hat{x}_0,f(\hat{x}_0)\right)\\ ... & ... & ...\\ q\left(\hat{x}_0\middle|\hat{x}_{N-1},f(\hat{x}_{N-1})\right) & ... & q\left(\hat{x}_{N-1}\middle|\hat{x}_{N-1},f(\hat{x}_{N-1})\right)\end{pmatrix},$$

$$Q_{12}=\begin{pmatrix} q\left(\tilde{x}_0\middle|\hat{x}_0,f(\hat{x}_0)\right) & ... & q\left(\tilde{x}_{N-1}\middle|\hat{x}_0,f(\hat{x}_0)\right)\\ ... & ... & ...\\ q\left(\tilde{x}_0\middle|\hat{x}_{N-1},f(\hat{x}_{N-1})\right) & ... & q\left(\tilde{x}_{N-1}\middle|\hat{x}_{N-1},f(\hat{x}_{N-1})\right)\end{pmatrix},$$

$$Q_{21} = \begin{pmatrix} q\left(\hat{x}_0 \middle| \tilde{x}_0, f\left(\tilde{x}_0\right)\right) & \dots & q\left(\hat{x}_{N-1} \middle| \tilde{x}_0, f\left(\tilde{x}_0\right)\right) \\ \dots & \dots & \dots \\ q\left(\hat{x}_0 \middle| \tilde{x}_{N-1}, f\left(\tilde{x}_{N-1}\right)\right) & \dots & q\left(\hat{x}_{N-1} \middle| \tilde{x}_{N-1}, f\left(\tilde{x}_{N-1}\right)\right) \end{pmatrix},$$

$$Q_{22} = \begin{pmatrix} q\left(\tilde{x}_0 \middle| \tilde{x}_0, f\left(\tilde{x}_0\right)\right) & \dots & q\left(\tilde{x}_{N-1} \middle| \tilde{x}_0, f\left(\tilde{x}_0\right)\right) \\ \dots & \dots & \dots \\ q\left(\tilde{x}_0 \middle| \tilde{x}_{N-1}, f\left(\tilde{x}_{N-1}\right)\right) & \dots & q\left(\tilde{x}_{N-1} \middle| \tilde{x}_{N-1}, f\left(\tilde{x}_{N-1}\right)\right) \end{pmatrix}.$$

The matrix element Q_{11} can be calculated with allowance for (2.9) and (2.10) by the formula

$$q\left(\hat{x}_j \middle| \hat{x}_k, y\right) = \sum_{\kappa \in K_j(\tau)} p\left(\hat{x}_\kappa(0) \middle| \hat{x}_k, y\right) \frac{P_0\left(\tau + \chi_\kappa\right)}{P_0\left(\chi_\kappa\right)}, \tag{2.11}$$

where $K_j(\tau) = \left\{ k : \Lambda\left(\hat{x}_k(0) + \left(1 - \hat{x}_k(0)\right) \frac{\gamma\tau}{\gamma\tau + \beta} \right) = \hat{x}_j \right\}$, $y = f\left(\hat{x}_k\right) \in Y_1$, χ_κ is the shift of the trajectory of new system conditions for the trajectory beginning $\hat{x}_\kappa(0)$.

Formula (2.11) remains correct for the matrix elements Q_{21} after $\hat{x}_k \in \hat{E}$ is replaced by $\tilde{x}_k \in \tilde{E}'$ and if $y = f\left(\tilde{x}_k\right) \in Y_2$.

The matrix elements Q_{12} can be calculated as follows

$$q\left(\tilde{x}_j \middle| \hat{x}_k, y\right) = \int_0^\tau \sum_{\kappa \in K_j(t)} p\left(\tilde{x}_\kappa \middle| \hat{x}_k, y\right) d\left(1 - \frac{P_0\left(t + \chi_\kappa\right)}{P_0\left(\chi_\kappa\right)}\right), \tag{2.12}$$

where $K_j(t) = \left\{ \kappa : \Lambda\left(\hat{x}_\kappa(0) + \left(1 - \hat{x}_\kappa(0)\right)\right) \frac{\gamma t}{\gamma t + \beta} = \hat{x}_j \right\}$, $y = f\left(\hat{x}_k\right) \in Y_1$, χ_κ is the shift of the trajectory of the new system conditions for the trajectory beginning $\hat{x}_\kappa(0)$.

Formula (2.12) is correct for the matrix elements Q_{22} after $\hat{x}_k$ is replaced by $\tilde{x}_k \in \tilde{E}'$ and if $y = f\left(\tilde{x}_k\right) \in Y_2$.

2.5 Basic set of failures

Complex system failure can occur as a result of failure of all subsystems in the system S or one specific set of subsystems. The duration of system downtime, as well as the cost of its restoring to an operational condition usually depends on the set of failed subsystems. Therefore, it is important to define all such sets in advance. We will assume that failure of any simple subsystem does not affect the failure probability of any other simple subsystem.

Simple subsystems wear out during operation and their failure probability increases. Failure of one or more subsystems does not necessarily result in the failure of the system S. However, there is a number M of sets of subsystems with the following property. Failure of all subsystems in a set from M leads to failure of the system S. None of the set from M contains subsystems "indifferent" to failure of the system S: failure of the system S occurs when all subsystems of the set fail, and after restoring any single subsystem of this set, the system S will not remain in a failure condition. If M contains all the different sets of subsystems from S that have the properties indicated above, then this set will be called a base set of sets leading to failure of the system S, or simply a basic set of failures. Note that sets of M can overlap.

Algorithm for compiling a basic set of failures. The method for finding the basic set of failures M which can be easily implemented by using the MAPLE mathematical program is proposed. Let the system S consist of n simple subsystems. The system reliability function is supposed to be known. Let us execute the following algorithm.

1. Find all subsets of a set of n elements. Denote it by C . Formally identify an element of the set C with the corresponding set of subsystems.
2. Form a set G , consisting of the elements from C , which correspond to a set of subsystems, the failure of which leads to failure of the system S . Denote by G_i , elements of the set G , which correspond to sets of i subsystems, $i=1,...,n$.

Note that, for some i the set $G_i = \varnothing$.

3. If $G_1 \neq \varnothing$, then $\tilde{G}_1 = G_1$.
4. If $G_1 = G_2 = ... = G_l = \varnothing$, $G_{l+1} \neq \varnothing$, then $\tilde{G}_{l+1} = G_{l+1}$.
5. Let $G_k \neq \varnothing$. Define the set $\tilde{G}_k$. Any subset $g \in G_k$ is excluded from G_k if there is a subset $h \in \tilde{G}_{k-j}$, $j=1,...,k-1$, such that $g \supset h$. The remaining elements form a set $\tilde{G}_k$. The union of the obtained non-empty sets $\tilde{G}_i$ forms the basic set of failures M .

The example of implementing this algorithm for the model can be found in Appendix Item 3.

The theoretical calculation of the failure probability of the given set $h \in M$ up to the moment of time t that leads to the failure of the system S is not difficult. However, applying this data to the model would make determining the condition rather cumbersome. The practical value of the model will be significantly lower due to the complexity of the implementation. Therefore, the method for accounting the sets of failures from M is proposed.

Suppose that failure of the system S occurs during a interval of duration τ . In this case, the only set from M is realized. Let us denote the cost of restoring the system S to a working condition by $g(h)$, $h \in M$, if the failure occurred as a result of failure of all subsystems in the set $h \in M$. Let us denote the number of elements in M by J . A selective distribution of the cost of restoring a failed system can be obtained from the available observations. Therefore, only intervals with failures and the beginnings of trajectories should be taken into account. The probability that the cost of restoration will be g_j , $j=1,...,J$, if the trajectory of the process starts in the condition $\hat{x}(0) \in \hat{E}$ and the system failure occurs is denoted by $p\left(g_j \middle| \hat{x}(0)\right)$.

2.6 Direct income function

Let us denote the average cost of restoring failed subsystems at the time of inspection by R; the cost of the action $f(x) \in Y$ determined by the decisive function f in the condition $x \in \tilde{E}$ by $r\left(f(x)\right)$; the income per unit of time that the operating system yields for one interval starting in the condition $x \in \tilde{E}$ by $\nu(x)$; the average duration of system downtime during preventive maintenance by δ; the average duration of system downtime when restoring from a failure condition by Δ; the cost of restoring the system from a failure condition if the failure occurs due to the realization of the set $h_j \in M$ by g_j .

The direct income function $w\left(x, f(x)\right)$, $x \in \tilde{E}$, $f(x) \in Y$, with the fixed decision function f is a column vector composed of $2N$ components. The component of this vector for a given $x \in \tilde{E}$ can be calculated by the formula

$$w\left(x, f(x)\right) = \sum_{k=0}^{N-1} \frac{\nu(\hat{x}_k) \cdot \tau}{\tau + \delta} \left(p\left(\hat{x}_k \middle| x, f(x)\right) - R - r\left(f(x)\right) \right) \frac{P_0(\tau + \chi_k)}{P_0(\chi_k)} +$$

$$+ \int_0^{\tau} \sum_{k=0}^{N-1} \sum_{j=1}^{J} \frac{\nu(\hat{x}_k) \cdot t}{t + \Delta} \left(p\left(\hat{x}_k \middle| x, f(x)\right) - R' - r\left(f(x)\right) - g_j \cdot p\left(g_j \middle| \hat{x}_k\right) \right) d\left(1 - \frac{P_0(t + \chi_k)}{P_0(\chi_k)}\right).$$

2.7 Optimality criterion. Optimization method

We assume that the inequality $a \ge b$ is true for the vectors a and b if a similar inequality $a_i \ge b_i$ is true for all components $a_i \ge b_i$. The inequality $a > b$ is satisfied if $a \ge b$ and $a \ne b$.

The case of an unlimited system operation time is being considered. The action policy is the selection of admissible actions at the moments of inspection. For each stationary policy $\pi = f^{(\infty)} = \{ f, f, ...\}$ we identify the corresponding average income per unit of time $\varphi(\pi)$ on an unlimited time interval [9]:

$$\phi(\pi) = \lim_{L \to \infty} \frac{1}{L} \sum_{l=0}^{L-1} Q^l(f) w(f).$$

The limit exists and the column vector $\varphi(\pi)$ is composed of the components of equal size if the matrix $Q(f)$ is regular.

A stationary policy π^* is optimal if the inequality $\phi(\pi^*) \ge \phi(\pi)$ is true for any stationary policy π. A stationary policy π_ε^* is ε-optimal if the inequality $\phi(\pi) - \phi(\pi_\varepsilon^*) \le \varepsilon$ is true for a given $\varepsilon > 0$ and any stationary policy π.

The search for an ε-optimal policy for controlling the reliability of a complex system can be implemented by a well-known method based on the Banach contraction principle [9]. Let us introduce the following auxiliary definitions and notations.

Let a seminorm $\tilde{p}(v) = \max_i v_i - \min_i v_i$, $1 \le i \le 2N$ be defined on a vector space V with elements v of dimension $2N$ [13]. The chosen optimization method requires a normalized space. To fulfill this condition we factorize V by $K = \{v : p(v) = 0\}$ to determine the factor-space $V' = V / K$ on which $\tilde{p}$ is the norm $\|\cdot\|$ [13].

Let us define the following operators on V.

$$F(f)v = w(f) + Q(f)v;$$

$$Uv = \max_f F(f)v.$$

Under the assumption that transition matrix is regular for any decision function f, the operator U on V' is contractive [9]. Those assumptions allow us to apply an optimization method based on the Banach contraction principle [9]. The algorithm stopping rule will be based on the following statement:

Statement [14]. Let $v \in V'$, $\|Uv - v\| = \varepsilon$, $Uv = F(f)v$. Then

1. The policy $\pi = f^{(\infty)}$ is ε-optimal.

2. $\min_x (Uv - v)(x) \le \varphi^* \le \max_x (Uv - v)(x)$.

Here φ^* is an average income per unit of time, which corresponds to an optimal policy.

Corollary [14]. Let $\pi^* = (f^*, f^*, ...)$ be the optimal policy. $v_k, v_{k+1} \in V'$ and f_{k+1} satisfy the conditions: $v_{k+1} = Uv_k = F(f_{k+1})v_k$ Let us denote $\varepsilon = \|v_{k+1} - v_k\|$. Then the decisive function f_{k+1} determines the ε-optimal policy $\pi_\varepsilon^* = (f_{k+1}, f_{k+1}, ...)$.

Optimization algorithm [9].

Select $\varepsilon > 0$.

Choose a starting vector $v_0 \in V'$ arbitrary.

k-th step of the algorithm $(k = 1, 2...)$.

1. Calculate $v_k = Uv_{k-1} = F(f_k)v_{k-1}$, $v_k \in V'$.
2. Check the inequality $\|v_k - v_{k-1}\| < \varepsilon$.

If it is true, then the policy $\pi = f_k^{(\infty)}$ is ε-optimal.
Otherwise, execute the $(k+1)$-th step of the algorithm.
The ε-optimal policy will be achieved for any given $\varepsilon > 0$ in a finite number of steps [9].

Note 1. It is convenient to choose a v_k with zero first component from the coset of the factor-space V'. In practice, this means that for any considered vector v, it is necessary to subtract the value of the first component from all components of the vector v in order to obtain $v' \in V'$.

3 Conclusion

The constructed mathematical model of a complex technical system according to its condition provides a higher level of adequacy and, therefore, allows finding a more effective policy for preventive maintenance and repair for an unlimited time interval in comparison with those proposed earlier in similar conditions. There is no universal mathematical model of a complex system capable to solve any problem. The goal of modeling should always be formulated first.

The article deals with a complex technical system composed of a finite number of simple subsystems which are, generally speaking, different. Each of the simple subsystems wears out during operation, and the probability of its failure increases. A failure of one of the subsystems does not affect the failure probability of any other subsystem. Subsystems are arbitrarily interconnected in terms of reliability. Thus, failure of one or more subsystems does not necessarily lead to failure of the entire system. We assume that the failure of the system leads to long downtime, highly expensive and, therefore, is better to be avoided. The work of subsystems as part of one complex system is to achieve a specified objective. To do that, a modeling approach in terms of a Markov decision-making process, and an optimization method based on the Banach contraction principle have been chosen.

To implement this approach using the available statistical information for each subsystem is proposed. If the volume of statistical information is insufficient, it can be supplemented by expert data. Lack of information reduces the level of model adequacy. On the other hand, using all available statistical information is not always justified due to the increasing model complexity. The practical value of the model can be significantly reduced due to its complexity. In the proposed article a compromise between the complexity of the model and its practical convenience has been sought. Some key points of complex system modeling for the given model example are presented in Appendix.

One of the possible directions for further development of the proposed approach to constructing the models for complex system maintenance according to its condition can be revising, clarifying, and generalizing the concept of action in the problem formulation. Modeling a complex technical system, the work of which is influenced by seasonal fluctuations can be another direction. The optimization method for such a system is proposed in [15]. Optimizing the interval of planned system inspection can be another possibility to improve the efficiency of model usage. The solution to this problem is proposed in [14].

Appendix

1. Let us consider a model example of the complex technical system S consisting of $n = 9$ simple subsystems. The subsystems are interconnected in the sense of reliability as shown in Fig. 2. The connections are denoted by a line crossing.

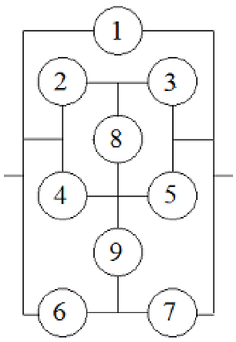

Fig. 2 The complex system example

$P^{(i)}(t)$, $i=1,\dots,9$, are subsystem reliability functions. The method for finding the reliability function $P(t)$ of the system S based on the total probability formula has been proposed in [8], namely,

$$P(t)=1-\left(1-P^{(1)}(t)\right)\left(1-PP(t)\right)$$

where:

$$PP(t)=\left[1-\left(1-P^{(2)}(t)\right)\left(1-P^{(4)}(t)\right)\right]\left(1-P^{(6)}(t)\right)\left[1-\left(1-P^{(3)}(t)\right)\left(1-P^{(5)}(t)\right)\right]\cdot$$
$$\cdot\left(1-P^{(7)}(t)\right)P^{(8)}(t)P^{(9)}(t)+$$
$$+\left[1-(1-PS_1)(1-PS_2))\right]P^{(8)}(t)\left(1-P^{(9)}(t)\right)+$$
$$+\left[1-(1-PS_3)(1-PS_4)\right]\left(1-P^{(8)}(t)\right)P^{(9)}(t)+$$
$$+\left[1-(1-PS_5)(1-PS_6)\right]\left(1-P^{(8)}(t)\right)\left(1-P^{(9)}(t)\right),$$

$$PS_1=\left[1-\left(1-P^{(2)}(t)\right)\left(1-P^{(4)}(t)\right)\right]\left[1-\left(1-P^{(3)}(t)\right)\left(1-P^{(5)}(t)\right)\right],$$

$$PS_2=P^{(6)}(t)P^{(7)}(t),$$
$$PS_3=P^{(2)}(t)P^{(9)}(t),$$
$$PS_4=\left[1-\left(1-P^{(4)}(t)\right)\left(1-P^{(6)}(t)\right)\right]\left[1-\left(1-P^{(5)}(t)\right)\left(1-P^{(7)}(t)\right)\right],$$
$$PS_5=1-\left(1-P^{(2)}(t)P^{(3)}(t)\right)\left(1-P^{(4)}(t)P^{(5)}(t)\right),$$

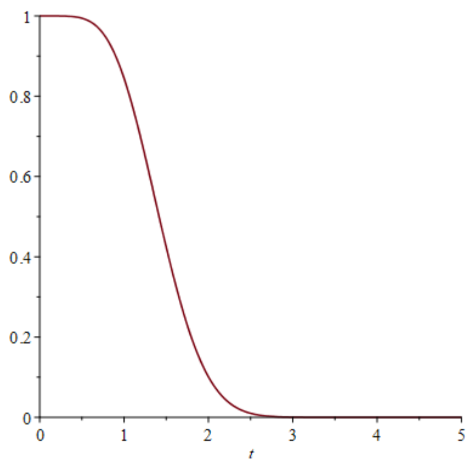

Fig. 3 The new system reliability function

Fig. 3 shows a graph of the reliability function $p0\_syst(t)$ of the new system S on the time interval $[0,5)$

The derivative $dp0\_syst(t)$ of the reliability function $p0\_syst(t)$ is found by using the following command:

$$dp0\_syst(t) := diff(p0\_syst(t), t);$$

Let us suppose that $\tau = 3$. The average operating time $T0$ of the new system on the time interval $[0,\tau)$ can be found with the command:

$$T0 := evalf(int(-t \cdot dp0\_syst(t), t = 0..\mathrm{tau}) + \mathrm{tau} \cdot p0\_syst(\mathrm{tau}));$$

$$T0 := 1.437222359\ .$$

The prediction of the reliability function $ptt\_syst(t)$ of the new system at the moment of time δ is found by the formula (2). In the MAPLE system, it is implemented with the command (δ replaced by tt):

$$ptt\_syst(t) := subs\left(p[1] = \left(\frac{p[1](t+tt)}{p[1](tt)}\right), p[2] = \left(\frac{p[2](t+tt)}{p[2](tt)}\right), p[3] = \left(\frac{p[3](t+tt)}{p[3](tt)}\right), p[4] = \left(\frac{p[4](t+tt)}{p[4](tt)}\right), p[5] = \left(\frac{p[5](t+tt)}{p[5](tt)}\right), p[6] = \left(\frac{p[6](t+tt)}{p[6](tt)}\right), p[7] = \left(\frac{p[7](t+tt)}{p[7](tt)}\right), p[8] = \left(\frac{p[8](t+tt)}{p[8](tt)}\right), p[9] = \left(\frac{p[9](t+tt)}{p[9](tt)}\right), p\_syst\right)$$

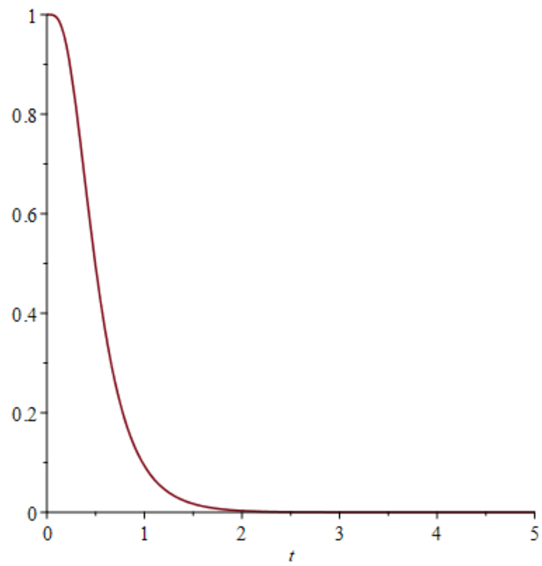

Fig. 4 The prediction of the system reliability function

Fig. 4 shows a graph of the prediction of the reliability function of a new system in the time interval $tt = 2$ over time $[0,5)$.

The derivative $dptt\_syst(t)$ of this function is found by the command:

$$dptt\_syst(t) := diff(ptt\_syst(t), t) :$$

Then we obtain the average operating time Ttt of the system, starting from the moment of time tt, over an interval of duration τ by using the command:

$$Ttt := int(-t \cdot dptt\_syst(t), t = (0.)..\mathrm{tau}) + \mathrm{tau} \cdot ptt\_syst(\mathrm{tau});$$

$$Ttt := 0.5653022542 \,.$$

The new system condition at a moment of time can be estimated by using the formula $x := \dfrac{T0 - Ttt}{T0}$. In this case $x := 0.6066702896$.

It is easy to obtain a prediction of the trajectory of the new system condition on the time interval $[0,\tau)$. To do this, we need to clear the variable tt by using the command $tt := 'tt';$. We define the function $Ttt(tt)$ and function $x(tt) = \dfrac{T0 - Ttt(tt)}{T0}$. The graph of condition changes of the new system S on a time interval $[0,\tau)$ can be obtained by using the command:

$$plot(x(tt), tt = 0\,..\mathrm{tau});$$

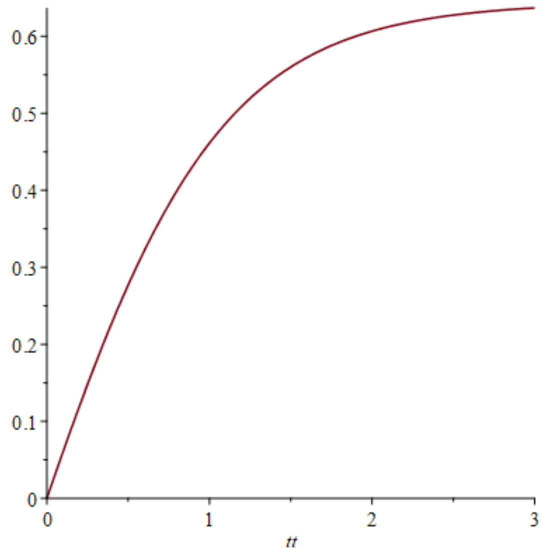

Fig. 5 The dependence of the system condition on time

Fig. 5 shows a graph of the predicted condition changes of the new system S on the time interval $[0,\tau)$, where $\tau = 3$.

3. The set C of all subsets of a set of $n = 9$ elements has cardinality $2^n = 512$. All subsets can be found in the MAPLE system by using the commands:

$$> with(combinat):$$

$$> C := choose(9);$$

Let us group the resulting subsets into rectangular matrices by i elements in the row, $i = 1,...,9$. We obtain matrices $C_i, i = 1,...,9$, with a different number of rows. These matrices can be obtained sequentially by using the command:

$$> convert(choose(9,i), Matrix);$$

The number of rows in the matrix C_i is equal to $\dfrac{n!}{i!(n-i)!}$. We assume that the k-th row of the matrix C_i is a set of subsystem numbers. From the rows of the matrix C_i, we compose the matrix G_i according to the following rule. A row of the matrix C_i is included in the matrix G_i, if a failure of all subsystems listed in that row results in a failure of the system . The MAPLE program, which solves the problem of finding matrices for the considered system S is presented below. It has been found that in the model example there are 84 sets of subsystems, the failure of which leads to the system failure.

```
#Sets of subsystems, the failures of which leads to the system failure
>restart;
>with(combinat);


>n:=9; #A number of subsystems
>p:=convert(array(1..n),Vector);#Vector for storing the subsystem
#reliabilities
>L:=0; #A number of found sets
>for i from 1 to n
       do
mi:=convert(choose(n,i),Matrix): #Matrix of sets of n elements by i
k_max:=n!/(i!*(n-i)!): #A number of all sets
for k from 1 to k_max
                  do
for l from 1 to n do p[l]:=1 od:
for l from 1 to I do p[mi[k,l]]:=0 od:
#System reliability calculation
p1:=(1-(1-p[2])*(1-p[4])*(1-p[6]))*(1-(1-p[3])*(1-p[5])*(1-p[7]))*p[8]*p[9]:
p2_s1:=(1-(1-p[2])*(1-p[4]))*(1-(1-p[3])*(1-p[5])):
p2_s2:=p[6]*p[7]:
p2:=(1-(1-p2_s1)*(1-p2_s2))*p[8]*(1-p[9]):
p3_s1:=p[2]*p[3]:
p3_s2:=(1-(1-p[4])*(1-p[6]))*(1-(1-p[5])*(1-p[7])):
p3:=(1-(1-p3_s1)*(1-p3_s2))*(1-p[8])*p[9]:
p4_s2:=p[6]*p[7]:
p4_s1:=1-(1-p[2]*p[3])*(1-p[4]*p[5]):
p4:=(1-(1-p4_s1)*(1-p4_s2))*(1-p[8])*(1-p[9]):
pp:=(p1+p2+p3+p4):
p_syst:=1-(1-p[1])*(1-pp):
      If p_syst=0 then L:=L+1 fi:
      If p_syst=0 then print(mi[k]) fi:
             end do:
  end do: print('A number of sets in G'L);
```

The basic set of failures M can be easily found by using the algorithm given in Section 2.5. The set M contains 8 sets of G_4, G_5, G_6 : $\{1,2,4,6\}$, $\{1,3,5,7\}$, $\{1,2,4,7,9\}$, $\{1,2,5,7,8\}$, $\{1,3,4,6,8\}$, $\{1,3,5,6,9\}$, $\{1,2,5,6,8,9\}$, $\{1,3,4,7,8,9\}$.

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