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Modeling of the viscous fluid flow around rotating circular cylinders with the lattice Boltzmann method at moderate Reynolds numbers

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In this work the task of the viscous fluid flow around both a circular cylinder which rotates with the constant speed in a plane channel and a lattice of rotating cylinders has been numerically solved by the lattice Boltzmann method. The method of setting the boundary conditions on the rotating cylinder boundary has been developed and tested. The comparison of obtained results with known numerical results obtained by other numerical methods has been made. Both stationary and periodic solutions have been investigated. The dependence of the computational grid resolution on the cylinder rotation speed for the predefined accuracy has been shown.

Keywords: *viscous fluid, rotating circular cylinder, lattice of cylinders, Boltzmann equation, Reynolds number.*

В роботі методом граткових рівнянь Больцмана чисельно розв’язувалася задача обтікання потоком в’язкої рідини кругового циліндра в плоскому каналі та решітки кругових циліндрів, що обертаються зі сталою швидкістю. Була розроблена та протестована методика задання граничних умов на границі циліндра, що обертається. Проведено порівняння отриманих результатів із відомими чисельними розв’язками, що були отримані іншими методами. Досліджувались як стаціонарні так і періодичні режими течій. Показана залежність розміру розрахункової сітки від швидкості обертання циліндру при заданій точності.

Ключові слова: *в’язка рідина, обертальний круговий циліндр, решітка циліндрів, рівняння Больцмана, число Рейнольдса.*

В данной работе с помощью методом решеточных уравнений Больцмана численно решалась задача обтекания потоком вязкой жидкости кругового цилиндра в плоском канале и решетки круговых цилиндров, которые вращаются с постоянной скоростью. Была разработана и протестирована методика задания граничных условий на границе вращающегося цилиндра. Проведено сравнение полученных результатов с известными численными решениями, полученными другими методами. Исследовались как стационарные, так и периодические режимы течений. Показана зависимость размера расчетной сетки от скорости вращения цилиндра при заданной точности.

Ключевые слова: *вязкая жидкость, вращающийся круговой цилиндр, решетка цилиндров, уравнение Больцмана, число Рейнольдса.*

1. Introduction

Rotating circular cylinder is a classical problem in fluid mechanics [1, 2]. During a cylinder rotation, a symmetric flow around it becomes disrupted and forms a circulation flow [2]. In this case, a flow rate increases on the one hand of a rotating cylinder and decreases on the other one. The speed difference entails the difference in pressure. Thus, the additional lift appears (the Magnus effect [3]).

In this paper, to simulate the fluid flow around a rotating cylinder the lattice Boltzmann method has been used. Usage of this method, based on the kinetic theory of gases, started in 1990 and is growing rapidly [4]. One of the advantages of this method are simplicity of understanding and programming, due to all stages being well understood processes of a particle collision and a particle transport which are described by the linear equations and therefore can be solved with the explicit schemes. There is also a possibility to parallelize the algorithm on CPU using the OpenMP technology and on GPU, using the CUDA technology. The usage of the CUDA technology provides a significant speed-up of computations, approximately in 50-70 times [5].

The lattice Boltzmann method is one of the new promising approaches in the computational fluid dynamics and it is widely used for the simulation of the conventional and multiphase flows [6], the multi-component flows [7], the flows with the free boundaries [8], the heat transfer [9] and the calculation of hydrodynamic coefficients [10]. The method has been already used for the flow simulation around a rotating circular cylinder built on complex boundaries [11-12], but clear and universal rules for setting the rotating boundary condition have not been given.

The aim of this work is to develop and test the boundary conditions needed to specify the rotation of a circular cylinder constructed along the edges of the computational grid cells with the lattice Boltzmann method. The obtained results have been compared with the similar results obtained with other numerical experiments.

2. The lattice Boltzmann method

For the fluid dynamics simulation with the lattice Boltzmann method, the pseudo particles described by the discrete particle density distribution function f_k [13-15] have been used. The particle displacement probability in one of the *k* directions has been described by the distribution function values. It should be noted that in the kinetic theory of gases, the particle density distribution function determines the density of the probability of finding the particle around the point of the six-dimensional phase space (coordinates and velocities) [16].

The computational field is divided by the square cells with the length *d*. There are nine values of the particle density distribution function in each cell. Thus, particles can move to one of the eight possible directions or stay at rest (fig.1). This model is called two-dimensional nine-vectors model of the lattice Boltzmann method (D2Q9) [13-15].

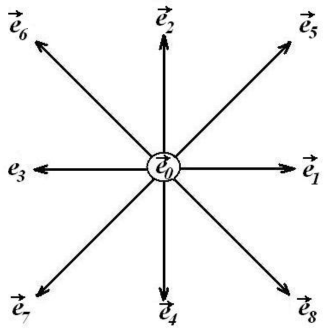


Fig. 1. Possible directions of the particle movement according to D2Q9 model

To determine the size of cells *d*, the fluid kinematic viscosity ν and the number of cells per length *N* are set. The time step is calculated according to the equation [17]:

$$\Delta t = \frac{1}{3}\frac{d^2}{\nu}\left(\tau - \frac{1}{2}\right) \tag{1}$$

where τ – nondimensional parameter of relaxation [17]. According to equation (1) and basing on the condition $\Delta t > 0$, the limitation of the relaxation parameter is $\tau > 0.5$.

Such modeling parameters as the lattice speed c and the lattice speed of sound c_s are defined as [18]:

$$c_s = \frac{1}{\sqrt{3}}c = \frac{1}{\sqrt{3}}\frac{d}{\Delta t} \tag{2}$$

The system of the discrete kinetic equations that describes the movement of pseudo particles is [17]:

$$f_k\left(\vec{r} + \vec{e}_k d, t + \Delta t\right) = f_k\left(\vec{r}, t\right) + \Omega_k,\ k = \overline{0,8}. \tag{3}$$

where Ω_k – collision operator [17] (the model of the collision integral from the integral Boltzmann equation);

$\vec{r} = (x, y)$ – coordinates;

t – time.

The model of the collision integral is presented in the BGK (Bhatnagar-Gross-Krook) approximation form [17], which is the linear relaxation to the local Maxwell equilibrium:

$$\Omega_k = \frac{f_k^{eq}\left(\vec{r}, t\right) - f_k\left(\vec{r}, t\right)}{\tau} \tag{4}$$

In the LBM for isothermal flows, the expansion of the Maxwell equilibrium distribution function by the powers of the velocity vector $\vec{u}$ has form [19]:

$$f_k^{eq}\left(\vec{r}, t\right) = w_k \rho\left(\vec{r}, t\right)\left(1 + \frac{\left(c\vec{e}_k, \vec{u}\left(\vec{r}, t\right)\right)}{c_s^2} + \frac{1}{2}\frac{\left(c\vec{e}_k, \vec{u}\left(\vec{r}, t\right)\right)^2}{c_s^4} - \frac{1}{2}\frac{\vec{u}\left(\vec{r}, t\right)^2}{c_s^2}\right) \tag{5}$$

where w_k – weights;

ρ – fluid density;

$\vec{u}$ – velocity vector.

The weights for the D2Q9 model are: $w_0 = \frac{4}{9}; w_{1-4} = \frac{1}{9}; w_{5-8} = \frac{1}{36}$ [17].

The conversion from the particle density distribution function to the fluid properties such as density ρ, velocity $\vec{u}$ and pressure p is performed according to equations [17]:

$$\rho\left(\vec{r},t\right)=\sum_{k=0}^{8} f_k\left(\vec{r},t\right); \qquad \vec{u}\left(\vec{r},t\right)=\frac{1}{\rho\left(\vec{r},t\right)}\sum_{k=0}^{8} c\vec{e}_k f_k\left(\vec{r},t\right); \quad p\left(\vec{r},t\right)=c_s^2\rho\left(\vec{r},t\right) \quad (6)$$

It is known that the disadvantage of the LBM is conditional stability [14, 15, 19-21]. The stability of the solution is affected by:

- c_s – the lattice speed of sound: as shown in [14], the method remains stable when: $c_s < \sqrt{1-U_{\max}^2}$, where $U_{\max}$ is the maximum speed value in the computational domain;
- τ – the relaxation parameter: to avoid the negative influence of the relaxation parameter on the numerical results, it is usually set as $\tau = 1$ [15];
- c – the lattice speed: as shown in [19, 20], the method remains stable when $M << 1$, where M is the lattice Mach number calculated by the equation:

$$M = \frac{U_{\max}}{c} \quad (7)$$

3. Setting the boundary conditions

There are several methods of setting the boundary conditions in LBM [15, 22-24] which can be chosen according to the task.

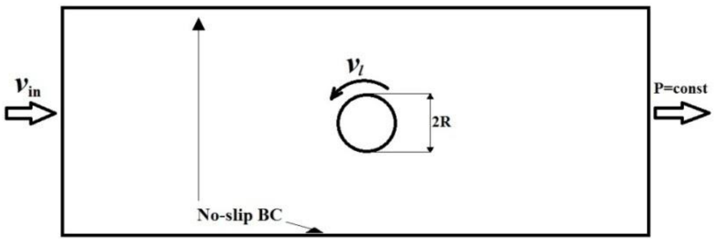


Fig. 3. Boundary conditions

In this work the following boundary conditions (BC) are used (fig. 3):

- No-slip BC. The method for setting the no-slip BC is described in [22] in details and it is implemented by setting the condition of the particles reflection from the boundary in the opposite direction.
- Inlet BC. Inlet BC. When the inlet BC is set, there are two levels that describe fluid: the macroscopic level and the microscopic one. Firstly, at the macroscopic level, the velocity components are set. The fluid density on the boundary can be calculated according to the method described in [24, 25] using the known distribution function values. As the lattice speed of sound is a variable value, the density is calculated according to the equation:

$$\rho(\vec{r},t) = \frac{\left(f_0+f_2+f_4\right)+2\left(f_3+f_6+f_7\right)}{1-\frac{1}{c}u_x(\vec{r},t)}$$

Then, at the microscopic level, the unknown distribution function values on the boundary can be calculated by formulas [25]:

$$f_1(\vec{r},t) = f_3(\vec{r},t) + \frac{2}{3c}\rho(\vec{r},t)u_x(\vec{r},t)$$
$$f_5(\vec{r},t) = f_7(\vec{r},t) + \frac{1}{2}\Big(f_4(\vec{r},t) - f_2(\vec{r},t)\Big) + \frac{1}{6c}\rho(\vec{r},t)u_x(\vec{r},t)$$
$$f_8(\vec{r},t) = f_6(\vec{r},t) + \frac{1}{2}\Big(f_2(\vec{r},t) - f_4(\vec{r},t)\Big) + \frac{1}{6c}\rho(\vec{r},t)u_x(\vec{r},t)$$

➢ Outlet BC. The fluid flow from a channel is implemented by the constant pressure boundary condition at the outlet of channel [25]. For this purpose, firstly set the u_x velocity component on the boundary which is equal to the velocity on the previous layer of the computational grid $u_x(N_x-1, j) = u_x(N_x-2, j)$. Here N_x is a number of cells along *x* axis. Velocity component u_y is set as zero: $u_y(N_x-1, j) = 0$ All nine values of the distribution function are recalculated according to the equation (5). Thus, there is the relaxation of the distribution function to the local equilibrium on the right boundary.

➢ Symmetric BC. Using the symmetric BC particles that move over the boundary not reflecting from it but transferring to the opposite boundary without changing their direction (fig. 4) [22].

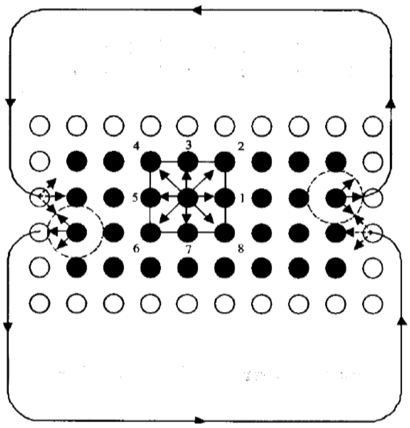


Fig. 4. The motion scheme of the particles. The symmetric boundary conditions are applied to the side walls

For setting the rotation of a cylinder the following technique is proposed. Consider a circular cylinder with radius *R*, which rotates with the constant linear speed v_l. A circumference shown in fig. 5 corresponds to the boundary layer of the cells with a fluid that are directly adjacent to the cylinder. Knowing the rotating speed of the cylinder the velocity components v_x, v_y in each cell with a fluid that borders the cylinder can be calculated. For this, knowing that the velocity vector is directed along tangent, we can use the property of the perpendicular vectors:

$$\bar{v} \cdot \bar{r} = 0 \qquad (8)$$

where $\bar{v} = (v_x, v_y)$ – the velocity vector at any point on the circumference;

$\bar{r}$ – the radius vector from an arbitrary point on the circumference to its center (AO vector in fig.5);

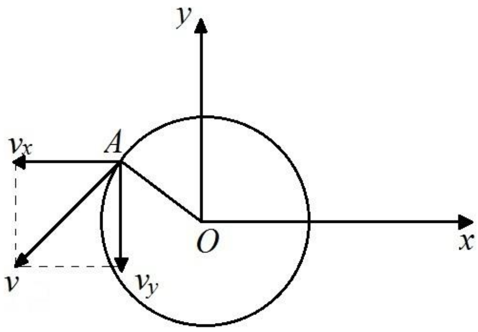


Fig. 5. The direction of the velocity vector and its components in any point of the circumference

The linear velocity of rotation v_l is a velocity vector module $\bar{v}$

$$v_l = \left|\bar{v}\right| = \sqrt{v_x^2 + v_y^2} \qquad (9)$$

Combining equations (8-9) we obtain the system of nonlinear equations for the each cell with a fluid from the circumference in fig. 5.

$$\begin{cases} \bar{v} \cdot \bar{r} = 0, \\ \left|\bar{v}\right|^2 = v_l^2. \end{cases} \qquad (10)$$

Solving the system (10) numerically by using the Newton method we got the velocity components values v_x, v_y for each fluid cell. For all fluid cells that border the cylinder we set all nine values of the particle density distribution function using equation (5) of the Maxwell equilibrium distribution function.

4. Results of the modeling

All calculations presented below have been obtained using original program written in C++ language in the development environment software Visual Community2015 using CPU parallel technology OpenMP.

4.1. Rotating circular cylinder modeling

To test the proposed method of setting the rotating conditions in LBM, the series of calculations of laminar flows around a rotating cylinder at Reynolds number $\mathrm{Re} = 200$ and at dimensionless rotating speed belonging to the range $v_l = 0{,}5 - 3$ have been conducted. The obtained results, namely, the flow patterns have been compared with the known experimental data [1-3].

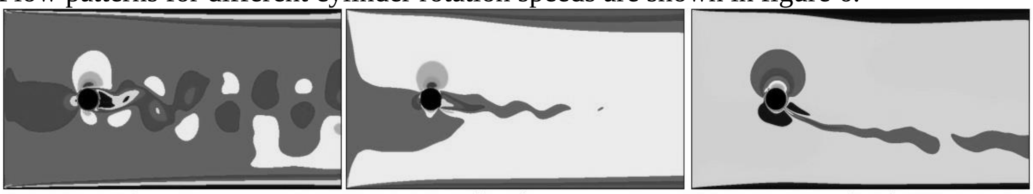
The results of the calculations have been obtained as periodic solutions and stationary circulating flows. Possible instability of a numerical solution is a feature of modeling such flows with the lattice Boltzmann method. When cylinder rotating speed increases, the lattice Mach number also increases according to formula (7). To avoid the increase of the lattice Mach number and, consequently, instability, the lattice speed c must be decreasing. Lattice speed c reduction can be achieved by grinding the computational grid, which leads to the increase in the simulation time.

Let us set the kinematic viscosity $\nu = 6,25 \cdot 10^{-5}$, the diameter of the cylinder $D = 0,125$, the fluid velocity at the entrance of the channel $U_{in} = 0,1$, the relaxation parameter $\tau = 0,55$, the simulation time $T = 20$ and the size of the domain 2×1. Thus, the Reynolds number is $\mathrm{Re} = 200$ and the blockage ratio is $B = 1/\mathrm{D} = 8$. Also the series of calculations with these parameters to get the lattice Mach number $M \approx 0,15$ have been conducted. A larger value M will result in increasing an error of the numerical solution, as shown in [19, 20]. The results of the numerical solutions are presented in the table 1.

Tab. 1. Dependence of the rotation speed of a cylinder on the modeling parameters

Rotation speed v_l	$v_l = 0,5$	$v_l = 1,0$	$v_l = 1,5$	$v_l = 2,0$	$v_l = 3,0$
Number of cells N	300	300	350	400	550
Time step Δt	0,00296	0,00296	0,00218	0,0025	0,00182
Lattice speed c	1,125	1,125	1,3125	1,5	2,0625
Mach number M	0,144	0,163	0,158	0,159	0,156
Solution time (min)	44	43	87	141	460

Flow patterns for different cylinder rotation speeds are shown in figure 6.



a) v_l=1 *b) v_l=2* *c) v_l=3*

Fig. 6 – Flows around a rotating circular cylinder at Reynolds number Re = 200 and at different rotation speeds

As shown in figure 6, when the fluid flows around a rotating circular cylinder the circulating current appears. A solution can be periodic or stationary depending on the cylinder rotation speed. It should be noted that the transition from the periodic to the stationary solution occurs at the rotation speed critical values $v_l^{kr} = 1,9$ for the Reynolds number Re=200. The same results have been shown in [3]. Thus, the obtained results fully correspond to the existing data.

As shown in table 1, the solution time increases with an increase of the rotation speed (fig. 7).

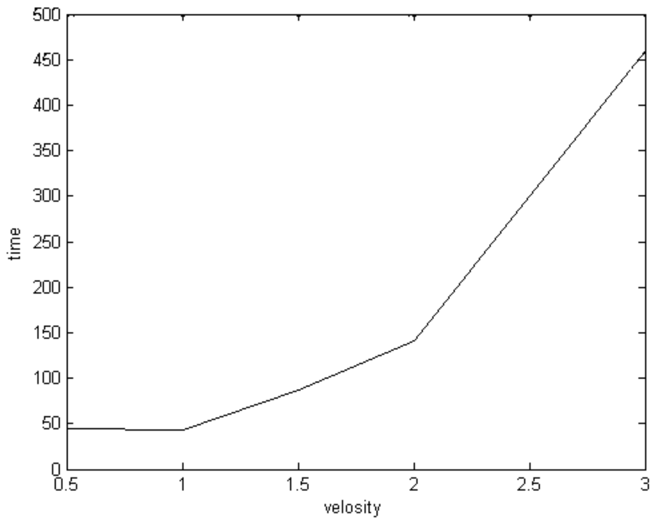
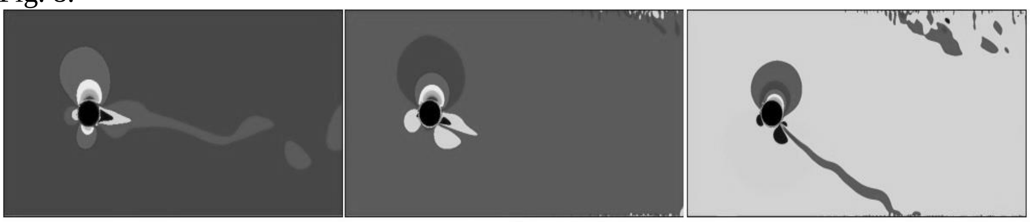


Fig. 7. Graph of the dependence of the solution time from the cylinder rotation speed at the Reynolds number Re=200

The reason for this growth is the grinding of the computational grid, required to obtain the lattice Mach number $M \approx 0{,}15$. Therefore, the calculations for the rotation speed $v_l > 3$ have not been carried out in this work. Nevertheless, these results can be obtained by using algorithm parallelization on GPU using CUDA technology, which could increase calculation speed up to 100 times depending on the computational capabilities of the graphics card [5].

4.2. Modeling a lattice of rotating circular cylinders

Let us make the series of similar calculations for a lattice of rotating cylinders. To do this, we define the symmetry boundary conditions on the top and the bottom boundaries of the domain, as has been shown in the previous section. The received flow patterns are corresponding to the flows at Reynolds number Re=200, shown in Fig. 8.



a) v_l***=1*** ***b)*** v_l***=2*** ***c)*** v_l***=3***

Fig. 8. Flow around a lattice of rotating circular cylinders at Reynolds number Re = 200 at different rotational speeds

As shown in Fig. 8, there is not the vortex separation from cylinders for the speed range $v_l = 1 - 3$ and the solutions are stationary. If the rotation speed is less than the velocity of the fluid flow, i.e. $v_l < 1$, there is the separation of vortices and the solution is periodic (fig. 9).

Fig. 9. Flow around a lattice of rotating circular cylinders at Reynolds number Re = 200 and $v_l = 0,5$

Thus, the flow pattern in the simulation of a lattice of rotating cylinders differs from the flow in a channel with the same parameters. Moreover, the critical rotation speed value at which the solution transits from the periodic to the stationary one for the case of the flow around a lattice of cylinders is less than for the case of the flow in a channel.

Conclusion

The paper presents the results of modeling the two-dimensional problem of a viscous fluid flow around a rotating circular cylinder in a plane channel and the flow around a lattice of rotating cylinders with the lattice Boltzmann method. The method of defining the boundary conditions for the rotation of cylinders for simple geometry, in which the cylinder is constructed along the edges of the cells of the computational grid has been proposed and tested. Both periodic and stationary regimes of the received flows have been shown. We have determined the critical number of the circular cylinder rotation speed, at which the transition from periodic solution (when the vortex track behind a cylinder is observed) to the stationary one occurs. Namely, $v_l^{kr} = 1,9$ for the Reynolds number Re=200 which corresponds to the results of other authors. The dependence of the cylinder rotation speed on the solution time has been shown. The data confirm the adequacy of the proposed method of setting the rotation conditions. Furthermore, this technique is much simpler in terms of description and programming. It should be noted that the lattice Boltzmann method is of great potential. We are planning to obtain a sustainable solution for large Reynolds numbers in further researches.

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