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DYNAMIC AND MATHEMATICAL MODELING OF VIBRATION PROCESSES IN GAS TRANSPORTATION EQUIPMENT

The article considers aspects of an integrated approach to vibration inspections of gas transportation equipment. A critical analysis of scientific literature and regulatory documents covering existing approaches to solving issues related to determining the actual technical condition of equipment, assessing its service life, and making decisions on extending its service life has been carried out. The application of vibroacoustic research to solve control problems has been analyzed by studying the scientific literature. Dynamic and mathematical models of gas transportation equipment have been developed, particularly models of the general level and harmonics of the vibration velocity spectrum of bearing housings, models of oscillations of the centrifugal supercharger housing at the rotor blade frequencies, models of the force generated by the turbine or compressor blade stage of a gas turbine unit transmitted to the rotor, and models of gas pressure pulsation in the pipe. The modeling has been performed using the developed dynamic and mathematical models. The analysis of the modeling results, corresponding to the results of experimental studies, has been presented.

Keywords: gas pumping equipment, vibration processes, technical condition of equipment, vibration speed, vibration amplitude.

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Statement of the problem

Gas transportation equipment (GTE) is a high-risk object where accidents can potentially occur. Maintenance of gas transportation equipment requires ensuring reliability and compliance of its parameters with regulatory requirements.

Vibration monitoring and diagnostics of process equipment are aimed at preventing and detecting accidents, and, accordingly, at ensuring the reliability of the gas transportation system.

One of the strategic directions of technical diagnostics development includes, definitely, industry safety in its general sense. The most important part of the safety management process is the detection of faults and the determination of their causes, which is the main task of vibration diagnostics.

Vibration studies are commonly used in determining the technical condition and service life; they are based on the use of vibration process models (VPMs) in technological equipment.

Depending on the purpose of diagnostics, the degree of study of vibration processes and the depth of diagnosis, the model describes the process equipment as a single dynamic system with a known (or determinable) structure, subject to the influence of external disturbing forces and internal disturbances. The main tasks of modeling include: building mathematical models of the random and deterministic components of vibration, dividing them into high-frequency and low-frequency parts, and building models of diagnostic signs of defects that cause vibration. Modeling is used to establish and study diagnostic features that characterize and determine the technical condition of technological systems; analysis of modeling results provides forecasting capabilities - predicting the occurrence of possible deviations, and is also the basis for developing methods and tools for determining the actual technical condition of equipment to prevent timely violations of its operating modes.

The above-mentioned determines the relevance of developing and using mathematical models for inspection of gas transmission equipment using vibration methods and ensuring high reliability of the results to determine the technical condition and extend the service life based on them. Maintaining the technical condition of the equipment at the appropriate level will ensure the uninterrupted and efficient functioning of the gas transportation system, which is critical for the country's energy security and economic stability. This critical analysis will allow us to identify existing gaps and uncertainties in the existing regulatory framework and propose specific changes and improvements to enhance the assessment.

Analysis of recent studies and publications

Regulatory documents define the methods and terms of vibration inspection [1-4] of gas transportation equipment.

Researchers both in Ukraine and in foreign countries are engaged in developing methods for diagnosing gas transmission equipment and its mathematical models as well as their application in technical diagnostic systems, taking into consideration various approaches [5-7].

Determining the technical condition and extending the service life of gas transportation equipment requires an integrated approach, and many scientific and technical publications are devoted to solving the problems arising in this regard.

In particular, publications cover various approaches to modelling vibration processes, determining and analysing the vibration characteristics of technological equipment and their application in technical diagnostics systems [8-15].

Paper [8] presents the results of a study of the causes of pipeline vibration and methods of its reduction using experimental and numerical models to assess the effectiveness of various approaches.

In their research, scientists propose various methods of monitoring and control of gas transmission equipment based on the analysis of input and output data to detect vibration defects [9], methods of vibration reduction and evaluation of their effectiveness based on the results of the analysis of vibrations caused by gas flow in pipelines of gas pumping stations [10-11].

The need to study the problems of vibration integrity and vibration risk management associated with the design, construction and operation of gas pipelines and technological gas transmission equipment has been proved [12].

Publications [13-14] emphasise the importance of using mathematical models to accurately determine the technical condition of equipment and optimise its operation.

In order to optimise the operation of technological equipment and ensure its safe and trouble-free operation, an urgent task is to create an effective mechanism for determining the technical condition [15].

There are currently a lot of software systems for researching the technical condition implementing various algorithms for vibration diagnostics. Basically, the presented algorithms can be divided into two groups. The first group includes algorithms related to the overall vibration level and implementing a probabilistic approach to defect recognition tasks.

The second group includes algorithms based on spectral analysis of vibration.

The modeling of gas transportation equipment faces several key challenges, which is caused, primarily, by the complexity and multifactorial aspects of the processes in the GTO, including the interaction between mechanical, hydrodynamic and thermal processes, complicating the creation of adequate mathematical models.

Vibration systems are often nonlinear, indicating that their behavior cannot be described by simple linear equations. It requires the use of complex mathematical methods and numerical approaches for modeling.

To develop effective models, accurate data on system and process parameters is essential. However, measurements of vibration characteristics can be complex and inaccurate, affecting the accuracy of models.

Modeling vibration processes requires significant computing resources, especially if numerical methods and simulations are used. However, it can be a problem for large and complex systems.

These problems require an integrated approach to modeling vibration processes, including the use of modern mathematical methods, numerical approaches and computing resources.

The purpose of the article is to develop a mathematical model of the rotary vibration of a gas compressor unit (GCU) and evaluate its applicability in technical diagnostics.

Summary of the main material

The mathematical model of the rotor harmonics of the GCU spectrum has been developed in the form of nonlinear differential equations of rotor vibrations.

$$\ddot{x}+\lambda_p^2 x+\nu(\lambda_p^2\nu^{-1}\sum_{k=1}^{\infty}C_{k+2}^k\left(1-\chi_0\right)^{-k}\times \times\cos^{-k}\varphi_0\cdot x^{k+1})=\varepsilon\omega^2\cos\omega t, \quad (1)$$

where x, η — relative vibration movements in the vertical and horizontal directions;

$\omega = 2pf$ — circular rotor speed;

λ_p — frequency of forced oscillations (FFO) of the rotor;

ε — relative rotor eccentricity;

ν — small parameter;

χ_0, φ_0 — the relative displacement of the stable equilibrium and the vibration phase of the bearing trunnion;

C_{k+2}^k — number of connections.

Applying the method of small parameter to solve differential equations and introducing the relationship between the stiffness coefficients of the rotor and the case using the transfer function from the equality of the dynamic forces acting on the case and the journal, an expression for the relative amplitude is obtained k - of the vibration velocity harmonic of the bearing case ξ_k, expressed through the amplitude of the vibration velocity of the first harmonic V_1, as follows:

$$\xi_k=\alpha_1\left(k+1\right)\cdot\alpha_2^k\cdot V_1^{k-1};$$
$$\alpha_1=\frac{\lambda_p^2\lambda_k^2\left(1-\chi_0\right)\cdot K_p\cdot\Delta_n\cdot\cos\varphi_0}{\left(\omega^3\cdot K_k\right)}; \quad (2)$$
$$\alpha_2=1/\left[2\left(1-\chi_0\right)\cdot\Delta_n\cdot\omega\cdot\cos\varphi_0\right];$$
$$\left(k=2,\ 3,\ ...\right),$$

where λ_k — PMC case;

K_p, K_k — stiffness coefficients of the rotor and body;

Δ_n — bearing clearance

The distribution of rotor harmonics of the vibration velocity levels of bearing housings of a defect-free GCU (at the standard measurement points) depends only on the amplitude of the first rotor harmonic of the spectrum. It has been theoretically established that the distribution density of the amplitude of the first rotor harmonic and rotor unbalance obeys Rayleigh's law. Based on the form of the distribution density, we obtained estimates of the permissible V_e^g and the limit $V_e^{гр}$ values of the overall vibration level of the GCU.

$$V_e^g=K_ш\cdot\sigma\left(V_1\right)\cdot\sqrt{1+4\left(\alpha/K_ш\right)^2\sigma^2\left(V_1\right)};$$
$$V_e^{гр}=1{,}5K_ш\cdot\sigma\left(V_1\right)\cdot\sqrt{1+7\left(\alpha/K_ш\right)^2\sigma^2\left(V_e\right)};$$
$$\alpha=0{,}35\cdot K_p\cdot\lambda_p^2\cdot\lambda_k^2\cdot\sqrt{\pi^2+6\chi_0^2}\times \times\Delta_n^{-1}\cdot\omega^{-5}\cdot\chi_0^{-1}\left(1-\chi_0\right)^{-3},$$

where $K_ш$ — noise factor;

$\sigma\left(V_1\right)$ — is the root mean square value (rms) of the amplitude of the first rotor harmonic.

The obtained expressions were used to normalize the vibration of GTK-10, GT-750-6, VCN 235, and BTDA units using experimental data. The value of the permissible level of the total vibration velocity $V_e^g=6{,}5$ mm/s in terms of the existing standards corresponds to the “good” grade.

Similarly to (3) the ratio for normalization of the limit and permissible V_k^g values of rotor harmonic amplitudes

$$V_k^g = \frac{K_{ш,k}\lambda_k^2 K_p\left(k+1\right)\sqrt{k!}\left(V_1^{гр}/K_ш\right)^k}{\left(4K_k\nu\Delta_n^{k-1}\omega^{k-1}\lambda_p^2\right)}; \quad (4)$$

$\left(k = 2, 3, ...\right)$,

where $K_{ш,k}$ — a coefficient similar to $K_ш$;

$V_1^{гр}$ — limit value V_1.

The obtained ratio makes it possible to normalize the maximum permissible levels of the amplitudes of rotary harmonics of the vibration velocity of GCU bearing housings with emerging defects, and to take the double value of the latter as the maximum levels of defective units. Coefficients α, α_1, α_2 і α_3 were obtained for all types of bearings of a defect-free GPA using experimental data by approximating the amplitudes of rotor harmonics by the relation (2).

Results of calculations of the approximation coefficients of the real spectra of the GT-750-6 unit at V_1 = 3,1 mm/s; V_2 = = 0,8 mm/s; V_3 = 0,3 mm/s; V_4=0,2 mm/s are equal to $\alpha_1 = 2,4$; $\alpha_2 = 0,11$. For the GTK-10 unit at V_1=5,5 mm/s; V_2 = 2,0 mm/s; V_3 = = 1,5 mm/s V_4=1 mm/s; are equal to $\alpha_1 = 2,2$; $\alpha_2 = 0,1$. The obtained values α_1 and α_2 are in good agreement with the calculated data.

When any of the rotary harmonics exceeds the limit level, a number of models of low-frequency oscillations of gas turbine generators have been developed to identify the main causes of the increased vibration level. For subharmonic oscillations of half the frequency with an increased clearance in the sliding bearing, a relation for the relative amplitude of subharmonic oscillations was obtained:

$$\xi_{1/2} = V_{1/2}/V_1 = \sqrt{\left(0,15e_1^{-1}-1\right)/2}, \quad (5)$$

where e_1 – is the relative amplitude of oscillations of the first rotational harmonic of the journal vibration movement in the bearing.

The increase in the vertical clearance in the bearing of the GT-750-6 unit in the process of wear of the antifriction surface of the liner leads to the limit of stability and sharp tuning to subharmonic resonance. In this case, at the average value of the amplitude of the vibration velocity of the first rotor harmonic $V_1 = 6,5$ мм/с the subharmonic amplitude is maximal and equal to $V_{1/2} = 4,6$ мм/с.

A further increase in the clearance in the plain bearing leads to a loss of stability and to self-oscillations of the rotor on the oil film. Relative amplitude of self-oscillations of bearing housings $\xi_a = V_a / V_1$ is presented in the form of the relative amplitude of oscillations of the bearing journal, which is calculated from the algebraic equation of the fourth degree relative to the square of the amplitude. For the unit, the calculations show that $V_a = 22$ мм/с, and the relative amplitude of self-oscillations is equal to $\xi_a \approx 2$.

When the tension is lost and a gap appears in one prismatic gasket, for example, in the vertical plane, the values of the stiffness coefficients in the vertical and horizontal directions are not the same. For this case, a system of differential equations with periodically varying coefficients (stiffness coefficients) was developed to describe quasi-harmonic vibrations.

$$\begin{aligned} &2\ddot{x} + x\left[\left(\lambda_в^2+\lambda_c^2\right)+\left(\lambda_в^2+\lambda_c^2\right)\cdot\cos\omega t\right]+ \\ &+\eta\left(\lambda_в^2-\lambda_c^2\right)\sin\omega t = 0; \\ &2\ddot{\eta} + \eta\left[\left(\lambda_в^2+\lambda_c^2\right)+\left(\lambda_в^2-\lambda_c^2\right)\cdot\cos\omega t\right]+ \\ &+x\left(\lambda_в^2-\lambda_c^2\right)\sin\omega t = 0, \end{aligned} \quad (6)$$

where $r_в = \lambda_в / \omega$;

$\lambda_в$ — Relative and absolute CMM of the bearing liner;

$r_c = \lambda_c / \omega$;

λ_c — Relative and absolute CFC of the rotor-bearing system.

System (6) can be reduced to the Mathieu equation (with known solutions of the Mathieu function). According to Floquet's theorem, the periodic solutions of equation (6) will be with frequencies ω and $\omega / 2$. The ratio between the amplitudes of oscillations is obtained from the solution of the Matthieu equation and is determined by the nonlinearity caused by the gaps between the liner gasket and the bearing housing. Further weakening of the rolling bearing fastening and the disappearance of tension on the sliding bearing liners leads to the slippage of the cage and the appearance of mechanical vibrations with frequencies that are multiples of the rotary harmonics described by nonlinear differential equations. It was found that in this case, the growth of the third rotary harmonic with a relative amplitude of $\xi_3 = V_3 / V_1$

$$\xi_3 = 1{,}5 \cdot 10^4 \cdot D^2 \cdot r_в^6 \cdot \omega^4 \times \times \Delta_в^{-1} \cdot K_в^{-2} \left(r_в^2 - 1\right)^{-2} \cdot \left(r_в^2 - 9\right)^{-1}, \quad (7)$$

where D — rotor imbalance;

$\Delta_в^{-1}$ — clearance in the bearing liner;

$K_в$ — stiffness coefficient of the liner.

For the rotor of the STD-4000 electric motor at $r_в = 3$ the amplitude of the third rotor harmonic is significantly higher than the first rotor harmonic. Clearance $\Delta_в = 100$ мкм with the amplitude of the first rotor harmonic $V_1 = 6$ мм/с for $r_в = 1/2$ is equal to $\xi_3 = 0{,}6$. A resonant case when $r_в = 1$, has also been sufficiently studied. For large values of the gap $\Delta_в$ the differential equation of oscillations is solved by the asymptotic method with a nonlinear restoring force , $F(X)$, composed of line segments, as follows

$$\xi_k = \nu \cdot K_a \cdot r_c^2 \cdot r_в^{-2} \cdot k \cdot \left(1 - k^2\right)^{-1},$$

where K_a — coefficient depending on the gap $\Delta_в$.

An assessment has been made ξ_k for $r_в = 1/2$ and it was obtained $\xi_2 = 0{,}48$; $\xi_3 = 0{,}24$.

Distortions in the rolling bearing cage, misalignment of two bearings, or misalignment of the rotors result in dual stiffness of the bearing journal, where the rotor stiffness values in two mutually perpendicular directions are different. According to Floquet's theorem, we have stable oscillations with a frequency of ω and 2ω. The amplitude of the second harmonic is comparable to the amplitude of the first harmonic and may exceed it. Combination oscillation frequencies represent oscillations excited at the sum or difference frequencies of two rotors, when two rotors with different rotational speeds ω_1 and ω_2 are interconnected by an intermediate bearing. To explain the origin of the frequencies $(\omega_2 - \omega_1)$ and $(\omega_2 + \omega_1)$ an acoustic analogy with combination tones is applied, which allows this phenomenon to be related to the nonlinear asymmetric elasticity of the system.

Expression for blade harmonic amplitudes $V_{лj}$ of the vibration spectrum of a centrifugal supercharger (CSD) for the general case is as follows:

$$V_{лj} = z_л \cdot K_{нв} \cdot K_Q \cdot \left(1 - \eta_н\right) \cdot R_н^{-1} \cdot K_k^{-1}; \quad (j = 1,\ 2,\ ...), \quad (8)$$

where $z_л$ — number of blades;

$K_{нв}$, K_Q, $K_н$ — coefficients;

$\eta_н$ — polytropical efficiency. WTC;

$R_н$ — the radius of the impeller of the ESP.

Defects occurring in the flowing part of the HPC are well understood and include

- Increased gas leakage through seals along the cover disk;
- erosion of the gas path;
- undercutting of the supercharger wheel blades;
- salt deposits in the interblade channel.

When deviations from the nominal pitch sizes, blade pitch angles, outlet edge thicknesses, velocity field, pressure, and flow angles occur at a fixed radius of the HPC wheel, the period is not the blade pitch, as in a homogeneous wheel, but in general the entire circumference. In this case, the entire low-frequency discrete spectrum appears in the inhomogeneous lattice, and the expression of the bearing housing vibration mathematically describes the phenomenon of amplitude modulation:

$$v_{лj}(t) = V_{лj} \left\{ \sin\left(jz_л\omega t\right) \sum_{s=1}^{\infty} \frac{K_m}{2} \sin\left[(jz_л\omega + s\omega)t + \sum_{s=1}^{\infty} \frac{K_m}{2} \sin\left[\left(jz_л\omega - s\omega\right)t\right] \right\}; \quad (9)$$
$$(j = 1,\ 2,\ ...);\ (s = 1,\ 2,\ ...),$$

where K_m - the partial modulation coefficient in the case of an undercut blade and a heterogeneous wheel is equal, respectively:

$$K_m = 2 / z_л;\ K_m = \sqrt{\pi / z_л} \cdot \delta\left(K_{нвk}\right),$$

where $\delta\left(K_{нвk}\right)$ — coefficient of variation.

Percentage ratio of upper and lower side frequency amplitudes $jz_л\omega \pm s\omega$ to the amplitude of the carrier blade frequency $jz_л\omega$ when the wheel channel is blocked with $z_л = 14$ is equal to $K_m / 2 = 7\%$. For a heterogeneous, defect-free wheel with different blade NPVs $d\left(K_{нвk}\right) = 0{,}3\%$ this ratio is equal to $K_m / 2 = 1{,}5\%$.

Similarly, to relation (9), expressions for the forces generated by the turbine or compressor blade stage and acting on the GTU rotor were obtained. The differential equations of the shaft-disk-blade oscillations for this case are obtained from the generalized Ostrogradsky-Hamilton principle, and the solution of the equations gives the value of the complex torque $\bar{M}_s$:

$$\bar{M}_s = \left[P_л \cdot \sum_{k=1}^{z_л} \left[\sum_{q=1}^{\infty} W_k\left(r_q\right) \cdot G_q \cdot \exp\left(i\varphi_k\right) \times \times \exp\left(-\frac{\sin 2\pi\left(k-1\right)}{z_л} \right) \right]_k \right] \cdot \sin\left(s\omega t\right); \quad (1)$$

$$\left(s = 1,\ 2,\ ...\right),$$

where $P_л$ — pressure forces acting on the blade;

W_k, φ_k — phase angle and dynamic coefficient;

r_q — is the relative blade NPV;

G_q — integrals [8].

Moving from the torque to the forces, expressions for various variants of the disturbing forces acting on the GTU rotor are obtained. A special example is the resonant oscillations of the blades of a heterogeneous wheel, when

$$\left(r_j\right)_k \approx 1 \ (k = 1,\ ..,\ z_л),$$

where $\left(r_j\right)_k$ — sequences of random disturbances of the relative natural frequencies of the blades along the circumference of the wheel.

Definitions for strength $(P(t))$ in this case is equal to:

$$P\left(t\right) = P_j \left[1 + 0,5\sqrt{\pi/z_л} \cdot \delta\left(w_k\right) \times \times \sum_{s=1}^{\infty} \sin\left(s\omega t\right) \right] \cdot \sin\left(jz_л\omega t\right); \qquad (11)$$

$$\left(j = 1,\ 2,\ ...\right);\ \left(s = 1,\ 2,\ ...\right),$$

where P_j — is the force amplitude of the *j*-th blade frequency.

To calculate the coefficient of variation $d\left(w_k\right)$ it is necessary to determine the mathematical expectation and variance of the dynamism coefficient w_k. For a close estimate, the dynamism coefficient is approximated by an exponential function, and the model uses a probabilistic approach to estimating the MTBF of turbocharger blades. If one blade breaks $K_m / 2 = z_л^{-1}$, and if, as a result of a crack, the blade's PEC at the non-resonant stage decreased and entered resonance $\left(r_j = 1\right)$, then $K_m / 2 = p / \delta_e z_л$, where δ_e — is the logarithmic decrement of the attenuation. For example, with the number of blades on the fifth stage of the GTK turbocharger -10 $z_л = 23$, when the blade breaks, we obtain the ratio of the sideband amplitude to the carrier frequency, which is equal to $K_m / 2 = 4,4\%$, and for the resonant blade of the first stage at $z_л = 29$ $K_m / 2 = 2$, i.e., the amplitude of the sideband frequency is twice as high as the carrier frequency $(z_л \cdot \omega)$. At resonant vibration of blades with the coefficient of variation of natural frequencies calculated from the results of the experiment, $\delta(\lambda_j) = 0,02$, for the fifth stage of the GTK-10 axial compressor, the ratio of sideband amplitudes to the carrier frequency is as follows $K_m / 2 = 13\%$.

To calculate the amplitudes of vibration of bearing housings at blade frequencies, a computer program was developed in the Machcad environment. The results of calculating the vibration of the GTK-10 turbocharger indicate that for the non-resonant degree of bearing housing vibration amplitude at blade frequencies $(jz_л\omega)$ are in the range $0,39 \div 1,57$ mm/s, and for the resonant stage $r_j = 1$ can reach the values 8,5 mm/s. An assessment of the change in blade NFR in the process of fatigue stress accumulation for different shank clamping forces showed that the reduction in the NFR of the elastic blade is 11%, which is significantly higher than the average difference in blade NFR along the stage.

A mathematical model of the gas flow pulsation generated by the supercharger was developed for the tubing of the HPC. The dependence of the gas flow pressure pulsation level $\left(p_д\right)$ on the flow rate and efficiency is as follows:

$$p_д = \gamma_н \cdot Q \cdot R_н \cdot \omega \cdot \left(1 - \eta_н\right) \cdot K_Q \times \times K_н \cdot \left(\frac{\sin\beta}{t_c \cdot \Delta_н} \right)^{1/2} \cdot \left(3 \cdot b_н \cdot z_л\right)^{-1}, \qquad (2)$$

where $\gamma_н$ — gas density;

Q — gas consumption;

t_c — pitch of the working blades;

$b_н$ — the width of the wheel of the central pumping station;

$\Delta_н$ — the gap between the body and the wheel of the HPC;

β — is the angle of exit of the relative speed of the HPC wheel.

The acoustic and cylindrical shell FEs were determined and the dynamic stresses and vibration velocity amplitudes of the pipe were calculated in various operating modes. Expression of the efficiency for the vibration velocity amplitude of the pipe liner at blade frequencies $\omega_л$:

$$V_{gk} = \frac{r_k^3 \cdot A_{gk}}{\left(\gamma_c \cdot h_m \cdot \omega_л \cdot \sqrt{\left(1-r_k^2\right)^2 + \left(\delta_e/\pi\right)^2 \cdot r_k^2}\right)}; \tag{13}$$

$$k = 1,\ 2,\ 3,\ ...$$

where γ_c — density of the pipe material;

h_m — pipe thickness;

r_k — is the relative PMC of the shell;

A_{gk} — amplitude of pressure pulsation (s.c.s.).

The calculations of the vibration velocity of the piping at the first blade frequency of the 650-22-2 supercharger showed that at the value of the blade frequency $f_л = 1480$ Гц a sharp resonance with a shell frequency of the order of $j = 10$, $q = 9$, and ring frequency $\overline{f}_k = 1642$ Гц has a slight recovery from the resonance with the blade frequency. The maximum vibration velocity amplitude is at $\delta_e = 0,1$ and values of losses in the supercharger wheel and it is within the limits of $3,9 \div 39$ мм/c, which is in good agreement with the experimental data. When determining the equivalent dynamic stresses of the shell according to the theory of the greatest linear deformation, a coefficient of reduction of the levels of high-frequency vibration velocity norms in comparison with low-frequency norms was obtained as follows:

$$K_m = \left[\left(n^2-1\right)/n^2 - \nu_n/2\sqrt{1-\nu_n^2}\right] \times \\ \times \sqrt{6\left(1-\nu_n^2\right)/\left(1+h_m/R_m\right)}, \tag{3}$$

where ν_n — Poisson's ratio;

n — order of the waveform.

The value of the coefficient (14) for the tubing of the CTD-25 unit at $h_m / R_m = 0,032$ and at $n \to \infty$ (taken with a margin) is equal to $K_m = 1,9$.

To register the leakage signal, two or three sensors are installed on the pipe and the delay time between the signals is determined by the mutual correlation function. At high gas flow rates and small slit diameters, the narrowband random process is close to harmonic, and the calculation of the mutual correlation function is difficult because the correlation function of each path is a cosine function, and it is impossible to separate individual paths. Logarithmization of such a signal seems to increase the broadband noise component. For example, if the ratio of noise to spectral density of a harmonic signal is 0.1, 0.01, and 0.001, then after logarithmization this ratio increases and is equal to 0.5, 0.33, and 0.25, and the gains for the cases under consideration are 5, 33, and 250. The kepstrum of a narrow-band random process and one that turns into a harmonic process is as follows:

$$K_{xx}(\tau) = -\left[\frac{\sin 2\pi B_ш \tau}{2\pi B_ш \tau} \frac{B_ш G_г}{\sigma_г^2} + \frac{\cos 2\pi f_г \tau}{\tau}\right]; \\ G_г = -\frac{4\sigma_г}{\sqrt{\pi}} \cdot \lg \frac{A_г^2}{4} + 4\left(B_ш^2 + f_p^2\right) \cdot \lg \exp, \tag{15}$$

where $B_ш$, f — noise frequency range and harmonic frequency;

$G_г$, $A_г$ — noise spectral density and amplitude of harmonic oscillations;

$\sigma_г$ — is the coefficient of approximation of a narrow-band random process by the Gaussian distribution

At $\sigma_г \to 0$ (narrowband random process turns into a harmonic process) $G_r \to 4 \cdot \lg(\exp)\left(B_ш^2 + f_г^2\right)$, and relation (15) describes the correlation function of an already broadband random process in the frequency range $B_ш$ with amplitude $G_г/\sigma_г^2$, and at $\sigma_г \to 0$ the value of the correlation function at zero tends to infinity. If the propagation delay time of elastic longitudinal waves is found using the eq. (15), then the change in the delay time of the vibroacoustic signal of a stressed gas pipeline is used to find the voltage $\sigma_д$ as follows:

$$\sigma_д = 2 \cdot E \cdot \bar{G}_m \cdot \Delta\tau\left(\tau_j - \tau_k\right)^{-1} + \sigma_m \cdot \left(1 - \bar{G}_m\right),$$

where E, $\bar{G}_m$ — modulus of elasticity and linear strengthening;

σ_m — the fluidity limits.

Audio power $(W_{кр})$, generated by the valve in the area of pre-critical pressure drops is proportional to the cube of the overpressure and the area of the valve slot:

$$W_{кр} = k_{кр} \cdot \left(P_н - P_k\right)^3 F_щ \cdot \gamma_{кр}^{-2} c_{зв},$$

where $k_{кр}$ — noise factor;

$P_н$, P_k — pressure at the inlet and outlet of the crane;

$F_щ$ — the area of the tap leakage gap;

$\gamma_{кр}$ — gas density;

$c_{зв}$ — sound speed.

In the area of supercritical pressure drops, the increase in sound power with an increase in the pressure drop across the valve is less intense. Approximately, the change in sound power in this region can be represented as follows:

$$W_{кр} = W_{кр}^* \left(P_н - P_к\right) \cdot \left(\Delta P^*\right)^{-1}, \quad (16)$$

where $W_{кр}^*$ — sound power of the valve noise at the pressure drop on the valve ΔP^*, which is the critical value.

Equation (16), based on the measured value of the vibration velocity level in dB, allows you to determine the area of the leakage gap of the crane $F_щ$ and, consequently, the leakage rate in the tap at a critical pressure drop.

To simplify the procedure for diagnosing and rapidly assessing the presence of leaks in cranes, it is proposed to assess the coherence function of two vibration signals of the crane body $\gamma_{xy}^2(f)$. At the same time, for a sealed tap $\gamma_{xy}^2(f) = 0$, and the task of determining the presence of a leak is based on the calculation of the mutual coherence function in the frequency range $12 \div 42$ kHz and determining the coefficient of technical condition of the crane:

$$K_{кр} = n^{-1} \sum_{k=1}^{n} \left| 1 - 0,5\sqrt{\gamma_{xy}^2(f_k)} \left(\sqrt{\gamma_{xy}^2(f_k)} + 1 \right) \right|.$$

Conclusions

A dynamic and mathematical model of the general level and rotational harmonics of the vibration velocity spectrum of the bearing housings of a gas turbine unit, a centrifugal blower, and a gas compressor unit has been developed. Analytical expressions of absolute and relative amplitudes of vibration velocity and an algorithm for calculating the amplitudes of rotary harmonics of GCU vibration velocity were obtained.

A dynamic and mathematical model of oscillations of the HPC casing at the rotor blade frequencies was developed. It was found that the amplitude of oscillations depends on the efficiency of the supercharger and is equal to 0.264 mm/s. The diagnostic signs of defects in the wheel and blades of the CBN were obtained in the form of the ratio of the amplitude of the vibration velocity of bearing housings at lateral frequencies to the amplitude of the blade frequency. For the 370 17 superchargers, this ratio varies between 1.5% and 7%.

A model of the force generated by the blade stage of a turbine or compressor of a gas turbine is presented, which is transmitted to the rotor. The differential equations of the joint oscillations of the blade-rotor-hull of a gas turbine are obtained. It is shown that the amplitude of oscillations reaches 8.5 mm/s. The diagnostic signs of defects in the stage blades in the form of the ratio of the amplitude of the vibration velocity of bearing housings at lateral frequencies to the amplitude of the blade frequency were established. For the GTK 10 unit, these ratios vary from 44 to 13%.

The analysis of gas pressure pulsation in the pipe, which depends on the flow rate and efficiency of the supercharger, was carried out. It was determined that for the 650 22 2 supercharger piping, the pressure pulsation is in the range from 0.01 to 0.1 MPa. The vibration amplitudes of the pipe caused by gas flow pulsations at the blade frequencies of the HPC were calculated, which for the 620 22 2 supercharger strapping are in the range of 3.9-39 m/s.

Conflict of interest

The authors declare that there is no conflict of interest regarding the publication of the manuscript. In addition, the authors fully complied with ethical standards, including plagiarism, data falsification, and double publication.

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ДИНАМІЧНЕ ТА МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ВІБРАЦІЙНИХ ПРОЦЕСІВ У ГАЗОТРАНСПОРТНОМУ ОБЛАДНАННІ

У статті розглянуто аспекти комплексного підходу до вібраційних перевірок газотранспортного обладнання. Здійснено критичний аналіз наукової літератури та нормативних документів, що охоплюють існуючі підходи до вирішення питань, пов'язаних з визначенням фактичного технічного стану обладнання, оцінкою його терміну служби та прийняттям рішень щодо продовження терміну його служби. Шляхом вивчення наукової літератури проаналізовано застосування віброакустичних досліджень для вирішення задач керування. Розроблено динамічні та математичні моделі газотранспортного обладнання, зокрема моделі загального рівня та гармонік спектра швидкості коливань корпусів підшипників, моделі коливань корпусу відцентрового нагнітача на частотах лопаток ротора, моделі сили, що створюється ступенем лопатки турбіни або компресорного агрегату газотурбінної установки, що передається на ротор, та моделі пульсацій тиску газу в трубопроводі. Моделювання виконано з використанням розроблених динамічних та математичних моделей. Представлено аналіз результатів моделювання, що відповідають результатам експериментальних досліджень.

КЛЮЧОВІ СЛОВА*:* газоперекачувальне обладнання, вібраційні процеси, технічний стан обладнання, швидкість вібрації, амплітуда вібрації.

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Конфлікт інтересів

Автори заявляють, що конфлікту інтересів щодо публікації рукопису немає. Крім того, автори повністю дотримувались етичних норм, включаючи плагіат, фальсифікацію даних та подвійну публікацію

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