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PLASMA WALL TRANSITION AND EFFECTS OF GEOMETRY IN PRESHEATH

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When plasma interacts with the wall of a conductor, electrons due to high mobility reach the wall first and develop negative potential on the wall and very near to the wall plasma is divided into sheath and presheath regions. The quasi-neutral plasma is shielded from the wall by a space charge sheath of the positive ions of the order of few electrons Debye's lengths (λ_D). At the sheath edge quasi neutrality breaks down from presheath side. In asymptotic limit $\varepsilon = \lambda_D/L \to 0$ varying area of geometry affects the structure of the presheath scale. In addition to geometry, collisions and ionization also affects the presheath structure. But the sheath region is universal and is independent of either of geometry, ionization rate and collision frequency. The region which play the role of a link between these two regions has characteristics of both regions and is known as intermediate region. Even in the absence of ionization source and collision expanding area of geometry can accelerates the ions towards the wall. The characteristic length of the geometric presheath depends on radius of curvature $R_c = A/A'$, where " A " is the area of geometry and " $A' = dA/dz$ ". If either of ionization or collisions is present along with the expanding area of geometry then dominant factor for the acceleration of ions in the presheath region is not the expanding area of geometry.
KEYWORDS: Debye's length, sheath, presheath, Bohm's criterion, ion acoustic speed, Boltzman's relation, Tonks Langmuir problem

When plasma interacts with the wall of a conductor, the electrons due to high mobility reach the wall first and develop negative potential on the wall and very near to the wall plasma is divided into two regions. The quasi-neutral plasma is shielded from the wall by a space charge sheath of the positive ions of the order of few electrons Debye's lengths (λ_D) [1]. Langmuir in history for the first time in 1929 used the terms plasma and sheath for a gas discharge [2]. Up to 1951 all the work on plasma wall transition was done assuming collisionsless plasma. Boyd in 1951 for the first time using diffusion controlled theory for collision dominated plasma, introduced Bohm's criterion [3]. Allen and Thonemann in 1954 confirmed Bohm's criterion in equality form and stated boundary conditions for sheath and the presheath region for the case of marginal form of Bohm's criterion. Harrison and Thompson in 1959 analytically solved Tonks Langmiur problem and find kinetic Bohm criterion [4]. According to this criteria for planar geometry at the sheath edge ions must have speed equal to ion acoustic speed $(v_i = c_S)$ in order to enter in the sheath region. This condition is known as Bohm's criteria in marginal form [1]. It was investigated by Riemann as well as by Sternberg that when the area expands or contracts then the standard Bohm's criterion must be changed and non-marginal form of Bohm's criteria is used [5]. The reason of the non-marginal Bohm's criterion is that as geometry contracts pressure increases, so density increases and velocity must be less than ion sound velocity at the sheath edge, in order to conserve steady state current density following towards the wall [4,6]. The region in which ions are accelerated to ion acoustic speed (c_S) is called the presheath region. In the presheath region number density of electrons and ions is equal, i.e, $(n_i = n_e)$. The presheath is dominated by either of the four basic mechanism which are collision of neutrals with ions, geometric current concentration, applied magnetic field, and ionization rate [7]. This paper clarifies the effects of collisions, ionization and especially of geometry on presheath. As the characteristic length of the geometric presheath depends on radius of curvature $R_c = A/A'$, where " A " is the area of geometry and $A' = dA/dz$ [8]. So, we can define three types of geometries depending on change in area of the geometry. If $A' = 0$ we have planar geometry, for $A' > 0$ we have widening geometry and for $A' < 0$ we have contracting geometry [1]. In this paper only widening geometry of spherical and cylindrical shape will be discussed. We have proved mathematically and graphically that the expanding area of geometry can accelerate charged particles. There are many practical applications in which effects of widening geometry are important. Some of these applications are plasma nozzle, plasma probe, limiters and diverters of tokamak, magnetoplasmadynamic thruster, Hall thruster, and the variable specific impulse magnetoplasma rocket (VASIMR) [9,10,11].

BASIC MODEL AND EQUATIONS

In this paper the effects of expanding area of geometry on one dimensional plasma (along z-axis) in front of a wall is discussed. Assuming the wall at $z = z_w$ and sheath edge at $z = x = 0$. As $z \to -\infty$ we have quasi neutral plasma [5]. On the left side of the sheath edge is the presheath and on right side is sheath. The ionization rate in the plasma depends on the source of ionization. There are collisions in the plasma of mean free path λ. Our case is time independent. We are assuming hot ions $(T_i \neq 0)$ in the absence of applied magnetic field. The electric field $\mathbf{E}$ is in z direction only. As shown in the Fig. 1 below.

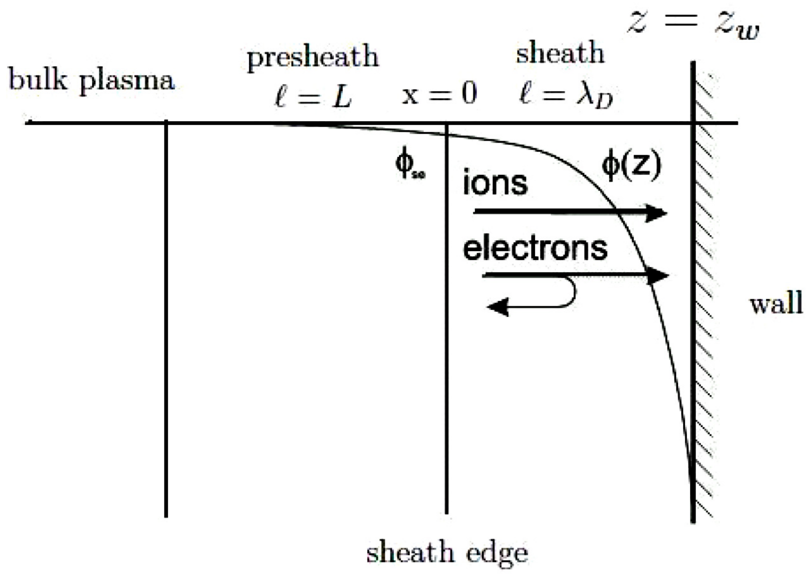

Fig. 1. Plasma wall interaction

For one dimensional case

$$\mathbf{v_i} = v_i \mathbf{e_z}, \qquad \nabla = \mathbf{e_z}\frac{d}{dz}, \qquad div = \frac{1}{A}\frac{dA}{dz} = \frac{A'}{A}. \tag{1}$$

Where A is the area, which specify the geometry of the plasma. For spherical shape we have $A = 4\pi(R \pm z)^2$ [12]. Here the plasma region is described by $z < 0$ and sheath edge is at $z = 0$. Momentum balance equation for such a system is of the form,

$$m_i v_i \frac{dv_i}{dz} + e\frac{d\phi}{dz} + \frac{\gamma_i k T_i}{n_i}\frac{dn_i}{dz} = -\left(v_{ci} + \frac{S_i}{n_i}\right) m_i v_i. \tag{2}$$

Equation of continuity, in one dimension along *z* axis is,

$$\frac{d}{dz}(n_i v_i) = S_i - \frac{A'}{A} n_i v_i. \tag{3}$$

Poisson's equation is,

$$\frac{d^2\phi}{dz^2} + \frac{A'}{A}\frac{d\phi}{dz} = -\frac{e}{\varepsilon_0}(n_i - n_e) \tag{4}$$

and Boltzman's relation for electrons is,

$$n_e = n_0 e^{(e\phi/kT_e)}. \tag{5}$$

Normalization

The normalization variables are,

$$s = \frac{z}{\ell}, \quad \varphi = -\frac{e\phi}{kT_e}, \quad \hat{n}_{i,e} = \frac{n_{i,e}}{n_0}, \quad u = \frac{v_i}{c_s}, \quad v(u) = \frac{L}{c_s} v_{ci},$$
$$\sigma = -\frac{L}{c_s}\frac{S_i}{n_s}, \quad a = \frac{L}{A}\frac{dA}{dz}, \quad \tau = \frac{\gamma T_i}{T_e + T_i}, \tag{6}$$

where

$$c_s = \sqrt{\frac{kT_e + \gamma kT_i}{m_i}}, \qquad \lambda_D = \sqrt{\frac{\varepsilon_0 kT_e}{n_0 e^2}}. \tag{7}$$

Using eqn. (6) and eqn. (7) our normalized set of equations is,
Boltzman's relation

$$\hat{n}_e = e^{-\varphi}. \tag{8}$$

Poission's equation.

$$\hat{n}_i - \hat{n}_e = \frac{\lambda_D^2}{\ell^2}\left(\frac{d^2\varphi}{ds^2} + a\frac{\ell}{L}\frac{d\varphi}{ds}\right). \tag{9}$$

Continuity equation.

$$\frac{d}{ds}\left(\hat{n}_i u\right) = \frac{\ell}{L}\left(\sigma - a\hat{n}_i u\right). \tag{10}$$

Momentum balance equation.

$$u\frac{du}{ds} - \frac{d\varphi}{ds} = -\frac{\ell}{L}\left(v + \frac{\sigma}{\hat{n}_i}\right)u - \tau\left(\frac{1}{\hat{n}_i}\frac{d\hat{n}_i}{ds} - \frac{1}{\hat{n}_e}\frac{d\hat{n}_e}{ds}\right). \tag{11}$$

Eqn. (8), eqn. (9), eqn. (10), and eqn. (11), i.e, Boltzman's relation, Poission's equation, continuity equation, and momentum balance equation respectively are basic equation of our model in normalized form. It is important for practical applications to study the transition from the presheath to sheath region for $\varepsilon = \lambda_D / L \to 0$ [1].

Asymptotic sheath theory

In inner region or sheath region scale length is Debye length $\ell_i = \lambda_D$. In this region the space coordinate changes to $\xi = s_i = z/\lambda_D$. So, Poisson's eqn. (9) for sheath region in limiting case $\varepsilon = \lambda_D / L \to 0$, is

$$\hat{n}_i - \hat{n}_e = \frac{d^2\varphi}{d\xi^2}. \tag{12}$$

Continuity eqn. (10) for sheath region in limiting case $\varepsilon = \lambda_D / L \to 0$, becomes,

$$\frac{d}{d\xi}\left(\hat{n}_i u\right) = o\left(\varepsilon\right) \tag{13}$$

and momentum eqn. (11) for sheath region in limiting case $\varepsilon = \lambda_D / L \to 0$, becomes,

$$u\frac{du}{d\xi} - \frac{d\varphi}{d\xi} + \tau\left(\frac{1}{\hat{n}_i}\frac{d\hat{n}_i}{d\xi} - \frac{1}{\hat{n}_e}\frac{d\hat{n}_e}{d\xi}\right) = o\left(\varepsilon\right). \tag{14}$$

If we plot φ verses ξ, at the sheath edge $\varphi \to 0$ [12].

Asymptotic presheath theory

In outer region or presheath region we have scale length $\ell_0 = L$. In this region the space coordinate is $x = s_0 = z/L$. Poission's eqn. (9) for the presheath region in limiting case $\varepsilon = \lambda_D / L \to 0$, is

$$\hat{n}_i - \hat{n}_e = o\left(\varepsilon^2 \frac{d^2\varphi}{dx^2}\right). \tag{15}$$

Continuity eqn. (10) for the presheath region becomes

$$\frac{d}{dx}\left(\hat{n}_i u\right) = \sigma - a\hat{n}_i u, \tag{16}$$

and momentum eqn. (11) for the presheath region in limiting case $\varepsilon = \lambda_D / L \to 0$, using eqn. (16) after simplification becomes,

$$\left(u-\frac{1}{u}\right)\frac{du}{dx}=a-\frac{\sigma}{\hat{n}_i}\left(u+\frac{1}{u}\right)-vu. \tag{17}$$

Eqn. (17) shows when $u=1$, we have a singular point and this point is known as sheath edge. It is the point at which quasi neutrality breaks down from the presheath side. If we plot φ verses x, at the sheath edge $\varphi\to\infty$ [13]. It has been observed that the asymptotic solutions of the potential profiles for plasma presheath and sheath for very small but finite value of λ_D/L do not match smoothly. The asymptotic presheath solution at the sheath edge turns into a singularity $d\phi/dz\to-\infty$ and on the other hand on sheath scale (z/λ_D) the sheath edge is characterized by $(d\phi/dz\to 0)$ [14]. In order to have smooth transition intermediate scale had been used in literature.

Intermediate scale

For intermediate scale, space coordinate is ζ and is defined as $\zeta=z/\ell_m$. Using $\varphi=0$, $u=1$, $\hat{n}_i=\hat{n}_e=1$,and $\frac{d\hat{n}_e}{d\varphi}=\frac{d\hat{n}_i}{d\varphi}=-1$ which are values of different variables at the sheath edge [8]. Hence at the sheath edge eqn. (17) can be written as,

$$\left(u-\frac{1}{u}\right)\frac{du}{dx}=a_s-2\sigma_s-v_s. \tag{18}$$

Where "s" denotes the value of different variables at the sheath edge. Using Boltzman's relation for electrons, "u"can be written as $u=1+\varphi$. At $u=1$ eqn. (18) shows a singularity. For singular transition right hand side of eqn. (18) can be taken equal to – 1. Substituting the value of $u=1+\varphi$ and after simplification eqn. (18) becomes

$$\varphi^2+x=o\left(\varepsilon^2\frac{d^2\varphi}{dx^2}\right). \tag{19}$$

Using Boltzman's relation, eqn. (14) which is the momentum equation for sheath region and Poission's equation, i.e, eqn. (12) we get

$$\frac{d^2\varphi}{d\xi^2}=\frac{\varphi^2}{1-\tau}+o(\varepsilon\xi). \tag{20}$$

The relation between space coordinates of sheath, presheath and intermediate scale is $\left(\zeta=\frac{z}{\ell_m}=\frac{L}{\ell_m}x=\frac{\lambda_D}{\ell_m}\xi\right)$. Using this relation eqn. (19) for intermediate scale is

$$\varphi^2+\frac{\ell_m}{L}\zeta=o\left(\frac{\lambda_D^2}{\ell_m^2}\frac{d^2\varphi}{d\zeta^2}\right), \tag{21}$$

and eqn. (20) is,

$$\frac{\lambda_D^2}{\ell_m^2}\frac{d^2\varphi}{d\zeta^2}=\frac{\varphi^2}{1-\tau}+o\left(\frac{\ell_m}{L}\zeta\right), \tag{22}$$

Eqn. (20) and eqn. (22) have uniform representation so, using this property and $w=\left(\frac{L}{\ell_m}\right)^{\frac{1}{2}}\varphi=-\left(\frac{L}{\ell_m}\right)^{\frac{1}{2}}\frac{e\phi}{kT_e}$, $\ell_m=(1-\tau_s)^{2/5}\lambda_D^{4/5}L^{1/5}$ we get [9],

$$\frac{d^2w}{d\zeta^2}=w^2+\zeta. \tag{23}$$

Equation (23) is universal differential equation for intermediate scale which had been widely used in literature to explain the variation of potential and electric field in intermediate scale. Eqn. (23) for inner expansion of outer solution gives presheath solution and for outer expansion of inner solution gives sheath solution [5,13,14]. So, far we have discussed some of the basic equations in plasma-wall transition which have already been developed to explain the mechanism of plasma wall transition. In the following section a new equation will be developed using equations of presheath and intermediate scale which will explain effect of geometry in presheath.

Generalized equation for Presheath

The generalized equation for presheath scale having the effect of geometry, ionization and collision can be developed using momentum balance equation for presheath. The reduced form of eqn. (17), using equation of continuity for the presheath region and eqn. (19) is

$$2\varphi \frac{d\varphi}{dx} = -2\sigma - v\left(1+\sqrt{-x}\right) + a. \tag{24}$$

On R.H.S of this equation first term is for ionizational effect, 2nd for collisional effect and third for geometrical effects in the presheath region.

RESULTS AND DISCUSSION

We can deduce some of the important results from equation (24). We are mainly focusing on geometrical effects with and without ionization and collision. These effects are discussed one by one with graphical presentation in the following sections.

Geometrical effects $(\sigma = 0,\ v = 0,\ a \neq 0)$

In the absence of ionization $(\sigma = 0)$, and collisions $(v = 0)$, the important factor is A'/A [10]. Change in area of geometry will affect only presheath region. Assuming cylindrical and spherical geometries the effect of change in area of geometry will be discussed in this section.

Spherical geometry

For spherical geometry of area "A" the presheath region in front of a spherical geometry have $a = -2\Big/\left(\frac{r}{L} - x\right)$. In this case eqn. (24) can be written as

$$\varphi = -\sqrt{2\ln\left(\frac{r}{L} - x\right) + c}. \tag{25}$$

The dependence of potential behavior on radius of spherical probes can be seen from the following plot (Fig. 2). We are using r_1 = 0.8 mm, r_2 = 0.4 mm, r_3 = 0.1 mm, r_4 = 0.04 mm, and L = 4.5 mm [15]. From the Fig. 2 it is obvious that at the sheath edge there is an electric field even in the absence of ionization and collision in plasma for spherical geometry. The value of potential at sheath edge decreases with the increase in radius of the sphere.

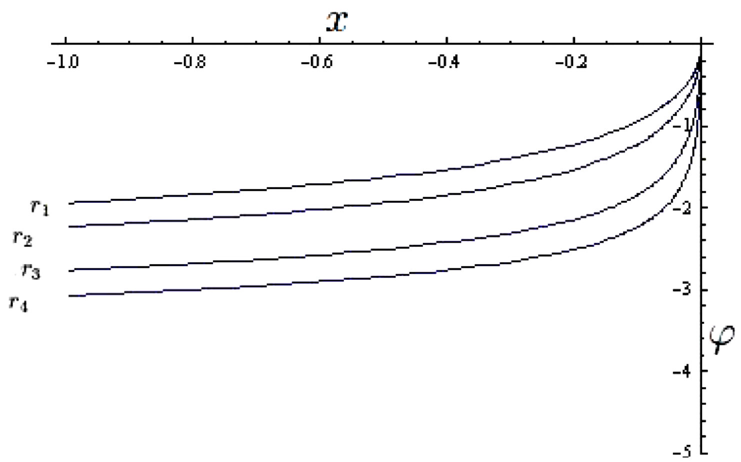

Fig. 2: Variation of φ with x for different values of radius of spherical probe

Cylindrical geometry

The presheath region in front of a cylinder of radius "r" and height "h" is described by, $A = 2\pi\left(r - z\right)h$. Considering geometrical effects and substituting $a = -1\Big/\left(\frac{r}{L} - x\right)$ for cylindrical geometry, eqn.(24) can be written as

$$\varphi = -\sqrt{\ln\left(\frac{r}{L} - x\right) + c}. \tag{26}$$

Using sheath edge conditions and for cylindrical probe of different values of radius and height (r_1, h_1)=(95 μm, 60 mm), (r_2, h_2)=(47.5 μm, 30 mm), (r_3, h_3)=(25 μm, 15 mm), and (r_4, h_4)=(8 μm, 7 mm) and L = 130 mm we get φ verses x as shown in Fig. 3 [16,17]. Again it is clear from the plot that at the sheath edge there is an electric field even in the absence of ionization and collision in plasma for cylindrical geometry. The value of potential at sheath edge decreases with the increase in radial size of the cylinder.

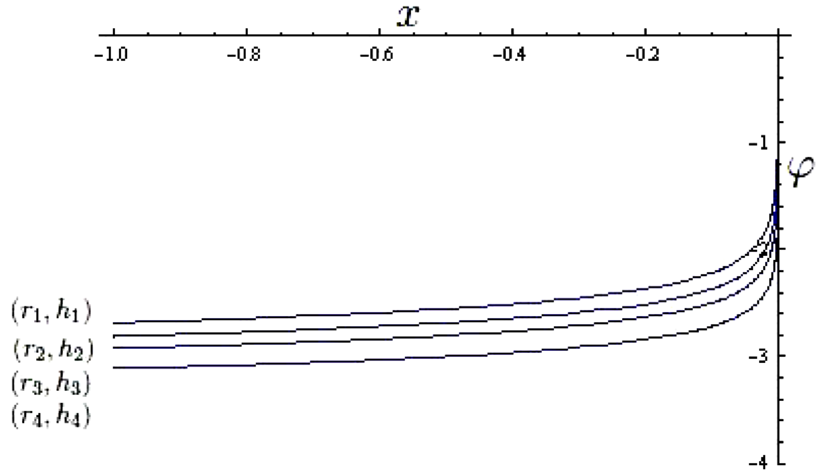

Fig. 3. Variation of φ with x for different values of radius and length of cylindrical probe

Collisionless case $(\nu = 0,\ \ \sigma = constant,\ a \neq 0)$

If we add effect of ionization in the presence of varying area of geometry then there will be no change in sheath region. Considering constant ionization rate $(\sigma = 1)$ and taking $\nu = 0$, eqn. (24) after integration for spherical geometry can be written as

$$\varphi = -\sqrt{-2x + 2\ln\left(\frac{r}{L} - x\right) + c}. \tag{27}$$

Where "c" is the constant of integration which can be calculated using boundary conditions and using characteristic of geometry. Whereas for cylindrical geometry eqn. (27) is

$$\varphi = -\sqrt{-2x + \ln\left(\frac{r}{L} - x\right) + c}. \tag{28}$$

In this case we can see from curves (b) in Fig. 4 and Fig. 5 that more potential penetrates into the presheath region from the wall. The main reason is that in this case more number of electrons gain energy from ionization source and reach the wall and develop more negative potential on the wall as compared to geometrical effects only.

No ionization case $(\sigma = 0,\ \ \nu = constant,\ a \neq 0)$

Now we are considering combined effect of geometry and colllisions on the structure of plasma presheath without any ionization source just like collision dominated plasma in fusion devices. Considering constant collisions frequency and taking $\sigma = 0$, eqn. (24) after integration using boundaries conditions for spherical geometry becomes

$$\varphi = -\sqrt{-x\left(1 + \frac{2}{3}\sqrt{-x}\right) + 2\ln\left(\frac{r}{L} - x\right) + c}. \tag{29}$$

For cylindrical geometry eqn. (29) becomes

$$\varphi = -\sqrt{-x\left(1 + \frac{2}{3}\sqrt{-x}\right) + \ln\left(\frac{r}{L} - x\right) + c}. \tag{30}$$

Again in this case we can see from the from curves (c) in Fig. 4 and Fig. 5 that more potential penetrates from wall into the presheath region due to small collisional cross section of electrons neutral collisions as compared to ions neutral collisional cross section. Hence electrons reach the wall in very short duration of time and develop more negative potential on the wall.

Combined effect of constant ionization rate, constant collision frequency and geometry

$\left(\sigma = constant,\ \ \nu = constant,\ \textbf{and}\ \ a \neq 0\right)$

If we consider constant collision frequency, constant ionization rate and take geometrical effects then after integration eqn. (24), using boundaries conditions for spherical geometry becomes

$$\varphi = -\sqrt{-x\left(3+\frac{2}{3}\sqrt{-x}\right)+2\ln\left(\frac{r}{L}-x\right)+c}. \tag{31}$$

Whereas for cylindrical geometry eqn. (31) becomes

$$\varphi = -\sqrt{-x\left(3+\frac{2}{3}\sqrt{-x}\right)+\ln\left(\frac{r}{L}-x\right)+c}. \tag{32}$$

From curves (d) in Fig. 4 and Fig. 5, we can see that the penetration of the potential of the wall in the case of combined effect of ionization, collision and geometry is more than as that of only geometrical effects, combined effect of geometry and either of the ionization and collision. This is due to the reason that electrons which are more mobile as compared to ions, in the presence of ionization source and collisions in plasma quickly reach the wall due to high energy as compared to without ionization source and collisions in plasma. Hence electrons develop more negative potential on the wall as compared to in the presence of geometrical effects only.

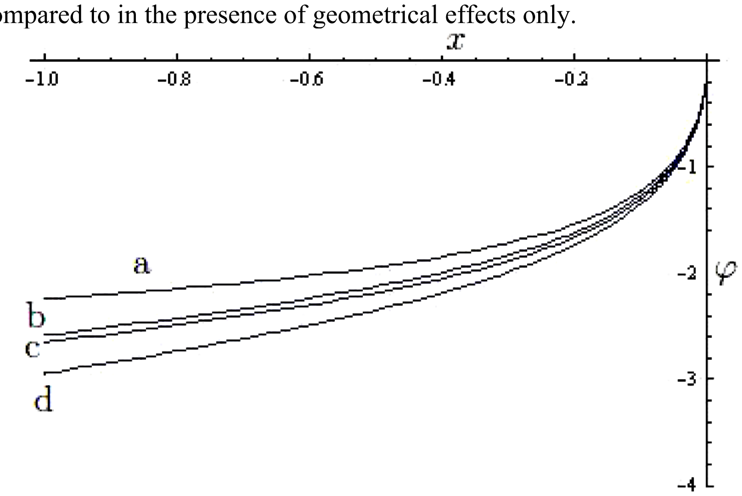

Fig. 4. Variation of φ with x for spherical geometry considering (a) Only geometry (b) Geometry and constant ionization rate (c) Geometry and constant collision frequency (d) Geometry, constant ionization rate and constant collision frequency

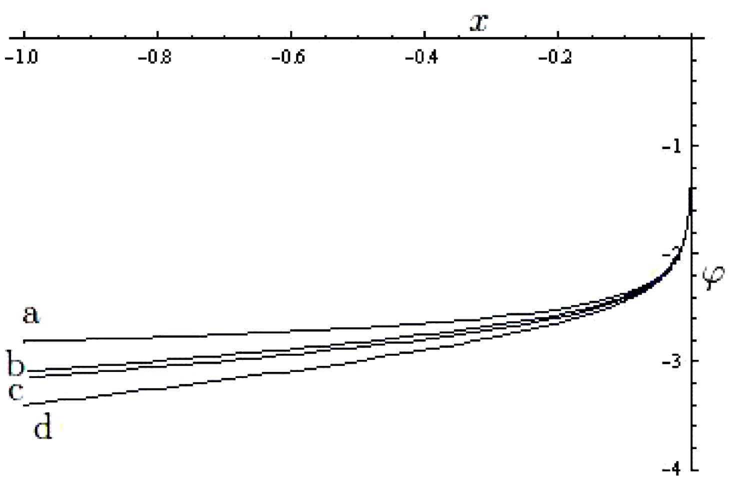

Fig. 5. Variation of φ with x for cylindrical geometry considering (a) Only geometry (b) Geometry and constant ionization rate (c) Geometry and constant collision frequency (d) Geometry, constant ionization rate and constant collision frequency