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CABLE FREE TRANSMISSION OF ELECTRICITY: FROM NIKOLA TESLA TO OUR TIME

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Model of Earth charge resonant oscillations excitations based on Tesla experiment, was offered. Solutions of d'Alembert wave equations for electric and magnetic potentials of the charged perfectly conductive sphere were found. Graphic analyses of perturbed potential distribution on the Earth surface was provided. It was shown that obtained solution provides adequate description of Tesla experiment on wireless transfer of electricity conducted in Colorado Springs in 1899 – 1900. In the very low-frequency area of the electro-magnetic oscillations spectrum it was offered to consider the Earth as capacitors battery, consisting of two put one into another perfectly conductive spheres, between which thin dielectric layers is placed. Own oscillation frequencies of the Earth charge were determined in this area. It was shown that these frequencies most precisely correspond to experimentally measured Schumann resonances.
KEYWORDS: Tesla's experiment, electricity on the Earth, cable free transmission, potential distribution, very low-frequency of the electro-magnetic oscillations, Schumann resonance frequency

БЕСПРОВІДНА ПЕРЕДАЧА ЕЛЕКТРИКИ: ВІД МИКОЛИ ТЕСЛА ПО ТЕПЕРІШНІЙ ЧАС

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Запропоновано модель збудження резонансних коливань заряду Землі, заснована на експериментах Тесли. Знайдено рішення хвильових рівнянь Даламбера для електричного і магнітного потенціалу зарядженої, ідеально провідної сфери. Надано графічний аналіз розподілу збуреного потенціалу на поверхні Землі. Показано, що отримане рішення адекватно описує експерименти Тесли по бездротовій передачі електрики, що проведені в Колорадо-Спрінгс в 1899 - 1900 р.р.
У наднизькочастотній області спектра електромагнітних коливань запропоновано розглядати Землю як конденсаторну батарею, що складається з двох вкладених одна в іншу ідеально проводять сфер, між якими знаходиться тонкий діелектричний прошарок. У цій області частот визначені власні частоти коливань заряду Землі. Показано, що ці частоти найбільш точно відповідають експериментально виміряним першим резонансам Шумана.
КЛЮЧОВІ СЛОВА: експеримент Тесли, електрика Землі, бездротова передача, розподіл потенціалу, наднизькочастотні електромагнітні коливання, резонанси Шумана

БЕСПРОВОДНАЯ ПЕРЕДАЧА ЭЛЕКТРИЧЕСТВА: ОТ НИКОЛЫ ТЕСЛА ДО НАШЕГО ВРЕМЕНИ

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Предложена модель возбуждения резонансных колебаний заряда Земли, основанная на экспериментах Теслы. Найдены решения волновых уравнений Даламбера для электрического и магнитного потенциала заряженной, идеально проводящей сферы. Дан графический анализ распределения возмущенного потенциала на поверхности Земли. Показано, что полученное решение адекватно описывает эксперименты Тесла по беспроводной передаче электричества, проведенные в Колорадо-Спрингс в 1899 - 1900 г.г. В сверхнизкочастотной области спектра электромагнитных колебаний предложено рассматривать Землю как конденсаторную батарею, состоящую из двух вложенных одна в другую идеально проводящих сфер, между которыми находится тонкая диэлектрическая прослойка. В этой области частот определены собственные частоты колебаний заряда Земли. Показано, что эти частоты наиболее точно соответствует экспериментально измеренным первым резонансам Шумана.
КЛЮЧЕВЫЕ СЛОВА: эксперимент Теслы, электричество Земли, беспроводная передача, распределение потенциала, сверхнизкочастотные электромагнитные колебания, резонансы Шумана

Idea of wireless transfer of electric energy appeared with physicists and engineers almost simultaneously with formation of electrical engineering science as and independent area of techniques (1870-1890). During this and further periods of formation and development, the electrical devices are based on wire electric energy transfer. It is obvious that transfer of a large quantity of energy through wires constitutes a complicated task.

In case of wireless transfer of electric energy the task becomes much more complicated. However some economic advantages of this method (economy of conducting metal, construction materials, simplification of circuits architecture) give a basis for research of such method of electric energy transfer.

Nikola Tesla is considered to be the pioneer in the field of wireless transfer of electric energy [1].

In Colorado-Springs laboratory (1899-1900) Tesla conducted test of wireless energy transfer system, using the giant high-frequency transformer with frequency $\omega_0 = 1.5 \cdot 10^5$ Hz and wave length $\lambda_0 = 2 \cdot 10^3$ m for the first time [2].

In his experiments he switched on 200 electric bulbs placed 26 miles (≈ 42 km) away from his laboratory. Power of each bulb constituted 50W, and the total energy consumption constituted 10 kW or 13,6 h.p. According to Tesla calculations coefficient of efficiency of energy transfer constituted 95%. He asserted that by means of 300-power vibrator he could switch-on dozen of electric garlands each consisting of 200 bulbs placed in different parts of the earth.

Very soon after this experiments Tesla moved to New York, on Long Island and further did not work on the wireless electricity transfer topic.

German electrical engineer Winfried Otto Schumann studied the transfer of electric signals in the very-low frequency area of frequency spectrum. In 1952 he used the model of resonator of the Earth-ionosphere cavity for description of such wave processes [3]. Schumann discovered the resonance frequency of electric signals, which were later called "Schumann resonances". These resonances constitute set of picks in the frequency spectrum which are created by standing electromagnetic waves between the Earth conducting surface and external conducting border of the ionosphere. Dimensional analyses using the light speed c and length of earth circle $2\pi R_E$ allowed him to determine the frequency order for minimally possible resonance $\omega_E = c/2\pi R_E = 7.5$ Hz.

It is interesting to note that long before this in Colorado-Springs, Tesla, while studying the thunderstorm activity of the local area, also discovered the resonance fluctuations of the Earth electric field in this frequency range. But there were continuation of the experiments as he started the experiments on the wireless electricity transfer.

Thus the described above experiments demonstrate the possibility of wireless transfer of electricity and electrical signals either in high-frequency or in very low-frequency range of frequencies.

At present time the topic of Schumann resonances attracts the interest of researches not only from the point of their theoretical description but also due to the possibility of their practical implementation. Schumann resonances theory is presented in monograph [4]. Numerical methods of such resonances description based on their presentation in the form of standing waves in the Earth ionosphere provide the results comparable with real data [5].

From the practical point of view, Schumann resonances can be used for transfer of electrical signals on the long distances. They are closely connected with global thunderstorm activity of the Earth and correspondingly can become an instrument for climate research. Schumann resonances can also be involved for the research of changes in the lower ionosphere of the Earth, earthquake forecasts, and research of other celestial bodies' properties.

At present time another type of wire transfer of electric energy becomes relevant – wireless charging and power supply units.

Phenomenon of electromagnetic induction is used in such systems for energy transfer from the source (transmitter) to the receiver, which is explained by appearance of the electric current in the closed circle under change of magnetic flux running through this circle. However the distance of electric energy transfer is small under this method and cannot be compatible with methods presented above.

Summarizing the mentioned above it can be stated that practical solution of the problem of wireless transfer of electricity on the long distances, despite rather well-grounded theoretical study of this issue, remains in the condition close to initial stage.

Aim of paper. The present paper offers physical maximally approached to Tesla experiment condition, model of resonance oscillations of the Earth charge. Estimation of resonance frequencies in the high-frequency and very-low frequency area of wave spectrum were conducted using the proposed model.

THEORETICAL MODEL

Let's consider the model for conducting the wireless transfer of electricity. In this model, in the equilibrium state the Earth constitutes the rigid ideally conductive sphere on which the negative charge is evenly distributed. The equilibrium state means absence of charges motion in the sphere. Assumption on the ideal conductivity of the sphere is based on a rather high level of ground conductivity. High conductivity of the ground was demonstrated in Tesla experiments for many times and also is confirmed with its use for ground connection of different electrical devices

Estimations of the Earth charge from different sources provide the following value, for example, $q_E = -5.7 \cdot 10^5$ κ[6], or $q_E = -6.0 \cdot 10^5$ κ[7,8].

When calculating in equilibrium state, we will model the Earth as ideally conducting rigid sphere with surface charge $\sigma_E = Q_E / 4\pi R_E^2$ taking as a basis the Earth charge $Q_E = -6.0 \cdot 10^5$ C. Here $R_E = 6.3710 \cdot 10^6$ m – is an average radius of the Earth [6].

As it follows from Gauss theorem in the equilibrium state, intensity of electric field is equal to $\vec{E}_E(r) = \frac{Q_E}{r^2}\frac{\vec{r}}{r}$ for radiuses $r > R_E$. Potential corresponding to this field is specified with the expression $\varphi_E(r) = Q_E / r$.

Charge oscillations on the sphere e $\tilde{q}(\vec{r},t) = q(\vec{r},t) - Q_E$ caused from outside, result in formation of perturbed scalar potential $\tilde{\varphi}(\vec{r},t) = \varphi(\vec{r},t) - \varphi_E(r)$ and corresponding disturbed current density $\tilde{\vec{j}}(\vec{r},t)$.

Let's determine the relation of the perturbed charge with the perturbed potential which was realized in Tesla experiments.

In this experiments the perturbed charges were created by transfer of part of the Earth charge which, in the simplified form, can be presented as spherical capacitor with capacity C_E and full charge Q_E on the attached to it spherical capacitor with significantly lower capacity C_{Sp} $(C_{Sp} << C_E)$ and charge Q_{Sp}. Spherical capacitors with capacity C_E and C_{Sp} are connected to the battery in-parallel as the negative plates of the capacitors are connected to the same contact. When the capacitors are connected in-parallel the total charge is constant and is equal Q_E and the potential difference on the capacitors is the same, so the expressions are true:

$$\begin{gathered}\frac{\tilde{q}(\vec{r},t)}{C_{Sp}} = \varphi_E(R_E) + \tilde{\varphi}(\vec{r},t), \\ \frac{Q_E - \tilde{q}(\vec{r},t)}{C_E} = \varphi_E(R_E) + \tilde{\varphi}(\vec{r},t)\end{gathered} . \tag{1}$$

Connection of perturbed charge with perturbed potential $\tilde{q}(\vec{r},t) = -C_E \tilde{\varphi}(\vec{r},t)$ follows from the second equality (1).

Dependence of oscillations of scalar $\tilde{\varphi}(\vec{r},t)$ and vector $\tilde{\vec{A}}(\vec{r},t)$ potentials on perturbed charge density and current density on the sphere with capacity C_E is described with D'Alembert equations [9]:

$$\begin{gathered}\Delta\tilde{\varphi}(\vec{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\tilde{\varphi}(\vec{r},t) = -4\pi\tilde{\rho}(\vec{r},t), \\ \Delta\tilde{\vec{A}}(\vec{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\tilde{\vec{A}}(\vec{r},t) = -\frac{4\pi}{c}\tilde{\vec{j}}(\vec{r},t),\end{gathered} \tag{2}$$

on condition of gauge invariance

$$\mathrm{div}\tilde{\vec{A}}(\vec{r},t) = -\frac{1}{c}\frac{\partial}{\partial t}\tilde{\varphi}(\vec{r},t),$$

where $\tilde{\rho}(\vec{r},t) = \rho(\vec{r},t) - \rho_E$, $\tilde{\rho}(\vec{r},t) = \tilde{q}(\vec{r},t)/V_E$, $\rho(\vec{r},t) = q(\vec{r},t)/V_E$, $\rho_E = Q_E/V_E$,V_E - the Earth volume, c - light speed, Δ - Laplace operator, t - time, $\vec{r}$ - spatial value.

Solution of equations (2) should correspond to the condition of potentials limitation on the infinity: $\left|\tilde{\varphi}(\vec{r},t), \tilde{\vec{A}}(\vec{r},t)\right| = O(1/r)$ under $r \to \infty$, and also their limitation on the sphere surface: $\left|\tilde{\varphi}(\vec{r},t), \tilde{\vec{A}}(\vec{r},t)\right| < \infty$ under $r = R_E$.

It follows from the gauge invariance that in conditions of Tesla experiments, the oscillated vector potential $\tilde{\vec{A}}(\vec{r},t)$ is of the same order with the oscillated scalar potential as $\left|\tilde{\vec{A}}(\vec{r},t)\right| / \left|\tilde{\varphi}(\vec{r},t)\right| \approx \lambda_0\omega_0/c = 1$.

Thus while studying the task of wireless transfer of electricity, space time change of vector potential $\tilde{\vec{A}}(\vec{r},t)$ should be described together with the change in space of the time of scalar potential $\tilde{\varphi}(\vec{r},t)$.

It can be shown from the continuity equation and condition of gauge invariance taking into account $\tilde{\rho}(\vec{r},t) = -C_E \tilde{\varphi}(\vec{r},t)/V_E$, that $\tilde{\vec{j}}(\vec{r},t) = -c \cdot C_E \cdot \tilde{\vec{A}}(\vec{r},t)/V_E$.

Based on the task symmetry, the vector potential has only the radial component i.e. $\tilde{\vec{A}}(\vec{r},t) = (\tilde{A}_r(\vec{r},t), 0, 0)$.

Thus the equation of radial projection of the vector potential is transferred in the following way:

$$\Delta \tilde{A}_r(\vec{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\tilde{A}_r(\vec{r},t) = 4\pi \frac{C_E}{V_E}\tilde{A}_r(\vec{r},t).$$

Thus in the result of usage of the connections between the oscillated potentials and charges oscillation found above, the equations (2) can be presented in the following way:

$$\Delta \Psi(\vec{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Psi(\vec{r},t) = 4\pi \frac{C_E}{V_E}\Psi(\vec{r},t) \tag{3}$$

where $\Psi(\vec{r},t) = \{\tilde{\varphi}(\vec{r},t), \tilde{A}_r(\vec{r},t)\}$ - function, which possesses the value either of the first or of the second expression in braces, $|\Psi(\vec{r},t)| = O(1/r)$ under $r \to \infty$, $|\Psi(\vec{r},t)| < \infty$ under $r = R_E$.

Distorted electric and magnetic field strengths are determined with simple differentiation of scalar and vector potentials: $\vec{H}(\vec{r},t) = \operatorname{rot}\tilde{\vec{A}}(\vec{r},t)$, $\tilde{\vec{E}}(\vec{r},t) = -\operatorname{grad}\tilde{\varphi}(\vec{r},t) - c^{-1}\partial\tilde{\vec{A}}/\partial t$.

SOLUTION FOR THE DISTORTED ELECTRIC POTENTIAL

Let us consider the equation (3) for the distorted electrical potential of the charged rigid sphere $\tilde{\varphi}(\vec{r},t)$ on condition $|\tilde{\varphi}(\vec{r},t)| < \infty$.

We suppose that its dependence on time in the form $\tilde{\varphi}(\vec{r},t) = \tilde{\varphi}'(\vec{r}) \cdot \exp(-i\omega_0 t)$.

Then the equation (3) for electric potential is transformed into Helmholtz equation [10] for the the perturbed potential:

$$\Delta \tilde{\varphi}'(\vec{r}) + k^2 \tilde{\varphi}'(\vec{r}) = 0 \tag{4}$$

where $k^2 = \frac{\omega_0^2}{c^2} - 4\pi \frac{C_E}{V_E} = \frac{\omega_0^2}{c^2}\varepsilon(\omega_0)$, $\varepsilon(\omega_0) = \left(1 - \frac{\Omega_e^2}{\omega_0^2}\right)$, $\Omega_e = \sqrt{4\pi c^2 C_E/V_E} \approx 81.189$ Hz – analogue of Langmuir frequency of sphere electron oscillation, the prime sign is further omitted in notation of distorted potential.

It should be noted that Ω_e frequency is calculated for ideally conducting sphere placed in vacuum when $C_E = R_E$.

In the real situation, the Earth can be presented as capacitors battery consisting of two put one into another spheres with a layer with dielectric capacity $\varepsilon(\omega_0)$ and thickness $d << R_E$ placed between them. Capacity of such capacitors battery will be equal to $C_{EB} = \varepsilon(\omega_0) S_E / 4\pi d$, where $S_E = 4\pi R_E^2$ - square of the Earth surface.

This precise definition as to the capacity of the Earth charge does not influence further discussions, but will be used below in order to match the obtained theoretical visions with experimental data.

Lets find the solution of equation (4) for the perturbed scalar potential $\tilde{\varphi}(\vec{r})$.

For this we will pass to spherical coordinates:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \tilde{\varphi}}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \tilde{\varphi}}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 \tilde{\varphi}}{\partial \varphi^2} + k^2 \tilde{\varphi} = 0 \tag{5}$$

where $r \ge R_E$, $0 \le \theta \le \pi$ - polar angle, $0 \le \varphi \le 2\pi$ - azimuth angle.

Expression for the perturbed potential we will present in the form of product of two functions depending on radial and angle variables (variable separation method):

$$\tilde{\varphi}(r,\theta,\varphi) = R(r)Y(\theta,\varphi) \tag{6}$$

After insertion of this expression into equation (6) we receive:

$$\begin{gathered}\frac{1}{R(r)}\frac{\partial}{\partial r}\left(r^2\frac{\partial R(r)}{\partial r}\right)+k^2r^2=-\frac{1}{Y(\theta,\varphi)\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial Y(\theta,\varphi)}{\partial\theta}\right)-\\-\frac{1}{Y(\theta,\varphi)\sin^2\theta}\frac{\partial^2 Y(\theta,\varphi)}{\partial\varphi^2}=\nu(\nu+1)\end{gathered}. \tag{7}$$

where $\nu(\nu+1)$ – is a constant.

Equations for functions, depending on angles follow from (7):

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial Y(\theta,\varphi)}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2 Y(\theta,\varphi)}{\partial\varphi^2}+\nu(\nu+1)Y(\theta,\varphi)=0 \tag{8}$$

Function depending on radius:

$$\frac{d}{dr}\left(r^2\frac{dR(r)}{dr}\right)+\left(k^2r^2-\nu(\nu+1)\right)R(r)=0 \tag{9}$$

First we will find the solutions of equation (8) for the angles.

In order to divide the variable we assume

$$Y(\theta,\varphi)=\Theta(\theta)\Phi(\varphi). \tag{10}$$

Then from (9) we receive:

$$\sin^2\theta\left[\frac{1}{\Theta(\theta)\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta(\theta)}{\partial\theta}\right)+\nu(\nu+1)\right]=-\frac{1}{\Phi(\varphi)}\frac{\partial^2\Phi(\varphi)}{\partial\varphi^2}=\mu^2 \tag{11}$$

where μ - real constant.

Thus from (11) we have two equations for determination of dependence of perturbed potential on angles θ and φ:

$$\sin^2\theta\left[\frac{1}{\Theta(\theta)\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta(\theta)}{\partial\theta}\right)+\nu(\nu+1)\right]=\mu^2 \tag{12}$$

$$\frac{\partial^2\Phi(\varphi)}{\partial\varphi^2}=-\mu^2\Phi(\varphi). \tag{13}$$

After insertion of $\cos\theta=x$, the equation (12) is transferred to the following form:

$$\left(1-x^2\right)\frac{d^2\Theta(x)}{dx^2}-2x\frac{d\Theta(x)}{dx}+\left[\nu(\nu+1)-\frac{\mu^2}{1-x^2}\right]\Theta(x)=0. \tag{14}$$

For real $\Theta(x)$, x under integral $\mu=m=0,1,2,3....$ and $\nu=n=0,1,2,3....$ the equation (14) has the solution in the form of attached Legendre polynomials of the n order, rank m :

$$P_{n,m}(x)=\left(1-x^2\right)^{\frac{m}{2}}\frac{d^mP_n(x)}{dx^m}=\left(1-x^2\right)^{\frac{m}{2}}\frac{1}{2^n n!}\frac{d^{n+m}}{dx^{n+m}}\left[\left(x^2-1\right)^n\right], \tag{15}$$

or in the integral form:

$$P_{n,m}(x)=\frac{(n+1)...(n+m)}{\pi i^m}\int_0^\pi\left(x+\sqrt{x^2-1}\cos\psi\right)^n\cos m\psi\, d\psi. \tag{16}$$

The attached Legendre polynomials are characterized with the following properties:

$$\begin{aligned}&P_{n,0}(x)=P_n(x), \quad m=0;\\&P_{n,m}(x)=0, \qquad m>n,\end{aligned} \tag{17}$$

where $P_n(x)$ - Legendre polynomial of the n order.

As it follows from (17) for the attached Legendre polynomials $m=0,1,2,3....,n$ on condition $m\le n$.

Equation (13) for integral value $\mu=m$ is integrated in elementary functions:

$$\Phi(\varphi) = C_1 \cos m\varphi + C_2 \sin m\varphi . \qquad (18)$$

Further we calculate that $C_2 = 0$.

Let us find the solution of equation (9) for radial dependence of perturbed potential.

Substitution of a new variable $R(r) = r^{-\frac{1}{2}} U(r)$ into equation (9) makes it the following:

$$\frac{d^2U(r)}{dr^2} + \frac{1}{r}\frac{dU(r)}{dr} + \left(k^2 - \frac{(n+1/2)^2}{r^2}\right)U(r) = 0 . \qquad (19)$$

Bessel and Neumann functions of the half-integer order [10, 11] constitute the solution of equation (18). Common solution of the original equation (8) has the following view:

$$R(r) = \left[A_1 J_{n+1/2}(kr) + A_2 N_{n+1/2}(kr)\right] / \sqrt{kr} \qquad (20)$$

where A_1 and A_2 - are the constants.

We consider that A_2 is equal to zero, as when argument kr tends to zero (on the sphere surface under $r = R_E$ it is possible when $k \to 0$) the solution should be limited.

Thus we will write the expression for the perturbed electric potential of the charged sphere:

$$\tilde{\varphi}(r,\theta,\varphi) = B_1 (kr)^{-\frac{1}{2}} J_{n+1/2}(kr) P_{n,m}(x) \cos m\varphi \qquad (21)$$

where: $B_1 = A_1 C_1$ - constant determined from border conditions for the solution dependence on radius ; $x = \cos\theta$.

SOLUTION FOR SCALAR AND VECTOR POTENTIALS. CONNECTION TO THE CONDITIONS OF TESLA EXPERIMENT

Scalar potential

Solution (20) should correspond to the border conditions on the sphere surface. As it follows from the experiments conducted by Tesla in Colorado-Springs, perturbations of the Earth charge can be schematically described in the following way: conductor was connected with one end to the specific point of the conductive sphere (the Earth) with capacitance C_E. Capacitor with capacity C_{Sp} was connected to the second part of this conductor with length $l << R_E$. By means of a transformer of his own construction Tesla transferred a specific part of the charge on the capacitor C_{Sp} and then again injected it on the capacitor C_E. Thus at the specific moments of transformer operation, potential was imposed in the specified point on the sphere surface C_E :

$$\tilde{\varphi}(R_E, \theta_0, \varphi_0) = \Phi_0 , \qquad (22)$$

where Φ_0 - amplitude of the imposed in Tesla experiment potential ($|\Phi_0| << |Q_E / R_E| \approx 10^9$ V).

Based on the border condition (22) the constant in the expression for the perturbed potential on the Earth surface (21) in the point with angle coordinates, for example, $\theta_0 = 0, \varphi_0 = 0$ is determined with the expression:

$$B_1 = \frac{\sqrt{kR_E}}{J_{1/2}(kR_E)}\Phi_0 \qquad (23)$$

Thus the solution (21) with a constant (23) describes distribution of the perturbed potential on the Earth surface.

It should be noted that setting of the coordinate of the point of application of the perturbed potential on the Earth surface is automatically determined the position of the pole of the spherical system of coordinates: $r = R_E, \theta_0 = 0, \varphi_0 = 0$.

It goes from the view of the border condition (22), in solution (21) it should supposed that $m = 0$. In this case the perturbed potential is described with Legendre polynomial and does not depend on azimuth angle φ.

Vector potential

We will consider the border conditions for vector potential have the view (22) where it is necessary to replace $\tilde{\varphi}(R_E, \theta_0, \varphi_0) \to \tilde{A}_r(R_E, \theta_0, \varphi_0)$ and $\Phi_0 \to J_0$ where J_0 - amplitude of the currency density in Tesla experiments in the point on pole with coordinates: $r = R_E, \theta_0 = 0, \varphi_0 = 0$.

Then we will write the solution for radial projection of the vector potential:

$$\tilde{A}_r(r,\theta,\varphi) = D_1(kr)^{-\frac{1}{2}} J_{n+1/2}(kr) P_{n,m}(x)\cos m\varphi, \tag{24}$$

where $D_1 = \dfrac{\sqrt{kR_E}}{J_{1/2}(kR_E)} J_0$.

Based on the experimental data, taken from [12], meanings of perturbed potential amplitudes and currency density can be estimated: $\Phi_0 \approx (3.5....4.0)\cdot 10^6$ V, $J_0 \approx 300/S_0$ A/mm^2 where S_0 current-carrying square in the cross-section of the conductor of the secondary coil of Tesla transformer (in mm^2)

Graphic analyses of distribution of the perturbed electric potential on the Earth surface

Let us build the diagram of potential distribution on the Earth surface in the spherical coordinates system Potential on the Earth surface will be presented in the form of sum of the Earth electrostatic potential and its perturbation:

$$\varphi(R_E,\theta,\varphi) = \varphi_E(R_E) + \tilde{\varphi}(R_E,\theta,\varphi). \tag{25}$$

In the process of graphic analyses, we will impose the electrostatic potential of the Earth $\varphi_E(R_E)$ with the unit radius sphere. We will consider that $|\tilde{\varphi}(R_E,\theta,\varphi)|/|\varphi_E(R_E)| << 1$.

In the numerical calculations, the potential in dimensionless units $\hat{\varphi}(R_E,\theta,\varphi)$ was imposed with the expression:

$$\hat{\varphi}(R_E,\theta,\varphi) = 1 + \tilde{\varphi}(R_E,\theta,\varphi)/\varphi_E(R_E) = 1 + Amp\cdot\tilde{\varphi}(R_E,\theta,\varphi). \tag{26}$$

The Figure presents the results of the Earth potential from different meanings n for the amplitude of perturbed potential $Amp = 0.1$.

Under $n = 0$ the perturbed potential does not depend on angles θ and φ and one sign relatively to the Earth equilibrium potential.

Dependence of the sign of the perturbed potential on the number $n \geq 1$ is presented on Fig.1 below.

It follows from Fig.1 that perturbed potential of the Earth on the upper pole of the sphere ($\theta_0 = 0, \varphi_0 = 0$) is determined with condition (22) i.e. is equal to Φ_0.

It goes from the figures that number of zones N of one sign on the sphere width is equal $N = n+1$, where n is a number of zeros of Legendre polynomials $P_n(x)$ in the interval $-1 \leq x \leq +1$ [10, 11].

n				
1	2	3	4	5

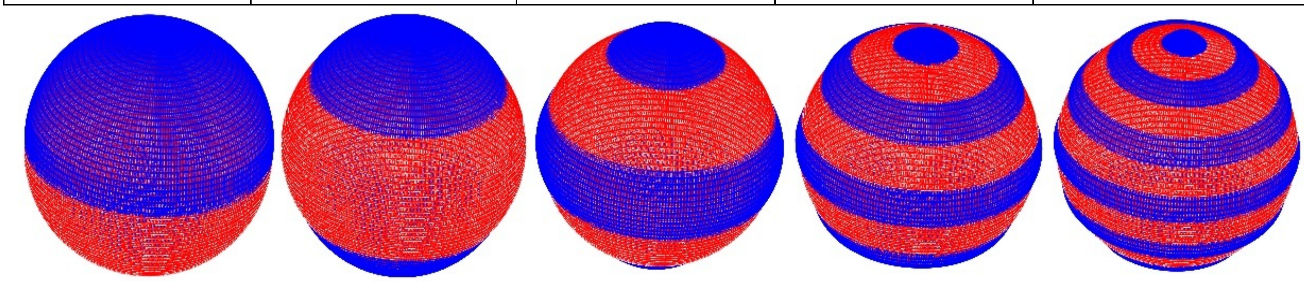

Figure. Dependence of sign of potential distribution on the Earth surface on the n number for $Amp = 0.1$.

In Tesla experiments the meaning of n number should be rather big: $n = n_0 >> 1$ as the Tesla generator wave length on the sphere width constitutes $\lambda_0 = 2$ km [2]. This wave length can be determined for relation $\lambda_0 = \lambda_{n_0} = 2\pi R_E/(n_0+1)$. It goes from this that n_0 is rather big : $n_0 \approx 2\pi R_E/\lambda_0 \approx 20015 >> 1$.

Thus in case of ideally conductive sphere the Tesla generator imposes the own charge oscillations placed on the conductive rigid sphere. Due to the task symmetry, the azimuth oscillations mode is equal to zero $m = 0$. Tesla generator frequency ω_0 is connected with wave length of the perturbed potential on the width λ_0 with relation:

$$\omega_0 = \omega_{n_0} = \frac{c}{\lambda_0} = \frac{c(n_0+1)}{2\pi R_E}. \tag{27}$$

In common case the frequencies and wave lengths of the own high-frequency oscillations of the ideally conductive charge sphere are determined from the equalities:

$$\omega_n = \frac{c}{\lambda_n} = \frac{c(n+1)}{2\pi R_E}. \tag{28}$$

Schumann resonances and Tesla experiments in wireless transfer of electricity

In capacitors battery the oscillation frequencies or Schumann resonances [3], $\tilde{\omega}_n$ slightly differ in relative numbers from the frequencies of the own oscillations of the conductive sphere, i.e. $\tilde{\omega}_n = \omega_n + \Delta_n$ where $|\Delta_n| << \omega_n$. The mentioned above divergence between the frequencies is presented in Table [13].

Table.

Comparison of Schumann resonances with the own frequencies of the ideally conductive sphere

Frequency (Hz)	Number of mode n				
	0	1	2	3	4
Schumann resonances[13], $\tilde{\omega}_n$	7.8	14.3	20.8	27.3	33.8
Sphere resonances, ω_n	7.5	15	22.5	30	37.5
Divergence, $\Delta_n = \tilde{\omega}_n - \omega_n$	0.3	-0.7	-1.7	-2.7	-3.7

Let us consider the reasons of divergences of resonance Schumann frequencies from the own frequencies of the conductive sphere.

For this the dielectric permittivity of the layer in spherical capacitor can be conditionally presented in the form:

$$\varepsilon(\tilde{\omega}_n) = \left(1 - \frac{\Omega_n^2}{\tilde{\omega}_n^2}\right) \tag{29}$$

where Ω_n - own oscillations frequencies of the capacitors battery, $\Omega_n^2 = (1+\alpha_n)\tilde{\omega}_n^2, \alpha_n << 1$.

Expression (29) describes the input of dielectric layer into the own frequencies of the ideally conductive sphere.

For the capacitors spherical battery on resonance frequencies we have:

$$\left(k_n^2\right)_B = \frac{\tilde{\omega}_n^2}{c^2} - \frac{4\pi C_{EB}}{V_E} = \frac{\tilde{\omega}_n^2}{c^2} - \frac{3\varepsilon(\tilde{\omega}_n)}{R_E d}. \tag{30}$$

For the ideally conductive sphere:

$$\left(k_n^2\right)_S = \frac{\omega_n^2}{c^2}. \tag{31}$$

Second summand in the right part (31) $4\pi C_E / V_E$ is not taken into consideration due to its smallness in relation to $4\pi C_{EB} / V_E$.

The wavelength of the ideally conductive sphere and capacitors battery is determined with a number of zeros of Legendre function. Equality $\left(k_n^2\right)_B = \left(k_n^2\right)_S = (n+1)^2 / (2\pi R_E)^2$ follows from this.

From the expressions (30) and (31), assuming that $\tilde{\omega}_n = \omega_n + \Delta_n$, where $|\Delta_n| << \omega_n$ we will determine the conditions under which the displaced, due to the presence of dielectric layer resonance frequencies of the ideally conductive ionosphere, correspond to Schumann resonances:

$$\Delta_n = -\frac{6\pi^2}{(n+1)^2} \frac{R_E}{d} \alpha_n \omega_n. \tag{32}$$

Assuming, for example, that the thickness of dielectric layer is small ($d << R_E$), from Table 1 and equality (32) we will determine the meaning of parameter α_n :

$$\alpha_n = \frac{1}{6\pi^2} \frac{d}{7.5R_E} (n-0.3)(n+1). \tag{33}$$

Expression for lowest Schumann resonances $\tilde{\omega}_n$ follows from (32) and (33):

$$\tilde{\omega}_n = \omega_n \left(1 + \frac{(0.3-n)}{7.5(n+1)} \right), \tag{34}$$

where $n = 0,1,2,3,\dots$.

Expression (34), unlike presented in [3] expression $\tilde{\omega}_n = 6.0\sqrt{n(n+1)}$, determines Schumann resonances, presented in Table 1, with a high level of accuracy.

Taking into account the daily frequency changes of the first harmonic of Schumann resonance, the figure 7,5 in the expression (34) can obtain the meanings in the ranges from 7 to 11 [14].

Thus, the present section provides determination of resonance of very low-frequency oscillations of the charged spherical capacitor with dielectric layer, which correspond to Schumann resonances.

CONCLUSIONS

Model of the Earth charge resonance oscillations excitation based on Tesla experiment was offered.

The task is directed on the research of D'Alembert equations for electric and magnetic potentials of the charged ideally conductive sphere. Based on the offered model connection of the perturbed charge and currency density with perturbed potentials was determined. It was shown that for high-frequency, harmonically changing in time disturbances of potentials the D'Alembert equations are transferred in Helmholtz equations, solutions of which in axial-symmetric case are expressed through Legendre polynomials. Substitution of border conditions into obtained solutions allow to describe the dependence of the Earth perturbed charged on the polar angle. Based on the found solution, graphic analyses of perturbed potential distribution on the ideally conductive Earth surface was provided. Obtained solution describes adequately Tesla experiments on the wireless electricity transfer, conducted in Colorado-Springs in 1899-1900.

In the very low-frequency spectrum area the Earth constitutes capacitors battery consisting of two placed one in another spheres, with a thin layer with dielectric permittivity $\varepsilon(\omega_0)$ and thickness $d << R_E$ placed between them. The own oscillations frequencies of the Earth charge were determined in this frequencies range. It was shown that value of these frequencies most precisely corresponds to experimentally measured Schumann resonances.

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