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CLASSIFICATION OF PARTICLES AT ARBITRARY QUANTITY OF GENERATIONS. II. LEPTONS

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The hypothesis on quark nature of the leptons is proposed. Leptons are compacted $\bar{q}^3$-systems. It ensures the equality of modules for the electric charges of the proton and the electron. The classification of particles based on the $U(N_f,g)\times SU(3,c)\times SU(4,fs)\times O(3)$ -group is proposed at arbitrary quantity N_f of the quark generations. The $U(N_f,g)$-group corresponds to the quark generations, the $SU(3,c)$-group describes the color variables, the $SU(4,fs)$-group corresponds to the variables in the spin (*s*) and flavor (*f*) spaces, and the $O(3)$-group describes the orbital excitations of quarks. In consequence of the Pauli principle leptons consist of antiquarks from 3 different generations. Minimal quantity of leptons with definite electric charge equal 20. Excited double charged l^{--}-leptons and $\bar{l}^{++}$-antileptons with the $J^p=\frac{1}{2}^+$ and $J^p=\frac{1}{2}^-$ are predicted, respectively. They can be resonances in $e^-\pi^-, e^-K^-, e^+\pi^+, e^+K^+$, $\mu^-\pi^-, \mu^-K^-, \mu^+\pi^+, \mu^+K^+$-systems.
KEY WORDS: leptons, quarks, excited leptons, lepton-meson resonances

КЛАСИФІКАЦІЯ ЧАСТИНОК ПРИ ДОВІЛЬНІЙ КІЛЬКОСТІ ПОКОЛІНЬ. ІІ. ЛЕПТОНИ

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Запропоновано гіпотезу про кваркову природу лептонів. Лептони являють собою щільно запаковані $\bar{q}^3$-системи. Це забезпечує рівність модулів електричних зарядів протона та електрона. Запропоновано класифікацію частинок на основі $U(N_f,g)\times SU(3,c)\times SU(4,fs)\times O(3)$ групи при довільній кількості поколінь кварків N_f Група $U(N_f,g)$ відповідає поколінням кварків, група $SU(3,c)$ описує кольорові змінні, група $SU(4,fs)$ - відповідає змінним в просторах спіну (*s*) та ароматів (*f*), і група $O(3)$ описує орбітальні збудження кварків. Внаслідок принципа Паули, лептони складаються з антикварків 3 різних поколінь. Мінімальна кількість лептонів з певним електричним зарядом дорівнює 20. Передбачено існування збуджених двічі заряджених l^{--}-лептонів та $\bar{l}^{++}$-антилептонів з $J^p=\frac{1}{2}^+$ та $J^p=\frac{1}{2}^-$,відповідно. Вони можуть бути резонансами в $e^-\pi^-, e^-K^-, e^+\pi^+, e^+K^+$, $\mu^-\pi^-, \mu^-K^-, \mu^+\pi^+, \mu^+K^+$-системах.
КЛЮЧОВІ СЛОВА: лептони, кварки, збуджені лептони, лептон-мезонні резонанси

КЛАССИФИКАЦИЯ ЧАСТИЦ ПРИ ПРОИЗВОЛЬНОМ КОЛИЧЕСТВЕ ПОКОЛЕНИЙ. II. ЛЕПТОНЫ

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Предложена гипотеза о кварковой природе лептонов. Лептоны являются плотно упакованными $\bar{q}^3$- системами. Это обеспечивает равенство модулей электрических зарядов протона и электрона. Предложена классификация частиц на основе группы $U(N_f,g)\times SU(3,c)\times SU(4,fs)\times O(3)$ – при произвольном количестве поколений кварков N_f. Группа $U(N_f,g)$ соответствует поколениям кварков, группа $SU(3,c)$ описывает цветовые переменные, группа $SU(4,fs)$ - соответствует переменным в пространствах спина (*s*) и ароматов (*f*), и группа $O(3)$ описывает орбитальные возбуждения кварков. Вследствие принципа Паули лептоны состоят из антикварков 3 разных поколений. Минимальное количество лептонов с определенным электрическим зарядом равно 20. Предсказано существование возбужденных дважды заряженных l^{--}-лептонов и $\bar{l}^{++}$-антилептонов с $J^p=\frac{1}{2}^+$ и $J^p=\frac{1}{2}^-$, соответственно. Они могут быть резонансами в $e^-\pi^-, e^-K^-, e^+\pi^+, e^+K^+$, $\mu^-\pi^-, \mu^-K^-, \mu^+\pi^+, \mu^+K^+$-системах.
КЛЮЧЕВЫЕ СЛОВА: лептоны, кварки, возбужденные лептоны, лептон-мезонные резонансы

As it is known the atoms consist of electrons, protons, and neutrons. A spin of all these particles equals ½. The electron belongs to leptons. The proton and the neutron belong to baryons (hadrons with the ½-spin). The leptons and the hadrons have got distinguish properties:

1. The quantity of the known leptons equals 12 ($e^-, \mu^-, \tau^-, \nu_e, \nu_\mu, \nu_\tau$ and their antiparticles) distributed in three generations. The quantity of hadrons (in particular, baryons) is very big.

2. The leptons participate in electroweak and gravitation interactions, whereas the hadrons participate in strong, electroweak, and gravitation interactions.

3. The leptons have not got excited states, whereas each hadron has a lot of excited states.

4. At perturbation (e.g., at the capture of a photon) leptons emit the photon and transit to initial state. At perturbation hadrons can transit in excited states, which can decay in other hadrons.

5. The leptons are point-like particles, i.e., their form factors are constants in the measured region of a momentum transfer. Form factors of hadrons have got essential dependences on momentum transfer. An existence of hadron form factors is related to a decrease of a probability of a transition of this hadron in initial state at a capture of a virtual photon

6. All the leptons are fermions and they have got the $\frac{1}{2}$ spin. Hadrons have got integer spin (mesons) and half-integer spin (baryons).

In spite of these distinctions of the leptons and the hadrons, the electric charge for any hadrons equals the product of the electron charge and the integer number with big accuracy of measurements. Therefore, it is of interest to study a problem of proportionality for the electric charges of leptons and hadrons [1].

The present paper is devoted to investigations of leptons in framework of the approach, which ensures in general case the proportionality of the electrical charges of hadrons and leptons. In particular case this approach must ensure the equality of the electrical charges for the proton and the positron.

ON ELECTRIC CHARGES OF PROTON AND ELECTRON

Let us consider a problem of the equality for the absolute values of the Q_e-electron charge and the Q_p-proton charge. It means that a value

$$Q_e + Q_p = \varepsilon_{ep} \tag{1}$$

must equal zero with any accuracy. It is known that in quantum theory of fields the interactions can change electric charges of particles and their masses (it is related to the renormalizations). Therefore, the bare charges and the masses (without interactions) distinguish from these physical values (with an inclusion of interactions).

In Ref. [2], using the gauge invariance, it is shown that a change of the electric charge is determined by interactions of the massless particles. The change of lepton charges can be determined by interactions of the leptons with the photons and possibly gravitons. But the change of hadron charges can be determined by interactions of the hadrons with the photons, the gluons, and possibly gravitons. Therefore, it may be assumed that at equal modules of bare charges the physical value ε_{ep} in (1) has got a non-zero quantity. For example, it may be assumed that the physical value ε_{ep} has got small magnitude related to gravitation interactions, which is essentially less than up-to-date accuracy of measurements. A non-zero value of ε_{ep} can be important in the astrophysics.

The elimination of divergences in the axial Adler-Bell-Jackiw anomaly [3] is important for a proof of a renormalization in the electroweak theory. This anomaly is determined by Feynman diagram corresponding to the loop with the fermion of the $\frac{1}{2}-$spin and with two photons as well as an axial neutral current for the I_3-third isospin component.

The contribution of a fermion to the ABJ-anomaly equals the product the $Q^2 I_3$-coefficient and the linearly divergent integral, where Q is the electric charge of the fermion. The I_3-value equal $-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}$ for the charged leptons $l^-(e^-, \mu^-, \tau^-)$, neutrinos, the *down*-quarks D_k, the *up*-quarks U_k, respectively. The contribution of a charged fermiom to the ABJ-anomaly gives the linear divergence, but a sum of contributions for the charged lepton, the neutrino, D_k-quark, and U_k-quark is proportional to $[0 - Q_e^2 + 3Q_p^2(\frac{4}{9} - \frac{1}{9})]/2 = (Q_p^2 - Q_e^2)/2$ for each generation. Thus, if $\varepsilon_{ep} \neq 0$ (even if ε_{ep} has got small magnitude, which is essentially less than up-to-date accuracy of measurements), then the ABJ-anomaly diverges linearly and the electroweak theory is non-renormalized. In consequence of good agreement of the electroweak theory with experimental data we must conclude that Q_p equals $-Q_e$ identically. In relation with this the question arises: what essence leads to the equality of the absolute values of the proton and the electron charges.

HYPOTHESIS ON QUARK NATURE OF LEPTONS

To explain the equality of the absolute values of the proton and the electron charges propose that these particles consist of similar particles. We shall consider the positron, as it has got positive electrical charge and the charges of the proton and the positron must equal identically. Then each particle in the positron must correspond to the particle in the proton with the same electrical charge. As the proton consists of three quarks the positron must consist of three quarks also. It may be proposed that other antileptons (leptons with positive electric charge) and antineutrinos consist of quarks. Then the electron and other leptons with negative electric charge as well as neutrinos consist of antiquarks.

In each generation a charged antilepton and an antineutrino exist. These particles have got the $\frac{1}{2}$-spin and they correspond to a representation of the 4-dimension of the $SU(4,sf)$-group. This representation may be identified with the antisymmetric representation for a q^3-system. Note that baryons, which belong to antisymmetric representation of the $SU(6,sf)$- (and $SU(4,sf)$-) group, are unknown [4]. Therefore, it may be assumed that the q^3-systems classified by the symmetric representation and mixed symmetry representation of the $SU(6,sf)$- (and $SU(4,sf)$-) group are baryons, whereas the q^3-systems classified by the antisymmetric representation of the $SU(6,sf)$- (and $SU(4,sf)$-) group are antileptons.

In Ref. [5] it is shown that the fourfold integrals corresponding to the Green functions of the Klein-Gordon and the Dirac equations diverge. For the $\frac{1}{2}$-spin particles the next generalization of the non-homogeneous Dirac equation is proposed in Refs. [5, 6]

$$\left(-i\hat{\partial}+m_1\right)\left(-i\hat{\partial}+m_2\right).....\left(-i\hat{\partial}+m_{N_f}\right)\Psi\left(x\right)=\chi\left(x\right). \tag{2}$$

The classical solution of the homogeneous equation (2) is given by

$$\Psi^{\alpha}\left(x\right)_{free}=\sum_{s}\sum_{k=1}^{N_f}\int d^4p\delta\left(q^2-m_k^2\right)\left[C_k u^{\alpha}{}_{k,s}\left(q\right)e^{-iqx}+\tilde{C}_k v^{\alpha}{}_{k,s}\left(q\right)e^{iqx}\right], \tag{3}$$

where α is the bispinor index, s corresponds to spin projection, $u^{\alpha}{}_{k,s}\left(q\right)$ and $v^{\alpha}{}_{k,s}\left(q\right)$ are the bispinors, C_k and $\tilde{C}_k$ are arbitrary constants. The Green functions (which are 4×4-matrixes) for this equation may be written as

$$\overline{S}\left(x\right)=\frac{1}{\left(2\pi\right)^4}\int\frac{\left(\hat{q}+m_1\right)\left(\hat{q}+m_2\right)....\left(\left(\hat{q}+m_{N_f}\right)\right)}{\left(-q^2+m_1^2\right)\left(-q^2+m_2^2\right)...\left(-q^2+m_{N_f}^2\right)}d^4q. \tag{4}$$

The N_f-number equals to the quantity of generations for the $\frac{1}{2}$-spin fermions and order of the equation (2). The integrals in (4) can converge at $N_f\geq5$. The study of a continuity of the causal Green functions (4) in all the space-time (and on the light cone too) has been shown that the quantity of fermion generations $N_f\geq6$ [7]. In Ref. [8] the classification of hadrons with respect to the $U(N_f,g)\times SU(3,c)\times SU(4,fs)\times O(3)$-group is proposed at arbitrary quantity N_f of fermion generations. The $U(N_f,g)$-group corresponds to the quark generations, the $SU(3,c)$-group describes the color variables, the $SU(4,fs)$-group corresponds to the variables in the spin (*s*) and flavor (*f*) spaces (which is subgroup of the known $SU(6,fs)$-group), and the $O(3)$-group describes the orbital excitations of quarks [4].

Propose that leptons (antileptons) are colorless, i.e., they are described by singlets of the $SU(3,c)-$group, which correspond to antisymmetric representations for q^3- ($\overline{q}^3-$) systems. Because a quantity of the leptons and the antileptons is small in a comparison with a quantity of baryons, it may be proposed that quarks and anriquarks in antileptons and in leptons, respectively, are in the $1s-$state with the $L=0$ orbital moment. It means that leptons (antileptons) are singlets of the $O(3)-$group, which correspond to symmetric representations for q^3- $(\overline{q}^3-)$ systems. Then from the Pauli principle follows that the antileptons (leptons) belong to the antisymmetric representations of the $U(N_f,g)-$group for q^3- $(\overline{q}^3-)$ systems. The dimension of these representations equals $N_A(N_f)=N_f(N_f-1)(N_f-2)/1{\bullet}2{\bullet}3$. Therefore, the minimal quantity of the generations for the antileptons (the leptons) equals 20 (for $N_f=6$ [8]). These leptons have got the $\frac{1}{2}$-spin.

The antileptons (e.g., the positron) and the proton have got the same spatial parity, as all the quarks are in the $1s-$state. Therefore the spatial parities of the leptons (e.g., the electron) and the proton must be opposite.

Denote the charged antileptons and the antineutrinos, which consist of the quarks from the number k_1,k_2,k_3 generations as $\overline{l}^+_{k_1k_2k_3}$ and $\overline{\nu}_{k_1k_2k_3}$, respectively, with the k_1,k_2,k_3 numbers disposed in increasing order. Denote corresponding the charged leptons and the neutrinos as $l^-_{k_1k_2k_3}$ and $\nu_{k_1k_2k_3}$, respectively. The antileptons consisting of the quarks from first, second, and third generations must exist among $N_A(N_f)$ antileptons. For such antileptons the M_0-value, which for baryons give the sum of quark masses (for example, in the mass formula (30) of Ref. [8]) is minimal among all the generations. Possibly these antileptons are the positron and the electron antineutrino.

The wave function of the charged antilepton (including the positron) may be written as:

$$\begin{aligned}\left|\overline{l}^+_{k_1k_2k_3},S_z=\frac{1}{2}\right\rangle=\frac{1}{6\sqrt{6}}\varepsilon_{c_1c_2c_3}\varepsilon^{N_f}{}_{g_1g_2g_3}(k_1,k_2,k_3)\Big|&U\uparrow_{c_1g_1}U\downarrow_{c_2g_2}D\uparrow_{c_3g_3}-U\downarrow_{c_1g_1}U\uparrow_{c_2g_2}D\uparrow_{c_3g_3}+\\&+U\downarrow_{c_1g_1}D\uparrow_{c_2g_2}U\uparrow_{c_3g_3}-U\uparrow_{c_1g_1}D\uparrow_{c_2g_2}U\downarrow_{c_3g_3}+\\&+D\uparrow_{c_1g_1}U\uparrow_{c_2g_2}U\downarrow_{c_3g_3}-D\uparrow_{c_1g_1}U\downarrow_{c_2g_2}U\uparrow_{c_3g_3}\Big\rangle,\end{aligned}\tag{5}$$

where c_1,c_2,c_3 are the indices of the $SU(3,c)$-group, the $\varepsilon_{c_1c_2c_3}$- tensor is the antisymmetric tensor of third rank. The g_1,g_2,g_3 indices are the numbers of the generations. The $\varepsilon^{N_f}{}_{g_1g_2g_3}(k_1,k_2,k_3)$- tensor is the antisymmetric tensor of the N_f-rank with six non-zero components, which correspond to the permutatons of the g_1,g_2,g_3-indices from $\varepsilon_{123\cdots N_f}$=1. For example, at N_f=6 and $k_1=2,k_2=4,k_3=5$ this tensor is given by $\varepsilon^6{}_{g_1g_2g_3}(2,4,5)=\varepsilon_{1g_13g_2g_36}$ and $\varepsilon^6{}_{245}(2,4,5)=\varepsilon^6{}_{524}(2,4,5)=\varepsilon^6{}_{452}(2,4,5)=-\varepsilon^6{}_{425}(2,4,5)=$ $-\varepsilon^6{}_{254}(2,4,5)=-\varepsilon^6{}_{542}(2,4,5)=1$. The wave function of the positively charged antilepton is determined by 216 terms.

The wave function of the antineutrino is given by:

$$\begin{aligned}\left|\overline{\nu}_{k_1k_2k_3},S_z=\frac{1}{2}\right\rangle=\frac{1}{6\sqrt{6}}\varepsilon_{c_1c_2c_3}\varepsilon^{N_f}{}_{g_1g_2g_3}(k_1,k_2,k_3)\Big|&-D\uparrow_{c_1g_1}D\downarrow_{c_2g_2}U\uparrow_{c_3g_3}+D\downarrow_{c_1g_1}D\uparrow_{c_2g_2}U\uparrow_{c_3g_3}-\\&-D\downarrow_{c_1g_1}U\uparrow_{c_2g_2}D\uparrow_{c_3g_3}+D\uparrow_{c_1g_1}U\uparrow_{c_2g_2}D\downarrow_{c_3g_3}-\\&-U\uparrow_{c_1g_1}D\uparrow_{c_2g_2}D\downarrow_{c_3g_3}+U\uparrow_{c_1g_1}D\downarrow_{c_2g_2}D\uparrow_{c_3g_3}\Big\rangle.\end{aligned}\tag{6}$$

The wave function (6) is derived from (5) by means of the changes $U\leftrightarrow D$ and by general sign similarly to the derivation of the wave function of the neutron from the wave function of the proton in the quark model of baryons [9,10].

The wave function of the neutrinos we derive from (6) by means of the substitutions $U\to\overline{U},D\to\overline{D}$:

$$\begin{aligned}\left|\nu_{k_1k_2k_3},S_z=\frac{1}{2}\right\rangle=\frac{1}{6\sqrt{6}}\varepsilon_{c_1c_2c_3}\varepsilon^{N_f}{}_{g_1g_2g_3}(k_1,k_2,k_3)\Big|&-\overline{D}\uparrow_{c_1g_1}\overline{D}\downarrow_{c_2g_2}\overline{U}\uparrow_{c_3g_3}+\overline{D}\downarrow_{c_1g_1}\overline{D}\uparrow_{c_2g_2}\overline{U}\uparrow_{c_3g_3}-\\&-\overline{D}\downarrow_{c_1g_1}\overline{U}\uparrow_{c_2g_2}\overline{D}\uparrow_{c_3g_3}+\overline{D}\uparrow_{c_1g_1}\overline{U}\uparrow_{c_2g_2}\overline{D}\downarrow_{c_3g_3}-\\&-\overline{U}\uparrow_{c_1g_1}\overline{D}\uparrow_{c_2g_2}\overline{D}\downarrow_{c_3g_3}+\overline{U}\uparrow_{c_1g_1}\overline{D}\downarrow_{c_2g_2}\overline{D}\uparrow_{c_3g_3}\Big\rangle.\end{aligned}\tag{7}$$

The wave function of the charged lepton (including the electron) may be presented as

$$\left|l^{-}{}_{k_1k_2k_3}, S_z = \frac{1}{2}\right\rangle = \frac{1}{6\sqrt{6}} \varepsilon_{c_1c_2c_3} \varepsilon^{N_f}{}_{g_1g_2g_3}(k_1,k_2,k_3) \left| \bar{U}\uparrow_{c_1g_1} \bar{U}\downarrow_{c_2g_2} \bar{D}\uparrow_{c_3g_3} - \bar{U}\downarrow_{c_1g_1} \bar{U}\uparrow_{c_2g_2} \bar{D}\uparrow_{c_3g_3} + \right.$$
$$+\bar{U}\downarrow_{c_1g_1} \bar{D}\uparrow_{c_2g_2} \bar{U}\uparrow_{c_3g_3} - \bar{U}\uparrow_{c_1g_1} \bar{D}\uparrow_{c_2g_2} \bar{U}\downarrow_{c_3g_3} + \tag{8}$$
$$\left. +\bar{D}\uparrow_{c_1g_1} \bar{U}\uparrow_{c_2g_2} \bar{U}\downarrow_{c_3g_3} - \bar{D}\uparrow_{c_1g_1} \bar{U}\downarrow_{c_2g_2} \bar{U}\uparrow_{c_3g_3} \right\rangle .$$

Now we consider the consequences of the antisymmetry of the representation of the $U(N_f, g)-$group for the set of quarks from the 1, 2, 3- numbers of the generations in antileptons:

$$\bar{l}^{+}_{123} = [ucb, ctd, tus], \bar{\nu}_{123} = [dst, bdc, sbu] . \tag{9}$$

From (9) it is seen that leptons and antileptons include antiquarks and quarks with big masses. Even the antileptons consisting of the quark from the 1, 2, 3- numbers of the generations (possibly the positron and the electron antineutrino) include the t-quark, which has got the $m(t) \approx 177 GeV$ mass [11]. Note that masses of $U_k -$quarks with $k \geq 3$ must be greater than $m(t)$. From (9) it can be seen that each antilepton include three U_k-quark and three D_k-quarks. Therefore it is of interest to derive the M_0-term, which for baryons give the sum of quark masses (for example, in the mass formula (30) of Ref. [8]). Using (5), the mass of the $l^{+}{}_{k_1k_2k_3}$-antilepton may be written as

$$M_0(\bar{l}^{+}_{k_1k_2k_3}) = \left\langle \bar{l}^{+}_{k_1k_2k_3} \left| \sum_i m_i \right| \bar{l}^{+}{}_{k_1k_2k_3} \right\rangle = \frac{6\times 6}{216} \varepsilon^{N_f}{}_{g_1g_2g_3}(k_1,k_2,k_3) \varepsilon^{N_f}{}_{g_4g_5g_6}(k_1,k_2,k_3)$$
$$\left\langle U\uparrow_{g_4} U\downarrow_{g_5} D\uparrow_{g_6} \left| m(U_{g_1}) + m(U_{g_2}) + m(U_{g_3}) + m(D_{g_1}) + m(D_{g_2}) + m(D_{g_3}) \right| U\uparrow_{g_1} U\downarrow_{g_2} D\uparrow_{g_3} \right\rangle =$$
$$= \frac{1}{3}\left[\sum_g m(U_{g_1}) + \sum_{g_2} m(U_{g_2}) + \sum_{g_3} m(D_{g_3}) \right] = \tag{10}$$
$$= \frac{1}{3}\left[2m(U_{k_1}) + 2m(U_{k_2}) + 2m(U_{k_3}) + m(D_{k_1}) + m(D_{k_2}) + m(D_{k_3}) \right].$$

In this formula are not written the color indices. The summation with respect of these indices gives the 6 factor. The m_i mass operators in the $\sum_i m_i$-operator are proportional to unique operator in the color and the spin spaces and they are diagonal operators in the space of generations. Therefore, such structure of the mass operators permits one to take the same results for six terms in (5). As analogy, the antineutrino masses are given by

$$M_0(\bar{\nu}_{k_1k_2k_3}) = \frac{1}{3}\left[2m(D_{k_1}) + 2m(D_{k_2}) + 2m(D_{k_3}) + m(U_{k_1}) + m(U_{k_2}) + m(U_{k_3}) \right]. \tag{11}$$

In particular, for the antileptons which consist of the quarks from the number 1, 2, 3 generations the M_0-values are

$$M_0(\bar{l}^{+}_{123}) = \frac{1}{3}\left[2m(u) + 2m(c) + 2m(t) + m(d) + m(s) + m(b) \right] \tag{12}$$
$$M_0(\bar{\nu}_{123}) = \frac{1}{3}\left[2m(d) + 2m(s) + 2m(b) + m(u) + m(c) + m(t) \right].$$

DIAMOND-GRAPHITE ANALOGY OF LEPTONS AND BARYONS

It is known that the diamond and the graphite consist of the carbon atoms. In spite of properties of them are different. The graphite is fairly soft material, but the diamond is the hardest material.

Pencils with graphite cores are used to write. It means that at small perturbation the graphite transits to other states. Indeed, in initial state we have got the pencil and the clean paper, but in a final state we have got the pencil and this paper with drawn line. The mass of the graphite core can change. It corresponds to a transition of the pencil graphite to other states. Therefore, the graphite is similar to hadrons, in particular – to baryons. The analogy of the graphite and the baryons (nucleons) can be continued. The carbon atoms in the graphite are located on the planes. The nucleons consist of the quarks from the same generation.

As the diamond is the hardest solid, the state of the diamond does not change at an interaction with other solids. Thus, the diamond is similar to the leptons (antileptons). In addition note that the carbon atoms in the diamond are located in the space and they yield the space lattice. It is the analogy of a proposition that the leptons (antileptons) consist of the quarks (antiquarks) from different generations.

EXCITED LEPTONS AND POSSIBLE EXPERIMENTAL TESTS

A consideration of the q^3-systems with the quarks in the $(1s)^2 1p-$states allows one to study the baryons with the negative parity [4]. In such systems the quark orbital moment equals one. The physical states belong to the representations with mixed symmetry of the $O(3)-$group. Then the uncolored baryons are described by the representations with mixed symmetry of the $SU(6,fs)-$group and the $SU(4,fs)-$group of the dimensions 70 and 20, respectively.

Let us study the leptons (antileptons) as the $\overline{q}^3-(q^3)-$systems with the quarks in the $(1s)^2 1p-$states. The expansion of the representations of the $SU(4,fs)-$groups for the mixed symmetry with a respect of the $SU(2,f)\times SU(2,s)-$group for the q^3-system is given by:

$$20 = 4\times\frac{1}{2}^{-} + 2\times\frac{3}{2}^{-} + 2\times\frac{1}{2}^{-} . \tag{13}$$

In the expansion (13) the representations of the $SU(2,s)$-group are presented by J^p-values. The product of the representations with mixed symmetry for the $SU(2,s)-$ and the $O(3)-$groups can be expanded as the sum of the symmetric, mixed symmetric, and antisymmetric representations. Consider the antisymmetric representation of the $SU(4,fs)\times O(3)-$group. This representation can be obtained as the product of the mixed symmetric representations of the $SU(4,fs)-$group and the $O(3)-$group. The product of the antisimmetric representations of the $U(N_f,g)-$, the $SU(3,c)-$, and the $SU(4,fs)\times O(3)-$ groups agrees with the Pauli principle. The expansion (13) is valid for the q^3-systems (i.e., for the antileptons). Similar expansion can be written for the $\overline{q}^3-$system (i.e., for the leptons), but in this case the parity must be positive.

The representation of the $SU(2,f)-$group of the dimension 4 corresponds to excited leptons (antileptons). Denote these excited antileptons (which consist of the quarks) and leptons as

$$\overline{l}^*(4) = \{\overline{l}^{*-}, \overline{l}^{*o}, \overline{l}^{*+}, \overline{l}^{*++}\}, \qquad l(4) = \{l^{*--}, l^{*-}, l^{*0}, l^{*+}\}. \tag{14}$$

The representation of the $SU(2,f)-$group of the dimension 4 is symmetric. So to obtain the antisymmetric representation of the $SU(4,fs)\times O(3)-$group we have to consider the antisymmetric representation of the $SU(2,s)\times O(3)-$group. This representation corresponds to $J=\frac{1}{2}$. Such, we predict that excited l^{*--}- leptons and l^{*++}- antileptons with the $\frac{1}{2}$-spin can exist. Note that the representation of the $SU(2,f)-$group of the dimension 4 can occur at other modes for products of the representations of the $U(N_f,g)\times SU(3,c)\times SU(2,s)\times O(3)$-group. As the representation of the $O(3)-$group corresponding to the quarks in the $(1s)^2 1p-$states has got mixed symmetry, the representations $U(N_f,g)\times SU(3,c)\times SU(2,s)$ must have got mixed symmetry too. In particular, for the symmetric representation of the $SU(2,s)$-group the $\overline{l}^*(4)$-antileptons and the $l^*(4)$-leptons can exist in two cases: the antisymmetric representation of the $U(N_f,g)$-group in the state of the color octet and the mixed symmetric representation of the $U(N_f,g)$-group in the state of the color singlet. In these cases the $\overline{l}^*(4)$-antileptons with $J^p=\frac{1}{2}^{-}$ and the $l^*(4)$-leptons with $J^p=\frac{1}{2}^{+}$ can exist. For the quarks from the first, the second, and the third generations the $\overline{l}^{*++}$- and the $\overline{l}^{*-}$-antileptons consist of the $u,c,t-$ and the $d,s.b-$ quarks, respectively. Therefore, we may assume that for the masses of the $\overline{l}^*(4)$-antileptons the inequalities $m(\overline{l}^{*-}) < m(\overline{l}^{*o}) < m(\overline{l}^{*+}) < m(\overline{l}^{*++})$ are valid. If the contributions of the color-magnetic and the color-electric interactions in the mass formula (30) of Ref. [8] are additional, then the relations

$$m(\overline{l}^{*+}) - m(\overline{l}^{*0}) = m(\overline{l}^{*0}) - m(\overline{l}^{*-}) = \tag{15}$$

$$=\frac{1}{3}[m(U_{k_1})+m(U_{k_2})+m(U_{k_3})-m(D_{k_1})-m(D_{k_2})-m(D_{k_3})]$$

may be valid for the antileptons consisting of the quarks from the k_1,k_2,k_3-numbers of the generations.

The analogy between the $\bar{l}^{*-}$-antileptons and the $\Delta-$baryons, which consist of the quarks in the $(1s)^2 1p-$states, may be proposed. Among the $\Delta-$baryons with negative parity the $\Delta(1620)$-resonance ($S_{31}(1620)$-resonance in the $\pi N-$system) with $J^p=\frac{1}{2}^-$ has got the minimal mass [12]. In quark models the mass difference between the $\Delta(1620)$-resonance and the nucleon mainly determined by the $\vec{L}\vec{S}-$interactions [4]. If we assume that the $\vec{L}\vec{S}-$interactions give additional contributions to the masses of excited baryons and antileptons, then we may expect that the masses of the $\bar{l}^{*o}-$ antileptons and the $\bar{l}^{*+}-$antileptons equal approximately 700 MeV. Such transitions between quarks as $u\to d\pi^+, d\to u\pi^-, u\to sK^+, s\to uK^-$, $u\to sK^{*+}, s\to uK^{*-}$ $u\to d\rho^+, d\to u\rho^-, u\to sK^{*+}, s\to uK^{*-}$ can occur in hadrons and in leptons. These transitions induce the $P_{33}(1232)\to N\pi, D_{13}(1520)\to N\pi, S_{11}(1535)\to N\pi, S_{31}(1620)\to N\pi$ decays. Similarly these transitions can induce the $l^*(4)\to l\pi$, $l^*(4)\to l\rho$, $\bar{l}^*(4)\to\bar{l}\pi$, $\bar{l}^*(4)\to\bar{l}\rho$ decays. One can expect that these decays must be induced by strong interactions. It is of interest to study the decays of the $\bar{l}^{*++}$-antilepton and the l^{*--}-leptons. As these excited antilepton and lepton ought to have got fairly large masses, they must be resonances. It can be used for the test of the hypothesis on quark nature of leptons.

For the tests of proposed hypothesis it is of interest to investigate the resonances in the $e^-\pi^-, e^-\rho^-, e^-K^-, e^-K^{*-}$, $\mu^-\pi^-, \mu^-\rho^-, \mu^-K^-, \mu^-K^{*-}$, $e^+\pi^+, e^+\rho^+, e^+K^+, \mu^+\pi^+, \mu^+\rho^+, \mu^+K^+, \mu^+K^{*+}$, $e^+K^{*+}, \mu^+\pi^+, \mu^+\rho^+, \mu^+K^+, \mu^+K^{*+}$-systems. The resonances in these systems can be investigated in hadron reactions. However, it can be more convenient to investigate these resonances in the reactions of the lepton-antilepton interactions:

$$e^+e^-\to\bar{l}^{*++}l^{*--},\qquad \mu^+\mu^-\to\bar{l}^{*++}l^{*--}. \tag{16}$$

ON STRONG INTERACTIONS OF LEPTONS

According to the quark nature of leptons it may be assumed that some strong transitions between quark systems, which usually considered as leptons or antileptons can become possible. Such strong transitions can lead to decays of $\pi^\pm-, K^\pm-$mesons into charged lepton (antilepton) and neutrino (antineutrino). These decays are well known. But they are weak. Therefore, non-zero amplitudes of such strong transitions will contradict to experimental data. Compare similar strong $e^+\to\pi^+\bar{\nu}_e$- and $p\to\pi^+n$- transitions. The spin structure of the amplitudes for these transitions is the same: $\bar{u}(p_2)\gamma_5 u(p_1)\varphi_\pi$. As usual assume that the π^+-meson is the $u\bar{d}$-pair and the proton and the neutron are the *uud*-system and the *ddu*-system, respectively. Then the $e^+\to\pi^+\bar{\nu}_e$- and the $p\to\pi^+n$- transitions are determined by a $u\to\pi^+d$-transition between the quarks. In the $p\to\pi^+n$- transition the proton becomes the neutron directly after an emission of the π^+-meson by the *u*-quark.

Assume that e^+- and $\bar{\nu}_e$- consist of the quarks from the first, the second, and the third generations. Then these antileptons are the antisymmetric combinations of different quarks (9). The *ucb*- and the *uts*- systems from the positron transit to the *cdb*- and the *tds*- systems from the antineutrino after the emission of the pion, respectively. But the the *ctd*-system from the positron cannot transits to the *usb*-system from the antineutrino at the emission of the pion. Thus, the positron does not transit immediately to the antineutrino after the emission of the pion. Therefore, it may be concluded that strong $e^+\to\pi^+\bar{\nu}_e$-transition (with the $\bar{u}(p_2)\gamma_5 u(p_1)\varphi_\pi$-amplitude) is forbidden. Similarly, strong $\mu^+\to\pi^+\bar{\nu}_\mu$-, $e^+\to K^+\bar{\nu}_e$-, $\mu^+\to K^+\bar{\nu}_\mu$-, $e^+\to\rho^+\bar{\nu}_e$-, $\mu^+\to\rho^+\bar{\nu}_\mu$-, $e^+\to K^{*+}\bar{\nu}_e$-, $\mu^+\to K^{*+}\bar{\nu}_\mu$-transitions must be forbidden. As result, these transitions are induced by the weak interactions.

It is of interest to consider the transitions between a baryon and a antilepton, such as $p\to e^+\gamma$, $p\to\mu^+\gamma$, $p\to e^+\pi^0$, $p\to\mu^+\pi^0$.These transitions have not been observed yet. Usually it is explained by a violation of the conservation laws for the baryonic and the leptonic numbers. The proton belongs to symmetric representation of the

$U(N_f, g)$-group $B^{\{g_1,g_2,g_3\}}$ and according to the hypotheses on quark nature of leptons the antileptons belong to the antisymmetric representation $\overline{L}^{[g_1,g_2,g_3]}$ of this group. Denote the representation of the $U(N_f, g)$-group for $q\overline{q}-$mesons as $M^{g_1}{}_{g_2}$. Then the amplitudes of the transitions of the *B*-baryons to the $\overline{L}-$antileptons and the *M*-mesons may be written as

$$T(B \to \overline{L}M) = B^{\{g_1g_2g_3\}}\overline{L}_{[g_4g_2g_3]}M^{g_4}{}_{g_1}A(B \to \overline{L}M) = 0, \tag{17}$$

where $A(B \to \overline{L}M)$ is a part of the amplitude corresponding to particle momenta, the color, the spin, and the flavor variables. The amplitudes of strong transitions (17) vanish as they express through the products of the symmetric and antisymmetric tensors in the space of generations. In particular, from (17) it follows that the $p \to e^+\gamma$ -, $p \to \mu^+\gamma$ -, $p \to e^+\pi^0$ -, $p \to \mu^+\pi^0$ -transitions are forbidden. Note that the amplitudes of radiative decays of the pronon can be derived from (17) by means of the model of vector dominance.

CONCLUSION

The proposed hypothesis on quark nature of leptons ensures the equality of the modules of the electron and the proton electric charges. The antisymmetry of the representations of the $U(N_f, g)$-group and the $SU(4, fs)$-group of q^3-systems for antileptons allows one to explain distinctions of the properties for leptons (antileptons) and hadrons. Indeed, the antisymmetric representation of the $SU(4, fs)$-group corresponds to two particles of the $\frac{1}{2}$-spin in the ground state (the $(1s)^3$-state), e.g., positron and antineutrino for q^3-systems. Next excited states occur for q^3-systems in $(1s)^2 1p$-state. Among excited states the leptons (antileptons) with double electric charge must exist. In consequence of big mass values for the quarks from the third generation and the antisymmetry of the representation of the $U(N_f, g)$-group the mass difference between the excited and the ground states have to be rather large.

Therefore, the leptons (antileptons) ought to behave at relatively small momentum transfer as point-like particles.

It may be assumed that the leptons (antileptons) can participate in strong interactions at high energies. It means that the strong interactions of the leptons (antileptons) are biased to the high-energy region. The leptons (antileptons) can be interpreted as strongly packed antibaryons (baryons).

It is of important to explain the mass spectrum of the baryons and the antileptons for the consistence of proposed classification of particles and the hypothesis on quark nature of leptons. There can be doubts in possibility to do this by means of the mass formula. It can be seen from the M_0-value for the nucleons and the leptons (antileptons). The M_0-value for the nucleons and $\Delta(1232)$ approximately equal $1.02GeV$ whereas according to (12) the minimal values of M_0 for the electron (positron) and electron neutrino (antineutrino) equal $M_0(e^-) \approx 2m(t)/3 = 118GeV$ and $M_0(\nu_e) \approx m(t)/3 = 59GeV$. Last values exceed essentially the experimental ones ($m(e) = 0.51MeV, m(\nu_e) < 5eV$). As it is known the color-magnetic interactions reduce the nucleon masses and enlarge the $\Delta(1232)$ masses. This allows obtain the values of the nucleon and the $\Delta(1232)$-isobar masses near to experimental ones. But it cannot be expected that taking into account of the color-magnetic interaction allows one to derive the values of the electron and the electron neutrino masses near to experimental values.

Assume that quarks interact by means of tensor mesons in an addition to the gluon exchange. Amplitudes of the quark interactions by means of the $J^p = 2^+$-meson exchange are proportional to the quark 4-momenta [13]. In the rest frame these amplitudes are proportional to the quark masses. Therefore, the $J^p = 2^+$-meson exchange may be just essential for heavy quarks (i.e. for leptons (antileptons)) and it can be invisible for hadrons consisting of light quarks (e.g., for the nucleons and the $\Delta(1232)$-isobars). In Refs. [14, 15] it is shown that interaction currents for higher spin particles (i.e. particles with the 2-spin) must obey the theorem on currents and fields as well as the theorem on continuity of current derivatives. In Ref. [13] the currents of the interactions of the $J^p = 2^+$-boson with the $\frac{1}{2}$-spin fermions obey the theorem on currents and fields.

In proposed hypothesis on quark nature of leptons the excited leptons (antileptons) with double electric charge are predicted. These excited leptons (antileptons) ought to be the meson-lepton (meson-antileptons) resonances. These mesons can be pseudoscalar or vector. The investigations such resonances may be very important for the test of proposed hypothesis.

Weak interactions of the quarks in higher orders can lead to such decays, as $\mu \to e\gamma, \mu \to e\gamma\gamma$. They can be investigated in the $\mu e \to \gamma\gamma -, \gamma e \to \gamma\mu -$ processes [16].

A situation with calculations of amplitudes for interactions of leptons and antileptons, represented as antiquark and quark systems, is similar to a situation with calculations of amplitudes of soft hadron processes in the quantum chromodynamics. As it is known, the QCD is the theory of strong interactions. But in the QCD the amplitudes of soft hadron reactions cannot be calculated. In particular, the coupling constants of the $\pi NN -, \pi N\Delta -$ interactions cannot be calculated by means of the QCD Lagrangian with known the constant of the quark-gluon interaction and the quark masses. On the other hand, amplitudes of some hadron reactions can be calculated with reasonable accuracy in models with hadrons represented as elementary particles. In such calculations the values of interaction constants and particle masses known from experimental data are used. Therefore, it is naturally that leptonic processes are described well in the framework of the electroweak theory with leptons and antileptons considered as elementary particles.

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