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LOCALITY OF QUANTIZED SCALAR FIELDS FOR GENERATIONS OF PARTICLES

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It is shown that an integral corresponding to the contribution of one particle to equal-time commutator of quantized scalar fields diverges in a reality, contrary to usual assumption that this integral vanishes. It means that commutator of scalar fields does not vanish for space-like intervals between the field coordinates. In relation with this divergence the generalization of the Klein-Gordon equation is considered. The generalized equation is presented as products of the operators for the Klein-Gordon equation with different masses. The solutions of derived homogeneous equations are sums of fields, corresponding to particles with the same values of the spin, the electric charge, the parities, but with different masses. Such particles are grouped into the kinds (or families, or dynasties) with members which are the particle generations. The commutator of fields for the kinds of particles can be presented as sum of the products of the commutators for one particle and the definite coefficients. The sums of these coefficients for all the generation equal zero. The sums of the products of these coefficients and the particle masses to some powers equal zero too, i.e., for these coefficients some relations exist. In consequence of these relations the commutators of the fields for the particle generations vanish on space-like intervals. Thus, the locality (the microcausality) is valid for the fields of the particle kinds. It is possible if the number of the generations is greater than two.
KEY WORDS: convergence of integrals, differential equations, generations of particles, microcausality principle, generations of particles, indefinite metrics

ЛОКАЛЬНІСТЬ КВАНТОВАНИХ СКАЛЯРНИХ ПОЛІВ ДЛЯ ПОКОЛІНЬ ЧАСТИНОК

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Показано ,що інтеграл відповідний внеску однієї частинки в одночасний коммутатор квантованих скалярних полів в дійсності розбігається, в протилежність звичайному припущенню, що цей інтеграл дорівнює нулю. Це означає, що коммутатор скалярних полів не дорівнює нулю для просторово-подібних інтервалів між координатами полів. В зв'язку з цією розбіжністю розглянуто узагальнення рівняння Клейна-Гордона. Узагальнене рівняння представляє собою добуток операторів рівняння Клейна-Гордона з різними масами. Розв'язки одержаних однорідних рівнянь представляють собою суми полів, відповідних частинкам з однаковими значеннями спіну, електричного заряду, парностей, але з різними масами. Такі частинки групуються в роди (або сім'ї, або династії) а їхні члени є покоління. Комутатор полів для поколінь частинок можна представити як суми добутків комутаторів для однієї частинки і визначених коефіцієнтів. Суми цих коефіцієнтів для всіх поколінь дорівнюють нулю. Суми добутків цих коефіцієнтів на маси частинок у деяких степенях теж дорівнюють нулю, тобто для цих коефіцієнтів існують деякі співвідношення. Внаслідок цих співвідношень комутатори полів для поколінь частинок стають рівними нулю на просторово-подібних інтервалах. Таким чином, локальність (мікропричинність) має місце для полів родів частинок. Це можливе, якщо кількість поколінь частинок більша двох.
КЛЮЧОВІ СЛОВА: збіжність інтегралів, диференціальні рівняння, принцип мікропричинності, покоління частинок, індефінітна метрика

ЛОКАЛЬНОСТЬ КВАНТОВАННЫХ СКАЛЯРНЫХ ПОЛЕЙ ДЛЯ ПОКОЛЕНИЙ ЧАСТИЦ

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Показано, что интеграл, соответствующий вкладу одной частицы, в одновременной коммутатор скалярных квантованных полей, в действительности расходится, в противоположность обычному предположению, что этот интеграл равен нулю. Это означает, коммутатор скалярных полей не равен нулю для пространственно-подобных интервалов между координатами полей. В связи с этой расходимостью рассмотрено обобщение уравнения Клейна-Гордона. Обобщенное уравнение представляет собой произведения операторов уравнения Клейна-Гордона с разными массами. Решения полученных однородных уравнений представляют собой суммы полей, соответствующих частицам с одними и теми же значениями спина, электрического заряда, четностей, но с разными массами. Такие частицы группируются в рода (или семьи, или династии), а их члены являются поколениями. Коммутатор полей для родов частиц можно представить как сумму произведений коммутаторов полей для одной частицы и определенных коэффициентов. Суммы этих коэффициентов соответствующих всем поколениям равны нулю. Суммы произведений этих коэффициентов на массы частиц в некоторых степенях также равны нулю, то есть для этих коэффициентов существуют некоторые соотношения. Вследствие этих соотношений коммутаторы полей для поколений частиц становятся равными нулю на пространственно-подобных интервалах. Таким образом, локальность (микропричинность) справедлива для полей поколений частиц. Это возможно, если количество поколений частиц больше двух.
КЛЮЧЕВЫЕ СЛОВА: сходимость интегралов, дифференциальные уравнения, принцип микропричинности, поколения частиц, индефинитная метрика

A locality of boson operators means that commutator of these operators vanishes for the space-like intervals. The locality principle is named as the microcausality principle too. As a rule, commutator is expressed through improper integrals, which must be convergent. We consider the well-known commutator of scalar fields with the m mass (for example, from Refs. [1-7]). It may be written as

$$\left[\varphi(x),\, \varphi^{+}(y)\right] = i\Delta(x-y,m), \tag{1}$$

where $\Delta(x-y,m)$ is next function

$$\Delta(x-y,m) = -\frac{i}{(2\pi)^3}\int \frac{d^3p}{2p_0}\left[e^{-ip(x-y)} - e^{ip(x-y)}\right],\ p_0 = \sqrt{\vec{p}^2 + m^2}\,, \tag{2}$$

$\varphi(x)$ is an operator of the scalar field

$$\varphi(x) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int \frac{d^3p}{2p_0}\left[e^{-ipx}a_p + e^{ipx}b_p^{+}\right], \tag{3}$$

and $\varphi^{+}(x)$ is its hermitian conjugated operator

The function $\Delta(x-y,m)$ has next properties: 1) it is a function of the $x-y$ coordinate difference; 2) it is the odd function of the $x-y$ difference; 3) it is relativistic invariant function and depends on $(x-y)^2$ and the sign of $x_0 - y_0$ for $(x-y)^2 \geq 0$ or on $(x-y)^2$ for $(x-y)^2 < 0$; 4) it vanishes for the space-like intervals (i.e., for $(x-y)^2 < 0$). Last property means that signals with superlight velocities do not exist. A test of the locality principle is more simple for the equal-time commutator in (1), (i.e., for $x_0 = y_0$ in (1), (2)). Then the $\Delta(\vec{x},m)$ function is expressed through the $I(\vec{x},m)$ integral:

$$\Delta(\vec{x},m) = \frac{1}{(2\pi)^3} I(\vec{x},m),\ I(\vec{x},m) = \int \frac{d^3p}{p_o} sin\vec{p}\vec{x}\,. \tag{4}$$

Usually it is assumed that the integral $I(\vec{x},m)$ vanishes since the integrand in (4) is the odd function of the $\vec{p}$ -momentum. But this assumption is true only for convergent integral. In relation with the integral (4) we consider simple integral

$$J = \int\limits_{-\infty}^{\infty} x\, cos\, xdx\,. \tag{5}$$

The integrands in (4), (5) are the odd functions of the integration variables and have similar asymptotical behaviors. It is well known that the integral (5) diverges. Indeed, according to usual method of a calculation of improper integral we have

$$J = \lim_{\substack{A\to\infty \\ B\to\infty}} \int\limits_{-A}^{B} x\, cos\, xdx = \lim_{A\to\infty}\left(-A\, sin\, A - cos\, A\right) + \lim_{B\to\infty}\left(B\, sin\, B + cos\, B\right) \tag{6}$$

As it is known an improper integral in infinite limits converges only in the case if finite limits exist at arbitrary A/B ratio and $A-B$ difference. The integral (5) vanish in the case $A=B$ only. It means that the improper integral (5) diverges and the main value of the integral (5) vanishes only. Therefore, we may expect that the integral (4) diverges also.

In the present paper the divergence of the integral (4) is studied in details. As this divergence leads to non-zero value of the commutator (1) on space-like intervals, it means that the locality principle can be violated. But it is well known that the locality principle (in particular, the absence of any signals with superlight velocities) is valid in reality. Therefore, we assume that the commutator (1) must be changed by the commutator of some proper fields, which vanishes on the space-like intervals. The present paper is devoted to the investigations of the total fields corresponding to generations of particles. We show that the commutator (1) can be changed by the sums of the commutators multiplied by some coefficients for generations of particles. Such sums vanish on the space-like intervals at the number of the generations, which is greater than two.

DIVERGENCE OF INTEGRALS FOR COMMUTATORS OF FIELDS ON SPACE-LIKE INTERVALS

The integral (4) is the triple improper integral. It converges only in the case of absolute convergence, i.e., if the corresponding integral of the module of integrand converges [8,9]. As $\left|sin\,\vec{p}\vec{x}\right| \geq sin^2\,\vec{p}\vec{x}$, we can write

$$\int\frac{d^3p}{p_0}\left|sin\vec{p}\,\vec{x}\right| > \frac{1}{2}\int\frac{d^3p}{p_0}\left(1-cos\,2\vec{p}\vec{x}\right) = I_1 + I_2. \tag{7}$$

The I_1 integral diverges as $\pi\Lambda^2$ at $\Lambda\to\infty$. We evaluate a divergence of the integral I_2 in a spherical frame. After integration with respect to angular variables we derive

$$I_2 = -\frac{\pi}{r}\int_0^\infty\frac{p\,sin\,2pr}{\sqrt{p^2+m^2}}dp = -\lim_{\Lambda\to\infty}\frac{\pi}{r}\int_0^\Lambda\frac{p\,sin\,2pr}{\sqrt{p^2+m^2}}dp, \tag{8}$$

where $r = \left|\vec{x}\right|$. For space-like intervals, r is not equal to zero. The integral (8) has not definite limit at $\Lambda\to\infty$. Indeed, the integrand in (8) is similar to $sin2pr$ and the limit of $cos2\Lambda r\,/\,2r$ does not exist at $\Lambda\to\infty$. Thus, the triple integral (4) does not converge also. Therefore, according to [8,9] the integral (4) diverges and commutator (1) may not vanish for space-like intervals.

To confirm non-zero value of the commutator (1) for the space-like intervals we explicitly derive the integral (4) in the cylindrical frame. Let $\vec{x} = (0,0,r)$, $\vec{p} = (p_1,p_2,p_3)$, $\vec{p}\vec{x} = p_3 r$, $p_1 = p_t\,cos\,\alpha, p_2 = p_t\,sin\,\alpha$, $d^3p = p_t dp_t dp_3 d\alpha$. The limits of integration are following: $0\leq\alpha<2\pi$, $0\leq p_t\leq\Lambda$, $-A\leq p_3\leq B$, at $\Lambda\to\infty$, $A\to\infty$, $B\to\infty$. Then the integral (4) is given by

$$\begin{gathered}I(\vec{x},m) = \int_0^{2\pi}d\alpha\int_0^\Lambda p_t dp_t\int_{-A}^{B}\frac{sin\,p_3 r}{\sqrt{p_t^2+p_3^2+m^2}}dp_3 = \\ \frac{2\pi}{r}\Lambda\left(-cos\,Br + cos\,Ar\right) - 2\pi\int_{-A}^{B}\sqrt{p_3^2+m^2}\,sin\,p_3 r dp_3\end{gathered} \tag{9}$$

The limit of first term in the right hand of (9) does not exist at $\Lambda\to\infty$, $A\to\infty$, $B\to\infty$. Using the identity $\sqrt{p_3^2+m^2} = \left(\sqrt{p_3^2+m^2}-\left|p_3\right|\right)+\left|p_3\right| = \frac{m^2}{\sqrt{p_3^2+m^2}+\left|p_3\right|}+\left|p_3\right|$ the integral in (9) may be written as

$$-2\pi m^2\int_{-A}^{B}\frac{sin\,p_3 r}{\sqrt{p_3^2+m^2}+\left|p_3\right|}dp_3 - \frac{2\pi}{r}\left(A\,cos\,Ar - B\,cos\,Br\right) - \frac{2\pi}{r^2}\left(sin\,Br - sin\,Ar\right) \tag{10}$$

The improper integral in (10) converges conditionally and the limits of next terms do not exist at $A\to\infty$, $B\to\infty$. Therefore, the integral (4) diverges. Moreover, the main value of the integral (4) does not vanish, as the limit of first term in the right hand of (9) does not exist at $\Lambda\to\infty$, $A\to\infty$, $B\to\infty$. In consequence of a divergence of the integral (4), we can write down $\int\frac{d^3p}{p_0}e^{i\vec{p}\vec{x}} \neq \int\frac{d^3p}{p_0}e^{-i\vec{p}\vec{x}}$.

We see that the commutator (1) does not vanish for the space-like intervals of coordinates in reality. It means that the locality principle is not valid for the fields corresponding to contributions of one particle and one antiparticle. From this fact we can conclude that the signals with superlight velocities can exist. But such signals have not been observed. Thus, we have contradiction between theoretical conclusions and the experimental data.

A situation with a convergence of the integrals (4) for the commutator (1) is similar to a situation with a situation with convergence of the integral for the Yukawa potential as well as the integrals for the Green functions of Klein-Gordon and Dirac equations.

PARADOX OF GREEN FUNCTIONS AND GENERALIZATIONS OF KLEIN–GORDON EQUATION

In [10,11] it is shown that the integral for the Yukawa potential as well as the integrals for the Green functions of Klein-Gordon and Dirac equations diverge. In relation with these divergences, the paradox of the Green functions has been formulated [10,11].

From the mathematical point of view the use of the Green functions of the Klein-Gordon and Dirac equations is incorrect, but these Green functions (calculated by some fashion) give fairly good description of different experimental data.

To avoid the divergences in the integrals for the Green functions of the Klein-Gordon and Dirac equations these equations and their Green functions have been generalized. So, instead of the Green function of the Klein-Gordon equation

$$D(x,m)=\frac{1}{(2\pi)^4}\int\frac{e^{-iqx}d^4q}{-q^2+m^2} \tag{11}$$

the following functions

$$\bar{G}(x)=\frac{1}{(2\pi)^4}\int\frac{e^{-iqx}d^4q}{\left(-q^2+m_1^2\right)\left(-q^2+m_2^2\right)....\left(-q^2+m_N^2\right)}=\frac{1}{(2\pi)^4}\int\frac{e^{-iqx}d^4q}{P_N\left(q^2\right)} \tag{12}$$

where $P_N\left(q^2\right)$ is the polynomial of the N degree with respect to q^2 has been proposed [10,11]. The Green function (12) corresponds to generalized non-homogeneous Klein-Gordon equation of the $2N$ degree

$$\left(\Box+m_1^2\right)\left(\Box+m_2^2\right)........\left(\Box+m_N^2\right)\Phi(x)=\eta(x), \tag{13}$$

where $\Phi(x)$ is the field and $\eta(x)$ is the current (the field source). We consider the case of the polynomial with real non-negative different roots $m^2{}_1<m^2{}_2<m^2{}_3<....<m^2{}_N$. Note that for the advanced, retarded and causal Green functions we must write the corresponding imaginary infinitesimal term to each m_k^2.

The general classical solution $\Phi_{cl}(x)$ of the linear equation (13) is the sum of the general solution of the corresponding homogeneous equation $\Phi(x)_{free}$ and partial solution $\Phi(x)_{nh}$ of non-homogeneous equation:

$$\Phi(x)_{free}=\int d^4q\sum_{k=1}^{N}\delta\left(q^2-m_k^2\right)\left[c_ke^{-iqx}+\tilde{c}_ke^{iqx}\right] \tag{14}$$

$$\Phi(x)_{nh}=\int\bar{G}(x-y)\eta(y)d^4y, \tag{15}$$

where c_k and $\tilde{c}_k$ are the arbitrary constants. Thus, $\Phi(x)_{free}$ is the sum of the terms corresponding to the particles with the same charge, parity, spin but with different masses. Each term in (14) corresponding to number k is the solution of the homogeneous Klein – Gordon equation as $\left(\Box+m_k^2\right)\left(c_ke^{-iqx}+\tilde{c}_ke^{-iqx}\right)\ \delta\left(q^2-m_k^2\right)=0$. In Ref. [10] it is shown that the case of equal masses in Eq. (13) must be excluded. It was shown that the functions $\Phi(x)_{free}$ are non- normalizable if at least two masses are equal. Thus the masses in the generalized Klein Gordon equation must be different. The $\Phi(x)$ fields we shall name as total fields, the fields for the particle generations, and the fields of a kind (or a family or a dynasty).

For the rational fraction in (12) the expansion can be written [10, 11]

$$\begin{gathered}\frac{1}{P_N\left(q^2\right)}=\frac{1}{\left(-q^2+m_1^2\right)\left(-q^2+m_2^2\right)...\left(-q^2+m_N^2\right)}=\sum_{k=1}^{N}\frac{A_k}{-q^2+m_k^2},\\ A_k=-\frac{1}{P_N'\left(m_k^2\right)}=\lim_{q^2\to m_k^2}\frac{-q^2+m_k^2}{P_N\left(q^2\right)},\quad A_k=(-1)^{k+1}\left|A_k\right|.\end{gathered} \tag{16}$$

The A_k coefficients obey the relations:

$$\sum_{k=1}^{N}A_km_k^{2l}=0,\ l=0,1,2,...,N-2, \tag{17}$$

$$\sum_{k=1}^{N}A_km_k^{2N-2}=(-1)^{N+1}, \tag{18}$$

Using the equalities (16) we can express the Green function (12) of Eq. (13) in terms of the Green functions (11)

$$\bar{G}(x) = \sum_{k=1}^{N} A_k D(x, m_k). \tag{19}$$

As the dimension of the time-space is equal to four, the integral (12) can be convergent at $N \geq 3$. Consequently, for each spinless particle two (or greater) particles with the same charges, isospin, C - and P parity, but different masses, must exist in addition. We may say that such particles are members of some set (a family or a kind or a dynasty). In (14,16) k is the number of the particle generation. We may assume that the number of members in kinds for the elementary particle is less than the number of member in kinds for the composite particle. Each particle belongs to some kind and some generation.

In Ref. [12] the causal Green functions of generalized Klein-Gordon equations (13)

$$\bar{G}(x)_c = \frac{1}{(2\pi)^4} \int \frac{e^{-iqx} d^4 q}{\left(-q^2 + m_1^2 - i\varepsilon\right)\left(-q^2 + m_2^2 - i\varepsilon\right)....\left(-q^2 + m_N^2 - i\varepsilon\right)} \tag{20}$$

have been considered. The expressions (11) and (19) have been used for calculation of the Green function (20). According to Ref. [1] the causal Green function (11) is expressed through the cylindrical $K_1(m\sqrt{x^2})$ function, which has singularities on the light cone ($x^2 = 0$). The series for the $K_1(m\sqrt{x^2})$ function has been used from Refs. [13, 14]. The use of the relations (17) at $l = 0,1$ allowed us to eliminate all the singularities. It has been shown that the integral (20) converges at $N \geq 3$ in all the space-time. Thus, it may be concluded that minimal quantity of the generations in the kinds of the spinless particles equals three. Consideration of the causal Green function for generalized Dirac equations has shown that similar minimal quantity of the spin-1/2 fermion equals six [12].

ELIMINATION OF DIVERGENCES IN COMMUTATORS OF TOTAL FIELDS

For one particle the imaginary (absorptive) part of the causal Green function (11) (with $x - y$ instead of x) can be expressed through the commutator of the scalar fields (1). Therefore, we can expect that integrals for the commutators of total fields will converge and these commutators will vanish for space-lire intervals. Using the formulae (1) and (19) the commutator of total fields may be written as

$$\left[\Phi(x), \Phi^+(y)\right] = i\sum_{k=1}^{N} A_k \Delta(x - y, m_k), \tag{21}$$

where the A_k coefficients are defined in (16). Notice that (21) can be derived also using the expansion for the total quantized scalar fields

$$\Phi(x) = \frac{1}{\sqrt{(2\pi)^3}} \sum_{k=1}^{N} \sqrt{A_k} e^{i\delta_k} \int \frac{d^3 p}{2\omega_k} \left[e^{-ipx} a_{pk} + e^{ipx} b^+{}_{pk}\right], \tag{22}$$

where $\omega_k = \sqrt{\vec{p}^2 + m_k{}^2}$, δ_k are some phases. The δ_k phases can give parameters of mixing for composite states. We use the indefinite metrics. The non-zero commutators of the a_{pk} (b_{pk}) annihilation operators for particles (antiparticles) and the $a^+{}_{pk}$ ($b^+{}_{pk}$) creation operators are the generalization of usual commutators for one particle:

$$\begin{gathered}
< \vec{p}_2, k_2 \mid \vec{p}_1, k_1 > = (-1)^{k_1+1} \delta_{k_1 k_2} 2\omega_{k_1} \delta(\vec{p}_1 - \vec{p}_2), \\
\left[a_{p_1 k_1}, a^+_{p_2 k_2}\right] = (-1)^{k_1+1} \delta_{k_1 k_2} 2\omega_{k_1} \delta(\vec{p}_1 - \vec{p}_2), \\
\left[b_{p_1 k_1}, b^+_{p_2 k_2}\right] = (-1)^{k_1+1} \delta_{k_1 k_2} 2\omega_{k_1} \delta(\vec{p}_1 - \vec{p}_2).
\end{gathered} \tag{23}$$

The commutator (21) can be expressed through the integral:

$$I(x) = \sum_{k=1}^{N} A_k I(\vec{x}, m_k) = \int d^3 p \sin \vec{p}\vec{x} \sum_{k=1}^{N} \frac{A_k}{\omega_k}. \tag{24}$$

We decompose the domain of integration in (24) on a sum of two domains: the D_{int} internal domain and the D_{ext} external domain. For example, the intervals of variations for the momentum components in the internal domain for different frames are following: 1) in the spherical frame $\left|\vec{p}\right| \leq R$, $R \gg m_N$; 2) in the cylindrical frame $0 \leq p_t \leq \Lambda$, $-A \leq p_3 \leq B$, $\Lambda \gg m_N$, $A \gg m_N$, $B \gg m_N$: 3) in the Descartes frame $-A_1 \leq p_1 \leq B_1$, $-A_2 \leq p_2 \leq B_2$, $-A_3 \leq p_3 \leq B_3$, $A_1, A_2, A_3, B_1, B_2, B_3 \gg m_N$. Similarly, the intervals of variations for the momentum components in the external domain for these frames are following: 1) in the spherical frame $\left|\vec{p}\right| \in [R,\infty)$; 2) in the cylindrical frame $p_t \in [\Lambda,\infty)$, $p_3 \in (-\infty,-A] \bigcup [B,\infty)$; 3) in the Descartes frame $p_1 \in (-\infty,-A_1] \bigcup [B_1,\infty)$, $p_2 \in (-\infty,-A_2] \bigcup [B_2,\infty)$, $p_3 \in (-\infty,-A_3] \bigcup [B_3,\infty)$.

The integrand in (24) is continuous function in the internal domain and integral is a restricted function of spatial coordinates. Therefore, in symmetrical internal domains these integrals vanish (e.g., in the spherical frame, in the cylindrical frame with $A = B$, and in the Descartes frame with $A_1 = B_1$, $A_2 = B_2$, $A_3 = B_3$). Now, we consider the integral (24) in the external domain. This integral can diverge on upper limits, i.e., at $p \to \infty$. Then the $1/\omega_k$ value can be expanded in the binomial series with respect to m_k / p, as $m_k / p \ll 1$. The integrand in (24) may be written at upper limits as:

$$sin\vec{p}\vec{x}\cdot\sum_{k=1}^{N}\frac{A_k}{\omega_k} = \frac{sin\vec{p}\vec{x}}{p}\sum_{k=1}^{N}A_k\left(1+\frac{m_k{}^2}{p^2}\right)^{-\frac{1}{2}} = \frac{sin\vec{p}\vec{x}}{p}\sum_{k=1}^{N}A_k\sum_{n=0}^{\infty}C^n_{-\frac{1}{2}}\left(\frac{m_k{}^2}{p^2}\right)^n = \\ = \frac{sin\vec{p}\vec{x}}{p}\left[\sum_{k=1}^{N}A_k - \frac{1}{2p^2}\sum_{k=1}^{N}A_k m_k{}^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2p^4}\sum_{k=1}^{N}A_k m_k^4 + \right], \quad (25)$$

where the $C^n_{-\frac{1}{2}}$ coefficients are the generalization of the binomial coefficients for non-natural β-numbers $C_\beta{}^n = \dfrac{\beta(\beta-1)(\beta-2)\cdots(\beta-n+1)}{n!}$. In consequence of the relations (17) ($\sum_{k=1}^{N}A_k = 0$ and $\sum_{k=1}^{N}A_k m_k{}^2 = 0$) the integrand (25) is given by at $N \geq 3$

$$\frac{sin\,\vec{p}\vec{x}}{p}\cdot\sum_{k=1}^{N}A_k\cdot\sum_{n=2}^{\infty}C^n_{-\frac{1}{2}}\left(\frac{m_k^2}{p^2}\right)^n. \quad (26)$$

Thus, the integral (24) in the external domain equals

$$I\left(\vec{x}\right)_{ext} = \int_{Dext}\frac{d^3p}{p}sin\,\vec{p}\vec{x}\sum_{n=2}^{\infty}C^n_{-\frac{1}{2}}\left(\frac{m_k^2}{p^2}\right)^n. \quad (27)$$

This integral converges. Therefore, the integral (24) converges in all the domain of the integration. Now, in consequence of that this integrand is odd function of the integration variables, the integral (24) and the commutator (21) vanish for space-like intervals at $N \geq 3$. Besides, we can write the important equality

$$\sum_{k=1}^{N}A_k\int\frac{d^3p}{\omega_k}e^{i\vec{p}\vec{x}} = \sum_{k=1}^{N}A_k\int\frac{d^3p}{\omega_k}e^{-i\vec{p}\vec{x}}. \quad (28)$$

CONCLUSIONS

We have shown that integral, corresponding to the commutator of usual scalar quantized fields on space-like intervals between the field coordinates [1-7], diverges. It is confirmed by explicit calculations. In consequence of this divergence, the commutator of scalar fields for one particle can be non-zero value. It means that the locality principle can be violated for these fields. But we believe that the causality principle must be valid for the commutator of the scalar fields. Therefore, we consider the total scalar fields, which correspond to the contributions of the kinds (or the families, or the dynasties) for spinless particles. The total fields are the solutions of generalized homogeneous Klein-Gordon equations proposed in [10,11]. We have shown that the commutator of the total scalar fields vanishes on space-like intervals between the field coordinates, when the number of generations is greater than or equals three. Thus, the locality principle is valid for this number of the generations in the kinds. Note that the same number of the generations in the kinds ensures the continuity of the causal Green functions for the generalized Klein-Gordon equations in all the space-time [12].

Besides the locality of the scalar fields, it is of interest study the locality for the spin-1/2 fields. Such studies are great of importance, as quarks have the spin ½. As it is known, hadrons consist of the quarks and the antiquarks.

The elimination of divergences in the commutator (21) of the total fields (22) is induced by the relations (17) and the use of the indefinite metrics. Possibly, the use of the relations (17) opens new capabilities for an elimination of some divergences in physical values. It is known that in the quantum theory of fields a lot of the infinity values exist. The divergences in physical values are eliminated by means of the renormalization procedures with infinity renormalization constants. Among such values the vacuum energy and the self energy operators (in particular, the polarization operator of photon for the contributions of charged particles) are important. All these values are related to the diverging integrals. Changes of fields for one particle on total fields (which include this particle) allow us to use the relations (17) and possibly to eliminate divergences. We may assume that some such values will have finite renormalization constants at the use of total fields. However, some values can be presented as sums of contributions for all particles from a kind with the same sign. As the example, we consider the polarization operator. In second order of a perturbation theory, with respect to the electromagnetic interactions, this operator is given by [7,15]

$$\Pi(q)_{\mu\nu} = -i\int d^4x e^{iqx} \left\langle 0 \middle| T^*(j(x)_\mu j(0)_\nu) \middle| 0 \right\rangle = \Pi(q)(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}), \tag{29}$$

where T^* is the covariant chronological product of the electromagnetic currents. In agreement with the optical theorem the imaginary part of the polarization operator is related to the total cross section of the electron-positron annihilation:

$$\sigma_{tot}(e^+e^- \to anything) = \frac{e^2}{q^4} Im\,\Pi(q). \tag{30}$$

Thus, the imaginary part of the polarization operator must be positive. In second order of the perturbation theory with respect to the electromagnetic interactions, the polarization operator is determined by the one-loop diagrams. The contribution of the total fields to the imaginary part of the polarization operator gives the sum of terms for the particles on their mass shells. These terms are positive, as they are products of positive value and the $A_k{}^2$-coefficient for the $k-$number of the generation determined in (16) and (22). For the spin-1/2 particles, instead of the $A_k{}^2$-coefficients, similar $B_k{}^2$-coefficients [10-12] must stand. Therefore, the contribution of the total fields to the imaginary part of the polarization operator must be positive.

For calculations of different values, in terms of the total fields, a lot of topics must be investigated. For example, the vacuum energy (and the density of the vacuum energy) for the total fields can be calculated with known the Lagrangians of the total fields and the energy-momentum tensors for the total fields. In relations with this we note that the Lagrangians for the total fields of the ½-spin have been derived in [16]. For calculations of the polarization operator and reaction amplitudes, the Lagrangians of interactions and the scattering matrix for total fields must be derived. In particular, the operators of the electric charge and currents of the electromagnetic interactions in terms of total fields have to be obtained.

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