

## Fundamental researches

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### QUANTUM, MOLECULAR AND CONTINUUM MODELING IN NONLINEAR MECHANICS OF VIRUSES

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(A – research concept and design; B – collection and/or assembly of data; C – data analysis and interpretation; D – writing the article; E – critical revision of the article; F – final approval of the article)

**Introduction.** Viruses are a large group of pathogens that have been identified to infect animals, plants, bacteria and even other viruses. The 2019 novel coronavirus SARS-CoV-2 remains a constant threat to the human population. Viruses are biological objects with nanometric dimensions (typically from a few tens to several hundreds of nanometers). They are considered as the biomolecular substances composed of genetic materials (RNA or DNA), protecting capsid proteins and sometimes also of envelopes.

**Objective.** The goal of the present review is to help predict the response and even deconstruction of viruses taking into account the influence of different environmental factors, such as, mechanical loads, thermal changes, electromagnetic field, chemical changes and receptor binding on the host membrane. These environmental factors have significant impact on the virus.

**Materials and methods.** The study of viruses and virus-like structures has been analyzed using models and methods of nonlinear mechanics. In this regard, quantum, molecular and continuum descriptions in virus mechanics have been considered. Application of single molecule manipulation techniques, such as, atomic force microscopy, optical tweezers and magnetic tweezers has been discussed for a determination of the mechanical properties of viruses. Particular attention has been given to continuum damage–healing mechanics of viruses, proteins and virus-like structures. Also, constitutive modeling of viruses at large strains is presented. Nonlinear elasticity, plastic deformation, creep behavior, environmentally induced swelling (or shrinkage) and piezoelectric response of viruses were taken into account. Integrating a constitutive framework into ABAQUS, ANSYS and in-house developed software has been discussed.

**Conclusion.** Link between virus structure, environment, infectivity and virus mechanics may be useful to predict the response and deconstruction of viruses taking into account the influence of different environmental factors. Computational analysis using such link may be helpful to give a clear understanding of how neutralizing antibodies and T cells interact with the 2019 novel coronavirus SARS-CoV-2.

**KEY WORDS:** virus, mechanics, atomic force microscopy, stress, deformation, damage, healing, modeling, simulation

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#### INTRODUCTION

In the past reviews [1, 2, 3, 4], we focused the attention of readers on the nonlinear biomechanics of hard and soft tissues of living organisms. Due to the appearance of a number of important research questions [5]

related to the recent outbreak of the 2019 novel coronavirus SARS-CoV-2 and fast spread of the disease COVID-19 around the world, we decided to present in this overview the applications of nonlinear mechanics to such biological structures as viruses. In this regard, the experimental, theoretical and

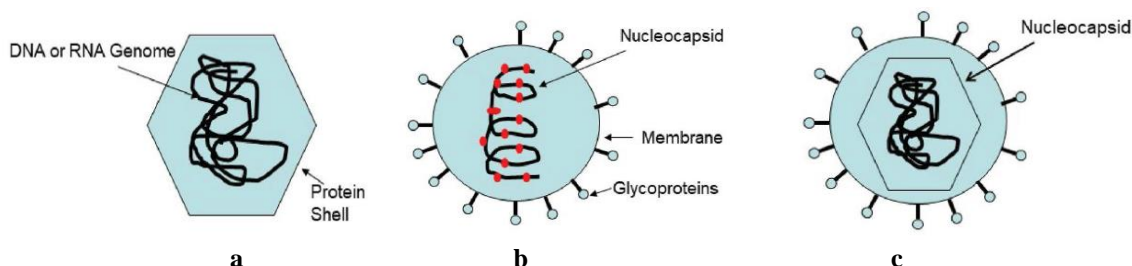
computational studies of viruses and virus-like structures have been analyzed using models and methods of nonlinear mechanics.

According to Schrödinger [6], the biosphere is not an isolated thermodynamic system. Therefore, appearance and mutation of such disease-causing microorganisms as viruses are closely related to the evolution of cellular life. The general aim of the present review is to help predict the response and even deconstruction of viruses taking into account the influence of different environmental factors. The environment is considered to be made up of those parts of the universal system with which the virus interacts [7]. Therefore, the influence of some environmental conditions like mechanical loads, thermal changes, electromagnetic field, chemical changes and receptor binding on the host membrane has been analyzed in the present review. These environmental factors have significant impact on the virus.

Viruses are a large group of pathogens that have been identified to infect humans, animals, plants, bacteria and even other viruses [8]. Viruses can be considered as the biomolecular substances composed of genetic materials (RNA or DNA), protecting capsid proteins and sometimes also of envelopes [9]. In other words, viruses consist of just a nucleic acid packaged within a protein shell and possibly also an envelope. Viral replication occurs by the inserting viral gene

in host cells, and an envelope of the new formed virus particle derives from the host cell membrane. Building of the viral envelope involves proteins (glycoproteins), carbohydrates and lipids.

Different viruses range in structure, size, complexity and mechanisms of entry. RNA viruses are mostly single-stranded (ss) while DNA viruses may be either single-stranded or double-stranded (ds). The sizes of the entire infectious virus particles (virions) are in the range of a few tens to several hundreds of nanometers [9]. RNA viruses include the common cold, hepatitis C, hepatitis E, human immunodeficiency virus type 1 (HIV-1), West Nile, Ebola, influenza, norovirus (NV), mumps, rabies, Dengue, SARS-CoV (or SARS-CoV-1), MERS-CoV, SARS-CoV-2, polio and measles [10]. DNA viruses include adenoviruses, herpesviruses, in particular, herpes simplex virus type 1 (HSV-1), parvoviruses, papillomaviruses, polyomaviruses and poxviruses [10]. Genetic structure of viruses may be linear or circular. Capsids may consist of only one or a few structural protein species, and they may be formed as single- or multilayered protein shells [11]. Geometry of virus capsids follows helical (spiral) shape or icosahedral (quasi-spherical) symmetry. Viral envelope can be considered as additional protective shell in viruses. Schematic of virion structure of different viruses is given in Fig. 1a, b and c.



**Fig. 1. Virion structure: Non-enveloped virus with icosahedral capsid structure (a), enveloped virus with helical nucleocapsid structure (b) and enveloped virus with icosahedral capsid structure (c) [10]**

Three highly pathogenic variants of coronaviruses (enveloped, positive-sense single-stranded RNA viruses) have emerged in the 21st century, such as, severe acute respiratory syndrome-related coronavirus (SARS-CoV, 2002), Middle East respiratory syndrome-related coronavirus (MERS-CoV, 2012) and severe acute respiratory syndrome-related coronavirus-2 (SARS-CoV-2, 2019)

[12]. Up to now, the novel coronavirus SARS-CoV-2, whose appearance was predicted 40 years ago in a fantastic novel [13], remains a constant threat to the human population. Schematic structure of the SARS-CoV-2 virion is presented in Fig. 2. It is seen (Fig. 2) that coronavirus particles comprise at least four structural protein species: spike (S), envelope (E), membrane (M) and

nucleocapsid (N) [12]. The S protein including a large globular S1 domain and a

rod-like S2 domain is responsible for receptor binding on the host cell surface.

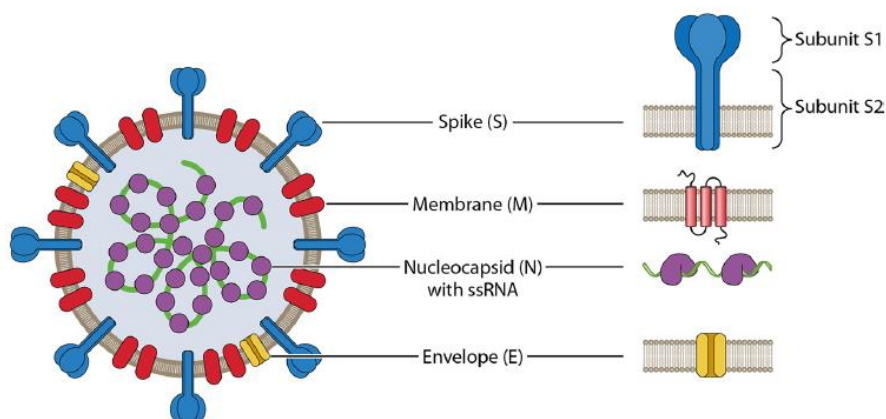


Fig. 2. Schematic SARS-CoV-2 structure and protein localization [12]

The atomic structure of viruses was studied in detail using X-ray crystallography and cryoelectron microscopy [14, 15]. After this it was established [16] that the most common simplifications for geometries of virions correspond to the sphere-like and rod-like particles, but spherocylinders, cones and other shell shapes are also available. For example, schematic of idealized capsid

models is given in Fig. 3a, b for the spherical cowpea chlorotic mottle virus (CCMV) and the ellipsocylindrical bacteriophage  $\phi 29$ . Here, the average dimensions for the idealized models are accepted. It is clear (Fig. 3) that the capsid of  $\phi 29$  virion can be considered as a thin shell [18, 19, 20, 21, 22] while the capsid of the CCMV virion as moderately thick shell [23, 24, 25].

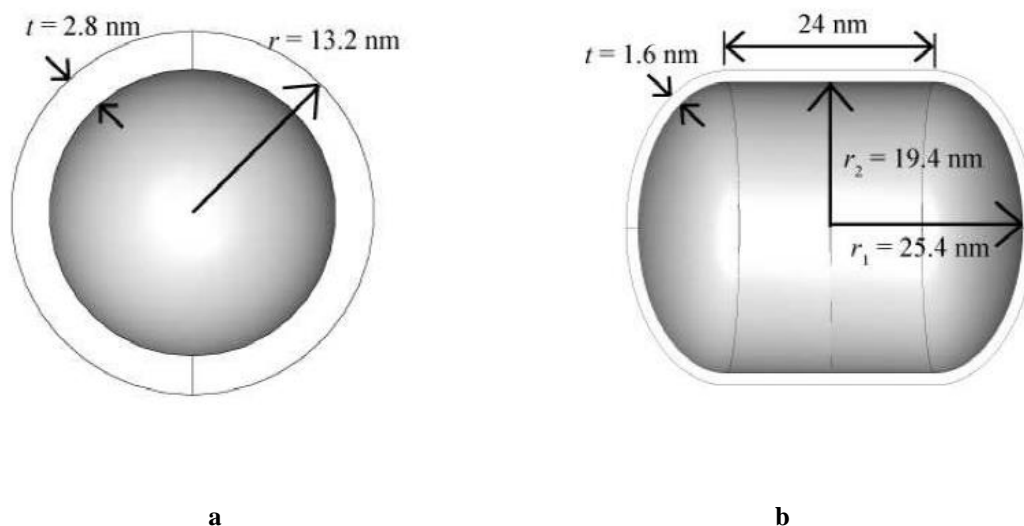
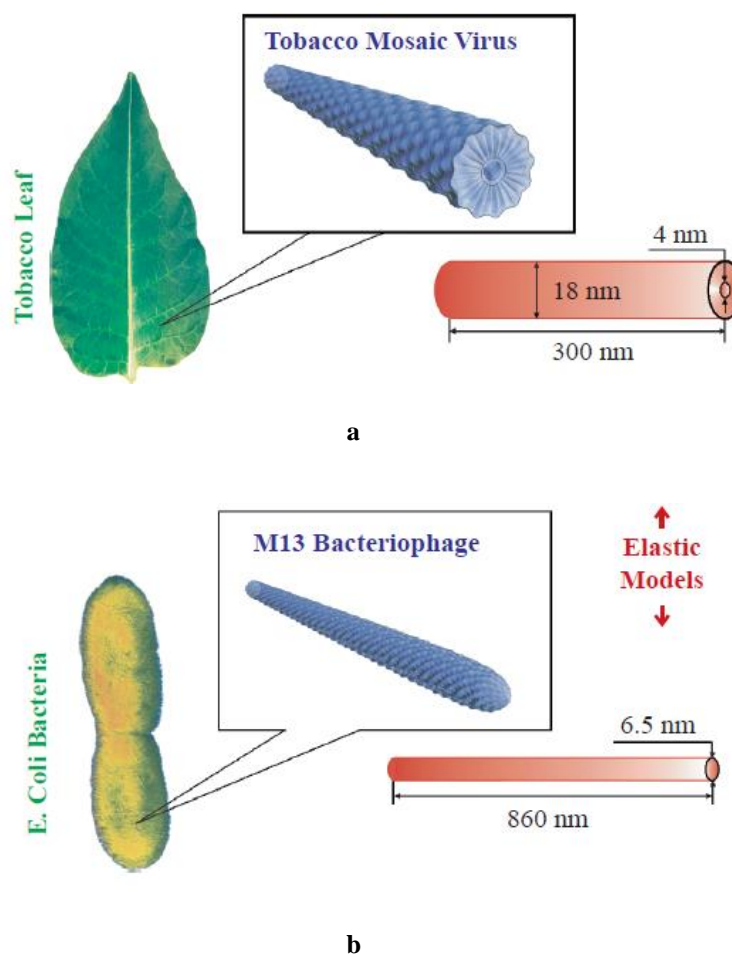


Fig. 3. Schematic structure and model dimensions for the CCMV virion (a), and  $\phi 29$  virion (b) [17]

Also, both tobacco mosaic virus (TMV) and M13 bacteriophage are rod-like particles of the cylindrical shape (Fig. 4a, b). M13 bacteriophage is 860 nm long and 6.5 nm in a

diameter, while TMV is 300 nm long, 18 nm in a diameter [26]. Thus, the TMV and M13 virions can be considered as thick-walled cylinders [27–29].

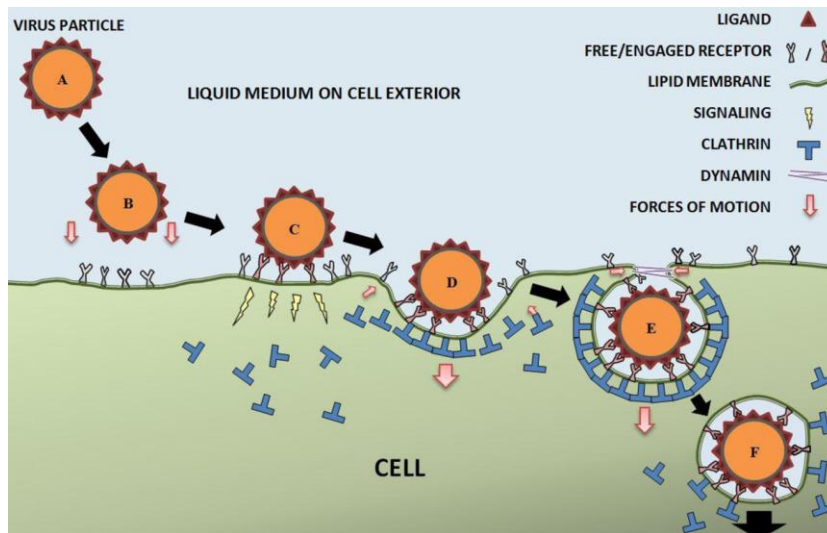


**Fig. 4. Schematic structure and model dimensions for the TMV virion (a), and M13 virion (b) [26]**

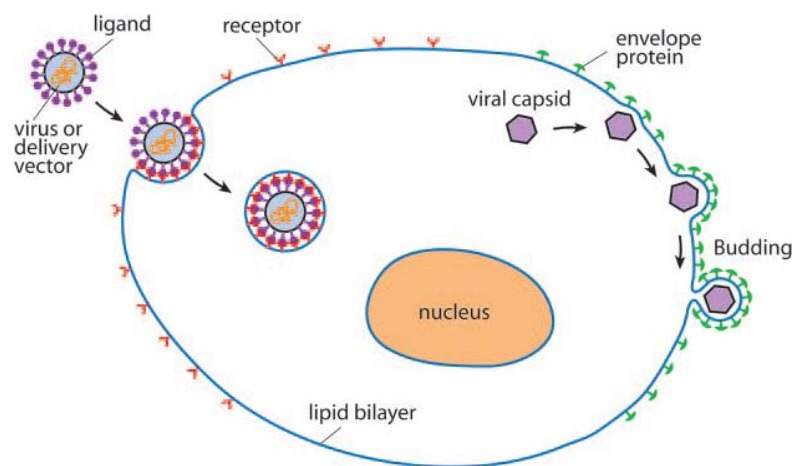
Additionally, the virion of a filamentous virus like Ebola can be considered approximately as a cylindrical particle. Furthermore, the Ebola virus has a diameter of about 80 nm, whereas its length can reach 1–2  $\mu\text{m}$  [30]. Thus, filamentous viruses are cylindrical and long. On contrary, HIV, Zika and SARS-CoV-2 virions are nominally spherical virus particles. For example, a diameter of SARS-CoV-2 with spike (Fig. 2) is around 120 nm [30]. Note that virus shape affects the mechanical properties of viruses [31].

Virus infectivity is defined as the capacity of viruses to enter the host cell and exploit its resources to replicate and produce new infectious virions, which may lead to infection and subsequent viral disease in the human host [32, 33]. In order to initiate infection, viruses must enter host cells and deliver their genetic material. The most of viruses take the receptor mediated endocytic pathways (Fig. 5a, b) to enter host cells [34]. In this case, the plasma membrane binds specific macromolecules and smaller particles

by means of cellular receptors, such as, sialic acids, integrins or cell adhesion molecules (Fig. 5a, b). Up to now, virus entry via endocytosis is not clear in detail. Depending on the virus, environment and cell type, viruses take a number of different endocytic pathways to gain entry. In general, there are several virus entry mechanisms with receptor binding on the host membrane. The first one (Fig. 5a) is related to clathrin and caveolin scaffolding in which the plasma membrane bending is assisted by clathrin/caveolin assembly [35]. The second one (Fig. 5b) is related to clathrin/caveolin-independent pathway in which membrane bending is driven by receptor-ligand binding [36]. A threshold virus particle radius, below which clathrin/caveolin independent endocytosis would not occur, was found to be about 12 nm for cylindrical and about 24 nm for spherical particles [37]. Thus, clathrin/caveolin-mediated endocytosis is the main virus' pathway from the extracellular space to the replication site [34].



a



b

**Fig. 5. Schematic illustration of the receptor-mediated endocytosis [34]: Stages of clathrin/caveolin-assisted pathway (a) [35], and clathrin/caveolin-independent pathway (b) [36]**

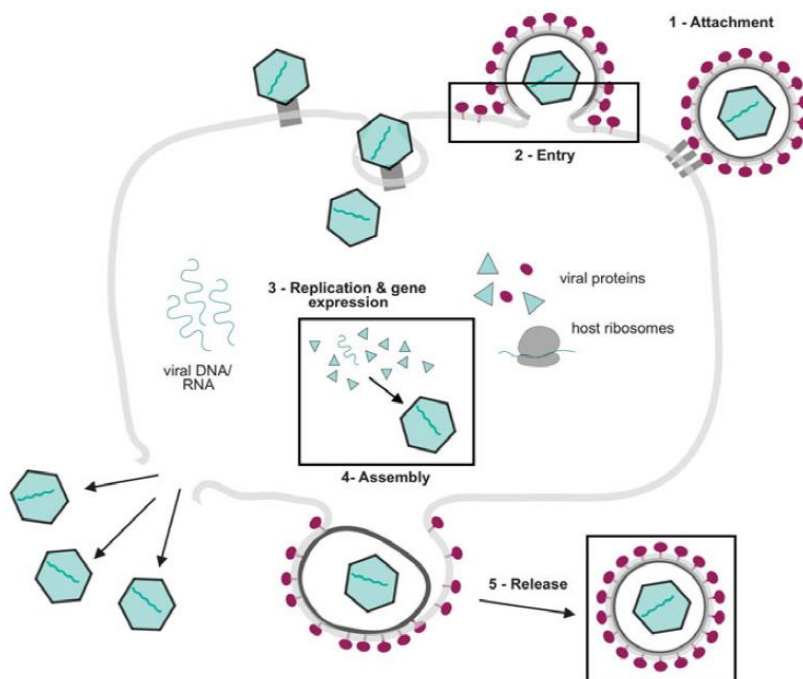
After replication and production of new infectious virions, viruses must package their newly replicated genomes for delivery to other host cells. So, application of optical tweezers to pull on single DNA molecules of the bacteriophage  $\phi 29$  virion, as they are packaged, shows that DNA-capsid complex under study is a force-generating motor [38]. The DNA is kept under high pressure (about 60 atm) inside the viral shell [38]. Furthermore, this large internal pressure may be available for initiating the ejection of the viral genome during infection. Thus, it is reasonable to expect the existence of external loads in virus entry mechanisms with receptor

binding on the host membrane [34]. The initial events when a virus binds to cell surface receptors were quantified using an atomic force and confocal microscopy setup [39]. A generalized viral replication cycle [40] is given in Fig. 6. Understanding of virus entry via endocytosis and viral replication is very important for the development of antiviral strategies.

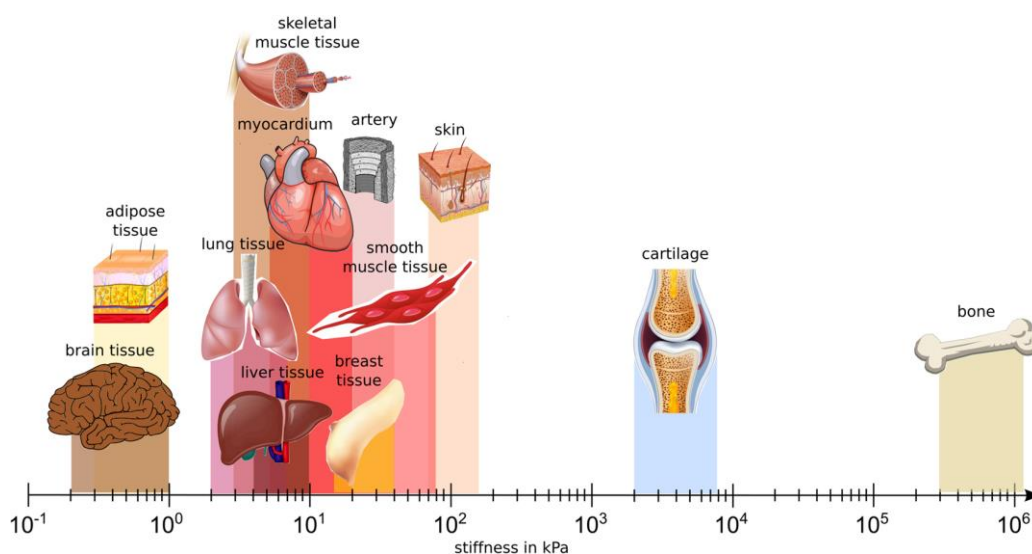
In 1997, Virus Mechanics was first introduced as a new research field by Falvo et al. [41] after determination of the Young's modulus (1.1 GPa) for the TMV (Fig. 4a). In this regard, atomic force microscopy (AFM) nanoindentation experiments, continuum

approximation of rod-like virus and elastic bending theory of beams with circular cross section under point loading created by the AFM tip were used. It is interesting to note that the Young's modulus of virus is comparable with that of hard plastics. Also, the measured Young's modulus of virus is in the same order versus the values measured for structural proteins (about 4 GPa ) [42], such as, collagen, actin or tubulin, and it is close to the values for bone (Fig. 7).

In the case of viral capsid proteins, there are strong covalent chemical bonds, and weaker noncovalent physical bonds that include electrostatic bonds (ion pairs and hydrogen bonds) and van der Waals bonds [42]. Therefore, deformation of viruses under mechanical loading arises from the stiffness of the bonds that hold the constituent atoms together.



**Fig. 6. Schematic viral replication cycle, showing the steps thought to be common to most viruses [40]**



**Fig. 7. Young's modulus of different tissues in the human body [43]**

In general, a complete consideration of the mechanical response in nanosized objects would benefit from quantum, molecular and continuum descriptions [42, 44, 45].

Computational analysis methods in quantum modeling (first principles based modeling or ab initio calculations) of viruses are related to solving the many-particle Schrödinger equation directly. In this case, the interaction between particles (atoms) is dictated by their quantum mechanical state. Unfortunately, understanding of the atomic-scale interactions between the SARS-CoV-2 spike protein and the ACE2 receptor is limited by low amount of atoms taken in simulations [46], because ab initio calculations are highly computationally intensive even using supercomputer clusters. So, about 2 300 atoms were taken in the computational analysis given in [46]. At the same time, the cryoelectron microscopy structure of the SARS-CoV-2 spike (S) glycoprotein in the postfusion state has been well studied and widely reported [47]. In order to reduce computational costs, many fragment-like approaches to quantum modeling have been developed [48], in which fragments appear as groups of atoms, and the state of the full system is computed. An important advance of fragment-based methods is an approximate solution of the Schrödinger equation considering the separate blocks derived for individual fragments instead of the consideration of a quantum many-atom system.

The core of molecular modeling techniques (molecular dynamics or MD) relies on solving the second Newton's law to extract the positions of each atom and to obtain mechanical properties of viruses by employing the energy approach. In this way, deformation of virus has been considered in response to the ability to alter its conformation under the action of temperature fluctuations. The obvious advantage of MD simulations is the ability to access larger number of atoms, as well as, larger length scales and longer time scales using supercomputer clusters. So, the results of numerically simulated deformation by the AFM tip are presented for southern bean mosaic virus using the Lennard-Jones potential [49, 50]. The solvated viral capsid system with more than 4.5 million atoms was equilibrated for 13 ns and then force-indentation simulations were performed [49].

The effect of calcium removal on the mechanical properties of this RNA plant virus has been also studied [50]. Elastic properties of sesbania mosaic virus were determined from equilibrium thermal fluctuations of the capsid surface in molecular dynamics [51]. Simulation system with more than 800 000 atoms was equilibrated for 30 ns. Molecular simulations on the satellite tobacco necrosis virus (STNV) capsid were performed with 1.2 million atoms on the 1  $\mu$ s time scale, exceeding any of the previous studies, and swelling of the viral capsid caused by calcium removal was predicted [52]. Molecular modeling technique was applied to generate equilibrium dynamics of the mature HIV-1 viral capsid [53]. Note that the HIV-1 retroviral capsid has a cone structure. Bending of the solvated viral capsid was numerically simulated for 100 ns at 2 fs time steps. Such capsid system with 64 million atoms is one of the largest biomolecular simulation systems in the world [54].

An alternative simulation technique in the MD is related to the consideration of virus as a system of coarse-grained particles instead of a system of atoms [55]. The simplification of a simulation system gives the possibility to reduce computational costs using coarse-grained models. In this way, several hundred atoms are represented by a single particle. Such level of coarse graining allows us to achieve the microsecond time scale. So, elastic deformation of the seven coarse-grained solvated viral capsid systems was simulated over time intervals of 1.5-25  $\mu$ s [56]. In this regard, the four plant viruses (satellite tobacco mosaic virus STMV, satellite panicum mosaic virus, STNV and brome mosaic virus), poliovirus, bacteriophage  $\phi$ X174, and reovirus were studied. It is interesting to note that poliovirus and reovirus infect humans, whereas bacteriophage  $\phi$ X174 infects bacteria. Coarse-grained models on 35 different viral capsids were developed to simulate the AFM nanoindentation experiments [57]. The mechanical response of capsids and breaking of protein bonds were studied. Considering capsid deformation by the AFM tip, it was found that the force-indentation curve includes the elastic part and the irreversible one. The relationship between the mechanical properties of viruses and their structure has been discussed.

The other alternative approach in the MD studies of viruses is associated with lattice description considering viral capsid as a system of its structural units (capsomers) [58, 59]. In this way, combined AFM nanoindentation experiments and computational modeling on subsecond timescales of the CCMV capsid are considered. Unlike the double-stranded DNA bacteriophages, which can actively package their genetic material to internal pressures up to 60 atm [38], the filled plant CCMV capsid is not packed under pressure. It was taken into account in lattice description of virus that the capsid of CCMV is an icosahedral protein shell which is comprised of 60 trimer structural units with pentameric symmetry at the 12 vertices (pentamer capsomeres) and hexameric symmetry at the 20 faces (hexamer capsomeres) of the icosahedron [58, 59]. Simulations were carried out in the isolated single pentamer and hexamer capsomers, as well as, in the full CCMV capsid at equilibrium. Computational studies show two dynamic regimes to be responsible for the CCMV capsid stiffening and softening. In particular, stiffening at low force (or indentation) is similar to Hertz response for solid bodies in contact. The large-amplitude out-of-plane displacements mediate the capsid bending which is in direct contact with collapse of the biological structure. It was numerically established that reversibility and irreversibility of indentation are correlated with the mechanical characteristics, i.e., elastic deformation and inelastic response due to local rearrangements of the capsid proteins. Such investigations can help to understand on how the CCMV can infect plant tissue through damaged cell walls that can be produced by either mechanical or biological means. For example, swelling of virus may be related to its mechanism of infectivity.

Continuum description of viruses is constructed using well developed models and methods of solid mechanics [27, 60, 61]. In 2004, Ivanovska et al. [62] determined the Young's modulus (around 1.8 GPa) for the bacteriophage  $\phi 29$  based on the AFM nanoindentation experiments, Hertz model and theory of elastic thin spherical shells. It is interesting to note that the Young's modulus of  $\phi 29$  is larger than that of the TMV [41]. Unlike [41], paper by Ivanovska et al. [62] was a start for the numerous studies of linear

elastic deformation for such viruses of the spherical shape as CCMV, bacteriophages  $\lambda$  and HK97, murine leukemia virus (MLV), minute virus of mice (MVM), STMV, Wiseana iridovirus (WIV), and hepatitis B virus (HBV) [63–71]. Nonlinear finite element analysis of spherical viruses using AFM nanoindentation was given in [17, 63, 64, 66, 69, 70, 72–76]. The mechanical response of viruses has been found to depend on the packed genome. The linear elastic bending theory of beams with circular cross section was applied to the rod-like TMV virion [77] by analogy with Falvo et al. [41]. Nonlinear finite element modeling of the TMV capsid at the AFM nanoindentation has been considered in [78]. The radial Young's modulus of the TMV [78, 79] determined using the Hertz model is comparable with the axial Young's modulus [41]. Thus, it is possible to assume the initial isotropy of the TMV. In general, continuum techniques are the preferable methods to simulate the mechanical behavior of viruses. At the same time, replacing a discrete structure of virus with a continuum media is the most critical part of modeling process.

Both MD and continuum modeling are employed for mechanical characterization of virus capsids taking into account prestresses and linear elasticity [80]. Also, molecular and continuum levels of description were applied to the nonlinear analysis of HK97, CCMV and HBV at the AFM nanoindentation [71, 81]. Multiscale simulations in Virus Mechanics are actually rare.

Regarding the simulation of the viral entry into a cell membrane via endocytosis (Fig. 5), such important aspects were involved as the contact mechanics of viruses onto a flexible membrane [30, 82, 83, 84, 85], effects of virus size and ligand density on the membrane wrapping [86], as well as, influence of receptor concentration gradient on the ligand-receptor binding [87]. The change of receptor density over the host membrane was described [37, 88, 89] by the Fick's second law [90, 91, 92], however, without consideration of diffusion induced stresses [93, 94, 95, 96, 97, 98]. Continuum modeling was used for adhesive contact between the virus and cell membrane, driven by adhesion [82, 83, 84] or driven by adhesion and, additionally, by external displacement (or force) [30, 85], however,



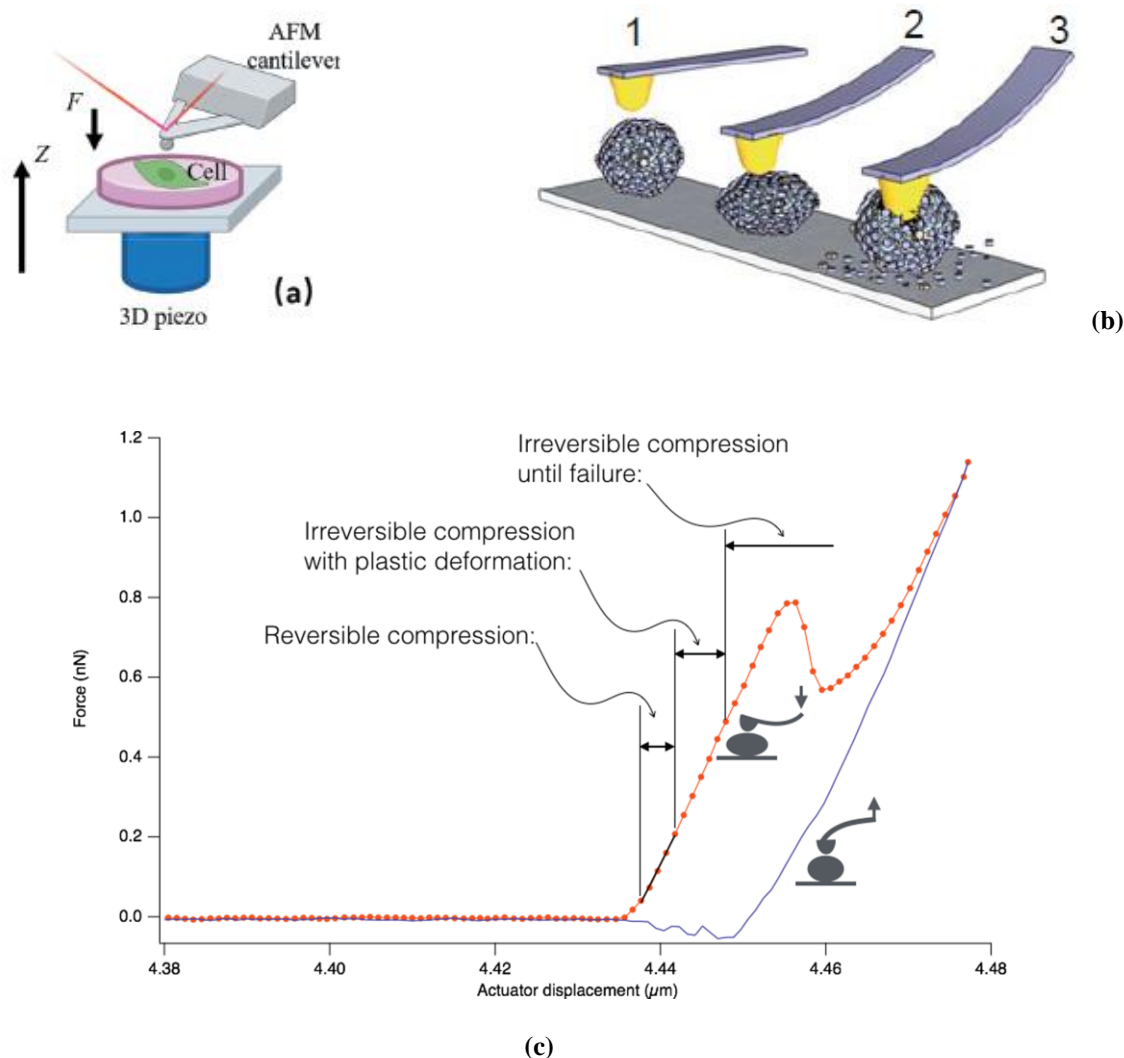
under the assumption of linear elasticity. Thus, a little is known about the nonlinear biomechanical interactions between ligand and receptor molecules. Therefore, up to now, antiviral strategy based on the nonlinear mechanics of viruses has not yet been developed.

## MATERIALS AND METHODS

Mechanically, virus behaves identically to any other elastoviscoplastic material in that it undergoes large deformation when subject to different environmental factors (mechanical load, light, magnetic field). In this regard, application of single molecule manipulation

techniques, such as, AFM, optical tweezers and magnetic tweezers has been given for a determination of the mechanical properties of viruses [99].

**AFM nanoindentation.** Since its invention in 1986, the AFM has been the most widely used scanning probe microscope for biological applications [100]. Unfortunately, a tutorial for laboratory training of Ukrainian students and young specialists in virology does not include the use of the AFM [101]. The majority of AFM experiments on viruses are performed by nanoindentation (Fig. 8a, b, c) [102, 103, 104].



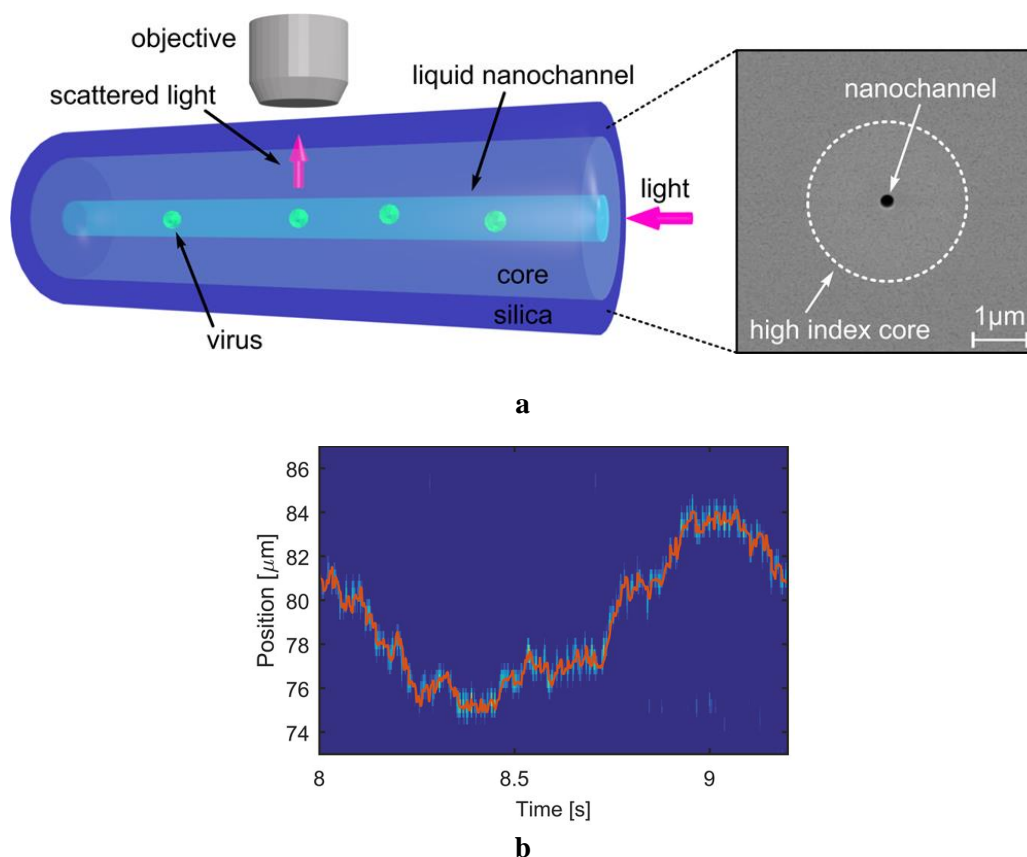
**Fig. 8. AFM nanoindentation: Schematics (a), three main phases of experiments (b): before contact (1), during indentation (2) and after breaking (3), and force–piezoactuator displacement curve for single bromo mosaic virus [102-104]**

The AFM uses sharp spherical (or conical) tip (often with radius of curvature about or

below 10 nm) mounted at the free end of a cantilever. The characteristic feature of AFM

nanoindentation test with probes attached to the end of cantilever beams is related to the principal opportunity for detection of attractive/repulsive forces between the probing tip and the sample surface in excess of 10 pN [99]. Taking into account that the indenter contacts the surface of the sample, the displacement of the cantilever base is translated into the indentation depth and the deflection of cantilever [104]. The indentation force  $F$  (Fig. 8a) can be determined by the deflection of the cantilever, and the indentation depth  $\delta$  can be calculated from the position of the 3D piezoelectric actuator. It is clear (Fig. 8c) that the force-displacement curve at loading reflects both elastic and plastic properties of virus. Also, the failure of virus occurs at indentations between 20 % and 30 % of the virus diameter and between 0.6 nN and 1.2 nN of the applied force [104]. Limitations of this technique are large high-stiffness probe and large minimal force [99].

**Optical tweezers.** Optical trapping (or optical tweezers) can manipulate individual virus particles using low-power near-infrared laser beams. In this way, light scattering imaging (Fig. 9 a, b) is the most often used method based on the measurement of the change in a signal or light scattering by individual virus particles at a given angle with time [105, 106]. The scattered light contains both elastic and inelastic components. Optical tweezers with the three dimensional manipulation and small low-stiffness probe have been developed to exert forces in excess of 0.1 pN [99]. Limitations of such technique are sample heating and photodamage. In one of the first applications of optical tweezers in Virus Mechanics [68], the Young's modulus of the wet STMV crystal was found to be 3.3 GPa. This value is comparable with that of the bacteriophage  $\phi$ 29 (around 1.8 GPa) by Ivanovska et al. [62].



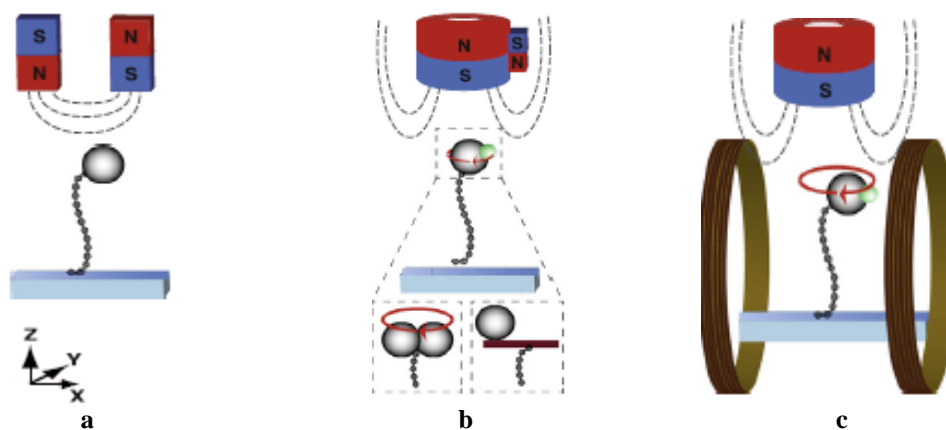
**Fig. 9. Light scattering of single virus particles in a nanofluidic optical fiber: Schematic of the apparatus and an SEM image of the fiber cross section (a), and detected position of a freely diffusing CCMV particle in water with time [105]**

**Magnetic tweezers.** Magnetic manipulation is extremely selective for the magnetic beads used as probes,

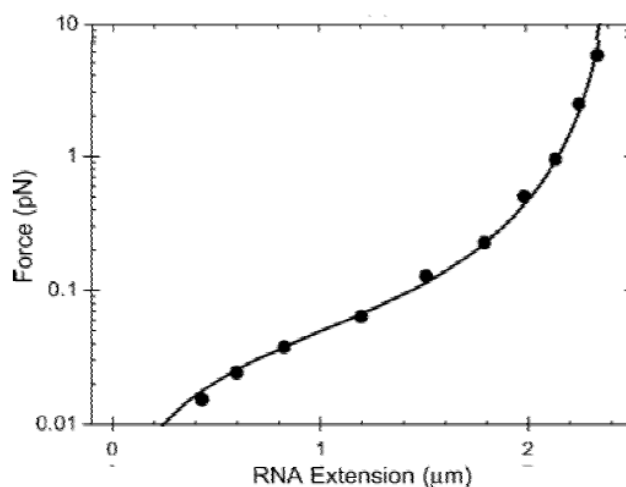
and the single molecules under study are tethered between a flow cell surface and magnetic beads. Magnetic tweezers (MT, Fig. 10a, b) and electromagnetic tweezers (MT designed with electromagnets, Fig. 10c) are the most straightforward of the three single molecule manipulation techniques to implement [99, 107]. Furthermore, novel kinds of this technique (Fig. 10 b, c) allow us to directly observe torque and twist of nucleoprotein complexes and viral genomes. Magnetic (electromagnetic) tweezers have been developed to exert forces in excess of  $10^{-3}$  (0.01) pN [99]. Also, the spatial and temporal resolutions of such technique are higher than optical tweezers and AFM.

Conventional MT (Fig. 10a) consist of two cubic permanent magnets that produce a horizontal magnetic field and induce the stretching of molecules in X, Y and Z directions [107]. As an example, Fig. 11 shows the experimental data for force-extension in Z direction of double-stranded RNA molecule [108].

The use of cylindrical magnets and additional side magnet in MT (Fig. 10b) gives the possibility to apply the magnetic torque. The advantage of electromagnets (Fig. 10c) is that force and rotation can be controlled by changing the current rather than moving the magnets. However, unlike optical tweezers, electromagnetic tweezers do not create a stable three-dimensional trapping potential [107]. Limitation of electromagnetic tweezers is hysteresis in the magnetic field as a function of current.



**Fig. 10. Schematics of magnetic and electromagnetic tweezers: Conventional MT (a), magnetic torque tweezers with cylindrical magnets and additional side magnet (b), electromagnetic torque tweezers with Helmholtz coils around cylindrical magnets (c) [107]**



**Fig. 11. Force-extension measurements using conventional MT for dsRNA [108]**

**Mechanical properties of viruses.**

Experimental data obtained by means of AFM, optical tweezers and magnetic tweezers can be used for nanomechanical characterization of the deformation response of viruses affected by different environmental factors (mechanical loads, chemical changes, thermal changes, electromagnetic field and receptor binding on the host membrane).

In particular, a reversible linear elastic regime was observed for AFM nanoindentation of virus capsids when the indentation did not exceed 20–30 % of the capsid thickness [109]. As known [102], in the case of spherical capsids and spherical probes, the relation between the indentation force  $F$  and the indentation depth  $\delta$  can be described mathematically by different contact models of continuum mechanics given in Table 1. Here,  $E$  is the Young’s modulus of a virus;  $\nu$  is the Poisson’s ratio of a virus;  $R$  is the probe radius;  $a$  is the contact area radius;  $h$  is the thickness of viral capsid;  $F_{AD}$  is the adhesion force;  $\chi_S$  is the correction parameter,  $\chi_S = \sqrt{R\delta}/h$ . Most of the studies of linear elastic deformation during AFM

nanoindentation refer to the application of the Hertz analysis [110]. However, the Hertz formulation of the contact problem (Table 1) leads to incompatibility of displacement fields due to the, first, geometrically linear formulation and, second, due to the ignoring the appearance of the tangential displacements [111, 112]. In other words, the Hertz analysis was done for frictionless and nonslipping boundary conditions under assumption that  $\delta \ll R$ . In contrary, the Sneddon model (Table 1) was developed for large indentation [113]. The Derjaguin–Müller–Toporov (DMT) [114] and Johnson–Kendall–Roberts (JKR) [115] formulations in Table 1 can be used in cases of small and large adhesion, respectively. The finite thickness correction (FTC) was used by Dimitriadis et al. [116] to introduce an approximation dependent on probe radius, thickness and indentation, as polynomial expansion of the correction parameter  $\chi_S$ , considering sample bound to substrate (FTC bound in Table 1) or free to move (FTC free in Table 1).

Table 1

**Interpretation of AFM nanoindentation measurements for spherical tip and spherical virus [102]**

| Model     | Equation(s)   |
|-----------|---|
| Hertz     | $F = \frac{4}{3} \frac{E\sqrt{R}}{(1-\nu^2)} \delta^{3/2}$  |
| Sneddon   | $F = \frac{E}{2(1-\nu^2)} [(a^2 + R^2) \ln(\frac{R+a}{R-a}) - 2aR]$ $\delta = \frac{1}{2} a \ln(\frac{R+a}{R-a})$                         |
| DMT       | $F = \frac{4}{3} \frac{E\sqrt{R}}{(1-\nu^2)} \delta^{3/2} + F_{AD}$   |
| JKR       | $F = \frac{4Ea^3}{3R(1-\nu^2)} - \sqrt{\frac{16EF_{AD}}{3(1-\nu^2)}} a^3$ $\delta = \frac{a^2}{R} - \sqrt{\frac{4F_{AD}(1-\nu^2)}{ER}} a$ |
| FTC bound | $F = \frac{4}{3} \frac{E\sqrt{R}}{(1-\nu^2)} \delta^{3/2} [1 + 1.133\chi_S^2 + 1.283\chi_S^2 + 0.769\chi_S^3 + 0.0975\chi_S^4]$           |
| FTC free  | $F = \frac{4}{3} \frac{E\sqrt{R}}{(1-\nu^2)} \delta^{3/2} [1 + 0.884\chi_S + 0.781\chi_S^2 + 0.386\chi_S^3 + 0.0048\chi_S^4]$             |

Thus, the Young’s modulus of a virus can be derived from the slope of the force-indentation curves taking into account that the indenter size, the Poisson’s ratio of a virus and the thickness of the viral capsid are known. Note also that the Poisson’s ratio of viruses can take a range of values from 0.3 to 0.5 [17, 62, 117–124]. Unfortunately, the Hertz model is affected by harsh limitation to be carefully considered upon application on viruses: indentation must be small compared with probe radius [102], i. e., it should be

$\delta \ll R$ . To get around this limitation, the solution of the theory of thin spherical shells by Reissner [125] was used [126], such as

$$F = \frac{4Eh^2}{\sqrt{3r}\sqrt{1-\nu^2}} \delta, \tag{1}$$

where  $r$  is the radius of the middle surface of viral capsid. Also, in the case of conical AFM tip and spherical virus the Hertz formulation [110] can be presented as follows [127]

$$F = \frac{2}{\pi} \frac{E \tan \alpha}{1-\nu^2} \delta^2, \tag{2}$$

where  $\alpha$  is the semivertical angle of a cone.

Table 2 summarizes the available information about the Young's moduli of viruses. It is an extension of the experimental data given in reviews [16, 109, 128].

Capturing the elastic effects of the presence of nucleic acids polymers (DNA and RNA) in viruses is a way to gain insight into the physicochemical aspects of virus infectivity [129]. Note that deformation by compressing, stretching and twisting is vital for genome packaging. Compression elements of genome resist pushing, while tension elements resist pulling.

Table 3 provides all the necessary information about the Young's moduli of genomes [130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140]. The freely jointed chain model and the worm-like chain model developed in the theory of entropic elasticity of genomes could not be applied to reproduce such data using different methods of measurement [141]. Therefore, the concepts of solid mechanics [27, 60, 61] were used with continuum description of genomes. In this way, the single molecule of DNA is considered as a straight rod of the circular cross section (initial radius  $r_0$  and initial length  $l_0$ ) under axial loading. Additionally,

the average dimensions of DNA are accepted, such as,  $r_0 = 1$  nm, as well as,  $l_0 = 50$  nm (long molecules [130, 131, 132, 133, 134, 135, 136, 139, 140]), and  $l_0 = 10$  nm (short molecules [137, 138]) in the idealized elastic model [130, 131, 135, 137, 140]:

$$F = \frac{\pi r_0^2 E}{l_0} \delta \quad (3)$$

Also, the Hertz-type contact models are used for the AFM nanoindentation [134, 136, 138]. It was found [131, 137] that ssDNA is more flexible and can reach a larger extension per base pair than dsDNA. Therefore, the Young's modulus of the dsDNA is greater than that of the ssDNA and the single-fixed dsDNA in which the end of one strand was left free [137]. It is seen (Table 3) that the values of Young's modulus obtained from the measurements on small fragments of DNA are significantly smaller than that determined from stretching experiments on long DNA. Additionally, experiments showed [107, 140] that the Young's modulus of dsRNA is approximately two times lower compared to that of dsDNA. The Poisson's ratio has been accepted for DNA and RNA to be in the range of 0.33–0.4 [141].

Table 2

Geometrical and elastic properties of viruses

| Virus     | Inner radius (nm) | Thickness (nm) | Geometry | Genome | Young's modulus (GPa) | Year | Reference | Comments    |
|-----------|-------------------|----------------|----------|--------|-----------------------|------|-----------|-------------|
| TMV       | 2                 | 7              | Cylinder | ssRNA  | 1.1                   | 1997 | [41]      | Axial       |
| TMV       | 2                 | 7              | Cylinder | ssRNA  | 6.8                   | 2007 | [77]      | Axial       |
| TMV       | 2                 | 7              | Cylinder | ssRNA  | 1.0                   | 2008 | [78]      | Radial      |
| TMV       | 2                 | 7              | Cylinder | ssRNA  | 1.3                   | 2020 | [79]      | Radial      |
| $\phi 29$ | 23.2              | 1.6            | Sphere   | dsDNA  | 1.8                   | 2004 | [62]      | Prohead     |
| $\phi 29$ | 23.2              | 1.6            | Sphere   | dsDNA  | 4.5                   | 2007 | [17]      | Prohead     |
| CCMV      | 10.5              | 3.8            | Sphere   | ssRNA  | 0.14                  | 2006 | [63]      | Empty WT    |
| CCMV      | 10.5              | 3.8            | Sphere   | ssRNA  | 0.19                  | 2006 | [63]      | Empty SubE  |
| CCMV      | 10.4              | 2.8            | Sphere   | ssRNA  | 0.28                  | 2007 | [17]      | Empty WT    |
| CCMV      | 10.4              | 2.8            | Sphere   | ssRNA  | 0.36                  | 2007 | [17]      | Empty SubE  |
| CCMV      | 10.5              | 3.5            | Sphere   | ssRNA  | 0.22                  | 2008 | [72]      | Native      |
| STMV      | 8.5               | 3              | Sphere   | ssRNA  | 3.3                   | 2007 | [68]      | Wet crystal |
| STMV      | 8.5               | 3              | Sphere   | ssRNA  | 10.0                  | 2007 | [68]      | Dry crystal |
| MLV       | 46                | 4              | Sphere   | ssRNA  | 1.0                   | 2006 | [66]      | Mature      |
| MLV       | 30                | 20             | Sphere   | ssRNA  | 0.23                  | 2006 | [66]      | Immature    |
| HIV-1     | 45                | 5              | Sphere   | ssRNA  | 0.44                  | 2007 | [120]     | Mature      |
| HIV-1     | 25                | 25             | Sphere   | ssRNA  | 0.93                  | 2007 | [120]     | Immature    |
| $\lambda$ | 29.5              | 1.8            | Sphere   | dsDNA  | 1.0                   | 2007 | [64]      |             |
| HSV-1     | 49.5              | 4              | Sphere   | dsDNA  | 1.0                   | 2009 | [117]     |             |

Continued of table 2

|            |       |     |        |       |              |      |       |                            |
|------------|-------|-----|--------|-------|--------------|------|-------|----------------------------|
| MVM        | 11.5  | 2   | Sphere | ssDNA | 1.25         | 2006 | [67]  | pH 4.0                     |
| HBV        | 11.9  | 2.4 | Sphere | dsDNA | 0.37         | 2008 | [70]  |                            |
| HBV        | 12    | 2.5 | Sphere | dsDNA | 0.26         | 2010 | [81]  |                            |
| WIV        | 66    | 4   | Sphere | dsDNA | 7.0          | 2008 | [69]  |                            |
| HK97       | 28.2  | 1.8 | Sphere | dsDNA | 1.0          | 2012 | [65]  |                            |
| NV         | 14.5  | 9   | Sphere | ssRNA | 0.03         | 2010 | [118] |                            |
| NV         | 10.85 | 2.7 | Sphere | ssRNA | 0.2          | 2011 | [119] |                            |
| Influenza  | 47.5  | 5   | Sphere | ssRNA | 0.045        | 2011 | [75]  |                            |
| Influenza  | 46.9  | 3.1 | Sphere | ssRNA | 0.03         | 2012 | [121] |                            |
| P2         | 26    | 4   | Sphere | dsDNA | 1.17         | 2021 | [122] |                            |
| P2         | 26    | 4   | Sphere | dsDNA | 0.87         | 2021 | [122] | Fully filled<br>2/3 filled |
| SARS-CoV-2 | 42.5  | 5   | Sphere | ssRNA | 0.2          | 2020 | [123] | Without spike              |
| SARS-CoV-2 | 48    | 4   | Sphere | ssRNA | 0.03 or 0.06 | 2021 | [124] | Without spike              |

Table 3

Elasticity of nucleic acid polymers

| Genome | Young's modulus (MPa) | Test                | Technics | Year | Reference | Comments               |
|--------|-----------------------|---------------------|----------|------|-----------|------------------------|
| dsDNA  | 346                   | Stretching          | Optical  | 1996 | [130]     | $\lambda$ DNA          |
| dsDNA  | 300                   | Stretching          | Optical  | 2002 | [131]     | $\lambda$ DNA          |
| ssDNA  | 240                   | Stretching          | Optical  | 2002 | [131]     | $\lambda$ DNA          |
| dsDNA  | 207                   | Nanoindentation     | AFM      | 2004 | [132]     | $\lambda$ DNA          |
| dsDNA  | 393                   | Twisting-stretching | Magnetic | 2006 | [133]     |                        |
| dsDNA  | 260                   | Nanoindentation     | AFM      | 2007 | [134]     | $\lambda$ DNA          |
| dsDNA  | 300                   | Stretching          | Optical  | 2009 | [135]     | $\lambda$ DNA          |
| dsDNA  | 138                   | Nanoindentation     | AFM      | 2019 | [136]     |                        |
| ssDNA  | 18                    | Nanoindentation     | AFM      | 2010 | [137]     | Small DNA              |
| dsDNA  | 45                    | Nanoindentation     | AFM      | 2010 | [137]     | Small single-fixed DNA |
| dsDNA  | 55                    | Nanoindentation     | AFM      | 2010 | [137]     | Small double-fixed DNA |
| ssDNA  | 75                    | Nanoindentation     | AFM      | 2014 | [138]     | DNA origami            |
| dsRNA  | 200                   | Stretching          | Optical  | 2011 | [139]     |                        |
| dsRNA  | 159                   | Stretching          | Optical  | 2013 | [140]     | 150 mM NaCl            |
| dsDNA  | 298                   | Stretching          | Optical  | 2013 | [140]     | 150 mM NaCl            |
| dsRNA  | 201                   | Stretching          | Optical  | 2013 | [140]     | 300 mM NaCl            |
| dsDNA  | 371                   | Stretching          | Optical  | 2013 | [140]     | 300 mM NaCl            |
| dsRNA  | 217                   | Stretching          | Optical  | 2013 | [140]     | 500 mM NaCl            |
| dsDNA  | 383                   | Stretching          | Optical  | 2013 | [140]     | 500 mM NaCl            |

The structural and atomistic defects may occur in the virus as the indicators of damage influenced by environment. Development of damage causes the reduction of virus stiffness. In this regard, the Young's modulus of virus at the current instant of time can be presented as

$$E = E_0(1 - \omega) \quad (4)$$

Here  $\omega$  is the Kachanov-Rabotnov damage parameter [27, 60, 61, 142] which is increasing with time from the initial value

$\omega = \omega_0$  at the reference instant to the final value  $\omega = \omega_*$  at the instant of rupture (or destructuration) of viruses,  $E_0$  is the Young's modulus of virus at the reference instant of time. In the simplest case it is possible to accept that  $\omega_0 = 0$  and  $\omega_* = 1$ . Also, healing of damage affected by the environmental factors may occur during the virus life. Therefore, in general, the Young's modulus of virus at the current instant of time can be written as follows

$$E = E_0[1 - \omega(1 - h)] \quad (5)$$

Here  $h$  is the healing parameter [143, 144, 145, 146] which is increasing with time from the initial value  $h = h_0$  at the reference instant to the final value  $h = h_*$ . In the simplest case we can accept that  $h_0 = 0$  and  $h_* = 1$ .

The Young's modulus defines the resistance of virus to elastic deformation. However, it is not enough to reproduce the nonlinear mechanical behavior of viruses. So, Fig. 12 shows the force-indentation curves of empty prohead of the  $\phi 29$  virus adsorbed on a modified glass surface [74]. The AFM nanoindentation tests were carried out with the initial prestress on upright and laid down proheads of  $\phi 29$ . Figure 13 demonstrates the nanoindentation studies under loading and unloading for empty capsid of CCMV with SubE protein mutant on a glass substrate [63]. The results of the AFM nanoindentation measurements [63] at increasing loading rates in the range of 6–6000 nm/s are given in Fig. 14 for empty CCMV (a) and HK97 (b) capsids.

Now, a number of comments need to be made in reference to Figs. 12–14. First, the nanoindentation results with the initial prestress given for one and the same location indicate that the  $\phi 29$  virus is stiffer in compression than in tension. In other words,  $\phi 29$  is excellent in resisting compression. One source of tension elements in virus is related to the natural ability of the protein-genome complexes to generate an opposing tension force that will return to its upstretched state [147]. The next source of tension elements is the chromatin-genome association that is highly extensible in the attempt to return to a state of maximal entropy [147]. Thus, viruses belong to a broad class of

natural and artificial materials with different behavior under tension, and compression [1, 2, 3, 27, 61, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160]. Second, the  $\phi 29$  shells are approximately two times stiffer along the short axis (particle lying on its side) than along the long one (upright particle). Obviously that stiffness of pentamers is different from that of hexamers. Also, the force-indentation curves exhibit large differences when the MVM capsids are compressed along the two-, three-, and fivefold symmetry axes [67, 161]. Thus, it is necessary to take into account the initial spherical anisotropy, and, in general, spherical virus may be a triclinic, monoclinic, orthotropic, tetragonal, transversely isotropic or cubic material [162]. Third, the experimental data under unloading show clearly that the total strain for viruses includes the permanent plastic part. Fourth, the force-indentation curves at the increasing loading rates give the possibility to reproduce the creep behavior of viruses. Hence, creep has been considered as a time dependent irreversible deformation process. Finally, fracture of viruses observed in experiments is related to the bond breaking. There are two types of fracture in viruses [128]. Brittle fracture observed in TMV, MVM and HSV-1 [128] occurs at the failure strain of about 0.05 [163]. Ductile fracture of SARS-CoV-2, CCMV and HBV [124, 128] corresponds to the failure strain of about 1.7 [124]. The failure mode of NV,  $\phi 29$ ,  $\lambda$  and HK97 [128] undergoes a transition from brittle fracture to ductile flow. Thus, deformation of viruses influenced by environment has been accompanied by small or large strains.

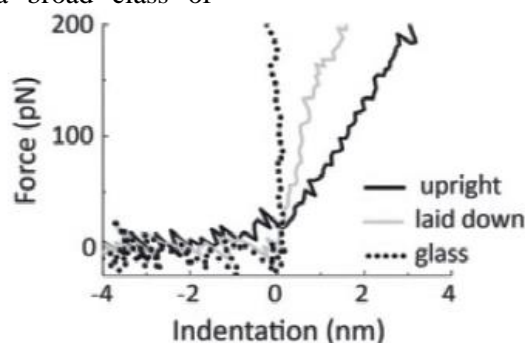


Fig. 12. The AFM nanoindentation experiments with the initial prestress for bacteriophage  $\phi 29$  on different locations: glass substrate (dotted), upright configuration (grey) and laid down configuration (black) [74]

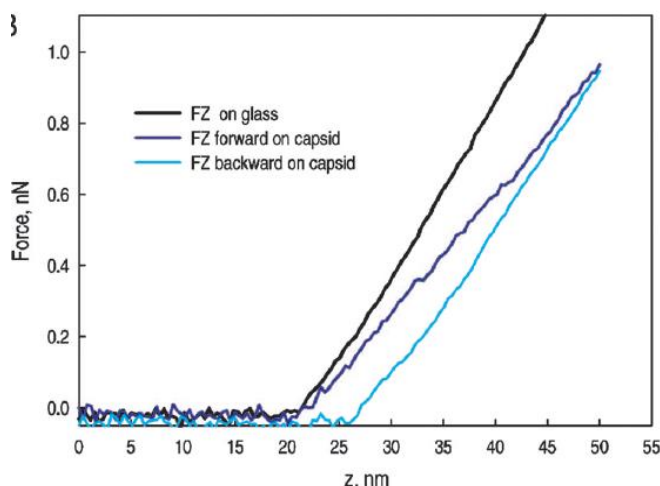


Fig. 13. The indenting force–piezoactuator displacement curves of empty mutant CCMV capsid [63]

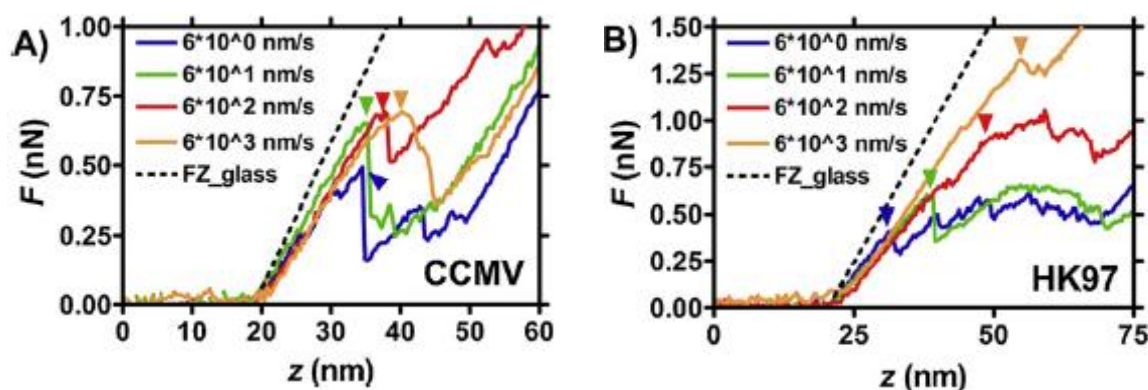


Fig. 14. The indenting force–actuator displacement curves at increasing loading rates: CCMV (a) and HK97 (b) [63]

Also, swelling occurs in viruses when there are changes in the pH of the solution, release of bound ions (such as,  $\text{Ca}^{2+}$ ) or different modifications of charge on the viral shells [164]. Such phenomenon was observed experimentally, and the radial expansion was found to be 8 % (STMV) [165], 10 % (CCMV) [166], 12 % (BMV) [167, 168] and 14 % (tomato bushy stunt virus) [169]. Electrostatic interaction among the negatively charged ssRNA/DNA molecule and the positively charged domains of the capsid proteins can be accepted as the basic physical mechanism of swelling in viruses [170, 171]. Environmentally induced shrinkage can also occur in viruses [172, 173, 174, 175]. Both swelling and shrinkage can be described taking into account the piezoelectric response of viruses [176, 177].

In general, the mechanical properties discussed above give the information which is necessary to find the relationship between the viral structure, environmental effect and biological function of viruses.

#### Constitutive modeling and simulation.

Let us consider virus as the isotropic material with different behavior in tension and compression. Then the initial (reference) configuration with coordinates  $x_i$  ( $i=1,2,3$ ) and the deformed (present) configuration with coordinates  $X_i$  will be used to identify the material point of virus at a given instant of time  $t$  ( $t \neq 0$ ) within the framework of solid mechanics at finite strains. Let  $u_i(x_1, x_2, x_3, t)$  are the components of the displacement vector of the material point at time  $t$  in directions  $x_1, x_2, x_3$ , respectively;  $u_i = X_i - x_i$ ;  $i=1,2,3$ .



The Cauchy-Green strain tensor can be defined as [178]:

$$\varepsilon_{ij} = 0.5(u_{i,j} + u_{j,i} + u_{k,i}u_{k,j}), \quad (6)$$

and it is easy to obtain the material time derivative of this tensor, such as

$$\dot{\varepsilon}_{ij} = 0.5 \left( \dot{u}_{i,j} + \dot{u}_{j,i} + \dot{u}_{k,i}u_{k,j} + u_{k,i}\dot{u}_{k,j} + u_{k,j}\dot{u}_{k,i} \right), \quad (i, j, k = 1, 2, 3) \quad (7)$$

Here the dot above the symbol denotes a material time derivative. The rate of strain tensor in the present configuration of virus can be written as follows [178]

$$d_{ij} = 0.5 \left( \frac{\partial \dot{u}_i}{\partial X_j} + \frac{\partial \dot{u}_j}{\partial X_i} \right) \quad (8)$$

Tensors  $\dot{\varepsilon}_{ij}$  and  $d_{ij}$  are connected by the relations [178]:

$$\dot{\varepsilon}_{ij} = \frac{\partial X_k}{\partial x_i} \frac{\partial X_l}{\partial x_j} d_{kl} \quad (9)$$

and

$$d_{ij} = \frac{\partial x_k}{\partial X_i} \frac{\partial x_l}{\partial X_j} \dot{\varepsilon}_{kl} \quad (10)$$

Let us assume the additive decomposition of the time derivative of the total Cauchy-Green strain tensor, as well as, of the rate of strain tensor in the present configuration of virus to a linear elastic part, nonlinear elastic part, plastic part, creep part and piezoelectric part, respectively, in the following form:

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^e + \dot{\varepsilon}_{ij}^{ne} + \dot{\varepsilon}_{ij}^p + \dot{\varepsilon}_{ij}^c + \dot{\varepsilon}_{ij}^{pe} \quad (11)$$

and

$$d_{ij} = d_{ij}^e + d_{ij}^{ne} + d_{ij}^p + d_{ij}^c + d_{ij}^{pe} \quad (12)$$

A connection between the second Piola-Kirchhoff stress tensor  $\tau_{ij}$  and the Cauchy stress tensor  $\sigma_{ij}$  in virus can be presented as [178]

$$\tau_{ij} = J \sigma_{kl} \frac{\partial x_i}{\partial X_k} \frac{\partial x_j}{\partial X_l} \quad (13)$$

and

$$\sigma_{ij} = J^{-1} \tau_{kl} \frac{\partial X_i}{\partial x_k} \frac{\partial X_j}{\partial x_l} \quad (14)$$

Here  $J$  is the Jacobian of deformation,  $J = \det \mathbf{F}$ ;  $\mathbf{F}$  is the matrix of the deformation gradients in the present configuration at time

$$t, \quad \mathbf{F} = \left[ \frac{\partial X_i}{\partial x_j} \right] = [\delta_{ij} + u_{i,j}], \quad (i, j = 1, 2, 3); \quad \delta_{ij} \text{ is the Kronecker delta.}$$

In order to describe the mechanical behavior of virus, it is necessary to formulate the constitutive equations in the rate form. Unfortunately, the material time derivative of the Cauchy stress tensor, i.e.  $\dot{\sigma}_{kl}$ , is not objective stress rate, because it is affected by rigid-body motions. Therefore, the Truesdell derivative  $\sigma_{kl}^{Tr}$  of the Cauchy stress tensor  $\sigma_{kl}$  will be used below, because it is an objective measure. As known [178], the material derivative of the second Piola-Kirchhoff stress tensor and the Truesdell derivative of the Cauchy stress tensor are connected by the relation

$$\dot{\tau}_{ij} = J \sigma_{kl}^{Tr} \frac{\partial x_i}{\partial X_k} \frac{\partial x_j}{\partial X_l} \quad (15)$$

with similar structure to (13). Then the relation between  $\sigma_{kl}^{Tr}$  and  $d_{ij}^e$  can be accepted as follows

$$\sigma_{kl}^{Tr} = A_{klrs}^* (d_{rs} - d_{rs}^{ne} - d_{rs}^p - d_{rs}^c - d_{rs}^{pe}). \quad (16)$$

Here  $A_{klrs}^*$  is the fourth-rank tensor of the elastic constants. By substituting (16) into (15) and using (10) we have

$$\dot{\tau}_{ij} = A_{ijkl} \left( \dot{\varepsilon}_{kl} - \dot{\varepsilon}_{kl}^{ne} - \dot{\varepsilon}_{kl}^p - \dot{\varepsilon}_{kl}^c - \dot{\varepsilon}_{kl}^{pe} \right), \quad (17)$$

where

$$A_{ijkl} = J A_{mnr s}^* \frac{\partial x_i}{\partial X_m} \frac{\partial x_j}{\partial X_n} \frac{\partial x_k}{\partial X_r} \frac{\partial x_l}{\partial X_s} \quad (m, n, r, s = 1, 2, 3) \quad (18)$$

It is easy to see that the condition of symmetry  $A_{ijkl} = A_{klij}$  takes place. Alternative based on the Jaumann stress derivative [178]

$$\sigma_{ij}^J = \sigma_{ij} - \Omega_{ir} \sigma_{rj} + \sigma_{ir} \Omega_{rj} \text{ with the spin tensor}$$

$$\Omega_{ij} = 0.5 \left( \frac{\partial \dot{u}_i}{\partial X_j} - \frac{\partial \dot{u}_j}{\partial X_i} \right)$$

instead of the Truesdell stress derivative in Eq. (16) does not provide the satisfaction of the symmetry condition  $A_{ijkl} = A_{klij}$ .

A constitutive relation between the kinematic tensor  $e_{kl}$  in the present configuration and the Kirchhoff stress tensor  $T_{kl}$  can be written in the following form [148, 149, 179]:

$$e_{ij} = e_0 \left( \frac{a I_1 \delta_{ij} + c T_{ij}}{T_2} + b \delta_{ij} \right). \quad (19)$$

Here  $T_{kl} = J \sigma_{kl}$ ;  $e_0 T_e = T_{ij} e_{ij}$ ;  $T_e = T_1 + T_2$ ;  $T_1 = b I_1$ ;  $T_2^2 = a I_1^2 + c I_2$ ;  $I_1 = T_{ii} \delta_{ii}$ ;  $I_2 = T_{ij} T_{ij}$ ;  $T_e$  is the Kirchhoff equivalent stress;  $e_0$  is the scalar function which depends on  $T_e$ , as well as, on the Kachanov-Rabotnov damage

parameter  $\omega$  and the healing parameter  $h$ , and which identifies for each physical state of virus (nonlinear elasticity, plasticity, creep);  $T_1$  and  $T_2^2$  are the linear and quadratic invariants of the Kirchhoff stress;  $a$ ,  $b$  and  $c$  are the material parameters.

Considering the particular case of the nonlinear elasticity of virus, the kinematic tensor  $e_{kl} \equiv d_{kl}^{ne}$  can be introduced as the nonlinear elastic part of the rate of strain tensor in the present configuration, and, additionally, it is necessary to define  $e_0$  as the function of  $T_e$ ,  $\omega$  and  $h$ , as well as, to specify the kinetic equations for  $\omega$  and  $h$ . In the simplest case of the nonlinear deformation with damage growth, but, without healing of damage, it is possible to accept [154, 180] in Eq. (19) that

$$e_0 = \frac{T_e^n \omega^k}{(1-\omega)^m} \quad (20)$$

with some material parameters  $n$ ,  $k$  and  $m$ .

For the plastic deformation of virus, it is necessary to assume that the kinematic tensor  $e_{kl} \equiv d_{kl}^p$  is the plastic part of the rate of strain tensor in the present configuration, as well as, to define [148, 179] the condition of loading and, additionally, the conditions when  $d_{kl}^p = 0$  in the cases of elastic deformation, unloading or neutral loading.

In the case of the creep deformation of virus, the kinematic tensor  $e_{kl} \equiv d_{kl}^c$  in Eq. (19) is the creep part of the rate of strain tensor in the present configuration, and the scalar function  $e_0$  in Eq. (19) should be defined [149, 181] using  $T_e$ ,  $\omega$  and  $h$ .

The piezoelectric part of the rate of strain tensor in the present configuration can be written in the following form [182]

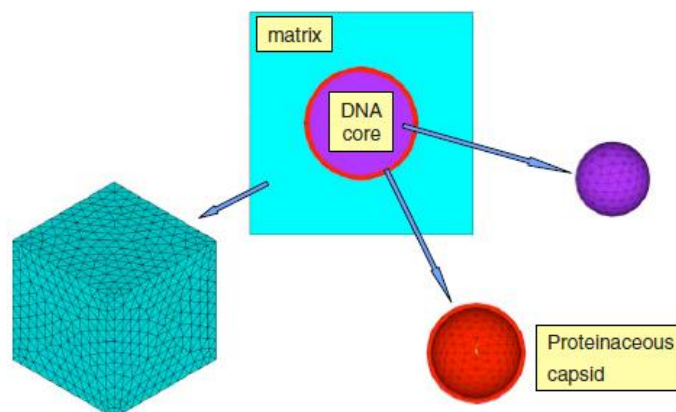
$$d_{ij}^{pe} = -S_{kij} \frac{\dot{\partial \phi}}{\partial X_k}, \quad (21)$$

where  $S_{kij}$  is the third-rank tensor of the piezoelectric constants, and  $\phi$  is the electric potential in virus.

Now, a number of comments need to be made in reference to the constitutive model given by Eqs. 11, 12, 17, 19–21. First, the material parameters of the model can be found by means of the identification methods [183, 184] using the experimental data discussed above under tension, compression and shear. Second, the present model can be extended to the case of the anisotropic plasticity/creep hardening under repeated

loading according to the approaches [185, 186, 187]. Third, the fatigue of viruses can be described through the evolution equations [188, 189, 190] of the fatigue damage. Finally, in the case of the initial anisotropy of viruses it is necessary to use the tensor relationship given in [191, 192, 193, 194] instead of Eq. (19). As marked recently [195], there is nothing more fundamental in nonlinear mechanics of anisotropic materials with tension-compression asymmetry than the constitutive framework [191–194] developed in the early 1980s. Furthermore, such constitutive framework was used in [196–207] for modeling of the natural and artificial materials, however, without referencing sources [191–194].

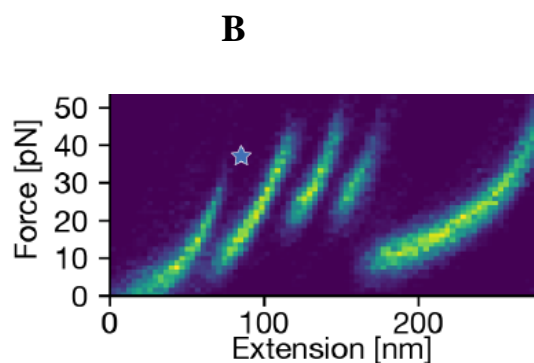
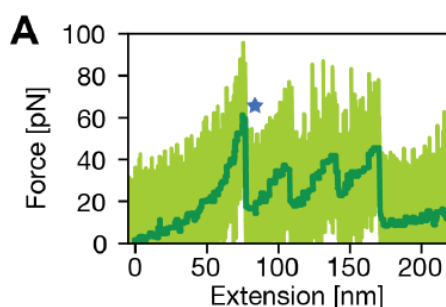
Analysis of stress distributions over time in viruses, damage/healing analysis and lifetime prediction studies of viruses are related to the consideration of the physically and geometrically nonlinear initial/ boundary value multiphysics problem [208]. A three-dimensional finite element model (Fig. 15) derived from the reconstruction of virus from CryoEM photos may help to effectively simulate the influences of environmental factors on the mechanical behavior of viruses. Then the constitutive framework given by Eqs. 11, 12, 17, 19–21 needs to be incorporated into ANSYS [90, 92, 96, 98], ABAQUS [208, 209] and in-house developed software [19, 21, 22, 28, 29, 210] in a form of the computer-based structural modeling tool. These software packages give the possibility to calculate the time-dependent multiaxial stress distribution, as well as, changes of damage parameter and healing parameter at a discrete site of virus under applied loading conditions (or piezoelectric impairments) as a function of virus parameters, environmental factors and virus infectivity, and additionally to predict the lifetime of virus. The identification of material parameters in a constitutive framework from the force-displacement data is related to a number of the methodological and numerical difficulties [211], and can be made using an optimization technique [183, 184] and finite element simulations. Note that the finite element studies show [17] that the highest stresses in the viral capsid can reach 760 MPa, although the tensile strength of typical proteins is 133 MPa [42]. Thus, the further research in this direction is necessary.



**Fig. 15. A representative volume element of the host cell (matrix) embedded with the spherical virus with genome (DNA or RNA) [69]**

In particular, the finite element analysis of the AFM nanoindentation data [212] for SARS-CoV-2 (Fig. 16 a) and SARS-CoV-1 (Fig. 16b) is critically important. It is seen (Fig. 16a, b) that the strength of SARS-CoV-2 is higher in comparison to SARS-CoV-1.

Lifetime prediction studies of viruses might help to give a clear understanding of how neutralizing antibodies and T cells interact with the 2019 novel coronavirus SARS-CoV-2.



**Fig. 16. The indenting force–piezoactuator displacement curves: SARS-CoV-2 (a) and SARS-CoV-1 (b) [212]**

## CONCLUSIONS

Knowledge of the mechanical response of viruses influenced by different environmental factors (mechanical loads, chemical changes, thermal changes, electromagnetic field and receptor binding on the host membrane) may assist medical specialists related to

development of novel treatment strategies or development of vaccines for COVID 19.

An interdisciplinary structural modeling tool based on the nonlinear mechanics of viruses provides new insight into the lifetime of viruses and gives the possibility to develop the antiviral strategy.

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**КВАНТОВЕ, МОЛЕКУЛЯРНЕ І КОНТИНУАЛЬНЕ МОДЕЛЮВАННЯ  
В НЕЛІНІЙНІЙ МЕХАНІЦІ ВІРУСІВ**

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A – концепція та дизайн дослідження; B – збір даних; C – аналіз та інтерпретація даних; D – написання статті;  
E – редагування статті; F – остаточне затвердження статті

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**Вступ.** Віруси – це велика група патогенів, які, як було встановлено, заражають тварин, рослини, бактерії та навіть інші віруси. Новий коронавірус 2019 року SARS-CoV-2 залишається постійною загрозою для населення. Віруси – біологічні об'єкти з нанометричними розмірами, зазвичай від кількох десятків до кількох сотень нанометрів. Вони розглядаються як біомолекулярні структури, що складаються з генетичного матеріалу (РНК або ДНК), білкової оболонки (капсида) із захисною функцією, а іноді і додаткової оболонки поверх капсида

**Мета.** Мета даного огляду – допомогти спрогнозувати реакцію і навіть деструкцію вірусів з урахуванням впливу різних факторів навколишнього середовища, таких як механічні навантаження, температурні зміни, електромагнітне поле, хімічні зміни та зв'язування з рецептором мембрани клітини-господаря. Ці фактори навколишнього середовища значно впливають на вірус.

**Матеріали та методи.** Дослідження вірусів і вірусоподібних структур проаналізовано з використанням моделей і методів нелінійної механіки. У зв'язку з цим розглянуті квантові, молекулярні і континуальні описи в механіці вірусів. Застосування методів маніпулювання окремими молекулами, таких як атомно-силової мікроскопія, оптичний пінцет і магнітний пінцет, обговорюється для визначення механічних властивостей вірусів. Особливу увагу приділено континуальній механіці пошкодження–загоєння пошкодження вірусів, білків і вірусоподібних структур. Також представлено конститутивне моделювання вірусів при великих деформаціях. Враховувалися нелінійна пружність, пластична деформація, повзучість, викликане навколишнім середовищем набухання (або усадка) і п'єзоелектричний відгук вірусів. Обговорюється інтеграція конститутивної моделі в ABAQUS, ANSYS і програмне забезпечення власної розробки

**Результати і висновки.** Зв'язок між структурою вірусу, навколишнім середовищем, інфекційністю і механікою вірусу може бути корисний для прогнозування відгуку і деструктуризації вірусів з урахуванням впливу різних чинників навколишнього середовища. Обчислювальний аналіз з використанням такого зв'язку може бути корисний для чіткого розуміння того, як нейтралізуючі антитіла і Т-клітини взаємодіють з новим коронавірусом 2019 року SARS-CoV-2.

**КЛЮЧОВІ СЛОВА:** вірус, механіка, атомно-силової мікроскопія, напруга, деформація, пошкодження, загоєння, моделювання, симуляція

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