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A comparative analysis of axisymmetric vibrations of conical and cylindrical fluid-filled elastic shells

K.G. Degtyarev[1], V.I.Gnitko[1], Yu.V.Naumenko[1], E.A. Strelnikova[1,2]
[1]A.N. Podgorny Institute for Mechanical Engineering Problems of the Ukrainian Academy of Sciences, 2/10,Pozharsky Str., Kharkiv, 61046, Ukraine
[2]V.N. Karazin Kharkiv National University, Freedom Square 4, Kharkiv, 61022, Ukraine
e-mail: strelea@ukr.net

This paper presents the comparison of low- frequency vibrations in liquid-filled cylindrical and truncated conical elastic shells. The liquid is supposed to be an ideal and incompressible one and its flow is irrotational. To evaluate a velocity potential the system of singular boundary integral equations has been obtained. The boundary element method is used for their numerical simulation. The vibration modes of the shells with liquids are determined as linear combinations of their natural vibration modes without liquids. Sloshing frequencies and modes of fluid-filled cylindrical and truncated conical shells are estimated. The solution of the hydro-elasticity problem is obtained using a combination of boundary and finite element methods. Shells with both rigid and elastic bottoms are considered. The illustrative examples are provided to demonstrate the accuracy and efficiency of the method.

Key words: *vibrations, ideal incompressible liquid, sloshing, cylindrical and conical shells, singular integral equations, boundary and finite element methods*

Розглянуто процеси коливання пружних оболонок обертання, частково заповнених рідиною. Припускається, що рідина є ідеальною та нестисливою, а її рух є безвихровим. В цих умовах існує потенціал швидкостей рідини. Для його обчислення із застосуванням методів теорії потенціалу та другої тотожності Гріна отримано систему сингулярних інтегральних рівнянь. Вважається, що форми коливань заповненої оболонки можна зобразити в вигляді ряду за формами коливань незаповненої оболонки. Для визначення цих базисних функцій застосовано одновимірний метод скінченних елементів. Форми коливань оболонки з рідиною без урахування плескань рідини складають другу систему базисних функцій. Для врахування впливу плескань знайдені форми коливань рідини у відповідному жорсткому резервуарі. Ці форми складають третю систему базисних функцій для знаходження шуканого потенціалу швидкостей. Для знаходження базисних функцій другої та третьої систем сформульовані мішані крайові задачі для рівняння Лапласа. Отримані при цьому системи граничних інтегральних рівнянь зводяться до одновимірних з невідомими густинами, що задані вздовж меридіану оболонки обертання. Числовий розв'язок отриманих систем здійснено за допомогою редукованого одновимірного методу граничних елементів. В роботі надано порівняння низькочастотних коливань циліндричних та конічних заповнених рідиною пружних оболонок. Розглянуті коливання заповнених та незаповнених рідиною оболонок з жорсткими та пружними стінками та днищами. Визначено частоти плескань рідини в жорстких заповнених циліндричних та усічених конічних оболонках. Розв'язок задачі гідро-пружності здійснено за допомогою підходу, заснованого на поєднанні методів скінченних та граничних елементів. Наведено ілюстративні приклади, що демонструють точність та ефективність методу. Отримані дані відносно найнижчих частот коливань системи «оболонка-рідина», що дозволяють провести ефективне відстроювання від небажаних резонансних частот.

Ключові слова: *коливання, ідеальна нестислива рідина, плескання, циліндричні та конічні оболонки, сингулярні інтегральні рівняння, методи граничних та скінченних елементів*

1 Introduction

Various fuel and liquid storage tanks, oil and propellant storage containers are widely used in different engineering areas. If such storages are subjected to surface shots caused by a terrorist act, an airplane crash or a seismic shockwave, this will be lead to a dangerous ecological catastrophe. So defining the strength characteristics of such elements is a topical engineering problem. These data allow evaluating the ultimate strength of a structure under shock or seismic actions, isolate spurious resonance frequencies, and identify the most hazardous zones from the viewpoint of stress concentration. However, studying the frequencies and natural vibration modes for structures interacting with a fluid is a challenging design problem. Complex experimental investigation of loading processes is difficult and sometimes impossible due to the various reasons. Hence mathematical modeling of physical processes with the help of advanced numerical procedures is a basic approach for these problems.

A lot of analytical and experimental research has been performed in the field of shells interacting with a fluid in last decade [1-6]. Most of these works have been devoted to the problem of flat plates, curved plates and circular cylindrical shells [1-3]. The dynamic behaviour of cylindrical shells without liquids has been studied in a considerable number of numerical, analytical and experimental investigations [4-6]. The case of a conical shell filled with a liquid has been successfully developed by Lakis et al. [7]. A free surface effect was neglected in [7], but in [8] it was shown that the supposition about the spectrum separation of frequencies of the elastic shell filled with the liquid and sloshing frequencies of the rigid shell with the same geometrical characteristics and filling level as for the elastic one is not always valid.

The main purpose of this paper is to study the influence of both sloshing and elasticity effects on vibrations of the fluid-filled tanks in the form of shells of revolution with an arbitrary meridian.

In this paper we demonstrate that dynamic characteristics of cylindrical and conical shells of equal heights and equal radii of free surfaces are differed drastically. The proposed method is based on representation of the velocity potential as a sum of two potentials. One of them corresponds to the problem of fluid free vibrations in the rigid shell and another one corresponds to the similar problem for an elastic shell with a fluid without including the gravitational component. The method allows us to obtain the natural frequencies and vibration modes for fuel tanks of different shapes.

2 Problem statement and mode superposition method for coupled dynamic problems

Free harmonic vibrations of fluid-filled elastic shells of revolution having arbitrary meridians are investigated. The shell is of uniform thickness h, and height H, made of homogeneous, isotropic material with elasticity modulus E, Poisson's ratio ν and mass density ρ_s. As the examples, fluid-filled cylindrical and truncated conical shells are considered, as shown in Fig. 1.

Denote the wetted part of the shell surface as σ and the free surface of a liquid as S_0. Let S_{bot} be the surface of the tank bottom. Let us denote the vector-function of shell displacements as $\mathbf{U}=(U_1,U_2,U_3)$. At first let us consider the free vibrations of the elastic shell without a liquid. The finite element method is applied by Ravnik *et al.* in [8] to evaluate natural frequencies Ω_k and modes $\mathbf{u}_k$, $k=\overline{1,N}$ of the shell of revolution without a liquid. After forming the global stiffness $\mathbf{L}$ and mass $\mathbf{M}$ matrices, the following equation of motion for the shell containing fluid has been obtained:

$$\mathbf{LU}+\mathbf{M\ddot{U}}=p\mathbf{n},$$

where $\mathbf{n}$ is an external unit normal to the shell wetted surface, the $p\mathbf{n}$ is the fluid dynamical pressure upon the shell, normal to its surface.

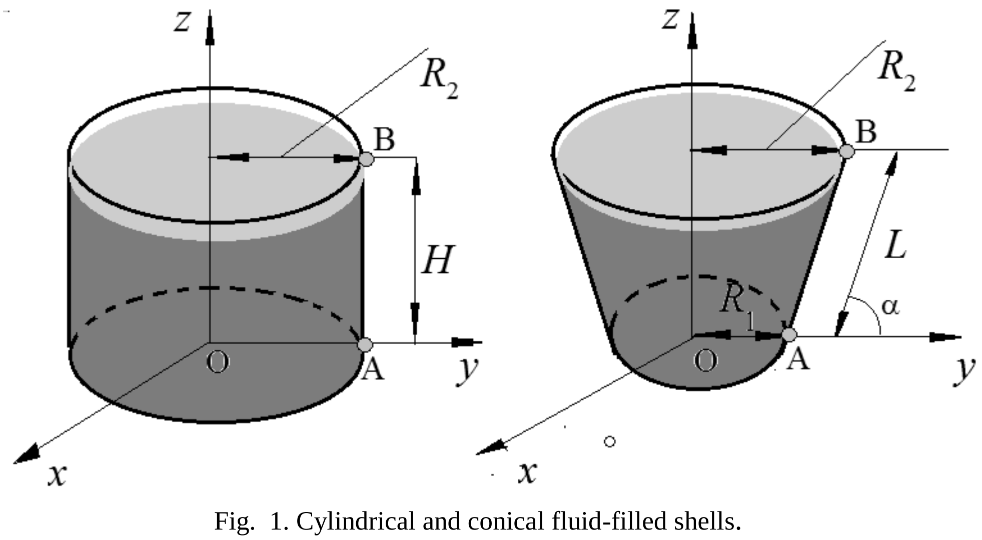

Fig. 1. Cylindrical and conical fluid-filled shells.

Consider the modes of fluid-filled shell vibrations in the form

$$\mathbf{U}=\sum_{k=1}^{N}c_k(t)\mathbf{u}_k\,, \tag{2.1}$$

where $c_k(t)$ are unknown coefficients, and $\mathbf{u}_k$ are the eigenmodes of the empty shell vibrations.

It is assumed here that a liquid is an ideal and incompressible one, and its motion, beginning from a state of rest, is irrotational. In these conditions, there exist a fluid velocity potential Φ

$$V_x=\frac{\partial\Phi}{\partial x};V_y=\frac{\partial\Phi}{\partial y};V_z=\frac{\partial\Phi}{\partial z},$$

that satisfies the Laplace equation. The liquid pressure p upon the shell walls is determined from the linearized Cauchy-Lagrange integral by the formula

$$p = -\rho_l\left(\frac{\partial\Phi}{\partial t} + gz\right) + p_0,$$

where Φ is the velocity potential, g is the acceleration of gravity, z is the vertical fluid point coordinate, ρ_l is a fluid density, p_0 is an atmospheric pressure. On the wetted surfaces of the shell the non-penetration boundary condition is set [9]. On the free surface, the following dynamic and kinematic boundary conditions must be satisfied

$$\left.\frac{\partial\Phi}{\partial \mathbf{n}}\right|_{S_0} = \frac{\partial\zeta}{\partial t}; \quad \left.p - p_0\right|_{S_0} = 0,$$

where the function ζ describes the shape and position of the free surface. Thus, for the velocity potential we have the following boundary-value problem [8]

$$\nabla^2\Phi = 0;\ \left.\frac{\partial\Phi}{\partial \mathbf{n}}\right|_{S_w} = \frac{\partial w}{\partial t};\left.\frac{\partial\Phi}{\partial n}\right|_{S_0} = \frac{\partial\zeta}{\partial t}; \quad \left.p - p_0\right|_{S_0} = 0;\left.\frac{\partial\Phi}{\partial t} + g\zeta\right|_{S_0} = 0. \tag{2.2}$$

Consider the potential Φ as $\Phi = \Phi_1 + \Phi_2$, as it is done in [10]. The series for potential Φ_1 can be written as

$$\Phi_1 = \sum_{k=1}^{N}\dot{c}_k(t)\varphi_{1k}.$$

Here time-dependant coefficients $c_k(t)$ are defined in equation (2.1). To determine functions φ_{1k} the following boundary value problems are formulated:

$$\Delta\varphi_{1k} = 0,\left.\frac{\partial\varphi_{1k}}{\partial \mathbf{n}}\right|_{\sigma} = w_k,\ \left.\varphi_{1k}\right|_{S_0} = 0,\ w_k = (\mathbf{u}_k, \mathbf{n}),\ k = \overline{1,N} \tag{2.3}$$

The solution of boundary value problems (2.3) is presented in [11].

To determine the potential Φ_2 we have the problem of fluid vibrations in the rigid shell including gravity effects.

Use the expansion

$$\Phi_2 = \sum_{k=1}^{M}\dot{d}_k(t)\varphi_{2k},$$

where $d_k.(t)$ are unknown coefficients, and functions φ_{2k} are natural modes of the liquid sloshing in a rigid tank. To obtain these modes the following boundary value problems are considered:

$$\Delta\varphi_{2k} = 0,\quad \left.\frac{\partial\varphi_{2k}}{\partial \mathbf{n}}\right|_{\sigma} = 0;\ \left.\frac{\partial\varphi_{2k}}{\partial t} + g\zeta\right|_{S_0} = 0;\ \left.\frac{\partial\varphi_{2k}}{\partial \mathbf{n}}\right|_{S_0} = \frac{\partial\varsigma}{\partial t};\ \iint_{S_0}\frac{\partial\varphi_{2k}}{\partial \mathbf{n}}dS_0 = 0,\quad k = \overline{1,N} \tag{2.4}$$

Finally, for the sum of potentials $\Phi = \Phi_1 + \Phi_2$ the following expression is valid:

$$\Phi = \sum_{k=1}^{N}\dot{c}_k(t)\varphi_{1k} + \sum_{k=1}^{M}\dot{d}_k(t)\varphi_{2k}. \tag{2.5}$$

The unknown function ζ takes the form

$$\zeta = \sum_{k=1}^{N}c_k(t)\frac{\partial\varphi_{1k}}{\partial n} + \sum_{k=1}^{M}d_k(t)\frac{\partial\varphi_{2k}}{\partial n}. \tag{2.6}$$

To define coupled modes of harmonic vibrations, suppose that $c_k(t) = C_k \exp(i\omega t)$; $d_l(t) = D_k \exp(i\omega t)$. Substituting these expressions into equations (5)-(6) and then into equations

$$\mathbf{LU} + \mathbf{M\ddot{U}} = p_d \mathbf{n} \quad \left.\frac{\partial \Phi}{\partial t} + g\zeta\right|_{S_0} = 0$$

results in the generalized eigenvalue problem where both elasticity and gravity effects are taken into account.

Both boundary value problems (2.3) and (2.4) are reduced to systems of singular integral equations by using boundary element method in its direct formulation, Brebbia *et al.* [12].

As it was shown in [3], the integral operators obtained in singular integral equations are of logarithmic singularities, and thus the numerical treatment of these integrals will also have to take into account the presence of this integrable singularity. The integrands are distributed strongly non-uniformly over the element, and standard integration quadratures fail in accuracy. So we treat these integrals numerically by the special Gauss quadratures [12] and apply the technique proposed by Naumenko *et al.* in [13].

3. Numerical simulation and discussion

Empty and fluid-filled isotropic cylindrical and truncated conical shells are considered. These shells are shown in Fig. 1. R_1 and R_2 are radii of the cone at its small and large edges, R_2 is also for cylinder radius, α is a semivertex angle of the cone, and H is the height of both cone and cylinder, L is the length of cone generatix. Both conical and cylindrical shells are referred to the cylindrical coordinate system (x,θ,z). For all following numerical simulation, the thickness of the shell and the Poisson's ratio are taken as h/R_1=0.01 and ν =0.3, semivertex angle $\alpha=45^o$, H/R_2=0.5, Young's modulus $E = 2{,}11\cdot10^6$ MPa, ρ_s=8000 kg/m^3 [6] ρ_l=1000 kg/m^3. The following boundary conditions for both shells are presented: clamped – free, i.e. shells of revolution are clamped at the ends A, and free at the ends B, Fig. 1.

For both shells we estimate sloshing frequencies, the frequencies of empty shells with rigid and elastic bottoms, and coupled hydro-elastic vibrations of fluid-filled conical and cylindrical shells.

3.1. Oscillations of empty shells with rigid and elastic bottoms.

First, we determine the requisite number of finite elements for a precise determination of the natural frequencies. The convergence is established when numbers of finite elements along the shell wall is equal to 60, along the bottom is 100 elements, the same numbers are used for boundary elements simulations in elastic tanks, the number of boundary elements along the free surface radius is 100 as well.

Then the numerical simulation of dynamic characteristics for both conical and cylindrical shells is provided.

In Table 1 the frequencies of empty cylindrical shells with rigid and elastic bottoms are presented, where n is the number of the mode.

Table 1: Frequency of axisymmetric oscillations of empty cylindrical shells, Hz

n	1	2	3	4	5	6(1)	7(2)	8	9(3)	10	11(4)	12(5)
rigid						817.07	844.14		1019.2		1448.0	1563
elastic	25.26	98.37	220.39	391.26	610.96	817.07	844.14	879.49	1019.2	1196.8	1448.0	1563

The lowest frequencies correspond to the shell with elastic bottom. The frequencies of the shell with rigid bottom are coincided with ones of the shell with the elastic bottom when the wall vibrations are dominant. It can be concluded that accounting for the bottom deformations leads to appearance of lowest frequencies.

Fig. 2 demonstrates the first four axisymmetric vibration modes denoted by numbers 1,2,3,4 of the cylindrical shell with the elastic bottom. There are any wall deformations for these modes, i.e. the bottom vibrations are dominant.

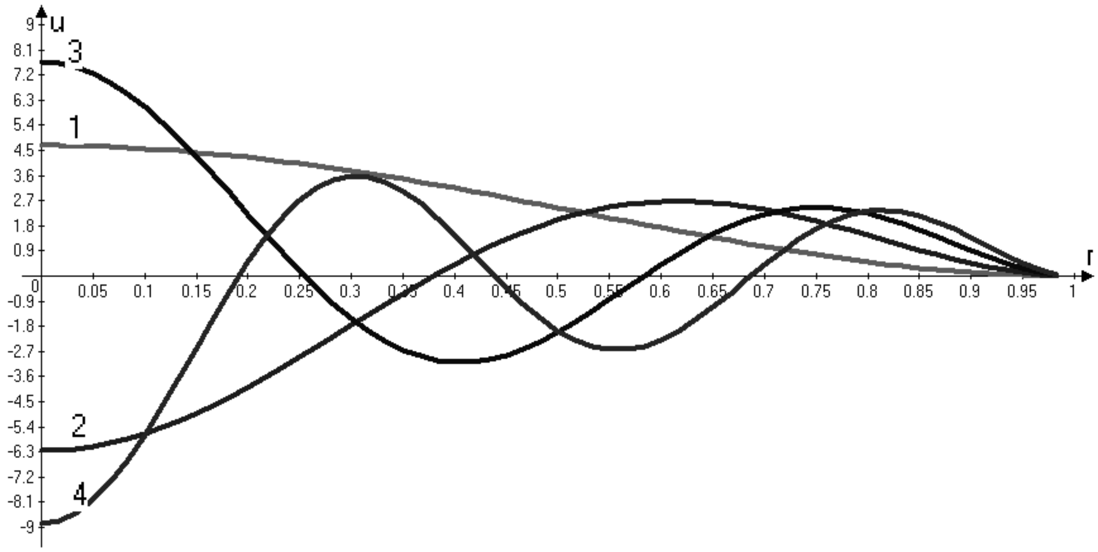

Fig. 2. Axisymmetric modes *m*=1,2,3,4 of cylindrical shell with elastic bottom

Fig. 3 demonstrates the first four axisymmetric vibration modes of the cylindrical shell with rigid bottom. These modes are coincided with axisymmetric modes for shells with elastic bottoms with numbers *m* = 6,7,9,11. So the wall vibration modes are not dominant for shells with elastic bottoms.

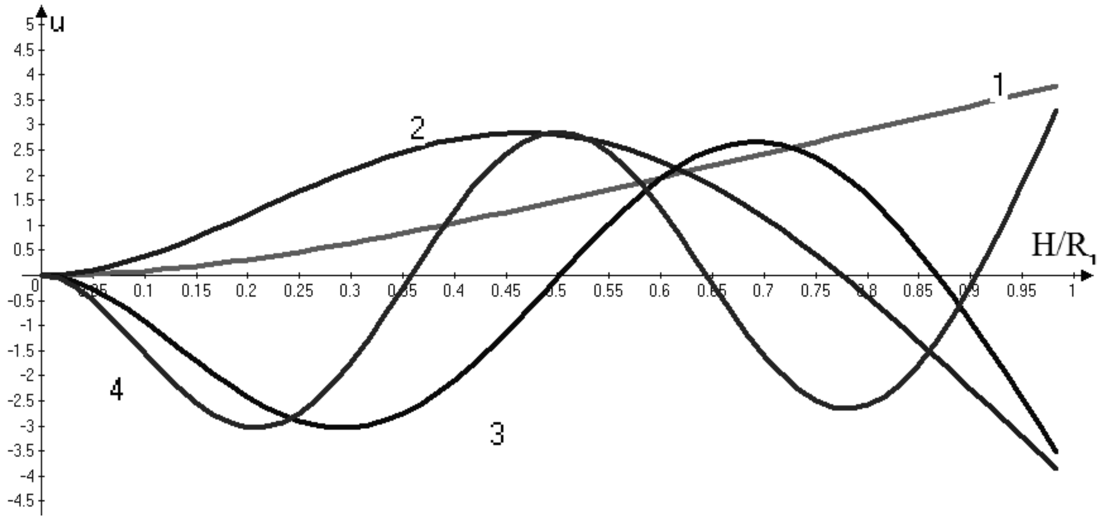

Fig. 3. Axisymmetric modes *m*= 6,7,9,11 of cylindrical shell with rigid bottom.

In Table 2 the frequencies of empty truncated conical shells with rigid and elastic bottoms are presented, where *m* is the number of the mode.

Table 2: Frequency of axisymmetric oscillations of empty conical shells, Hz

m	1	2	3(1)	4(2)	5(3)	6(4)	7	8(5)	9(6)	10	11(7)	12(8)
rigid			559.4	675.8	707.28	824.91		1001.1	1274.7		1679.0	1994.0
elastic	101.0	393.4	559.5	675.8	708.68	824.94	881.58	1001.1	1274.7	1565.0	1679.0	1995.3

Comparing results of Tables 1-2 we can conclude that frequencies of empty truncated cones differ essentially from cylinder ones. The lowest frequency of the cylindrical shell is nearly four times less than that of the truncated conical shell. The lowest frequencies of both shells correspond to dominant bottom vibrations.

Fig. 4 demonstrates the first axisymmetric vibration modes (*m*=1,2,3,4) of the cylindrical shell with the elastic bottom. For the first two modes there is no any wall deformation, i.e. the bottom vibrations are also dominant.

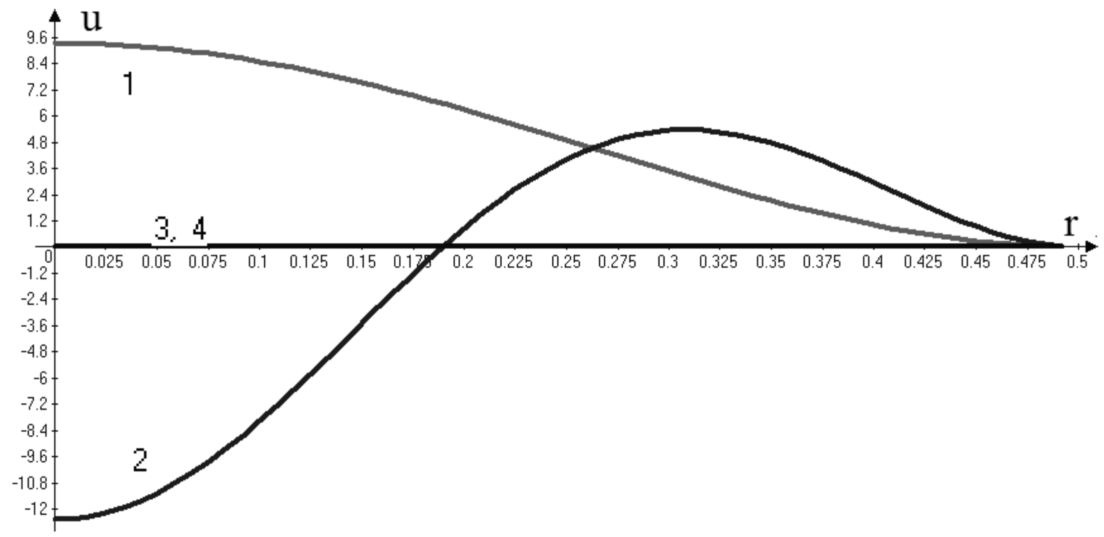

Fig. 4. Axisymmetric modes m=1,2,3,4 of conical shell with elastic bottom

Unlike cylindrical shells the only two first modes are bottom dominant.

Fig. 5 demonstrates the first four axisymmetric vibration modes of the truncated conical shell with rigid bottom. These modes are coincided with axisymmetric modes of the shell with the elastic bottom with numbers $m = 3,4,5,6$.

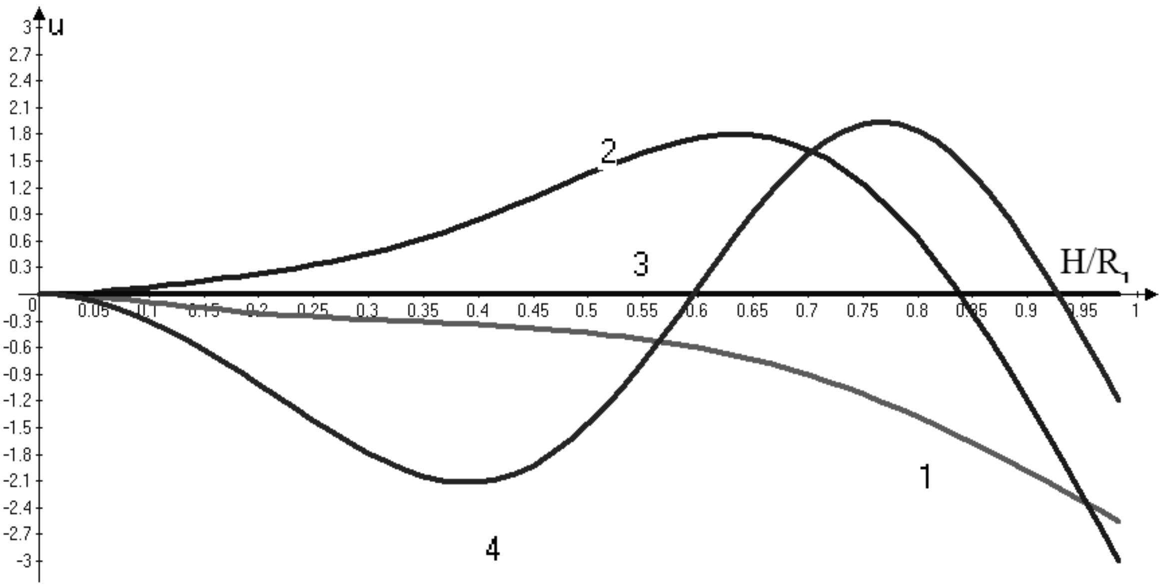

Fig. 5. Axisymmetric modes m=3,4,5,6 of conical shell with rigid bottom

It should be noted that the third mode is the torsion one, and it does not affect the fluid-structure interaction because an ideal fluid produces only a normal pressure on a moistened body.

3.2. Low frequency sloshing modes for fluid- filled cylindrical and conical shells.

Linear sloshing in the rigid Λ-shape conical shell with R_1 = 0.5m and R_2= 1.0m, H=0.5m and $\alpha=\pi/4$ is considered. The cylindrical shell is of R_2= 1.0m and H=0.5m. The sloshing frequencies are calculated accordingly to Degtyarev *et al.* [14]. The total number of boundary elements along the shell meridians, as well as, the radii of free surfaces is 240 for both cylindrical and truncated conical shells. Below we demonstrate that sloshing frequencies of rigid cylindrical and conical shells differ. Both shells are of equal height (H=0.5m), and the radius of cone R_2 =1m is equal to the cylinder radius. The comparison of the results for n=0 (axisymmetric modes) is shown in Table 3.

Table 3: Axisymmetric sloshing frequencies, Hz

m	Conical shell	Cylindrical shell	
	Numerical solution	Numerical solution	Analytical solution
1	5.3534	5.9989	5.9965
2	7.8068	8.2898	8.2842
3	9.6034	9.9922	9.9932
4	11.1044	11.438	11.426
5	12.4246	12.710	12.705
6	13.6180	13.872	13.865

Obtained results testify the accuracy of the proposed method. The lowest frequencies of cone and cylinder differ drastically, but with increasing the number *m* difference become smaller.

Fig. 6 demonstrates the first sloshing modes of the cylindrical and conical shells.

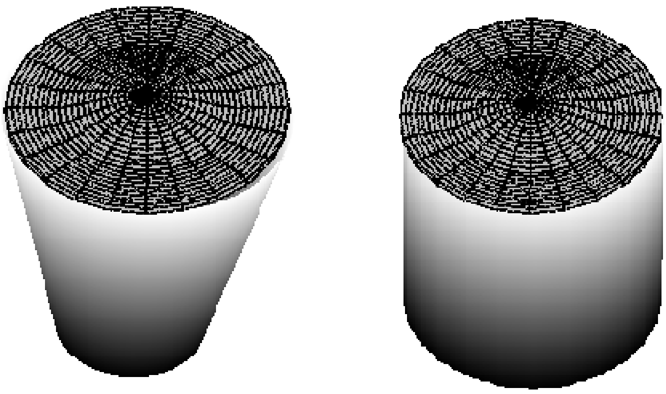

Fig.6. Axisymmetric sloshing modes of conical and cylindrical shells

It should be noted that the sloshing modes are the same for conical and cylindrical shells.

3.3. Vibrations of elastic fluid-filled truncated conical shells

Free vibrations of elastic cylindrical and conical shells coupled with liquid sloshing are under consideration. The results have been obtained for axisymmetric modes (n=0) and $m=\overline{1,12}$. The frequencies of empty and fluid-filled cylindrical and truncated conical shells have been considered. The results of numerical simulation are shown in Table 4.

Table 4: Comparison of axisymmetric frequencies for conical and cylindrical shells, Hz

m	Cylindrical shell		Conical shell	
	empty	fluid-filled	empty	fluid-filled
1	25.268	10.382	101.07	48.62
2	98.372	50.701	393.49	252.68
3	220.39	133.15	559.52	471.10
4	391.26	260.56	675.88	571.08
5	610.96	412.38	708.68	637.51
6	817.07	472.56	824.94	680.67
7	844.14	623.22	881.58	708.68
8	879.49	681.49	1001.1	859.20
9	1019.2	792.71	1274.7	1142.87
10	1196.8	956.08	1565.0	1250.17
11	1448.0	1163.75	1679.0	1519.97
12	1563.0	1289.83	1995.3	1565.55

From the results obtained we can conclude that the frequencies of fluid-filled shell vibrations differ drastically from the frequencies of empty ones for both cylindrical and truncated conical shells.

But when the circumferential wave number increases the difference becomes smaller gradually. The frequencies of the cylindrical shell are smaller than those of the conical shell. The frequencies ω near 10Hz may be considered as the most dangerous for the cylindrical shell. The results of tables 3 and 4 testify it. For example, ω=10.382Hz corresponds to n=0 and m=1 for the elastic shell; ω=9.9922 Hz corresponds to n=0 and m=3 for sloshing in the rigid shell. It can be the reason for the stability loss in shell structures.

Conclusion

The free vibration analysis of the elastic cylindrical and truncated conical elastic shell coupled with the liquid sloshing has been carried out. The combination of reduced finite and boundary element methods has been used. The analysis consists of several stages and each represents the separate task. The frequencies and modes of the empty shell vibrations have been defined at the first stage. The displacement vector, that is the solution of the coupled problem, is sought as the linear combination of natural modes of the empty shells. So the frequencies and free vibrations modes of the fluid-filled elastic shell without accounting for the gravity force have been defined. The frequencies and free vibrations modes of the liquid in the rigid shell under the gravity force have been estimated. These two problems have been solved by using the reduced boundary element method. This method substantially reduces the computing time for the analysis and reveals the new qualitative possibilities in modeling the dynamic behavior of shells. The difference in the dynamical characteristics between elastic truncated conical and cylindrical shells has been established. The frequencies of fluid-filled shell vibrations differ drastically from frequencies of empty ones for both cylindrical and truncated conical shells. The obtained results can be used as the basis for the further research of non-axisymmetrical vibrations of shells, as well as, dynamical characteristics of structures subjected to an intensive loading in case of interaction with a fluid.

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Kirill Degtyarev, leading engineer, A.N. Podgorny Institute for Mechanical Engineering Problems of the Ukrainian Academy of Sciences, 2/10,Pozharsky Str., Kharkiv, 61046,
email: kdegt89@gmail.com

Vasyl Gnitko, PhD, senior researcher, A.N. Podgorny Institute for Mechanical Engineering Problems of the Ukrainian Academy of Sciences, 2/10,Pozharsky Str., Kharkiv, 61046,
email: gnitkovi@gmail.com

Yury Naumenko, post-graduate, A.N. Podgorny Institute for Mechanical Engineering Problems of the Ukrainian Academy of Sciences, 2/10,Pozharsky Str., Kharkiv, 61046,
email: strelea@ukr.net, ORCID 0000-0001-9058-6727

Elena Strelnikova, DSc, Prof., leading researcher, A.N. Podgorny Institute for Mechanical Engineering Problems of the Ukrainian Academy of Sciences, 2/10,Pozharsky Str., Kharkiv, 61046,
email: elena15@gmx.com, ORCID 0000-0003-0707-7214

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