

УДК 658.51.012

## The statement of the task of optimal control of the production line using the additional time of equipment operation

V.D. Khodusov<sup>1</sup>, O.M. Pihnastyi<sup>2</sup><sup>1</sup> Kharkiv National University named after V.N. Karazin, Kharkov-22, Svobody Square, 4, 61022, Ukraine<sup>2</sup> National Technical University "Kharkiv Polytechnic Institute" Pushkinskaya 79-2, Kharkov, 61102, Ukraine, e-mail: pihnastyi@gmail.com

The production line of an enterprise with a line method of organizing production is considered as a dynamic distributed system. The technological route for manufacturing products in many modern enterprises contains several hundreds of technological operations and in the inter-operating reserve of each there are thousands of products awaiting processing. Technological routes of different parts for the products of the same type intersect. Therefore the distribution of objects of labor along the technological route has a significant impact on the throughput capacity of the production line. To describe such systems, a new class of production line models (PDE-model) has been introduced. Models of this class use partial differential equations to describe the behavior of production line flow parameters. In this article, a PDE-model of the production line is built, the flow parameters of which depend on the load factor of the process equipment for each operation. For the description of a distributed dynamic system, the PDE model of the production line was used. The single-shift mode of operation of a production enterprise is considered as a basic mode of operation.

**Keywords:** production line, PDE-model of production, balance equations, work in progress.

Виробнича лінія підприємства з поточним методом організації виробництва є складною динамічною розподіленою системою. Розподіл заготовок і напівфабрикатів уздовж технологічного маршруту істотно впливає на пропускну здатність виробничої лінії, може викликати зупинку технологічного обладнання через відсутність заготовок в накопичувальному бункері або в результаті його переповнення. Для опису розподілених виробничих систем введений новий клас моделей (PDE-модель). Моделі цього класу використовують диференціальні рівняння в приватних похідних, дозволяють визначити поведінку поточних параметрів уздовж виробничої лінії в довільний момент часу для заданої технологічної позиції обробки деталі. У даній роботі досліджується модель потокової лінії, синхронізація технологічних операцій на якій досягається за допомогою зміни коефіцієнта завантаження технологічного обладнання. Передбачається, що однозмінний режим роботи виробничого підприємства є основним режимом його функціонування. Підвищення коефіцієнта завантаження технологічного обладнання досягається за рахунок збільшення роботи устаткування для операцій з мінімальною продуктивністю. Для забезпечення безперервного одне змінного режиму роботи виробничої лінії технологічні операції синхронізуються протягом календарного дня. Для побудови оптимальної програми управління введений критерій якості, який враховує витрати виробничого підприємства, які потрібні для синхронізації технологічних операцій. При цьому враховується, що вартість однієї години функціонування технологічного обладнання залежить від виду технологічної операції. Для побудови диференціальних зв'язків при постановці завдання управління використано одно моментне рівняння, яке визначає взаємозв'язок лінійної щільності заготовок вздовж технологічного маршруту і темпу обробки заготовок на технологічній операції в довільний момент часу.

**Ключові слова:** виробнича лінія, PDE-модель виробництва, балансові рівняння, незавершене виробництво.

Производственная линия предприятия с поточным методом организации производства является сложной динамической распределенной системой. Распределение заготовок и полуфабрикатов вдоль технологического маршрута оказывает существенное влияние на пропускную способность производственной линии. Для описания распределенных производственных систем введен новый класс моделей (PDE-модель). Модели этого класса используют дифференциальные уравнения в частных производных, позволяют определить поведения поточных параметров вдоль производственной линии в произвольный момент времени. В настоящей работе исследуется модель поточной линии, синхронизация технологических операций на которой достигается посредством изменения коэффициента загрузки технологического оборудования. Предполагается, что односменный режим работы производственного предприятия является основным режимом его функционирования. Для обеспечения непрерывного односменного режима работы производственной линии технологические операции синхронизируются в течение календарного дня. Для построения оптимальной программы управления введен критерий качества, который учитывает затраты производственного предприятия, которые требуются для синхронизации технологических операций. При этом учитывается, что стоимость одного часа функционирования технологического оборудования зависит от вида технологической операции. В качестве дифференциальных связей при постановке задачи управления использовано одномоментное уравнение, которое определяет взаимосвязь линейной плотности заготовок вдоль технологического маршрута и темпа обработки заготовок на технологической операции в произвольный момент времени.

**Ключевые слова:** поточная линия, PDE-модель производства, уравнения баланса, незавершенное производство.

### 1 Introduction

The production line is required to process a batch of products on the enterprise [1]. For the process following factors are identified:

- a) the sequence of operations and their technological parameters;
- b) the equipment necessary to perform the operation, its working parameters and the layout scheme;
- c) the properties of the object of labor and the laws of the transfer of resources to objects of labor as a result of the impact of equipment.

It is assumed that the production technology during the production cycle does not change or the parameters which characterize the operation remain unchanged. The duration of the shift is fixed at eight hours. Reducing the duration of the production cycle is possible by changing the mode of equipment utilization. The amount of equipment load in the processing of a batch of products will be characterized by the shift factor of the equipment during the day  $K_{Sm}$ . We assume that the cost of one hour of equipment operation is different for each operation and depends on the time of day. The flow parameters of the model of the controlled production process in the two-step description are inter-operating reserve characterized by density  $[\chi]_0(t, S)$ , and by the flow of objects of labor  $[\chi]_1(t, S)$  on the technological route [2-5]. To describe the behavior of flow parameters in space and time, we use the one-dimensional coordinate space  $(t, S)$  [6, 7]. The coordinate  $S$  determines the place of the object of labor in the technological route. The introduced one-dimensional coordinate space  $(t, S)$  allows us to construct compact models for controlling the parameters of a production line. We divide the coordinate axis  $OS$  into segments  $\Delta S_m \in [S_{m-1}, S_m]$ . Coordinates  $S_{m-1}$  and  $S_m$  characterize the beginning and end of the  $m$ -th operation,  $m = 1..M$ . At the same time, we assume that,  $S_0 = 0$ ,  $S_M = S_d$ , where  $S_d$  is the cost of production.

Let the function  $z_c(t, S_m)$  determine the value of the excess costs of the resources required for the use of additional equipment within one hour of the  $m$ -th operation. The dependence of the function  $z_c(t, S_m)$  on time implies that during the production cycle, the value of the excess costs of resources required to perform the operation on additional equipment may vary over time. By the use of additional equipment we mean the usage of backup equipment in the required time or the main equipment in additional time (shift factor  $K_{Sm} < 3$ ) [8].

One of the approaches to synchronizing the processing performance of objects of labor in different operations of the production line is to use the main equipment in additional time between the main technological shifts (control of the shift of technological equipment for a given technological operation). If the time used by the equipment during single-shift operation is selected as the time axis of the state space, then the state of the backlog during the period between the end of the shift and the beginning of another shift in the case of using the main equipment during the second and the third shift will change abruptly by the number of processed products during the second and the third shift.

Let us introduce the distribution density of the value of the excess costs of technological resources required to use additional equipment within an hour for a technological route in the interval  $[0; S_d]$ :

$$\int_{S_{m-1}}^{S_m} \omega_z(t, S) dS = z_c(t, S_m), \quad \int_0^{S_d} \omega_z(t, S) dS = \sum_{m=1}^M \int_{S_{m-1}}^{S_m} \omega_z(t, S) dS = \sum_{m=1}^M z_c(t, S_m).$$

We introduce a function  $W_\delta(t, S)$  that characterizes the work of additional equipment (main equipment in the second or third shift, shift factor  $K_{Sm} > 1$ ).

Define  $\int_{t_1}^{t_2} W_\delta(t, S) dt$  as the number of additional equipment (located in the vicinity  $S$  of the technological route coordinates) hours for a period of time  $\Delta t = (t_2 - t_1)$ . As a result of the inclusion of additional equipment that ensures the processing of objects of labor in the second and the third shift at a rate equal to the rate of operation of the main equipment  $[\chi]_{1\psi}(t, S)$ , the overall rate of movement of objects of labor at the point of the technological route with the coordinate  $S$  increases by  $[\chi]_{1\psi}(t, S) \cdot W_\delta(t, S)$ . During  $\Delta t = (t_2 - t_1)$  the work of the additional equipment, an additional flow of objects of labor with the total number of units

$$\int_{t_1}^{t_2} [\chi]_{\text{II}\psi}(t, S) \cdot W_{\delta}(t, S) dt$$

will pass through the point of the technological route with the coordinate  $S$ .

The excess costs  $C_{cb}(T_d)$  required to ensure the operation of additional equipment located on the segment  $\Delta S_m \in [S_{m-1}, S_m]$  and used to perform the  $m$ -th operation for the duration of the production cycle  $T_d$ , and the integral is defined:

$$C_{cb}(T_d) = \int_0^{T_d} \int_0^{S_d} W_{\delta}(t, S) \cdot \omega_z(t, S) dS dt$$

### Relevance and practical significance

The scientific novelty consists in the development of a method for designing control systems for the parameters of the production line in enterprises with a continuous method of organizing production based on the PDE model of the object control.

In this case, the object control, the production line is represented by a dynamic system with distributed parameters along the technological route. The optimal control of the parameters of the production line is sought in the form of superpositions of delta functions.

The proposed method of designing a system for controlling the flow parameters of a production line can be used as the basis for designing highly efficient production flow control systems for enterprises manufacturing semiconductor products for the automotive industry.

### Production Line Model

Line parameters for continuous production flow with a sufficiently large number of operations satisfy the balance equation system:

$$\frac{\partial [\chi]_0(t, S)}{\partial t} + \frac{\partial [\chi]_{\text{II}}(t, S)}{\partial S} = 0, \quad [\chi]_{\text{II}}(t, S) = [\chi]_{\text{II}\psi}(t, S). \quad (1)$$

The normative tempo  $[\chi]_{\text{II}\psi}(t, S)$  of processing objects of labor for a production line is set at each point of the technological route and for each point in time. The parameters of the model of the controlled production process in the two-step description are the inter-operational reserves, which characterize  $[\chi]_0(t, S)$  the density of distribution of objects of labor along the technological route and their rate of movement  $[\chi]_{\text{II}}(t, S)$  [1].

We can control the value of the flow parameters  $[\chi]_0(t, S)$ ,  $[\chi]_{\text{II}}(t, S)$  by regulating the place of activation of additional equipment in the technological route and the duration of activation (change in the coefficient of the loading of the process equipment  $K_{Sm} > 1$ ).

Control function  $U_{\delta}(t, S) = W_{\delta}(t, S)$  determines the duration of additional equipment activation in the specified place of the technological route with the coordinate  $S$  at the time  $t = t_q$ , where  $t_q$  is the end time of the  $q$ -th shift ( $q = 1, 2, 3, \dots$ ). The planning interval for the line in question is equal to the interval of three shifts (daily planning interval) with one-shift operation of the main equipment. As an additional, the main equipment is used, which processes  $K_{Sm} > 1$  the objects of labor in the second and the third shift with the pace  $[\chi]_{\text{II}\psi}(t, S)$ .

The behavior of the parameters  $[\chi]_0(t, S)$ ,  $[\chi]_{\text{II}}(t, S)$  of the production line is limited by the initial conditions of the distribution of objects of labor along the technological route and the purpose of management:

$$[\chi]_0(0, S) = [\chi]_{00}(S), \quad [\chi]_0(T_d, S) = [\chi]_{0T_d}(S),$$

as well as the boundary conditions determining the receiving of raw materials from the warehouse, materials for the first operation and the output of finished products from the last operation:

$$[\chi]_0(t, 0) = [\chi]_{0S}(t), \quad [\chi]_{\text{II}}(t, 0) = [\chi]_{\text{II}S}(t).$$

In the absence of the activation of additional equipment parameters  $[\chi]_0(t, S)$ ,  $[\chi]_1(t, S)$ ,  $[\chi]_{1\psi}(t, S)$  in the moment  $t_q$  the end of the  $q$ -th shift and the start of the  $(q+1)$ -th shift are continuous functions of time  $t$  with the continuous derivatives of the  $n$ -th order:

$$\begin{aligned} [\chi]_0(t_{q-}, S) &= [\chi]_0(t_{q+}, S), & \frac{\partial^n [\chi]_0(t_{q-}, S)}{\partial t^n} &= \frac{\partial^n [\chi]_0(t_{q+}, S)}{\partial t^n}, \\ [\chi]_1(t_{q-}, S) &= [\chi]_1(t_{q+}, S), & \frac{\partial^n [\chi]_1(t_{q-}, S)}{\partial t^n} &= \frac{\partial^n [\chi]_1(t_{q+}, S)}{\partial t^n}, \\ [\chi]_{1\psi}(t_{q-}, S) &= [\chi]_{1\psi}(t_{q+}, S), & \frac{\partial^n [\chi]_{1\psi}(t_{q-}, S)}{\partial t^n} &= \frac{\partial^n [\chi]_{1\psi}(t_{q+}, S)}{\partial t^n}. \end{aligned}$$

The notation  $t_{q-}$  and  $t_{q+}$ , means that the functions  $[\chi]_0(t, S)$ ,  $[\chi]_1(t, S)$ ,  $[\chi]_{1\psi}(t, S)$  are considered in the infinitely small neighborhood to the left and to the right of  $t_q$ . We assume that the regulatory parameters characterizing the operation during the production cycle  $T_d$  remain unchanged in time:

$$[\chi]_0(t, S) = [\chi]_0(0, S) - \frac{\partial [\chi]_{1q}(S)}{\partial S} t, \quad [\chi]_1(t, S) = [\chi]_{1\psi}(t, S) = [\chi]_{1q}(S). \quad (2)$$

Condition

$$\left. \frac{\partial [\chi]_{1q}(S)}{\partial S} \right|_{S=S_m} < 0$$

corresponds to the increase in the density of inter-operating reserve for a technological  $m$ -th operation during the cycle time  $T_d$  for the case when there is no activation of additional equipment in the second and third shifts. In this case, the rate of processing of parts at the technological operation will be considered as a constant over time. The average daily rate can be changed depending on the shift value of the process equipment. The time value corresponds to the time  $t_q$  of completion of works in the  $q$ -th shift under the single-mode operation mode of the main equipment (duration of the work shift, hours):

$$\Delta t_q = (t_q - t_{q-1}) = 8.$$

The number of inter-operating reserve at the moment of time corresponding to the completion of work in the  $q$ -th shift is equal to their number at the beginning of the work of the  $(q+1)$ -th shift (2). The increase in the density of inter-operating reserves with time will lead to overflow of the capacity of the inter-operating bunker located in the point of the technological route defined by the coordinate  $S_m$ , and ultimately, to the overflow of bunkers on the section  $[0, S_m]$  of the technological route with the subsequent stopping of the production line.

The conditions for the occurrence of the drive overflow process and the study of the evolution of its development for the technological route section  $[0, S_m]$  are described in detail in [4, 6, 9-12].

To ensure the smooth operation of the production line, it is necessary to synchronize the rate of processing of objects of labor in individual operations within the time interval between the beginning of the  $t_q$ -th and the beginning of the  $t_{q+1}$ -th work shift.

We supplement equations (1) with the control function  $U_\delta(t, S)$ :

$$\begin{aligned} \frac{\partial [\chi]_0(t, S)}{\partial t} + \frac{\partial [\chi]_{1q}(S)}{\partial S} &= - \frac{\partial ([\chi]_{1q}(S) \cdot U_\delta(t, S))}{\partial S}, & (3) \\ U_\delta(t, S) = W_\delta(t, S) &= W(t, S) \cdot \delta(t - t_q), \quad t_q = 8 \cdot q \quad W(t_q, S) \leq 8, \end{aligned}$$

determining the duration of the activation of equipment in the position  $S$  at the time between the end of the  $q$ -th and the beginning of  $(q+1)$ -th shift ( $q = 1, 2, 3, \dots$ ). The control of the flow parameters carried

out as a result of the use of additional equipment at the moment of time  $t_q$  between the end of the  $q$ -th shift and the beginning of the  $(q+1)$ -th shift is determined through the Dirac delta function  $\delta(t-t_q)$  [13, 14]. The equation for changing the density of inter-operating reserve (3) can be integrated over time:

$$[\chi]_0(\tau, S) = [\chi]_0(0, S) - \frac{\partial [\chi]_{1q}(S)}{\partial S} \tau - \sum_{q=1}^k \frac{\partial}{\partial S} ([\chi]_{1q}(S) W(t_q, S)), \quad (4)$$

where the replenishment time points of the production line are  $0 < t_1 < t_2 < \dots < t_q < t_{q+1} < \dots < t_k \leq \tau$  (hour) with the duration of the work the additional equipment  $W(t_1, S), W(t_2, S), \dots, W(t_k, S)$  (hour).

Replenishment time moments  $t_q$  and the duration of additional equipment operation  $W(t_q, S) \leq 8$  (h) depend on the choice of control. The total number of objects of labor in the inter-operational reserve of the technological operation bounded by the coordinates of the technological route  $S_{m-1}$  and  $S_m$  is the quantity:

$$\int_{S_{m-1}}^{S_m} ([\chi]_0(\tau, S) - [\chi]_0(0, S)) dS = -([\chi]_{1q}(S_m) - [\chi]_{1q}(S_{m-1})) \tau - \sum_{q=1}^k ([\chi]_{1q}(S_m) \cdot W(t_q, S_m) - [\chi]_{1q}(S_{m-1}) \cdot W(t_q, S_{m-1})),$$

where

$$\int_{S_{m-1}}^{S_m} ([\chi]_0(\tau, S) - [\chi]_0(0, S)) dS - \text{changing backlog } m\text{-th operation for time } \tau;$$

$[\chi]_{1q}(S_{m-1}) \tau$  - the number of objects of labor, which arrived at the  $m$ -th operation with  $(m-1)$ -th during the time  $\tau$  (h);

$[\chi]_{1q}(S_m) \tau$  - the number of objects of labor, which took from  $m$ -th operation to  $(m+1)$ -th in time  $\tau$  (h);  $[\chi]_{1q}(S_{m-1}) \cdot W(t_q, S_{m-1})$  - the number of objects of labor, which arrived at the  $m$ -th operation with the  $(m-1)$ -th as a result of the work of additional equipment during the time  $W(t_q, S_{m-1})$  between the beginning of the  $q$ -th and the beginning of the  $(q+1)$ -th shift;

$[\chi]_{1q}(S_m) W(t_q, S_m)$  - the number of objects of labor, which went with the  $m$ -th operation on the  $(m-1)$ -th as a result of the work of additional equipment during the time  $W(t_q, S_m)$  between the beginning of the  $q$ -th and the beginning of the  $(q+1)$ -th shift.

The discontinuity of the function  $[\chi]_0(t, S)$  corresponding to the value  $[\chi]_{1q}(S_m) W(t_q, S_m)$ , is determined by the work of the additional equipment with the capacity  $[\chi]_{1q}(S_m)$  included between  $t_q$  and  $t_{q+1}$  the shift for processing the subject of labor for a time  $W(t_q, S_m)$ .

### Statement of the task of optimal control of the production line

In a general form, the task of building an optimal program for controlling flow parameters  $[\chi]_0(t, S)$ ,  $[\chi]_{1l}(t, S)$ , when additional equipment is used, can be formulated as follows: determine the state of parameters  $[\chi]_0(t, S) \in G_0, [\chi]_{1l}(t, S) \in G_1$  for each route point  $S \in [0, S_d]$  during the period of time  $t \in [0, T_d]$  while managing  $U_\delta(t, S) \in G_U$  the additional equipment operation time

$$U_\delta(t, S) = W_\delta(t, S) = W(t, S) \delta(t - t_q), \quad t_q = 8 \cdot q,$$

delivering a minimum of functionality

$$C_{cb} = \int_0^{T_d} \int_0^{S_d} U_\delta(t, S) \cdot \omega_z(t, S) dS dt \rightarrow \min, \quad (5)$$

with differential connections

$$\frac{\partial [\chi]_0(t, S)}{\partial t} + \frac{\partial [\chi]_{1q}(S)}{\partial S} = - \frac{\partial ([\chi]_{1q}(S) \cdot U_\delta(t, S))}{\partial S},$$

with restrictions along the trajectory on the phase variables  $[\chi]_0(t, S)$  determined by the storage capacity

$$0 \leq [\chi]_0(t, S), \quad [\chi]_0(t, S) \leq [\chi]_{0G}(S),$$

with constraints along the trajectory on the control [7]

$$0 \leq U_\delta(t, S), \quad 0 \leq \int_{t_q - 0.5\Delta t_q}^{t_q + 0.5\Delta t_q} U_\delta(t, S) dt \leq U_{\delta G}(S) = 8, \quad \Delta t_q \rightarrow 0, \quad (6)$$

initial conditions

$$[\chi]_0(0, S) = [\chi]_{00}(S),$$

final state (control goal)

$$[\chi]_0(T_d, S) = [\chi]_{0T_d}(S)$$

and boundary conditions

$$[\chi]_{11}(t, 0) = [\chi]_{11q}(0), \quad [\chi]_{11}(t, S_d) = [\chi]_{11q}(S_d),$$

where  $t_q = 8q$  is the end time of the  $q$ -th shift;  $\delta(t - t_q)$ - delta function. Control is carried out in the time interval between shifts

$$t \in \left[ t_q - \frac{\Delta t_q}{2}, t_q + \frac{\Delta t_q}{2} \right], 0 < t_1 < t_2 < \dots < t_q \dots \leq T_d \quad .$$

Control  $U_\delta(t, S)$  should be understood as a certain impulse, which is an idealization of ordinary control, during a shift concentrated in a neighborhood of a point  $t_q$  [13]. The value of the function  $U_\delta(t, S)$  at the time point  $t$  specifies the number of hours of operation of the additional equipment at the location of the technological route with a coordinate  $S$ .

Condition (6) for different parts of the route is recorded under the assumption that the duration of the work of additional equipment cannot exceed the duration of the technological shift equal to eight hours.

The tempo of processing of objects of labor along the technological route is a given function  $[\chi]_{1q} = [\chi]_{11q}(S)$  from the coordinate  $S$ . The tempo of movement of objects of labor  $[\chi]_{11}(t, 0) = [\chi]_{11q}(0)$ ,

coming in the form of raw materials and materials to the production line, and the tempo  $[\chi]_{11}(t, S_d) = [\chi]_{11q}(S_d)$  final parts leaving the last operation, do not depend on time, consistent with the plan of supplying the production with raw materials and the plan of shipments of final products. Functional (5) defines the criterion for the quality of control of flow parameters during the production cycle  $T_d$ , reflects the excess costs for all technological operations associated with the use of additional equipment. It is assumed that the cost of an hour of operation of additional equipment is set different for each operation and depends on the time, determined by the function. In determining the optimal control program, we assume that the intervals between shifts are adjacent to each other  $\omega_z(t, S)$ . We assume that the main equipment works continuously during the work shift. To ensure the continuity of the flow of items of labor on the technological route

$$[\chi]_{0G}(S) \geq [\chi]_0(t, S) \geq 0,$$

the operation of the additional equipment must be controlled in such a way that during the shift in the interval  $t \in ] t_q, t_{q+1}[$  the main equipment functions continuously. Strict equality

$$[\chi]_{0G}(S) \equiv [\chi]_0(t, S), \quad [\chi]_0(t, S) \equiv 0,$$

which corresponds to filling the bunker with objects of labor and emptying it, is allowed only in the time interval

$$t \in \left[ t_q - \frac{\Delta t_q}{2}, t_{q+1} - \frac{\Delta t_q}{2} \right].$$

Integrating the balance equations in the specified time interval, we get:

$$[\chi]_0 \left( t_{q+1} - \frac{\Delta t_q}{2}, S \right) = [\chi]_0 \left( t_q - \frac{\Delta t_q}{2}, S \right) - \frac{\partial [\chi]_{1q}(S)}{\partial S} \cdot 8 - \frac{\partial}{\partial S} \int_{t_q - \frac{\Delta t_q}{2}}^{t_q + \frac{\Delta t_q}{2}} [\chi]_{1q}(S) \cdot U_\delta(t, S) dt,$$

that allows us to write the condition of inadmissibility of overflow of the inter-operational bunker

$$[\chi]_{0G}(S) - [\chi]_0 \left( t_q - \frac{\Delta t_q}{2}, S \right) + \frac{\partial [\chi]_{1q}(S)}{\partial S} \cdot 8 + \frac{\partial}{\partial S} \int_{t_q - \frac{\Delta t_q}{2}}^{t_q + \frac{\Delta t_q}{2}} [\chi]_{1q}(S) \cdot U_\delta(t, S) dt \geq 0$$

and the condition of inadmissibility of the emptiness of the inter-operational bunker

$$[\chi]_0 \left( t_q - \frac{\Delta t_q}{2}, S \right) - \frac{\partial [\chi]_{1q}(S)}{\partial S} \cdot 8 - \frac{\partial}{\partial S} \int_{t_q - \frac{\Delta t_q}{2}}^{t_q + \frac{\Delta t_q}{2}} [\chi]_{1q}(S) \cdot U_\delta(t, S) dt \geq 0.$$

The function  $[\chi]_0(t_q, S)$  determines the distribution of objects of labor according to the technological route at the moment of time  $t = t_q$ , corresponding to the beginning of the  $q$ -th shift.

Control  $U_\delta(t, S)$  with restrictions on both phase variables and control, ensures the achievement of the control target with the minimum value of the integral (5) and with given differential constraints, is the optimal program for the flow parameters of the production line. In the absence of control, the equation of relations is (1). Changing the number of objects of labor in the area of the production line during the operation is only possible due to the receiving of objects of labor from the previous operation

and their leaving for the next as a result of technological processing. Term  $\frac{\partial([\chi]_{1q}(S) \cdot U_\delta(t, S))}{\partial S}$

containing control  $U_\delta(t, S)$ , plays the role of a source or destination [1] of objects of labor in the considered element of the volume of phase space. The initial conditions and the management goals determine the distribution of objects of labor along the route at the initial and the final point in time.

### Conclusion

Achievement of the production system with the initial distribution of objects of labor  $[\chi]_{00}(S)$  along the technological route in a precise line over the duration of the production cycle  $T_d$  of the final state  $[\chi]_{0T_d}(S)$  can be implemented in a variety of ways, each of which is called a control program. Among the technical problems of managing the state of the production line, finding the most optimal program for the use of resources (optimal control) is crucial. The mathematical reflection of this fact is that the control of the parameters of the production line should be chosen from the condition of minimum integral (5).

This paper shows that along with traditional models for controlling the parameters of production flow lines, control models associated with the use of partial differential equations (PDE models) play an important role. A PDE-model for controlling the parameters of a production line has been considered, taking into account the restrictions on the volume of inter-operating bunkers along the technological route.

The developed model allows us to determine the schedule of activating and deactivating the process equipment. A method for controlling the synchronization of technological operations of an industrial production line is considered. The quality criterion of the production system has been set, which allows building optimal control over the parameters of a production line operating in multi-shift mode. It is shown that partial differential equations that act as differential constraints for phase variables are replaced by a system of equations for the coefficients of decomposition of production line parameters, which allowed us to obtain the control function in the form of time dependence and position (coordinates) in the technological route.

#### ЛІТЕРАТУРА

1. Пигнастый О. М., Ходусов В. Д. Расчет производственного цикла с применением статистической теории производственно-технических систем. *Доклады Национальной академии наук Украины*. 2009. №12. С. 38-44. <https://doi.org/10.13140/RG.2.2.36267.54562>
2. Азаренков Н. А., Пигнастый О. М., Ходусов В. Д. Кинетическая теория колебаний параметров поточной линии. *Доклады Национальной академии наук Украины*. 2014. №12. С. 36-43. <https://10.15407/dopovidi2014.12.036>
3. Pinxten J., Waqas U., Geilen M., Basten A. Online scheduling of 2re-entrant flexible manufacturing systems. *ACM Transactions on Embedded Computing Systems*. 2017. Vol. 16. P. 1–20. <https://doi.org/10.1145/3126551>
4. Ancona F., Cesaroni A., Coclite G., Garavello M. On the Optimization of Conservation Law Models at a Junction with Inflow and Flow Distribution Controls. *SIAM Journal on Control and Optimization*. 2018. Vol. 56. P. 3370–3403. <https://doi.org/10.1137/18M1176233>
5. Kaihara T., Katsumura Y., Suginishi Y., Kádár B. Simulation model study for manufacturing effectiveness evaluation in crowdsourced. *CIRP Annals*. 2017. Vol. 66. Issue 1. P. 445–448. <https://10.1016/j.cirp.2017.04.094>
6. Власов В. А., Тихомиров И. А., Локтев И. И. Моделирование технологических процессов изготовления промышленной продукции. Томск : Изд-во ГТТУ, 2006. 300 с.
7. Михайлов В. С. Теория управления. Киев : Высшая школа, 1988. 312 с.
8. Коробецкий Ю. П. Имитационные модели в гибких системах. Луганск: Изд-во ВНУ им. В. Даля, 2003. 280 с.
9. Пигнастый О. М. Задача оптимального оперативного управления макропараметрами производственной системы с массовым выпуском продукции *Доклады Национальной академии наук Украины*. 2006. № 5. С. 79–85. <https://doi.org/10.13140/RG.2.2.29852.28802>
10. Tiacci L. Simultaneous balancing and buffer allocation decisions for the design of mixed-model assembly lines with parallel workstations and stochastic task times. *International Journal of Production Economics*. 2015. Vol.162. P. 201–215. <https://doi.org/10.1016/j.ijpe.2015.01.022>
11. Kück M., Broda E., Freitag M., Hildebrandt T., Frazzon E. Towards adaptive simulation-based optimization to select individual dispatching rules for production *Winter Simulation Conference (WSC)*. Las Vegas, NV, USA, 2017. P.3852–3863. <https://doi.org/10.1109/WSC.2017.8248096>
12. Falco M., Mastrandrea N., Mansoor W., Rarità L. Situation Awareness and Environmental Factors: The EVO Oil Production *New Trends in Emerging Complex Real Life Problems*. Springer, 2018. P. 209–217. [https://doi.org/10.1007/978-3-030-00473-6\\_23](https://doi.org/10.1007/978-3-030-00473-6_23)
13. Дыхта В. А., Самсонюк О. Н. Оптимальное импульсное управление с приложениями. Москва: Физматлит, 2000. 255 с.
14. Иваненко Д. Д., Соколов А. А. Классическая теория поля (новые проблемы). Москва: гос. изд-во технико-теорет. лит., 1951. 480 с.

#### REFERENCES

1. O.M. Pihnastyi and V. D. Khodusov “The calculation of a production cycle with application of the statistical theory of industrial-technical systems.” Reports of the National Academy of Sciences of Ukraine. vol.12. p.p.. 38-44. 2009. <https://doi.org/10.13140/RG.2.2.36267.54562> [in Russian]
2. N.A. Azarenkov, O.M. Pihnastyi and V.D. Khodusov “Kinetic theory of fluctuations of the parameters of a production line”. *Reports of the National Academy of Sciences of Ukraine*. vol.12. p.p.. 36-43. 2014. <https://10.15407/dopovidi2014.12.036> [in Russian]



3. J. Pinxten, U. Waqas, M. Geilen and A. Basten “Online scheduling of 2re-entrant flexible manufacturing systems.” *ACM Transactions on Embedded Computing Systems*, vol. 16, p.p. 1–20, 2017. <https://doi.org/10.1145/3126551>
4. F. Ancona, A. Cesaroni, G. Coclite, and M. Garavello “On the Optimization of Conservation Law Models at a Junction with Inflow and Flow Distribution Controls.” *SIAM Journal on Control and Optimization*, vol. 56, p.p. 3370–3403, 2018. <https://doi.org/10.1137/18M1176233>
5. T. Kaihara, Y. Katsumura, Y. Suginishi and B. Kádár “Simulation model study for manufacturing effectiveness evaluation in crowdsourced.” *CIRP Annals*, vol. 66, Issue 1, p.p. 445–448, 2017. <https://10.1016/j.cirp.2017.04.094>
6. V. A. Vlasov, I. A. Tihomirov and I. I. Loktev *Modelirovanie tehnologicheskikh processov izgotovleniya promyshlennoj produkcii*. Tomsk: izd-vo GTPU, 2006. [in Russian]
7. V. S. Mihajlov *Teoriya upravleniya*. Kiev: Vysshaja shkola, 1988. [in Russian]
8. Ju. P. Korobeckij *Imitacionnye modeli v gibkikh sistemah*. Lugansk: izd-vo VNU im. Dalja, 2003. [in Russian]
9. O.M. Pihnastyi “ The task of optimal operational control of macroparameters of a production system with mass production” *Доклады Национальной академии наук Украины*, vol. 5, p.p. 79–85, 2006. [in Russian]<https://doi.org/10.13140/RG.2.2.29852.28802>
10. L. Tiacci “Simultaneous balancing and buffer allocation decisions for the design of mixed-model assembly lines with parallel workstations and stochastic task times.” *International Journal of Production Economic*, vol.162, p.p. 201–215, 2015. <https://doi.org/10.1016/j.ijpe.2015.01.022>
11. M. Kück, E. Broda, M. Freitag, T. Hildebrandt and E. Frazzon. “Towards adaptive simulation-based optimization to select individual dispatching rules for production” *Winter Simulation Conference (WSC). Las Vegas, NV, USA, 2017*, p.p. 3852–3863. <https://doi.org/10.1109/WSC.2017.8248096>
12. M. Falco, N. Mastrandrea, W. Mansoor and L. Rarità “Situation Awareness and Environmental Factors: The EVO Oil Production” *New Trends in Emerging Complex Real Life Problems*. Springer, p.p. 209–217. 2018. [https://doi.org/10.1007/978-3-030-00473-6\\_23](https://doi.org/10.1007/978-3-030-00473-6_23)
13. V. A. Dyhta and O. N. Samsonjuk *Optimal'noe impul'snoe upravlenie s prilozhenijami*. Moskva: Fizmatlit, 2000. [in Russian]
14. D. D. Ivanenko and A. A. Sokolov *Klassicheskaja teoriya polja (novye problemy)*. Moskva: gos. izd-vo tehniko-teoret lit. 1951. [in Russian]

**Ходусов Валерій Дмитрович** - доктор фізико-математичних наук, професор; Харківський національний університет імені В.Н. Каразіна, Харків-22, площа Свободи, 4, 61022; e-mail: [vdkhodusov@karazin.ua](mailto:vdkhodusov@karazin.ua); ORCID: <http://orcid.org/0000-0003-1129-3462>.

**Khodusov Valery Dmitrievich** – Doctor of Physical and Mathematical Sciences, Professor; Kharkiv National University named after V.N. Karazin, Kharkov-22, Svobody Square, 4, 61022; e-mail: [vdkhodusov@karazin.ua](mailto:vdkhodusov@karazin.ua); ORCID: <http://orcid.org/0000-0003-1129-3462>.

**Ходусов Валерій Дмитрієвич** - доктор фізико-математических наук, професор; Харьковський національний університет імені В.Н. Каразін, Харків-22, площа Свободи, 4, 61022; e-mail: [vdkhodusov@karazin.ua](mailto:vdkhodusov@karazin.ua); ORCID: <http://orcid.org/0000-0003-1129-3462>.

**Пігнастій Олег Михайлович** - доктор технічних наук, професор; Національний технічний університет "Харківський політехнічний інститут" Пушкінська 79-2, 1 поверх, Харків, Україна, 61102; e-mail: [pihnastyi@gmail.com](mailto:pihnastyi@gmail.com); ORCID: <http://orcid.org/0000-0002-5424-9843>.

**Pihnastyi Oleh Mikhailovich**– Doctor of Engineering Science, Professor; National Technical University "Kharkiv Polytechnic Institute" Pushkinskaya 79-2, 1st Floor, Kharkov, Ukraine, 61102; e-mail: [pihnastyi@gmail.com](mailto:pihnastyi@gmail.com); ORCID: <http://orcid.org/0000-0002-5424-9843>.

**Пігнастій Олег Михайлович** - доктор техніческих наук, професор; Национальный технический университет "Харьковский политехнический институт" Пушкинская 79-2, 1 этаж, Харьков, Украина, 61102; e-mail: [pihnastyi@gmail.com](mailto:pihnastyi@gmail.com); ORCID <http://orcid.org/0000-0002-5424-9843>.