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A method synthesis of selection function scalar convolutions for the multi-objective decision-making problems

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The statement of the estimation problem for selection functions and sought values in multi-objective problems is considered. The authors propose a method of synthesis of selection function scalar convolutions in multi-objective problems of mathematical model identification, optimization and decision-making. The types of scalar convolutions of selection functions have been obtained, which are specific to some practical problems of such kind. Application of the scalar convolutions as a tool for solution synthesis with the help of regularizing algorithms provides the stable effective estimates of values sought when data are a priori uncertain.

Key words: *mathematical modeling, stochastic optimization, computational methods.*

Розглядається постановка задачі оцінювання функцій вибору та шуканих величин в багатокритеріальних задачах при апріорній невизначеності даних. Запропоновано метод синтезу скалярних згорток функцій вибору для багатокритеріальних задач ідентифікації математичних моделей, оптимізації та прийняття рішень. Отримано види скалярних згорток функцій вибору для ряду практичних завдань цього типу. Застосування отриманих скалярних згорток при синтезі рішень за допомогою регулярізіруючих алгоритмів забезпечить отримання стабільних ефективних оцінок величин, що потребують пошуку при апріорній невизначеності даних.

Ключові слова: *математичне моделювання, стохастична оптимізація, обчислювальні методи.*

Рассматривается постановка задачи оценивания функций выбора и искомых величин в многокритериальных задачах при априорной неопределенности данных. Предложен метод синтеза скалярных сверток функций выбора для многокритериальных задач идентификации математических моделей, оптимизации и принятия решений. Получены виды скалярных сверток функций выбора для ряда практических задач этого типа. Применение полученных скалярных сверток при синтезе решений с помощью регуляризирующих алгоритмов обеспечивает получение стабильных эффективных оценок искомых величин при априорной неопределенности данных.

Ключевые слова: *математическое моделирование, стохастическая оптимизация, вычислительные методы.*

1. The problem statement and actuality

Let us consider as an object of study the processes of selection functions and sought values estimation in multi-objective problems of mathematical model identification, optimization and decision-making in the case of the a priori uncertain data.

In accordance to the Law of Requisite Variety (W. Ross Ashby's law), the creation of a problem resolving system requires that the system's variety is greater than one of the problem being addressed, or the system is able to produce such a variety. The Ashby's law obviously implies that introduction of information redundancy may be chosen by decision-makers (DM) as one of the ways to improve the quality of estimation in the course of the above problems' solution synthesis.

As it is known, when structuring the above named problems, whose data are random variables, an uncertainty appears in the choice of metrics for estimating the selection functions and sought values (parameters, control variables or state variables). Thus, for solving ill-defined problems of such kind, the regularizing algorithms should be used that will provide the stable effective estimates of sought values, while the mathematical models based on them will possess the property of robustness.

Already a lot of works were published, which were devoted to description of the methods of selection functions and sought values estimation in multi-objective problems of mathematical model identification, optimization or decision-making during technical and medical-biological systems designing, improvement, and diagnosing.

For structuring the problems of multivariate observation classification, the A. N. Kolmogorov's concept of power averages [1] has being widely used. According to this concept, for partitions quality estimation, a number of functionals are proposed: the interclass distance (e.g., the Mahalanobis type distance), the measure of concentration of points in each partition, the measure of intraclass scattering (e.g., generalized intraclass dispersion).

As a computational method for the synthesis of quasi-solutions of ill-defined problems, the decision regularization method by A. N. Tikhonov [2] has being often applied, in which the smoothing functional serves for a scalar convolution of selection functions. This smoothing functional contains as a supplemental summand some stabilizing functional (fitness function), which, on the one hand, allows taking into account the decision maker's system of preferences, and on the other hand, ensures the method correctness. When choosing the stabilizing functional, one has to take into account a number of conditions [2]; in particular, this functional should be continuous, non-negative, and convex.

The big amount of publications devoted to mathematical issues both in multi-objective optimization, formulated as stochastic problems, and in robust optimal design can be found in proceedings of World Congress of Structural and Multidisciplinary Optimization (WCSMO).

The following locally stochastic methods (including those based on self-organization) are used as computational methods for solution synthesis in stochastic optimization problems:

- stochastic quasi-gradient algorithms [3];
- evolutionary (genetic, immune) algorithms [4-8];
- multi-agent methods (simulation of motion in migrating flights or in ant and bee colonies).

In proceedings of Genetic and Evolutionary Computation Conference (GECCO), one can find many publications devoted to evolutionary methods in stochastic optimization issues.

The analysis of the existing literature sources shows that the development of estimation methods for selection functions and sought values in multi-objective problems of mathematical model identification, optimization and decision-making, especially in the case of the a priori uncertain data, faces a number of mathematical issues:

- shaping of the decision-maker's system of preferences: the generalized choice functions (the scalar convolutions), the system limitations, and the correctness set;
- structuring of the regularizing algorithms of the quasi-solutions synthesis.

It should be noted, that in the most of works devoted to the selection functions and sought values estimation in the above listed problems, there is no analysis of significance of nonlinear model variables based on their correlation and measurement accuracy.

Thus, there is a need to improve existing and develop new mathematical methods for selection functions and sought values estimation in multi-objective problems of mathematical model identification, optimization and decision-making in the case of the a priori uncertain data.

This article is devoted to the development of such method of selection function scalar convolution synthesis in multi-objective problems, which, in order to improve the quality of estimation, takes into account the measurement accuracy. The case of the pairwise correlation between the values of participating variables is considered.

2. Statement of the problem and a method of selection functions and sought values estimation in multi-objective problems with the a priori uncertain data

Let X^0 is a vector of random variables (the model parameters, control variables, and state variables), whose dimension is M; F^0 is a vector of random variables (measurement data, selection functions), whose dimension is I. The values F^0 can be found using the initial mathematical model (IMM) of the subject of investigation, presented in the form $F^0 = F(X^0)$, where F is a vector function.

Let us define the projections of X^0 and F^0 as the random variables with normal distribution having given mathematical expectations, standard deviations, and correlation matrices. This allows considering X^0 and F^0 as systems of several random variables with the multidimensional normal distribution.

In accordance with the A. N. Kolmogorov's concept of power averages, we will use the Student's statistics as criterion for testing the hypothesis of equality of distribution centers for representative samples of two multidimensional general populations t, and the multidimensional analogue of Romanovsky criterion for testing the hypothesis of equality of covariance matrices Ro:

$$t = \sqrt{\frac{n_\alpha}{2} MD^2}, \tag{2.1}$$

where n_α is the dimension of samples from the general populations;

MD - Mahalanobis distance;

$$Ro = \frac{\left|\chi^2 - k\right|}{\sqrt{2k}}, \; k = n_\alpha - 2, \tag{2.2}$$

where $\chi^2 = \frac{n_\alpha}{N}\left(\sigma^0\right)^T R\sigma^0$ is the multidimensional analogue of Pearson's chi-squared test;

N is dimension of X^0 (or F^0);

$\sigma^0 = \left\{ \frac{\sigma_n}{\sigma_n^*} \right\}$, $n = 1..N$;

σ_n, σ_n^* - standard deviations of variables $x_n \in X^0$ (sign * marks target values);

R - correlation matrix.

The difficulty of developing quality criteria in the problems of optimization and decision-making lies in identification of such formal relational systems, which would be isomorphic (or, at least, homomorphic) to the given empirical relational system. This correspondence is usually defined by a coordinating scale of relational systems, which in our case can be represented by a cortege of the form:

$$\left\{ F^0, \phi, t_F, \xi, Ro_F, \zeta, \psi_t, \psi_{Ro} \right\}, \tag{2.3}$$

where F^0, t_F, Ro_F - carriers of relational systems;

ϕ, ξ, ζ - binary relations of priority;

ψ_t, ψ_{Ro} - isomorphisms between F^0 and t_F (2.1), F^0 and Ro_F (2.2), respectively.

Expression of the form $\hat{F}\phi\hat{F}^*$, where $\hat{F} = \left(M\left[F^0\right], \sigma_F^0\right)$, $\hat{F}^* = \left(F^*, 1\right)$, means that solution $\hat{F}$ is equivalent to the target values $\hat{F}^*$, if the predicate value is true:

$$\left(\left|M\left[F^0\right] - F^*\right| < \varepsilon_F\right) \vee \left(\left|\sigma_F^0 - 1\right| < \varepsilon_\sigma\right)$$

Relation of the form $t_F \xi t_{crt}$ means that if $t_F \le \varepsilon_t < t_{crt}$, where $t_{crt} = t_{n_\alpha + n_\alpha - 2, \alpha}$, then one can assume with the reliability $P = 1 - \alpha$ that divergence of averages $M\left[F^0\right]$ and F^* is not significant (is random). Similarly, $Ro_F \zeta Ro_{crt}$ means that if $Ro_F \le \varepsilon_{Ro} < Ro_{crt}$, where $Ro_{crt} = Ro_{n_\alpha, n_\alpha, \alpha}$, then, with the reliability ` $P = 1 - \alpha$, it can be assumed that accuracy of estimates of averages, which are compared, is the same.

Mappings ψ_t and ψ_{Ro} define the correspondence between the properties of the samples F^0 described informally and the properties of criteria t_F and Ro_F, which have formal definitions.

Thus, having in mind reduction of the problem overall dimension, we have aggregated the variables with the help of introduced quantitative scale (2.3) (the scalar convolutions of selection functions are obtained).

Further, assuming the isomorphism of maps ψ_t and ψ_{Ro}, let us estimate parameters of distributions represented by samples X^0 and F^0, which is equivalent to looking up a solution in such form:

$$\hat{X} = \left(M\left[X^0\right], \sigma_X^0\right).$$

Let us find the density of probability distribution of several joint events:

$$\rho(t_X, Ro_X, t_F, Ro_F) = \rho(t_X, Ro_X) \cdot \rho(t_F / t_X, Ro_X) \cdot \rho(Ro_F / t_F, t_X, Ro_X), \quad (2.4)$$

where $\rho(t_X, Ro_X) = \rho(Ro_X / t_X) \cdot \rho(t_X)$.

Then, according to Bayes' formula, the expression for the a posteriori probability density (t_X, Ro_X) after n_α measurements of F^0 looks as:

$$\rho(t_X, Ro_X / t_F, Ro_F) = \frac{\rho(t_X, Ro_X, t_F, Ro_F)}{\rho(t_F, Ro_F)},$$

where, by the formula of total probability:

$$\rho(t_F, Ro_F) = \rho(t_X) \cdot \rho(t_F, Ro_F / t_X) + \rho(Ro_X) \cdot \rho(t_F, Ro_F / Ro_X). \quad (2.5)$$

Let us consider some possible hypotheses, which, if adopted, would simplify the equality (2.5).

In the first case, taking into account that the IMM is deterministic, we have:

$$\rho(t_F, Ro_F) = \rho(t_X) + \rho(Ro_X). \quad (2.5a)$$

In the second case, assuming that the IMM is deterministic and Ro_F does not depend on X^0 (the homoscedasticity hypothesis), we have:

$$\rho(t_F, Ro_F) = c_\rho \text{ (the uniform distribution)}, \quad (2.5b)$$

or

$$\rho(t_F, Ro_F) = \rho(\Delta F^0 / F^0, \varepsilon, \theta) \text{ (the noise model)}, \quad (2.5c)$$

where ε, θ are the hyperparameters of the noise model.

Let us define the log-likelihood function

$$L(\hat{X} / t_F, Ro_F) = -\ln \rho(t_X, Ro_X / t_F, Ro_F) = -\ln \rho(t_X) - \ln \rho(Ro_X / t_X) - \ln \rho(t_F / t_X, Ro_X) - \ln \rho(Ro_F / t_F, t_X, Ro_X) + \ln \rho(t_F, Ro_F). \quad (2.6)$$

Further, let us take into account that the statistics t_F and Ro_F are random variables whose distribution laws tend to normal one with mean equal to zero and variance equal to unity, when $n_\alpha \to \infty$.

Obviously, the final form of the scalar convolution of selection functions (2.6) will depend on the accepted hypotheses, for example (2.5a) - (2.5c). Thus, in (2.5a)-case, which is typical for the identification problems, we have

$$L(\hat{X} / t_F, Ro_F) = \frac{1}{2}(t_F^2 + Ro_F), \quad (2.6a)$$

in (2.5b)-case, typical for decision-making, we have

$$L(\hat{X} / t_F, Ro_F) = \frac{1}{2}(t_F^2 + Ro_F + t_X^2 + Ro_X) + C_L. \quad (2.6b)$$

Hence, the estimation problem $\hat{X} = (M[X^0], \sigma_X^0)$ can be reduced to multi-objective problem of stochastic optimization with mixed conditions (MPSOMC), whose quasi-solution, according to the principle of maximum likelihood, is:

$$\hat{X} = \arg\inf_{\hat{X} \in D_X} L(\hat{X} / t_F, Ro_F), \quad (2.7)$$

where D_X is a correctness set, which, in the general case, is defined by the DM's system of preferences.

3. The results of synthesis of selection function scalar convolutions for the multi-objective problems with the a priori uncertain data

Let us consider some particular cases of scalar convolutions of selection functions for a number of practical problems of mathematical model identification, optimization, and decision-making.

The identification problem. Let us assume that $R_F = E, n_\alpha = 2$. Using (2.6a), we obtain:

$$L\left(\hat{X} / t_F , Ro_F\right) =$$
$$\frac{1}{2PI}\sum_{p=1}^{P}\sum_{i=1}^{I}\left[4\left(\frac{\Delta_{f_i,p}}{\sigma^*_{f_i}}\right)^2 \cdot \left(1+\sigma^0_{f_i,p}\right)^{-2} + \left(\sigma^0_{f_i,p}\right)^2\right],$$

where $\Delta_{f_i} = f_i - f_i^*, \sigma^*_{f_i} = \frac{\Delta^0_{f_i}\sqrt{n_\alpha}}{300} f_{i,\max}$;

$\Delta^0_{f_i}$ is the relative error of f_i, in percentage.

It is the form shown in (3.1) that should be used as the scalar convolution of selection function in the case of the identification problem.

$$E = \frac{1}{2PI}\sum_{p=1}^{P}\gamma^{P-p}\sum_{i=1}^{I}\left\{f_{fit}\left[4\left(\frac{\Delta_{f_i,p}}{\sigma^*_{f_i}}\right)^2\left(1+\sigma^0_{f_i,p}\right)^{-2}\right] + \beta\cdot f_{fit}\left[\left(\sigma^0_{f_i,p}\right)^2\right]\right\}, \quad (3.1)$$

$$\gamma = 0{,}95..0{,}99$$

where f_{fit} is a fitness function (FF);

$f_{fit}(d) = 1 - \exp(-C\cdot d)$, $C > 1$, d is an argument of FF ($d > 0$);

β is a regularization parameter.

MPSOMC. Let us assume that $R_F = R_X = E$. Using (2.6b), we obtain:

$$L\left(\hat{X} / t_F , Ro_F\right) = \frac{1}{2I}\sum_{i=1}^{I}\left[4\left(\frac{\Delta_{f_i}}{\sigma^*_{f_i}}\right)^2\left(1+\sigma^0_{f_i}\right)^{-2} + \frac{\left|\chi^2_{f_i} - k\right|}{\sqrt{2k}}\right] +$$
$$+\frac{1}{2M}\sum_{m=1}^{M}\left[4\left(\frac{\Delta_{xm}}{\sigma^*_{xm}}\right)^2\left(1+\sigma^0_{xm}\right)^{-2} + \frac{\left|\chi^2_{xm} - k\right|}{\sqrt{2k}}\right] + C_L,$$

where $\Delta_{f_i} = M_\alpha[f_i] - f_i^*$, $\chi_{f_i}^2 = n_\alpha \dfrac{M_\alpha\left[(f_i - M_\alpha[f_i])^2\right]}{\left(\sigma_{f_i}^*\right)^2}$;

$$\Delta_{xm} = M_\alpha[x_m] - x_m^*,\ \chi_{xm}^2 = n_\alpha \frac{M_\alpha\left[(x_m - M_\alpha[x_m])^2\right]}{\left(\sigma_m^*\right)^2};$$

x_m^*, σ_m^* are the values of mathematical expectation and standard deviation of variable x_m for the prototype.

It is the form shown in (3.2) that should be used as the scalar convolution of selection function in the MPSOMC-case

$$E = \frac{1}{2I}\sum_{i=1}^{I}\left\{ f_{fit}\left[4\left(\frac{\Delta_{f_i}}{\sigma_{f_i}^*}\right)^2\left(1+\sigma_{f_i}^0\right)^{-2}\right] + \beta\cdot f_{fit}\left(\frac{\left|\chi_{f_i}^2 - k\right|}{\sqrt{2k}}\right)\right\} + \\ + \frac{1}{2M}\sum_{m=1}^{M}\left\{ f_{fit}\left[4\left(\frac{\Delta_{xm}}{\sigma_{xm}^*}\right)^2\left(1+\sigma_{xm}^0\right)^{-2}\right] + \beta\cdot f_{fit}\left(\frac{\left|\chi_{xm}^2 - k\right|}{\sqrt{2k}}\right)\right\}. \quad (3.2)$$

Obviously, another cortege of the form (2.3) can contain the alternative statistics, as well as their distribution laws, depending on the additional information and assumed heuristics.

Let us define a measure of informativeness of a system as its average entropy:

$$S\left(F^0\right) = \int s\left(F^0\right)\cdot\varphi\left(dF^0\right),$$

where $s\left(F^0\right) = -\rho\left(F^0\right)\log_2\rho\left(F^0\right)$;

$\varphi\left(dF^0\right)$ is an a priori probability measure $F^0 = F\left(X^0\right)$.

Let us choose statistics H instead of statistics Ro, where H is the mutual information: some statistical function of two random variables, which describes the amount of information about one of these variables contained in another one. Then the mutual system information H_F can be obtained from the formula:

$$H_F = h_F\left(F^0, t_F, t_X, H_X\right) = S\left(F^0\right) - S\left(F^0 / t_F, t_X, H_X\right).$$

Let us take as a basis such heuristic, according to which the statistics H_F should satisfy the distribution law of the form:

$$\rho\left(H_F / t_F, t_X, H_X\right) \sim \exp\left(-\beta\cdot H_F\right).$$

Then, in the case (2.5a) instead of (2.6a), we obtain:

$$L\left(\hat{X} / t_F, H_F\right) = \frac{1}{2}t_F^2 + \beta\cdot H_F. \quad (3.3)$$

Let us assume that $R_F = E$. In this case $H_F = \ln\left[\prod_{i=1}^{I}\left(\sigma_{f_i}^0\right)^2\right] + C_H$.

Then using (3.3), we obtain

$$L\left(\hat{X}/t_F, H_F\right) = \frac{1}{2PI}\sum_{p=1}^{P}\sum_{i=1}^{I}\left\{\begin{matrix} 4\left(\dfrac{\Delta_{f_i,p}}{\sigma^*_{f_i}}\right)^2\left(1+\sigma^0_{f_i,p}\right)^{-2} + \\ \beta\cdot\ln\left(\sigma^0_{f_i,p}\right)^2 \end{matrix}\right\} + C_H,$$

and it is the form shown below that should be used as the scalar convolution of selection function

$$E = \frac{1}{2PI}\sum_{p=1}^{P}\gamma^{P-p}\sum_{i=1}^{I}\left\{ f_{fit}\left[\begin{matrix} 4\left(\dfrac{\Delta_{f_i,p}}{\sigma^*_{f_i}}\right)^2\left(1+\sigma^0_{f_i,p}\right)^{-2} + \\ \beta\cdot\ln\left(\sigma^0_{f_i,p}\right)^2 \end{matrix}\right]\right\}, \gamma = 0{,}95..0{,}99. \quad (3.4)$$

Note that, due to the monotonicity of functions $f_{fit}(d)$ and $\ln(d)$, the solutions of the problem (2.7) obtained using either (3.1) or (3.4) coincide.

Thus, the types of scalar convolutions of selection functions are obtained for some practical optimization and decision-making problems with the a priori uncertain data.

Methodological preciousness of this work is that the synthesis of scalar convolutions of selection functions has been formalized, whereas previously they were entirely heuristic (see. for example [9-12]).

4. Results and conclusions

The authors gave further development to the theory of selection functions and sought values estimation in multi-objective problems of mathematical model identification, optimization and decision-making.

The method of synthesis of selection function scalar convolutions has been proposed for the mentioned above problems under condition of the a priori uncertain data.

For a number of practical problems of named kinds, the types of scalar convolutions of selection functions have been obtained, whose application for solution synthesis with the help of regularizing algorithms would provide the stable effective estimates of sought values in the case of the a priori uncertain data.

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