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Regularization of the electrostatics problem for three spheres and an electrostatic charge

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A numerical-analytical algorithm for investigation of the electrostatic potential of a sphere with a circular hole and a charge surrounded by ribbon spheres is constructed. The method of inversion of the integral operator and semi-inversion of the matrix operator of the Dirichlet problem for the Laplace equation is used. A system of the second kind with a compact operator in space ℓ_2 is obtained. Particular variants of the problem are considered.
Keywords: spheres; hole; electrostatics; linear system of the second kind; compact.

Резуненко В. О. **Регуляризація задачі електростатики для трьох сфер та електростатичного заряду.** Побудований чисельно-аналітичний алгоритм дослідження потенціалу сфери з круговим отвором і заряду, оточених стрічковими сферами. Використаний метод обернення інтегрального оператора і напівобернення матричного оператора задачі Діріхле для рівняння Лапласа. Отримано система другого роду з компактним оператором в просторі ℓ_2. Розглянуто окремі варіанти задачі.
Ключові слова: сфери; отвір; електростатика; лінійна система другого роду; компакт.

Резуненко В. А. **Регуляризация задачи электростатики для трёх сфер и электростатического заряда.** Построен численно-аналитический алгоритм исследования потенциала сферы с круговым отверстием, окружённой ленточными сферами. Электростатический заряд размещён между сферами. Использован метод обращения интегрального оператора и полуобращения матричного операторов задачи Дирихе для уравнения Лапласа. Получена система второго рода с компактным оператором в ℓ_2. Рассмотрены частные варианты задачи.
Ключевые слова: сферы; отверстие; электростатика; линейная система второго рода; компакт.

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1. Introduction

Among the problems of electrostatics on classical surfaces, the interest in problems on a sphere with various geometric and physical properties has not faded for more than a hundred years. So, at present, the parts of the sphere, made

up of conductive tapes, can be a model of microcircuit blocks, connecting nodes and adapters of various wave-guides, electrical appliances, and charge storage devices. New nanomaterials with complex molecular structures can be models of thin conductive spherical ribbons [1, 2, 3, 4, 12]. In this work, an algorithm is constructed for calculating the electrostatic field of a sphere with a circular hole and an electrostatic charge placed between two closed spheres composed of conductive ribbons.

The algorithm is based on the application of a variant of the regularization method for the non-self-adjoint Dirichlet problem for the Laplace equation, proposed and developed in the works of Kharkiv mathematicians [17, 18, 16, 14, 13]. In this work, a system of linear algebraic equations of the second kind with a compact matrix operator in the Hilbert space ℓ_2 of complex sequences is obtained.

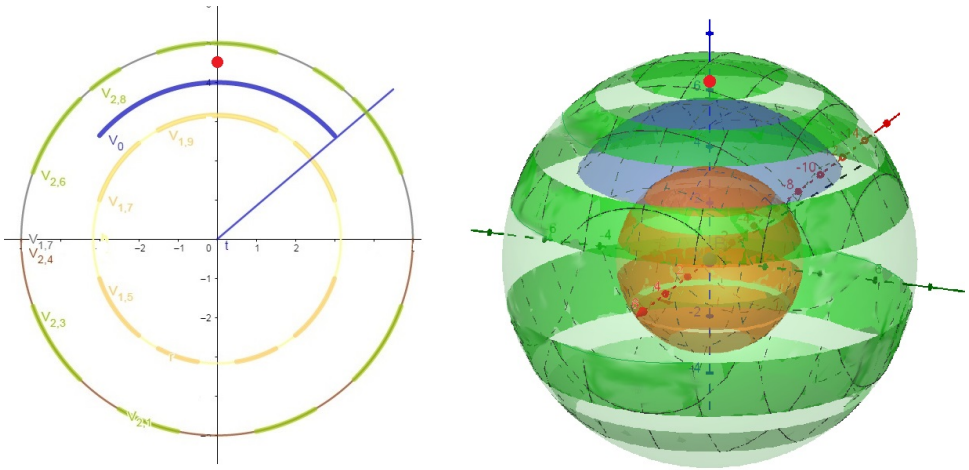
2. Statement of the problem

The task has several parameters. Let's consider on them in detail. Let the origin of the Cartesian and spherical coordinate systems be aligned with the center of the sphere with a circular hole. Let the radius of the sphere $r = r_0$ and the hole in the sphere formed by a plane cut, and the polar cut angle of the sphere is θ_0. At the hole, angle θ varies from θ_0 to π. Let another closed sphere of radius r_1 be located inside the sphere with a hole. The third closed sphere of radius r_2 encloses the sphere with a hole. In this case, respectively, $a_1 < a_0 < a_2$. Closed spheres are made up of infinitely thin ribbons. There are infinitely thin partitions between the ribbons. The partitions lie in planes parallel to the plane XOY. The polar angles of the partitions are $\theta_{1,k}$, $k = 1, 2, ..., N$ for a sphere of radius r_1. The sphere of radius r_2 has other ribbons. The partitions between ribbons have polar angles $\theta_{2,m}$, $m = 1, 2, ..., M$. The number of ribbons is limited. All three spheres are equipped with independent potentials. Let V_0 be the potential of the sphere with a hole, $V_{1,k}$ be the potential of the k -ribbon of the inner sphere, and $V_{2,m}$ be the potential of m-ribbon of the outer sphere. We assume that the electrostatic charge is located between the outer sphere and the sphere with a hole on the OZ axis at point b_0, $a_0 < b_0 < a_2$. Assume the potential of the charge $V_3 \neq 0$. Pic. 1 (left) shows a cross section of three spheres by a vertical plane passing through the OZ axis. In Pic. 1 (right) a three-dimensional model of the spheres is given.

Here we note that this work differs from work [5] by the geometry of the problem and the presence of an electrostatic charge. The total electrostatic field must satisfy Maxwell's equations and material equations $rot\,\vec{E} = 0$, $div\,\vec{D} = \rho_0$, $\vec{D} = \varepsilon\vec{E}$, where ρ_0 is the density of charges on the surface of the conductors, ε is the dielectric constant of the medium.

By the assumption, the magnetic permeability of the medium $\mu = 0$ and the magnetic field is absent $\vec{H} = 0$, the magnetic induction $\vec{B} = 0$.

Let us take into account the connection between the electrostatic field $\vec{E}$ and its scalar potential $\vec{E} = -grad\,U$ and proceed to the scalar problem for the potential U. The total potentials must satisfy the Laplace equation, must be equal to the given potentials at the spherical boundaries are equal V_0, V_1, V_2,



Pic. 1. Spheres and an electrostatic charge.

respectively. Its partial tangential derivatives must be continuous at the hole in the open sphere $r = a_0,\ \theta \in (\theta_0, \pi]$, must satisfy the condition of bounded electrostatic energy in any limited volume of three-dimensional space. except for the volume, containing the given electrostatic charge $\int_W |grad\, U|^2 dW < \infty$, have the required singularity at the point where the charge is placed, decrease at infinity as $O(r^{-1}),\ r \to \infty$. It is required to find the electrostatic field of the three spheres and the charge placed between them. In this statement, the problem is well-posed – it has a stable unique solution [9, 15].

3. Functional equations

Let us represent the 3-dimensional space by the sum of four spherical domains $G_1 = \{r \le a_1\},\ G_2 = \{a_1 < r < a_0\},\ G_3 = \{a_0 < r < a_2\},\ G_4 = \{r > a_2\}$ for which $\theta \in [0, \pi],\ \varphi \in [0, 2\pi]$. In the Laplace equation, we separate the spherical variables and represent the charge potential and the unknown secondary potentials by Fourier - Legendre power series in the corresponding domains.

$$V_3 = \left\{ \begin{matrix} V_{3,1} \\ V_{3,2} \end{matrix} \right\} = \sum_{n=0}^{\infty} \left\{ \begin{matrix} R_n r^n, & r < b_0 \\ H_n r^{-n-1}, & r > b_0 \end{matrix} \right\} P_n(\cos\theta), \tag{1}$$

$$\left. \begin{matrix} U_1 \\ U_2 \end{matrix} \right\} = \sum_{n=0}^{\infty} \left\{ \begin{matrix} A_n r^n, & 0 \le r < a_1 \\ F_n r^{-n-1}, & r > a_2 \end{matrix} \right\} P_n(\cos\theta), \tag{2}$$

$$\left. \begin{matrix} U_3 \\ U_4 \end{matrix} \right\} = \sum_{n=0}^{\infty} \left\{ \begin{matrix} B_n r^{-n-1}, & a_1 < r < a_0 \\ C_n r^n, & a_1 < r < a_0 \end{matrix} \right\} P_n(\cos\theta), \tag{3}$$

$$\left. \begin{matrix} U_5 \\ U_6 \end{matrix} \right\} = \sum_{n=0}^{\infty} \left\{ \begin{matrix} D_n r^n, & a_0 < r < a_2 \\ E_n r^{-n-1}, & a_0 < r < a_2 \end{matrix} \right\} P_n(\cos\theta), \tag{4}$$

where $P_n(\cos\theta)$ are Legendre polynomials of the 1st kind of order zero of integer degree n of the argument $\cos\theta,\ 0 \le \theta \le \pi$.

Note that the method of separation of variables and Fourier-Legendre series for problems of mathematical physics on a sphere were first applied by Debye in 1908–1909. Since then, the method has been effectively used for a wide range of problems, including for problems on sphero-conical and other surfaces [9, 2, 14, 8, 10]. The coefficients of series (1)–(4) are sought in the Hilbert space ℓ_2, ensuring the fulfilment of the condition that the energy integral is finite. Note that the potential V_3 (1) charge is known, and the potentials V_1 and V_2 of the ribbon spheres are known, and the potential V_0 of the sphere with a hole is also known. First, we construct an algorithm for finding the coefficients of series (3)–(4) for potentials U_3,..,U_6. We use the boundary conditions on spherical surfaces for $r = a_0,\ r = a_1,\ r = a_2,\ 0 \le \theta \le \pi$. We use the completeness and orthogonality of the Legendre polynomials $P_n(\cos\theta)$ with weight $\sin\theta$ in $L_2(0,\pi)$ and change from the equalities of the Fourier series to the equalities of their coefficients. As a result, from the boundary conditions, we obtain a system of 3 linear equations with four sequences of unknowns $B_n,\ C_n,\ D_n,\ E_n,\ n = 0, 1, 2...$ (2)–(4)

$$\left\{\begin{array}{lcl} B_n a_1^{-n-1} + C_n a_1^n &=& V_{1,n}^{(1)}, \\ D_n a_2^n + E_n a_2^{-n-1} &=& V_{2,n}^{(2)}, \\ B_n a_0^{-n-1} + C_n a_0^n &=& D_n a_0^n + R_n^{(1)}, \end{array}\right. \tag{5}$$

where the values $V_{2,n}^{(2)},\ V_{1,n}^{(1)},\ R_n^{(1)}$ are known. To find the coefficients, for example, E_n we express from (5) the coefficients $B_n,\ C_n,\ D_n$ through $E_n,\ n \ge 0$. We transform respectively, system (5) and calculate the determinant of the new system. As a result, we get the determinant Δ_n:

$$\Delta_n = -a_2^n \left[\frac{1}{a_1}\left(\frac{a_0}{a_1}\right)^n + \frac{1}{a_0}\left(\frac{a_1}{a_0}\right)^n\right]. \tag{6}$$

The determinant Δ_n is nonzero, since by condition $a_2 > 0,\ 0 < a_1 < a_0$. The system has a unique solution. It is solvable, for example, according to Cramer's rule. So we get, in particular, the coefficients B_n of the potential U_3 (3):

$$B_n = \frac{1}{\Delta_n}\Big\{V_{1,n}^{(1)}(-a_0^n a_2^n) + V_{2,n}^{(2)} a_0^n a_1^n$$

$$-E_n a_1^n a_2^{-n-1} a_0^n + E_n a_2^n a_0^{-n-1} a_1^n + R_n^{(1)} a_1^n a_2^n\Big\}, \tag{7}$$

where $V_{1,n}^{(1)}$ are known, and $V_{2,n}^{(2)} = V_{1,n}^{(1)} - H_n,\ R_n^{(1)} = R_n a_0^n$. Functional equations for finding the coefficients E_n are obtained from the boundary conditions on a sphere with a hole at $r = a_0$

$$U_4 + U_5 = -V_{3,1} + V_0, \qquad 0 \le \theta < \theta_0, \tag{8}$$

$$\frac{\partial[U_4 + U_5]}{\partial r} = \frac{\partial[-V_{3,1} + U_2 + U_3]}{\partial r}, \quad \theta_0 < \theta \le \pi. \tag{9}$$

Now, in (8)–(9) we replace the potentials by series (3), (4). Then we substitute the coefficients $B_n,\ C_n,\ D_n$ in the series, particularly, by (5), (6), (7). As a result, we obtain a paired system of functional equations:

$$\sum_{n=0}^{\infty}\left\{E_n a_0^{-n-1}[1-\mu_n^{(0)}]+V_{2,n}^{(2)}\mu_n^{(0)}+R_n^{(1)}\right\}P_n(\cos\theta)=V_0,\quad 0\le\theta<\theta_0, \quad (10)$$

$$\sum_{k=0}^{\infty}(2k+1)\left\{E_n a_0^{-n-1}[1-\varepsilon_n^{(0)}]-L_n\right\}P_n(\cos\theta)=0,\quad \theta_0<\theta\le\pi, \quad (11)$$

where

$$\mu_n^{(0)}=\left(\frac{a_0}{a_2}\right)^{2n+1},\quad \varepsilon_n^{(0)}=\frac{-\left(\frac{a_1}{a_2}\right)^{2n+1}+\left(\frac{a_1}{a_0}\right)^{2n+1}}{1-\left(\frac{a_1}{a_0}\right)^{2n+1}}, \quad (12)$$

$$L_n=\left(V_{2,n}^{(1)}\varepsilon_n^{(1)}-V_{2,n}^{(1)}\varepsilon_n^{(2)}+H_n\varepsilon_n^{(2)}a_2^{-n-1}-R_n\varepsilon_n^{(2)}\right)a_0^{-1},$$

$$\varepsilon_n^{(1)}=\left\{\frac{a_0^n}{a_1^{n+1}}\left[1-\left(\frac{a_1}{a_0}\right)^{2n+1}\right]\right\}^{-1},\quad \varepsilon_n^{(2)}=\left(\frac{a_1}{a_2}\right)^n\left\{\frac{a_0^n}{a_1^{n+1}}\left[1-\left(\frac{a_1}{a_0}\right)^{2n+1}\right]\right\}^{-1}.$$

In (10)–(12) all quantities are infinitesimal $\mu_n^{(0)},\ \varepsilon_n^{(0)},\ \varepsilon_n^{(1)},\ \varepsilon_n^{(2)},\ n\to\infty$. The series in (10), (11) belong to $L_2(0,\pi)$. System (10), (11) is prepared for transformation into the system of algebraic equations of the second kind (SLAE-II). The transformation is based on the method of regularization of paired summator and integral functional equations, which is close to the method of the Riemann - Hilbert problem.

4. System of linear algebraic equations of the second kind

The system of functional equations (10), (11) can be transformed into SLAE-II in several ways.

Let us choose a variant leading to SLAE-II with the least compact matrix operator in the norm in ℓ_2 [8, 11].

Note that (10), (11) is a system of the first kind. Despite the fact that it can be relatively easily transformed into a system of the 11th kind, it is also not effective for direct application of both analytical and numerical methods. Let us redesignate in (10), (11)

$$E_n=Y_n a_0^{n+1}[1-\varepsilon_n^{(0)}]^{-1},\quad T_n=V_{2,n}^{(2)}\mu_n^{(0)}+R_n^{(1)} \quad (13)$$

and introduce the smallness parameter

$$\varepsilon_n^{(3)}=\frac{-\varepsilon_n^{(0)}+\mu_n^{(0)}}{1-\varepsilon_n^{(0)}},\quad \varepsilon_n^{(3)}=\overline{o}(q^{2n-1}),\quad 0<q<1,\quad n\to\infty. \quad (14)$$

We obtain the system of equations

$$\sum_{n=0}^{\infty}\left\{Y_n\left(1-\varepsilon_n^{(3)}\right)+T_n\right\}P_n(\cos\theta)=V_0,\quad 0\le\theta<\theta_0, \tag{15}$$

$$\sum_{n=0}^{\infty}(2n+1)\left[Y_n-L_n\right]P_n(\cos\theta)=0,\quad \theta_0<\theta\le\pi. \tag{16}$$

which is convenient for further transformations.

Let us use that the series in (15), (16) belong to the space $L_2(0,\pi)$. Substitute the Mehler-Dirichlet integral representation in the series (15) instead of the Legendre polynomials and change the order of summation and integration. We obtain a homogeneous Volterra integral equation of the 1st kind

$$\int_0^{\theta_0} f(\varphi)\left(\cos\varphi-\cos\theta\right)^{-\frac{1}{2}}\,d\varphi=0, \tag{17}$$

which has a unique solution $f(\varphi)=0,\ \varphi\in[0,\theta_0)$. This, instead of (15), we obtain a functional equation for the complete orthogonal system $\cos\left(n+\frac{1}{2}\right)\varphi$, $n=0,1,2,...$ in $L_2(0,\pi)$:

$$\sum_{n=0}^{\infty}\left\{Y_n\left(1-\varepsilon_n^{(3)}\right)+T_n\right\}\cos\left(n+\frac{1}{2}\right)\varphi=V_0\cos\frac{\varphi}{2},\quad 0\le\theta<\theta_0. \tag{18}$$

In (16), we replace $(2n+1)P_n(\cos\theta)$ by the difference $(P'_{n+1}(\cos\theta)-P'_{n-1}(\cos\theta))(\sin\theta)$ and reduce by $\sin\theta$. Then we integrate the series in (16) term by term. In this case, the integration constant vanishes, since the polynomials $P_{n+1}(x),\ P_{n-1}(x)$ have the same parity and $P_{n+1}(0)-P_{n-1}(0)=0$. Now, instead of the Legendre polynomials, we substitute another (on the half-interval $(\theta_0,\pi]$) integral representation and using the uniform convergence of the integrated series, we change the order of summation and integration. We obtain a homogeneous Volterra integral equation of the 1st kind, similar to (17) on the half-interval $(\theta_0,\pi]$. This integral equation has a trivial solution on $(\theta_0,\pi]$. The functional equation (16) is thus transformed into the equation

$$\sum_{n=0}^{\infty}\left[Y_n-L_n\right]\cos\left(n+\frac{1}{2}\right)\varphi=0,\quad \theta_0<\varphi\le\pi. \tag{19}$$

Let us single out the main part in (18) and, together with (19), prepare a system of functional equations in trigonometric functions for the semi-inversion:

$$\sum_{n=0}^{\infty}Y_n\cos\left(n+\frac{1}{2}\right)\varphi$$

$$=\begin{cases}\sum\limits_{m=0}^{\infty}\left(Y_m\varepsilon_m^{(0)}-T_m\right)\cos\left(m+\frac{1}{2}\right)\varphi+V_0\cos\frac{\varphi}{2}, & 0\le\varphi<\theta_0,\\ \sum\limits_{k=0}^{\infty}L_k\cos\left(k+\frac{1}{2}\right)\varphi, & \theta_0<\varphi\le\pi.\end{cases} \tag{20}$$

Let us invert the left-hand side of the Fourier series (20). As a result, we obtain a system of algebraic equations of the second kind (SLAE-II)

$$Y_n = \sum_{m=0}^{\infty} Y_m \varepsilon_m^{(0)} \alpha_{n,m}^{(0)} - \sum_{m=0}^{\infty} T_m \alpha_{m,n}^{(0)} + V_0 \alpha_{n,0}^{(0)} + \sum_{k=0}^{\infty} L_k \alpha_{n,k}^{(1)}, \tag{21}$$

where

$$\alpha_{n,m}^{(0)} = \frac{1}{\pi}\left[\frac{\sin(n+m+1)\theta_0}{n+m+1} + \frac{\sin(n-m)\theta_0}{n-m}\right], \quad n \neq m, \tag{22}$$

$$\alpha_{n,n}^{(0)} = \frac{1}{\pi}\left[\frac{\sin(2n+1)\theta_0}{2n+1} + \theta_0\right], \quad \alpha_{n,m}^{(1)} = \delta_{n,m} - \alpha_{n,m}^{(0)}, \tag{23}$$

where $\delta_{n,m}$ is the Cronecer's symbol. Let us write (21) in the following matrix form

$$Y = MY + Z. \tag{24}$$

Since the matrix operator of system (24) is compact in ℓ_2 and the right column belongs to ℓ_2 and unity is not an eigenvalue of the operator, then system (24) has a unique solution in ℓ_2.

The obtained SLAE-II (24) has a wider region of effective solvability both in the parameter $r = a_0$ and in the parameter θ_0. This is due to the application of integral equation of the Volterra type [8] on various intervals and new choice of the small parameters.

Since elements $\alpha_{n,m}^{(0)}$ are bounded by the value $2\theta_0/\pi$, then the system is solvable by the method of successive approximations for $0 \leq \theta_0 << \pi$. For the numerical solution of the system, for example, with the selection of the main diagonal, it is necessary to re-designate the unknowns to accelerate the convergence of the solution $Y_n^{(1)} = Y_n/(n+1)$, $n = 0, 1, 2, 3....$ We note, in particular, that the coefficients A_n, F_n, $n = 0, 1, 2, ...$ of the Fourier - Legendre series of potentials U_1, U_2 (2) are in explicit form. For this, it is sufficient to use the boundary conditions at $r = r_1$ and at $r = r_2$ and take into account that θ belongs to the (complete) segment $[0, \pi]$. An important particular variant of the problem statement will be the lack of charge in the area G_3. In this case, the final SLAE-II will also change. So, on the right-hand side of SLAE-II, the quantities T_n and L_n acquire the limiting form: $V_{2,n}^{(2)}$ instead of T_n (13), $V_{2,n}^{(1)}\varepsilon_n^{(1)} - V_{2,n}^{(1)}\varepsilon_n^{(2)}$ instead of L_n (12).

5. Conclusions

An efficient algorithm for calculating the electrostatic field of a complex structure containing three nested spheres, among which the inner sphere has a circular hole is constructed. A point charge is placed between the outer sphere and the open sphere. The algorithm is based on the analytical method of semi-inversion of the matrix operator of the problem in the space ℓ_2.

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Регуляризація задачі електростатики для трьох сфер та електростатичного заряду

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Побудований чисельно-аналітичний алгоритм дослідження потенціалу сфери з круговим отвором, оточеної зовнішньої і внутрішньої замкненими стрічковими сферами. Число стрічок на сферах довільно. Стрічки на сферах розділені непровідними нескінченно тонкими перегородками. Перегородки знаходяться в площинах, паралельних площині зрізу сфери з отвором. Кожна стрічка має свій незалежний потенціал. Електростатичний заряд розміщений між зовнішньою сферою і сферою з отвором на осі структури. Повні потенціали повинні задовольняти, зокрема, рівнянь Максвелла

з урахуванням відсутності магнітних полів, задовольняти граничним умовам, мати необхідну особливість в точці розміщення заряду. Для вирішення поставленого завдання спочатку використані метод часткових областей і розділення змінних в сферичної системі координат. При цьому для рядів Фур'є застосовуємо степеневі функції і поліноми Лежандра цілих порядків. З граничних умов, використовуючи допоміжну систему 3-х рівнянь з 4-ма невідомими, отримана парна система функціональних рівнянь першого роду відносно коефіцієнтів рядів Фур'є. Система неефективна для вирішення прямими методами. Застосовані метод обернення інтегрального оператора Вольтерра і напівобернення матричного операторів задачі Діріхле для рівняння Лапласа. Метод заснований на ідеях аналітичного методу задачі Рімана - Гільберта. При цьому використані інтегральні уявлення для поліномів Лежандра. Отримано система лінійних алгебраїчних рівнянь другого роду з компактним матричним оператором у гільбертовому просторі ℓ_2. Система ефективно вирішується чисельно для довільних параметрів задачі і аналітично для граничних параметрів задачі. Розглянуто окремі варіанти завдання.
Ключові слова: сфери; отвір; електростатика; лінійна система другого роду; компакт.

Regularization of the electrostatics problem for three spheres and an electrostatic charge

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A numerical-analytical algorithm for investigation of the potential of a sphere with a circular hole, surrounded by external and internal closed ribbon spheres, is constructed. The number of ribbons on the spheres is arbitrary. The ribbons on the spheres are separated by non-conductive, infinitely thin partitions. The partitions are located in planes parallel to the shear plane of the sphere with a hole. Each ribbon has its own independent potential. An electrostatic charge is placed between the outer sphere and the sphere with a hole on the axis of the structure. The full potential must satisfy, in particular, Maxwell's equations, taking into account the absence of magnetic fields, satisfy the boundary conditions, have the required singularity at the point where the charge is placed. To solve this problem, we first used the method of partial domains and the method of separating variables in a spherical coordinate system. In this case, for the Fourier series, we use power functions and Legendre polynomials of integer orders. From the boundary conditions, using an auxiliary system of 3 equations with 4 unknowns, a pairwise system of functional equations of the first kind with respect to the coefficients of the Fourier series is obtained. The system is not effective for solving by direct methods. The method of inversion of the Volterra integral operator and semi-inversion of the matrix operators of the Dirichlet problem for the Laplace equation are applied. The method is based on the ideas of the analytical method of the Riemann - Hilbert problem. In this case, integral representations for the Legendre polynomials are used. A system of linear algebraic equations of the second kind with a compact matrix operator in the Hilbert space ℓ_2 is obtained. The system is effectively solvable numerically for arbitrary parameters of the problem and analytically for the limiting parameters of the problem. Particular variants of the problem are considered.
Keywords: spheres; hole; electrostatics; linear system of the second kind; compact.

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