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Modeling of the discrete flows interaction considering correlation between flow elements

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In the paper, the authors consider the simulation of flows interaction in the frame of extended model of discrete stochastic flows with correlation between elements. In the random arrival problem, the average delay time for a single element, which depends on differing clustered structures of major flow, is obtained using computer simulation system for crossing process modeling developed by the authors. The difference between models with and without correlation between elements as well as their application scopes are considered. The applicability of presented in this paper results to creation of intelligent transportation system is.

Key words: *stochastic discrete flow, cluster flow structure, average delay time*

У статті розглядається комп'ютерне моделювання взаємодії потоків в рамках розширеної моделі дискретних стохастичних потоків с урахуванням кореляції між елементами. Середній час очікування для одного елемента в задачі випадкового прибуття на перетин залежно від різної кластерної структури головного потоку знаходиться за допомогою розробленої авторами системи комп'ютерного моделювання процесу перетину. Обговорюється відмінність між моделями з урахуванням і без урахування кореляції між елементами в потоці і областю застосування цих моделей. Розглядається можливість застосування результатів, наведених у статті, в інтелектуальних транспортних системах.

Ключові слова: *стохастичний дискретний потік, кластерна структура, розподіл часових інтервалів, пропускна здатність.*

В статье рассматривается компьютерное моделирование взаимодействия потоков в рамках расширенной модели дискретных стохастических потоков с учетом корреляции между элементами. Среднее время ожидания для одного элемента в задаче случайного прибытия на пересечение в зависимости от различной кластерной структуры главного потока находится с помощью разработанной авторами системы компьютерного моделирования процесса пересечения. Обсуждается различие между моделями с учетом и без учета корреляции между элементами в потоке и областью применения этих моделей. Рассматривается возможность применения результатов, приведенных в статье, в интеллектуальных транспортных системах.

Ключевые слова: *стохастический дискретный поток, кластерная структура, распределение временных интервалов, пропускная способность.*

1. Introduction

With the help of computer simulation one can predict such negative phenomena as queuing and capacity reduction in complex transportation systems. Considering possibility of queues formation, intersections are the most critical components of the network.

Priority-controlled intersection is the mostly used type of intersections in road transportation systems [1-4]. Let us suppose that in the intersection (Fig.1) the possibility of crossing (merging) for element of minor flow is restricted if the time gap between major flow elements is less than some value α , and element of minor flow has to stop and give way. Intervals less than α will be called critical. The elements of major flow have the priority and can pass through without stopping at the intersections.

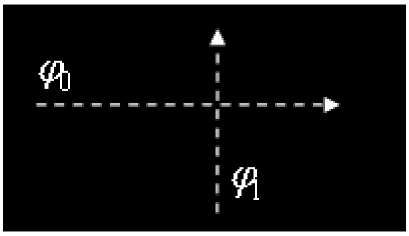

Fig.1 Model of crossing flows with different priorities φ_0 and φ_1

The classic problem is capacity determination of that kind of intersection. Capacity describes the maximum possible throughput of an intersection under predefined conditions. In [1-4] the authors show that capacity of the priority-controlled intersection in a transportation system is determined by the traffic flow rate and time headway distribution of the flow. It is often assumed that time gaps between elements are distributed independently, i.e. there is no correlation between them.

But there are some problems, in which existence of such flow element groups where all time gaps are less than critical value becomes important. Distribution of these groups in the flow must be included into consideration as well. According to Kerner [5], these groups or "moving jams" in discrete flows can be formed due to stochastic nature of a flow in the long roads or at the intersections of different kind.

Let us define as "clusters" [1] the groups of flow elements having only critical time gaps between them, and the "clustered structure" of a flow as a combination of two characteristics: the intercluster intervals distribution and the cluster size distribution.

On assumption that there is no correlation between cluster elements, the cluster size distribution in the flow can be found as:

$$p_{cl}(n) = (1-\omega)\omega^{n-1} \quad (1),$$

where ω – the probability of critical time gap appearance, n – the group size.

However, it will be shown bellow that $p_{cl}(n)$ can significantly vary depending on the right side of equation (1) while time headway distribution remains fixed.

Time headway distribution does not allow us to find out what is the clustered structure of the traffic flow without taking into consideration additional parameters that should describe the correlation between elements.

This paper is devoted to priority-controlled intersection modeling in the bounds of clustered flow model. The average delay time for element of minor flow before merging (crossing) in random arrival problem was calculated using for traffic flow description both clustered and non-clustered model. The authors measure the influence of the flow clustered structure on their interaction characteristics in two ways: with the help of analytical calculations and using computer system for crossing process simulation.

2. The average delay time in the random arrival problem

Let us consider the model of priority-controlled intersection with two crossing flows depicted in Figure 1: φ_0 - the major flow with time headway distribution ρ_0 and φ_1 – the minor flow; size of elements in the flows is equal to zero, critical interval for crossing – α. It is supposed that the minor flow level is so low that the probability of queuing (more than one element in the intersection simultaneously) tends to zero.

At first, let us obtain the average delay time in the non-clustered flow model using time headway distribution.

The element of minor flow randomly arriving to the intersection finds the time gap x between major flow elements. Let us define the distribution of x as modified time headway distribution - $\rho_0^*(x)$. According to Ventcel $\rho_0^*(x)$ can be obtained from ρ_0 as shown in [7]:

$$\rho_0^*(x) = (1/\bar{x})\int_x^{\infty} \rho_0(x)dx \qquad (2),$$

where $\bar{x}$ – the mean of ρ_0.

Then the average delay time can be calculated using the formula (3):

$$\overline{\tau_w} = \overline{\tau_\alpha^*} + \omega_\alpha^* \frac{\overline{\tau_\alpha}}{1-\omega_\alpha} \qquad (3),$$

Where:

$$\overline{\tau_\alpha^*} = \int_0^{\alpha} x\rho_0^*(x)dx \qquad (4)$$

$$\overline{\tau_\alpha} = \int_0^{\alpha} x\rho_0(x)dx \qquad (5)$$

$$\omega_\alpha = \int_0^{\alpha} \rho_0(x)dx \qquad (6)$$

$$\omega_\alpha^* = \int_0^{\alpha} \rho_0^*(x)dx = (\alpha/\bar{x})(1-\omega_\alpha) + 1/\bar{x}\int_0^{\alpha} x\rho_0(x)dt \qquad (7)$$

As we can see from (3), the average delay time depends only on time headway distribution of the major flow.

Now let us consider again the problem of element random arrival to the intersection, this time describing the major flow in terms of the flow model, which takes into account its clustered structure.

For clustered structure description the authors have used the composite model presented by May [6]. Intercluster intervals are distributed exponentially and intracluster intervals have the normal distribution (the intercluster intervals are larger than the critical interval size). p_1 - is the ratio of summarized intracluster intervals to total time of intervals in the major flow.

The probability for element of minor flow to fall within the cluster of size *n* is determined by time the cluster of that size occupies the intersection and is proportional to the cluster length in terms of time – $n\overline{\tau_\alpha}$. The delay time such element needs to wait until cluster of size *n* passes the intersection is proportional to $n\overline{\tau_\alpha}/2$.

So, theoretical evaluation of the average delay time before the element of the minor flow crosses (merges) the intersection with the major flow yields a value proportional to cluster size squared (8):

$$\overline{\tau_{wcl}} \sim p_1 \sum_n (n\overline{\tau_\alpha})^2 / 2 \qquad (8),$$

It is possible to change cluster size distribution and consequently $\sum_n (n\overline{\tau_\alpha})^2 / 2$ keeping p_1 untouched. The time, during which the cluster of size n occupies the intersection, can greatly vary depending on cluster size distribution in the flow. Therefore, based on the formula (8), one can conclude that in the case of fixed p_1 the average delay time depends on cluster size distribution as well.

3. The simulation of the discrete flows' intersection.

Since it is hard to calculate the average delay time in clustered flow model, the authors have extended previously developed computer modeling system adding there the generator of flows with different clustered structure [8]. In this simulation system, the algorithm for calculation of average delay in random arrival problem is implemented. The algorithm is based on the Monte Carlo method.

In the simulation experiment, the time delay for element of minor flow randomly arriving to intersection was obtained according to both clustered flow model (Monte Carlo simulation) and non-clustered one (formula 3 – calculated numerically). Time headway distribution of the major flow was a constant and consequently p_1 was always the same as well. Nothing was changing but the cluster size (the authors were considering the case of same size for all clusters). Figure 2 shows the generated time headway distribution according to clustered flow model with parameters p_1 = 0.3, mean of intercluster interval $\overline{t_\alpha}$ =0.75, exponent shift = 0.75.

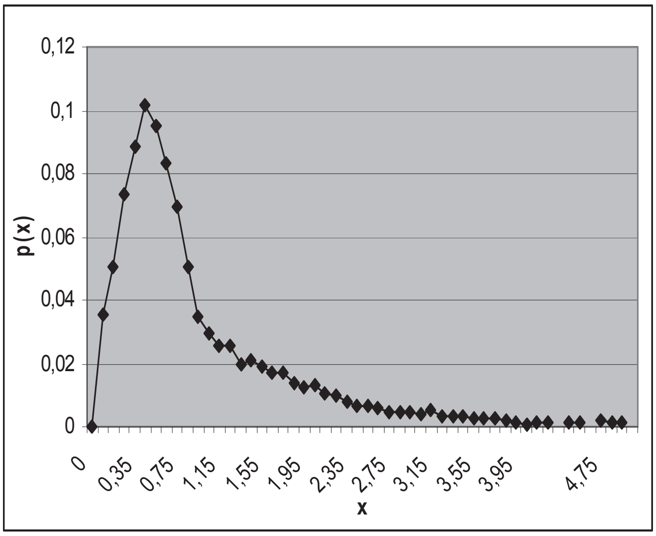

Fig.2 Time headway distribution generated according to the clustered flow model

Time headway distribution of that shape (exponential tail and great part of small intervals) frequently occurs in real traffic flows in the city road network [6].

The series of experiments were carried. The number of series performed for each cluster size n from 2 to 10 was $N=10^6$. The shape of distribution and value of p_1 did not vary when n was changing. Evaluation results for average delay time in dependence of different sizes of clusters in the major flow, with fixed distribution in both models, is shown on Figure 3.

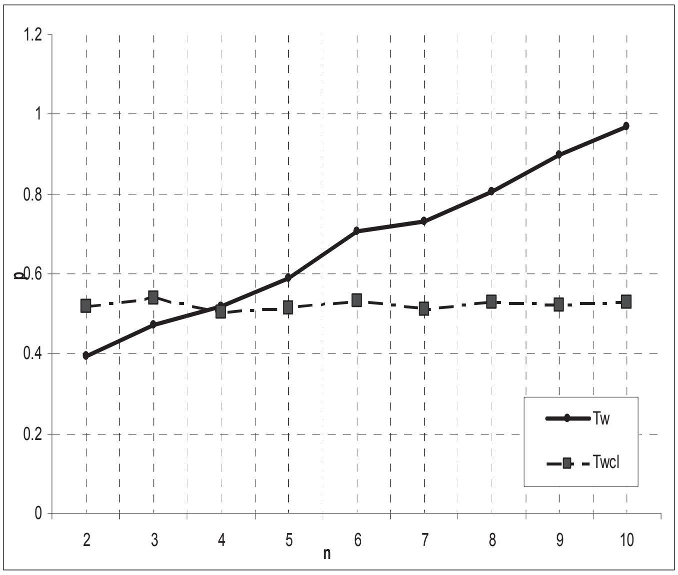

Fig.3 Average delay time for different cluster size (n) in major flow calculated by computer simulation - solid curve for clustered model and for non-clustered model - dotted

Average delay time for different cluster size (n) in major flow calculated by computer simulation: solid curve – for clustered model and dash-dotted one – for non-clustered.

4. Conclusions

In this paper, simulation of flows interaction is considered using the extended model of discrete stochastic flows. In the model, besides the time headway distribution and intensity of a flow, two characteristics of its clustered structure were taken into account: the distribution of a gap between the elements of the cluster and the cluster size distribution in the flow. It is shown that, in the random arrival problem, the difference in clustered structure can significantly influence the delay time for flow element. The authors have developed a computer modeling system, which allows obtaining the value of the average delay time for a single element, depending on the major flow clustered structure. The results presented in this paper can be used when creating an intelligent transportation system [9].

REFERENCES

1. Ning Wu A universal procedure for capacity determination at unsignalized (priority-controlled) intersections / Wu Ning. – Transportation Research part B 35. – Issue 3, 2001. – 33 p.
2. Denos C. Gazis. Traffic theory. – Kluwer academic publishers, Dodrecht, 2002. – P. 259.
3. C. Daganzo. Fundamentals of transportation and traffic operations. Elsevier science ltd, Oxford, UK, 2003, 339 p.
4. Highway Capacity Manual 2010. Transportation Research Board, Washington, D.C.. 2010.
5. B.S. Kerner, Introduction to Modern Traffic Flow Theory and Control: The Long Road to Three-Phase Traffic Theory, Springer, New York, 2009, 265 p
6. May, Adolf. Traffic Flow Fundamentals. – Prentice Hall, Englewood Cliffs, NJ, 1990. – P. 464.
7. Ventcel, E.S. & Ovcharov, L.A. 1988. Probability theory and its engineering applications. Moscow: Nauka
8. Didenko I. Simulation of the discrete traffic flow movement through network node / Didenko I., Baiev O., Lazurik V. // Bulletin Kherson National Technical University. – 2013. – vol. 2(47). – P. 125-128.
9. Tarnoff, Philip John, Bullock, Darcy M, Young, Stanley E, et al. "Continuing Evolution of Travel Time Data Information Collection and Processing", Transportation Research Board Annual Meeting 2009 Paper #09-2030. TRB 88th Annual Meeting Compendium of Papers DVD

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