UDC 519.24

The Sequences with Stationary differences

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This paper studies nonstationary random sequences with stationary increments. General representations are obtained for their correlation function and correlation differences. The general case is studied for non-stationary sequence, which is the solution of difference equation with stationary right-hand side. Derived spectral representations prove that such sequences are harmonizable. The general representation of solution correlation function is obtained for equation, the right-hand side of which is a non-stationary sequence of finite non-stationarity rank.

Key words: *difference equation, stationary increment, the correlation function, the correlation difference, spectral expansion, harmonizable, non-stationarity rank.*

У статті вивчаються нестаціонарні випадкові послідовності зі стаціонарними приростами. Для них отримані загальні зображення кореляційної функції і кореляційних різниць. Досліджено загальний випадок нестаціонарної послідовності, що є рішенням різницевого рівняння зі стаціонарною правою частиною. Отримано спектральні зображення, що доводять гармонізуємість таких послідовностей. Для рівняння, права частина якого є нестаціонарна послідовність кінцевого рангу нестаціонарності, отримано загальне зображення кореляційної функції розв'язку.

Ключові слова: *різницеве рівняння, стаціонарний приріст, кореляційна функція, кореляційний різниця, спектральне зображення, гармонізуємість, ранг нестаціонарності.*

В статье изучаются нестационарные случайные последовательности со стационарными приращениями. Для них получены общие представления корреляционной функции и корреляционных разностей. Исследован общий случай нестационарной последовательности, являющейся решением разностного уравнения со стационарной правой частью. Получены спектральные представления, доказывающие гармонизируемость таких последовательностей. Для уравнения, правая часть которого является нестационарной последовательностью конечного ранга нестационарности, получено общее представление корреляционной функции решения.

Ключевые слова: *разностное уравнение, стационарное приращение, корреляционная функция, спектральное разложение, гармонизируемость, ранг нестационарности.*

Introduction

Linear random difference equations with discrete variables, the right side of which is a random function having effect of discrete white noise [1], are widely used in modeling of random processes with discrete times. The type of non-stationarity of equation, the right side of which is a random sequence that belongs to one of random sequence classes, have not been studied yet.

To study sequences in *Hilbert* space, an operator concept was suggested and adequate non-stationarity characteristics were introduced in [2,3].

The related close subjects see also [4-6].

1. Cauchy function

For further investigation, we will use the representation of linear difference equations of order k with constant coefficients in the form:

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$$\begin{cases} L_k\, x(n) = \sum_{j=0}^{k} \alpha_j\, \Delta_j\, x(n) = f(n) \\ \alpha_k = 1 \quad , \quad \Delta_0 x(n) = x(n) \end{cases} \qquad (1.1)$$

where n is a natural number, x is the unknown function, and $\alpha_j \left(j = \overline{0\, ,\, k} \right)$ are known real numbers;
or in equivalent form:

$$\begin{cases} L_k\, x(n) = \sum_{j=0}^{k} a_j\, x(n+j) = f(n) \\ a_k = 1 \end{cases} \qquad (1.2)$$

Definition:

For each given natural number m, the Cauchy function $\Phi(n,m)$ is defined as solution of homogenous linear difference equation with constant coefficients

$$\begin{cases} L_k\, \Phi(n,m) = \sum_{j=0}^{k} \alpha_j\, \Delta_j\, \Phi(n,m) = 0 \\ \alpha_k = 1 \quad , \quad \Delta_0 \Phi(n,m) = \Phi(n,m) \end{cases} \qquad (1.3)$$

under such initial conditions

$$\Phi(n,m)\Big|_{n=m} = 0 \;,\; \Delta_1 \Phi\Big|_{n=m} = 0 \;,\; \ldots \;, \Delta_{k-2} \Phi\Big|_{n=m} = 0 \;,$$
$$, \Delta_{k-1} \Phi\Big|_{n=m} = 1 \qquad (1.4)$$

Let us consider some examples of Cauchy function for linear difference equations with constant coefficients [7].

Example 1

Let us consider the difference equation

$$\Delta_1\, x(n) + \alpha_0 x(n) = f(n) \qquad (1.5)$$

The expression for its Cauchy is following: $\Phi(n,m) = \left(1-\alpha_0\right)^{n-m}$.

Example 2

In the case of linear difference equation of second order

$$\Delta_2\, x(n) + \alpha_1\, \Delta_1\, x(n) + \alpha_0\, x(n) = f(n) \qquad (1.7)$$

using the equality $\Delta_k\, x(n) = \sum_{j=0}^{k} (-1)^{k-j}\, C_k^j\, x(n+j)$ we can transform (1.7) to the form

$$x(n+2) + (\alpha_1 - 2)\, x(n+1) + (1 - \alpha_1 + \alpha_0)\, x(n) = f(n) \qquad (1.8)$$

which is a linear difference equation of second order with constant coefficients.
Cauchy function $\Phi(n,m)$ is that solution of difference equation

$$\Phi(n+2,m) + (\alpha_1 - 2)\Phi(n+1,m) + (1 - \alpha_1 + \alpha_0) = 0 \qquad (1.9)$$

which satisfies this two initial conditions

$$\Phi(n,m)\big|_{n=m}=0 \qquad , \qquad \Delta_1\,\Phi(n,m)\big|_{n=m}=1$$

$$\begin{cases} c_1(m)\lambda_1^{\,n}+c_2(m)\lambda_2^{\,n}=0 \\ c_1(m)\left(\lambda_1^{\,m+1}-\lambda_1^{\,m}\right)+c_2(m)\left(\lambda_2^{\,m+1}-\lambda_2^{\,m}\right)=1 \end{cases}$$

Thus, for $\Phi(n,m)$ we have $\qquad \Phi(n,m)=\dfrac{1}{\lambda_1-\lambda_2}\lambda_1^{\,n-m}+\dfrac{1}{\lambda_2-\lambda_1}\lambda_2^{\,n-m}$

Remark:

If the roots λ_1 and λ_2 are complex, they can be written in the form $\lambda_1=\rho e^{i\theta}$ and $\lambda_2=\rho e^{-i\theta}$, and making this substitution, we bring the Cauchy function to the form

$$\Phi(n,m)=\rho^{n-m-1}\frac{\sin\left((n-m)\theta\right)}{\sin\theta}=c\sin\left((n-m)\theta\right)$$

Further we prove several general theorems.

Theorem 1

Cauchy function $\Phi(n,m)$ for linear difference equation (1.3) of order k with homogenous constant coefficients, which satisfies the initial conditions (1.4), is a function of the difference $(n-m)$.

Theorem 2

The difference equation (1.1) has such a solution:

$$x(n)=\sum_{j=1}^{n-1}\Phi(n-1\,,\,j)\,f(j) \qquad (1.11)$$

where Cauchy function $\Phi(n,m)$ is a solution of homogenous difference equation (1.3) with initial conditions (1.4) [7].

Application of theorem (2) to both examples (1) and (2)

Due to mentioned above, we can transform solution of equation (10) to the form

$$x(n)=\sum_{j=1}^{n-1}(1-\alpha_0)^{\,n-1-j}f(j)$$

Solution of the equation (12) gets the form:

$$x(n)=\sum_{j=1}^{n-1}\left(\frac{1}{\lambda_1-\lambda_2}\lambda_1^{\,n-1-j}+\frac{1}{\lambda_2-\lambda_1}\lambda_2^{\,n-1-j}\right)f(j)$$

If n_0 is any integer then solution of non-homogenous equation (1.1) has the form

$$x(n)=\sum_{j=n_0}^{n-1}\Phi(n-1\,,\,j)\,f(j) \qquad (1.12)$$

where $\Phi(n,m)$ is Cauchy function.

So we have found that if $\lambda_k, \ldots, \lambda_2, \lambda_1$ are the roots of characteristic equation and they all are different then the general form of solution for (1.1) is

$$\begin{cases} x(n) = \sum_{j=1}^{k} \gamma_j \ \lambda_j^{n-n_0} + \sum_{j=n_0}^{n-1} \Phi(n-1\,,\,j)\, f(j) \\ \gamma_j = \dfrac{1}{\prod\limits_{\substack{\ell<i \\ \ell=j}} (\lambda_i - \lambda_\ell)} \end{cases} \tag{1.15}$$

It is obvious that if $\left(j=\overline{1,k}\right)$ $|\lambda_j|<1$, then for $n_0 \to -\infty$ we obtain:

$$x(n)_{stat} = \sum_{j=-\infty}^{n-1} \Phi(n-1\,,\,j)\, f(j) \tag{1.16}$$

This solution of difference equations (1.1) is called *steady state*.

We can consider each of non-homogenous stationary equations (1.1) and (1.2) as stochastic linear difference equation, the right side of which is a stochastic function $f(n)$ that behaves as discrete white noise $[1]$.

In this paper, we consider $f(n)$ as random sequence belonging to one of the random sequence classes.

2. Analytic approach to difference equations with random right side of the form $L_k \ \eta(n) = \xi(n)$

Let L_k be a linear difference operator having real constant coefficients, and let us consider the difference equation

$$L_k \ \eta(n) = \xi(n) \tag{2.1}$$

We have shown above that if all roots of characteristic equation for difference equation (2.1) are different and their absolute value is less than one, then the stationary solution will be given by:

$$\eta(n)_{stat} = \sum_{j=-\infty}^{n-1} \Phi(n-1, j)\, \xi(j) \tag{2.2}$$

where $\Phi(n,m)$ is Cauchy function that satisfies the difference equation (1.3) and initial conditions (1.4).

When the correlation function for equation right side is given the correlation function for solution has the following appearance

$$K_{\eta\eta}(n,m) = \sum_{j,\ell=-\infty}^{n-1,m-1} \Phi(n-1, j)\ \overline{\Phi(m-1,\ell)}\ K_{\xi\xi}(j,\ell) \tag{2.3}$$

and according to the theorem (1):

$$\Phi(n-1, j) = \Phi(n-1-j) \tag{2.4}$$

Thus the relationship (2.3) takes form:

$$K_{\eta\eta}(n,m) = \sum_{j,\ell=-\infty}^{n-1,m-1} \Phi(n-1-j)\,\overline{\Phi(m-1-\ell)}\,K_{\xi\xi}(j,\ell) \qquad (2.5)$$

In addition, since the sequence $\xi(n)$ is stationary, then:

$$K_{\xi\xi}(j,\ell) = K_{\xi\xi}(j-\ell)$$

Consequently, the correlation function of the sequence $\eta(n)$ takes the form:

$$K_{\eta\eta}(n,m) = \sum_{j,\ell=-\infty}^{n-1,m-1} \Phi(n-1-j)\,\Phi(m-1-\ell)\,K_{\xi\xi}(j-\ell) \qquad (2.6)$$

Using the following transformation: $n-1-j=n_1 \;\Rightarrow\; j=n-1-n_1$

$$m-1-\ell=m_1 \;\Rightarrow\; \ell=m-1-m_1$$

we obtain: $K_{\eta\eta}(n,m) = \sum_{n_1,m_1=+\infty}^{0,0} \Phi(n_1)\,\overline{\Phi(m_1)}\,K_{\xi\xi}(n-m-(n_1-m_1)) = f(n-m)$

which means that in this case the sequence $\eta(n)$ is stationary too.

Therefore, we have proved the following theorem.

Theorem 3:

If L_k is linear difference operator with real constant coefficients, and the sequence $\xi(n)$ is stationary, then the steady-state solution of difference equation, is also stationary.

According to the theorem (1), the equation (2.1) has solution given by the following expression:

$$\eta(n) = \sum_{j=1}^{n-1} \Phi(n-1-j)\,\xi(j)$$

Consequently, for initial conditions, we have:

$$K_{\eta\eta}(n,m) = \sum_{j,\ell=1}^{n-1,m-1} \Phi(n-1-j)\,\overline{\Phi(m-1-\ell)}\,K_{\xi\xi}(j,\ell)$$

For the case when coefficients of operator L_k are constant and roots of characteristic equation are different, we obtain the following expression for Cauchy function:

$$\Phi(n,m) = \Phi(n-m) = \sum_{p=1}^{k} \gamma_p\,\lambda_p^{n-m} \quad ; \quad \gamma_p = \frac{1}{\prod\limits_{\substack{j<i \\ j=p}} (\lambda_i - \lambda_j)}$$

Now we will shall consider in general the case when $\xi(n)$ is a stationary sequence. In this case, the correlation function of sequence $\eta(n)$ takes the form

$$K_{\eta\eta}(n,m) = \sum_{j,\ell=1}^{n-1,m-1} \sum_{p,q=1}^{k} \gamma_p \overline{\gamma_q}\,\lambda_p^{n-1-j}\,\overline{\lambda_q^{m-1-\ell}} \int_0^{2\pi} e^{i\lambda(j-\ell)}\,dF(\lambda)$$

or $$K_{\eta\eta}(n,m) = \int_0^{2\pi} \sum_{p,q=1}^{k} \gamma_p \overline{\gamma_q} \sum_{j=1}^{n-1} \lambda_p^{n-1-j} e^{i\lambda j} \sum_{\ell=1}^{m-1} \overline{\lambda_q^{m-1-\ell}} e^{-i\lambda\ell} dF(\lambda)$$

Supposing that: $\chi_p(n,\lambda) = \sum_{j=1}^{n-1} \lambda_p^{n-1-j} e^{i\lambda j}$

we get: $k_{\eta\eta}(n,m) = \int_0^{2\pi} \theta(\lambda,n)\overline{\theta(\lambda,m)}\, dF(\lambda)$

where: $\theta(\lambda,n) = \sum_{p=1}^{\kappa} \gamma_p \chi_p(n,\lambda), \ \Delta F(\lambda) = \|\Delta E_\lambda\|^2$

E_λ is the solution of equation

$$\chi_p(n,\lambda) = \sum_{j=1}^{n-1} \lambda_p^{n-1-j} e^{i\lambda j} = \lambda_p^{n-1} \sum_{j=1}^{n-1} \left(\frac{e^{i\lambda}}{\lambda_p}\right)^j =$$

$$= \lambda_p^{n-1} \frac{\left(\frac{e^{i\lambda}}{\lambda_p}\right)^n - \frac{e^{i\lambda}}{\lambda_p}}{\frac{e^{i\lambda}}{\lambda_p} - 1} = \frac{e^{i\lambda n} - e^{i\lambda}\lambda_p^{n-1}}{e^{i\lambda} - \lambda_p}$$

Supposing that $F'(\lambda)$ exists and equals to $f(\lambda)$, we can conclude that the correlation function can be written in the form:

$$K_{\eta\eta}(n,m) = \int_0^{2\pi} \tilde{\theta}(\lambda,n)\overline{\tilde{\theta}(\lambda,m)}\, d\lambda$$

where $\tilde{\theta}(\lambda,n) = \theta(\lambda,n)\sqrt{f(\lambda)}$

(i. e. the sequence $\eta(n)$ is harmonizable [10]).

So we have proved the following theorem:

Theorem 4:

Let L_k be a linear difference operator with constant coefficients, all roots of characteristic equation $L_k \eta(n) = 0$ are different, and sequence $\xi(n)$ where $L_k \eta(m) = \xi(n)$ is stationary. Then the correlation function of process $\eta(n)$ takes form:

$$K_{\eta\eta}(n,m) = \int_0^{2\pi} \tilde{\theta}(\lambda,n)\overline{\tilde{\theta}(\lambda,m)}\, d\lambda$$

where: $$\tilde{\theta}(\lambda,n) = \theta(\lambda,n)\sqrt{f(\lambda)}$$

Application:

Let $\tilde{\xi}(n)=\sqrt{2\pi}\,\tilde{\theta}(n,\xi_0(w))$ be a random sequence, where $\xi_0(w)$ is uniform over the interval $[0,2\pi]$. Then the correlation function of this sequence is:

$$\mathrm{M}\,\tilde{\xi}(n)\ \overline{\tilde{\xi}(m)} = 2\pi\int_0^{2\pi}\tilde{\theta}(n,\lambda)\ \overline{\tilde{\theta}(m,\lambda)}\,P(\lambda)\,d\lambda$$

where $P(\lambda)=\dfrac{1}{2\pi}$, thus it will be: $\mathrm{K}_{\tilde{\xi}\tilde{\xi}}(n,m) = \mathrm{K}_{\eta\eta}(n,m)$

This means that the two sequences $\xi(n)$ and $\tilde{\xi}(n)$ are unitary equivalent $[2,3]$.

3. The Relation between correlation differences of two sequences *ξ*(*n*) and *η*(*n*) in case of stationary solution

Let us rearrange the stationary equation $L_k\,\eta(n) = \xi(n)$ in the form

$$L_k^{\eta(n)}\,\eta(n) = \xi(n)$$

and suppose that $\Phi(n-m)$ is Cauchy function of L_k difference operator having constant real coefficients. Then we find that the correlation function of sequence $\xi(n)$ can be written in the form:

$$L_k^{\eta(n)}\ L_k^{\eta(m)}\left\langle \eta(n),\eta(m)\right\rangle = \left\langle \xi(n),\xi(m)\right\rangle$$

and thus
$$L_k^{\eta(n)}\ L_k^{\eta(m)}\ \mathrm{K}_{\eta\eta}(n,m) = \mathrm{K}_{\xi\zeta}(n,m)$$

The steady state solution of this equation is

$$\mathrm{K}_{\eta\eta}(n,m) = \sum_{p=-\infty}^{n-1}\sum_{q=-\infty}^{m-1}\Phi(n-1-p)\ \overline{\Phi(m-1-q)}\ \mathrm{K}_{\xi\zeta}(p,q)$$

we can obtain the same representation if we write stationary solution of difference equation of the form:

$$\eta(n) = \sum_{p=-\infty}^{n-1}\Phi(n-1,p)\ \xi(p)$$

Let $\mathrm{K}_{\eta\eta}(n,m)=\left\langle \eta(n)\ ,\ \eta(m)\right\rangle_{H_\eta} = \mathrm{M}\,\eta(n)\,\overline{\eta(m)}$

then $\mathrm{K}_{\eta\eta}(n,m)=\sum_{p=-\infty}^{n-1}\sum_{q=-\infty}^{m-1}\Phi(n-1-p)\,\overline{\Phi(m-1-q)}\ \mathrm{K}_{\xi\xi}(p,q)$

Replacing n with $n+1$ in equation: $L_k^{\eta(n)}\ \ \eta(n) = \xi(n)$

we have: $L_k^{\eta(n)}\,L_k^{\eta(m)}\ \mathrm{K}_{\eta\eta}(n,m) = \mathrm{K}_{\xi\xi}(n,m)$

$$L_k^{\eta(n)}\,L_k^{\eta(m)}\ \mathrm{K}_{\eta\eta}(n+1,m+1) = \mathrm{K}_{\xi\xi}(n+1,m+1)$$

therefore $L_k^{\eta(n)}\,L_k^{\eta(m)}\ W_{\eta\eta}(n,m) = W_{\xi\xi}(n,m)$

Consequently the steady state will be:

$$W_{\eta\eta}(n,m) = \sum_{p=-\infty}^{n-1}\sum_{q=-\infty}^{m-1}\Phi(n-1-p)\,\overline{\Phi(m-1-q)}\ W_{\xi\xi}(p,q)$$

Setting $W_{\eta\eta}(n,m) = \varphi(n)\overline{\varphi(m)}$ we get that:

$$W_{\eta\eta}(n,m) = \sum_{p,q=-\infty}^{n-1,m-1} \Phi(n-1-p) \;\; \overline{\Phi(m-1-q)} \;\; \varphi(p)\overline{\varphi(q)} = \psi(n) \;\; \overline{\psi(m)}$$

where $\psi(n) = \sum_{p=-\infty}^{n-1} \Phi(n-1-p) \; \varphi(p)$

So we have proved the following theorem.

Theorem 5

In steady state of difference equation $L_k \eta(n) = \xi(n)$, the correlation differences of two sequences $\eta(n), \xi(n)$ are tied by relation $W_{\eta\eta}(n,m) = \psi(n)\overline{\psi(m)}$

where $\psi(n) = \sum_{p=-\infty}^{n-1} \Phi(n-1-p) \; \varphi(p)$ and $\Phi(n,m)$ are Cauchy Functions of the difference operator L_k with real constant coefficients.

Definition

The non-stationary random sequence $\xi(n)$ is called a dissipative sequence if all the quadratic forms of this type

$$\sum_{n,m=0}^{N} W(n,m)\lambda_n \overline{\lambda_m}$$

are non-negative

From this definition, it follows that if the random sequence is dissipative then the quadratic forms will be non-increasing sequence OT p.

$$\sum_{n,m=0}^{N} K(n+p,m+p)\lambda_n \overline{\lambda_m}$$

In a special case, the sequence $K(n,n)$ is non-increasing sequence, from which it follows that the limit exists.

$$\lim_{n\to\infty} K(n,n) = \sigma_\infty^2$$

It is possible that $1)\sigma_\infty^2 = 0; \;\; 2)\sigma_\infty^2 > \infty$

In the first case, the sequence is called asymptotically damped, and in the second case is called asymptotically undamped

Retuning to equation(3.1), in addition to what we have supposed above, we assume that $\xi(n)$ is non-stationary random sequence of the order r, dissipative, and convergent damped [3].

Easily, we find that the correlation function can be expressed by relation:

$$\mathrm{K}_{\xi\xi}(n,m) = \sum_{\alpha=1}^{r} \sum_{r=0}^{\infty} \psi_\alpha(n+\tau)\overline{\psi_\alpha(m+\tau)}$$

Using (3.5) we obtain:

$$\mathrm{K}_{\eta\eta}(n,m) = \sum_{\alpha=1}^{r}\sum_{r=0}^{\infty}\sum_{j,\ell=-\infty}^{n-1,m-1}\Phi(n-1-j)\ \overline{\Phi(m-1-\ell)}\ \psi_\alpha(j+\tau)\ \overline{\psi_\alpha(\ell+\tau)}$$

$$= \sum_{\alpha=1}^{r}\sum_{r=0}^{\infty}\widetilde{\psi_\alpha}(n+\tau)\,\overline{\widetilde{\psi_\alpha}(m+\tau)}$$

where

$$\widetilde{\psi_\alpha}(n+\tau) = \sum_{j=-\infty}^{\mathrm{n}-1}\Phi(n-1-j)\,\psi_\alpha(j+\tau)$$

By rearrangement $(j = n-1-n_1)$ $n-1-j = n_1$

we get: $$\widetilde{\psi_\alpha}(n+\tau) = \sum_{\mathrm{n}_1=0}^{\infty}\Phi(n_1)\,(n-1-n_1+\tau)$$

Easily we can see that the non-stationarity order of sequence $\eta(n)$ equals to r

Application:

Let $x(n+1) = a\,x(n) + f(n)$ be the first order difference equation.
The solution of this equation, which meets the initial conditions.

$$x(n_0)\big|_{n=n_0} = x_0$$

is given by the relation $x(n) = a^{n-n_0} \quad x_0 + \sum_{j=n_0}^{n-1} a^{n-(j+1)}\ f(j)$

If $|a|<1$ the stationary solution takes this form

$$x_{stat}(n) = \sum_{j=-\infty}^{n-1} a^{n-1-j}\ f(j) = a^{n-1}\sum_{j=-\infty}^{n-1}\frac{f(j)}{a^j}$$

This series is convergent when n_0 tends to $-\infty$ if

$$\lim_{j\to-\infty}\frac{f(j+1)}{a^{j+1}}.\frac{a^j}{f(j)} = \frac{1}{a}\lim_{j\to-\infty}\frac{f(j+1)}{a^{j+1}} < 1$$

which means $\lim\limits_{j\to-\infty}\dfrac{f(j+1)}{f(j)} < \alpha$

Easily we find that the correlation function of this sequence is given by expression $\mathrm{K}_{xx}(n,m) = \sum_{p=-\infty}^{n-1}\sum_{q=-\infty}^{m-1} a^{n+m-p-q-2}\ \mathrm{K}_{ff}(p,q)$

If $\mathrm{K}_{ff}(p,q) = \sum_{j=0}^{\infty}\phi(p+j)\ \overline{\phi(q+j)}$

then $\mathrm{K}_{xx}(n,m) = \sum_{j=0}^{\infty}\psi(n,j)\ \overline{\psi(m,j)}$

where $\psi(n,j) = \sum_{p=-\infty}^{n-1} a^{n-p-1}\ \phi(p+j)$

Using equality $p+j=\ell$ for substitution we get

$$\psi(n,j)=\sum_{j=-\infty}^{n+j-1} a^{n+j-\ell-1}\ \phi(\ell)$$

Consequently, if it is $\psi(n,j)=\psi(n+j)$, therefore the non-stationary order equals to 1.

Conclusion

In this study a new class of non-stationary sequences was introduced. An operator approach was applied, harmonizability of mentioned sequences was shown, and general representations are obtained for their correlation function and correlation differences. This makes possible modeling of random sequences having different nature of non-stationarity, as well as sequence restoration only by its spectrum.

One can use such sequences in investigations of transient modes of discrete control systems in both cases: when noise or useful signals are non-stationary random sequences.

The authors express deep acknowledgment to Prof Yantsevich Artem A for valued and useful guidance offered during this work.

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Надійшла у першій редакції 25.10.2014, в останній - 30.10.2014.