East Eur. J. Phys. Vol.3 No.1 (2016) 23-34

PACS: 11.10.Jj,40.Dw,11.90.+1,13.75.Lb,11.10.Gh

CONSISTENT MODEL FOR INTERACTIONS OF HIGHER-SPIN FERMIONS WITH 0- AND 1/2 - SPIN PARTICLES AND πN - SCATTERING

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Received February 8, 2016

It is shown that the currents for the interactions of the higher spin fermions must obey the theorem on currents and fields as well as the theorem on continuity of current derivatives. In consequence of the theorem on continuity of current derivatives the current components must decrease at $|p_\nu| \to \infty$, where p is the momentum of the higher spin fermion. The decrease of the currents is ensured by the form factors. The form factor in the vertex function of the interaction of the higher spin fermion with the 0 - and 1/2 - spin particles is derived in agreement with the theorem on continuity of current derivatives. The proposed model of the currents is used for the calculations of the contributions of the higher spin nucleon resonances $N^*(J)$ (J is the spin of higher spin fermion) to the s - channel amplitudes of the elastic πN-scattering. It is shown that these contributions to the amplitudes decrease at least as s^{-8} at the square of the energy $s \to \infty$.
KEY WORDS: Higher-spin fermions, differential equations, convergence of integrals, form factors, nucleon resonances, πN-scattering

НЕСУПЕРЕЧЛИВА МОДЕЛЬ ВЗАЄМОДІЇ ВИСОКОСПІНОВИХ ФЕРМІОНІВ З ЧАСТИНКАМИ ІЗ СПІНОМ 0 І 1/2 ТА πN-РОЗСІЮВАННЯ

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Показано, що струми взаємодій високоспінових ферміонів повинні задовольняти теоремі про струми та поля а також теоремі про неперервність похідних струмів. Внаслідок тереми про неперервність похідних струмів компоненти струмів повинні спадати при $|p_\nu| \to \infty$, де p - імпульс високоспінового ферміона. Спадання струмів забезпечується форм-факторами. Одержано форм-фактор у вершинній функції взаємодії високоспінового ферміона з частинками зі спіном 0 та 1/2, які узгоджуються з теоремою про неперервність похідних струмів. Запропонована модель для струмів використана для обчислення внесків високоспінових нуклонних резонансів $N^*(J)$ (J - спін високоспінового ферміона) в s - канальні амплітуди пружного πN-розсіювання. Показано, що ці внески в амплітуди спадають по меншій мірі як s^{-8} при квадраті енергії в с.ц.м. $s \to \infty$.
КЛЮЧОВІ СЛОВА: високоспінові ферміони, диференціальні рівняння, збіжність інтегралів, форм фактори, нуклоні резонанси, πN - розсіювання.

НЕПРОТИВОРЕЧИВАЯ МОДЕЛЬ ВЗАИМОДЕЙСТВИЯ ВЫСОКОСПИНОВЫХ ФЕРМИОНОВ С ЧАСТИЦАМИ СО СПИНОМ 0 И ½ И πN-РАССЕЯНИЕ

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Показано, что токи взаимодействий высокоспиновых фермионов должны удовлетворять теореме о полях и токах а также теореме о непрерывности производных токов. Вследствие теоремы о непрерывности производных токов компоненты токов должны убывать при $|p_\nu| \to \infty$, где p - импульс высокоспинового фермиона. Убывание токов обеспечивается форм-факторами. Получен форм-фактор в вершинной функции взаимодействия высокоспинового фермиона с частицами обладающими спином 0 и 1/2, в согласии с теоремой о непрерывности производных токов. Предложенная модель токов использована для вычисления вкладов высокоспиновых нуклонных резонансов $N^*(J)$ (J-спин высокоспинового фермиона) в s-канальные амплитуды упругого πN-рассеяния. Показано, что эти вклады в амплитуды убывают, по меньшей мере как s^{-8} при квадрате энергии в с.ц.м. $s \to \infty$.
КЛЮЧЕВЫЕ СЛОВА: высокоспиновые фермионы, дифференциальные уравнения, сходимость интегралов, форм-факторы, нуклонные резонансы, πN - рассеяние.

Higher – spin hadrons are investigated theoretically and experimentally more than fifty years. It is known they consist of the quarks and the antiquarks similarly to the nucleons and the pions. Unfortunately the soft reactions involving hadrons cannot be described fairly well in QCD. Therefore, models for amplitudes of hadron reactions are of importance. As it is known reactions with the π - mesons and (or) the nucleons at low and intermediate energies can be described with reasonable accuracy in frameworks of such approaches as isobar models and dispersion relations. These approaches are based on the quantum field theory. The π - mesons and the nucleons are considered in these approaches as elementary particles. Therefore, the consideration of higher spin hadrons ($J \geq 1$) similarly to elementary particles can be assumed also. Investigations of the higher spin particles show their qualitative distinctions from low-spin particles (i.e., the 0 – spin and the 1/2 – spin particles). For example, it can be seen from the papers [1-7] and the reviews [8, 9]. Calculations of cross-section for the production of higher spin hadrons (instead of low-spin particles) give a theoretical power energy growth of the cross-sections for higher spin particles production in comparison with a non-increase of the cross-sections for the low-spin particles only. In particular, if the vertexes of the higher spin hadrons include constants (without form factors) then HSF – resonance contributions to the s - channel amplitudes of elastic reactions give power energy growth of corresponding cross sections (which are parts of total cross sections for the same initial states). But it is known [10] that according to experimental data the total cross-sections at high energies approach to some constants or increase as $\ln s$, where s is a square of a total energy in CMS. It is a contradiction, as the part cannot be greater than the whole one. As a rule, a power of s in amplitudes for the HSF contributions increases with J. To eliminate the energy growth of the amplitudes the interactions in initial and final states are considered [11] or the form factors in interaction vertexes are introduced (as a generalization of coupling constants). The form factors of monopole and dipole types are often used. But these form factors give diverging integrals for the contributions of HSF to the amplitudes corresponding to the loop diagrams. Thus, the form factors with other analytical dependences are needed. Besides, a strong distinction between theoretical predictions and experimental data on the cross-sections for a production of the higher spin fermions (HSF) exists. For example, the rule has been formulated on the basis of these data: the higher spin nucleon resonances are formed but not produced. It means that the higher-spin nucleon resonances $N^*(J)$ are well observed at their excitation in πN - and γN - interactions. But the cross-sections of the $N^*(J)$ (with an exception for $\Delta(1232)$) production together with another particle are fairly small. These characteristic distinctions between the theoretical results for the low-spin particle interactions indicate on the necessity of a modification of the existing approaches to description of the higher-spin particle interactions. It may be expected that the general properties for the interactions of higher-spin particles of any mass must exist in the addition to the properties of the low-spin particle interactions.

The contributions of the higher spin hadrons are considered at relatively high energies in the Regge pole model [12, 13]. It is known that particles of different spin with the same values of the electric charge, the isospin, the strangeness, and parities are on the trajectories, which are approximately straight on the plot of a square of a mass and an angular momentum. It allows predict the masses of the higher spin hadrons. The Regge pole model describes well the differential cross-sections of binary reactions at relatively high energies and small modules of momentum transfer.

In consequence of the relativistic invariance the vertex functions (related to the interaction lagrangian) for higher-spin particle interactions are the scalar product of the field tensor for the higher-spin boson (or spin-tensor for HSF) and corresponding current tensor (on spin-tensor).

We use the Rarita-Schwinger formalism (e.g. see Ref. [12]). For HSF such vertex functions give the non-homogeneous Dirac equation

$$\left(i\hat{\partial}-M\right)U(x)_{\mu_1\ldots\mu_l}=\chi(x)_{\mu_1\ldots\mu_l}, \tag{1}$$

where $U(x)_{\mu_1\ldots\mu_l}\equiv U(x)^l_\mu$ is the field spin-tensor of HSF, M is the HSF mass, $\chi(x)_{\mu_1\ldots\mu_l}=\chi(x)^l_\mu$ is the current spin-tensor, $J=l+\frac{1}{2}$. As it is known the Klein-Gordon equation and the Dirac equation are not used immediately in the calculations of the amplitudes for the particles of the 0-spin and the ½-spin, respectively. In such calculations vertex functions and propagators are used. The propagators are the causal Green functions of these equations. Similarly to this, the HSF propagators in the calculations of the amplitudes correspond to the causal Green functions of the Eq. (1). Besides, an action of the $\left(i\hat{\partial}+M\right)$-operator on the homogeneous Dirac equation in (1) leads to the homogeneous Klein-Gordon equation, which corresponds the relativistic relation between the energy, 3-momentum, and the M mass of the free HSF.

It is known that the field spin-tensor must obey next auxiliary conditions: (i) the four-dimension divergences and traces vanish; (ii) their convolution with γ - matrices vanishes too. Usually for the current spin-tensors it is assumed that they obey the symmetry condition only. We name the approaches with such current as usual ones. In Refs.[14, 15] it is shown that the usual approaches have some shortcomings: (i) the algebraic inconsistencies of the equation systems; (ii) the power divergences of the loop amplitudes; (iii) the ambiguities of the vertex functions for virtual HSF; (iv) the contradictions to the experimental data in wide energy regions. As these shortcomings are in usual approaches to the interactions of any spin and mass HSF, we assume that the interaction currents must have got some general properties in addition to the symmetry condition. In Refs.[16, 17] for the higher – spin bosons and in Ref. [18] for HSF have been shown that the interaction currents must obey two theorem: (i) theorem on currents and fields: (ii) theorem on continuity of current derivatives. We denote the current spin – tensors, which obey these theorems, as $j(x)_{\mu_1...\mu_l} \equiv j(x)^l_\mu$ and name them as physical currents. Accordingly to the theorem on currents and fields the physical current spin-tensors must obey the same conditions as the field spin-tensors:

$$\partial_{\mu_k} j(x)_{\mu_1...\mu_l} = 0, \qquad p_{\mu_k} j(p)_{\mu_1...\mu_l} = 0, \tag{2}$$

$$g_{\mu_i\mu_k} j(x)_{\mu_1...\mu_l} = 0, \qquad g_{\mu_i\mu_k} j(p)_{\mu_1...\mu_l} = 0, \tag{3}$$

$$\gamma_{\mu_k} j(x)_{\mu_1...\mu_l} = 0, \qquad \gamma_{\mu_k} j(p)_{\mu_1...\mu_l} = 0, \tag{4}$$

coordinate representation   momentum representation

where $i,k = 1,2,...,l$. Note that, as the consequence of Esq. (2) (the current conservation), the contributions of the HSF propagator terms including the HSF momentum p_{μ_i} or p_{ν_k} to the products of the HSF propagator and the physical currents vanish. Thus, the current conservation (2) allows to avoid (Refs. [14-20]) one source of the power divergences which exists in usual approach. In Refs [21, 22] the model for the interaction of the higher – spin boson with two spinless particles, which obeys the theorem on currents and fields as well as the theorem on continuity of current derivatives has been proposed. Using this model in Ref. [15] it is shown that the contribution of the virtual higher – spin boson and spinless particle to self – energy operator for spinless particle gives finite result in one – loop approximation, whereas usual approaches give for this operator the power divergences. Note that, as it is known, the $\lambda\varphi^3$ - theory gives the logarithmic divergence for the self – energy operator in one-loop approximation.

In paper [14] the model for HSF interaction with the 0 – and $\frac{1}{2}$ - spin particles is proposed. This model obeys the theorem on currents and fields. The consequences of this theorem can be tested in πN - scattering. The model [14] and usual approaches give different sets of the partial amplitudes corresponding to the off – mass- shell $\Delta(1232)$. In the model [14] the off – mass- shell $\Delta(1232)$ contributes to the P_{33} - and D_{33} amplitudes only, whereas in usual approaches $\Delta(1232)$ contributes to $S_{31}, -P_{31}, -P_{33},$ and D_{33} - amplitudes. In the usual approach the off – mass-shell $\Delta(1232)$ gives the most contribution to the S_{31} - amplitude.

According to the partial wave analyses the energy dependence of the S_{31} - amplitude agrees better with the zero contribution of $\Delta(1232)$. Thus, we may conclude that theorem on currents and fields is valid.

The present paper is devoted to modification of existing approaches (usual approaches) to eliminate their shortcomings. We try to achieve a mathematical correctness and do not use any experimental data. In Refs [14, 15, 21] it is shown that the divergences in usual approaches related to the higher spin particle propagators can be avoided by means of the theorem on currents and fields. The divergences in usual approaches related to the vertex functions possibly can be avoided by means of the theorem on continuity of current derivatives. In present paper the model for simplest interactions of HSF is proposed. We study the validity of the theorem on continuity of current derivatives [18] and apply the derived currents to investigate the contributions of the HSF resonances $N^*(J)$ to the s-channel amplitudes of πN-scattering at high energies. This paper is the continuation of [14].

THEOREM ON CONTINUITY OF CURRENT DERIVATIVES

The physical current in the momentum representation $j(x)_{\mu_1...\mu_l} \equiv j(x)^l_\mu$ has been derived in Ref. [18] by means of the projection operator $\Pi(p)_{\mu_1...\mu_l,\nu_1...\nu_l} \equiv \Pi(p)^l_{\mu,\nu}$. We use the modification of the projection operator from Refs. [23, 24] proposed in Refs. [18, 19]

$$j(p)^l_\mu = \left(p^2\right)^l \Pi(p)^l_{\mu,\nu}\, \eta(p)^l_\nu . \tag{5}$$

The currents $j(x)^l_\mu$ are the Fourier transformations of $j(p)^l_\mu$. We consider the HSF moving along the z - axis, $p = (p_0, 0, 0, p_3)$. Then the current is given by

$$j(x_0,x_3)^l_\mu = \int_{-\infty}^{\infty} dp_0 \int_{-\infty}^{\infty} dp_3 e^{i(p_0x_0 - p_3x_3)} j(p_0,p_3)^l_\mu . \tag{6}$$

The integrals (6) are the improper integrals depending on parameters x_0, x_3. These integrals must converge at arbitrary x_0, x_3. However, the current spin-tensors $j(x_0,x_3)^l_\mu$ have the partial derivatives of some degree (e. g. in the differential equations for the HSF interactions). In this case the integrals with the integrands including the products of $j(p)^l_\mu$ and some number of the p- components have to converge in addition also. If we shall demand that $j(x)^l_\mu$ and their partial derivatives are continuous functions in the space-time then the convergence of these integrals must be uniform. Thus, the existence of the $j(x)^l_\mu$ and their partial derivatives leads to the asymptotic decreasing of $j(p)^l_\mu p_{\nu_1}...p_{\nu_m}$ with p_0, p_3. Using the Weierstrass test for uniform convergence of integrals for $J(x)^l_\mu$ and their partial derivatives we conclude that the next integrals have to converge:

$$\int_{-\infty}^{\infty} dp_0 \int_{-\infty}^{\infty} dp_3 \left| j(p_0,p_3)^l_\mu \right| \cdot |p_0|^{m_0} |p_3|^{m_3}, \tag{7}$$

where m_0 and m_3 are non-negative integer numbers, $m_0 + m_3 = m(j)$. The integer number $m(j)$ is determined by the maximal degree of the partial derivatives.

In Ref. [18] $m(j)$ has been derived from the differential equations for the HSF interactions. Indeed to derive the consistent equations system including the physical currents $j(x)^l_\mu$ we act on the equation in system (1) by the operator $(-\Box)^l \Pi(x)^l_{\mu,\nu}$. Then using (5) and the properties of the projection operator we derive

$$(-\Box)^l \left(i\hat{\partial} - M \right) U(x)^l_\mu = j(x)^l_\mu . \tag{8}$$

From the theory for the system of the linear differential equations it is known that we can derive the set of the non-homogeneous linear differential equations with the same left sides for each unknown function. But the maximal degree of the derivatives in each such equation is more than the degree in the system. In particular for the system of the ordinary differential equations the degree of each equation in the set for each unknown function equals to the sum of the degrees of all the differential equations in the system. But for the system of partial differential equations such results is not valid [25]. It can be seen on the example for the system of the Dirac equations, where the equations in the system have first degree, but the equations in the set (the non-homogeneous Klein-Gordon equations) have second degree. As second example we consider the telegraph equations, (equations for the current strength $I(x,t)$ and the voltage $U(x,t)$). These differential equations can be writed as the system

$$\begin{aligned} &\frac{\partial U}{\partial x} + RI + L\frac{\partial U}{\partial t} = 0, \\ &\frac{\partial I}{\partial x} + GU + C\frac{\partial U}{\partial t} = 0, \end{aligned} \tag{9}$$

where R, L, G, C are constants. This is the system of homogeneous partial differential linear equations of the first degree. But from (9) the set of two differential equations can be derived:

$$\frac{\partial^2 U}{\partial t^2} + \left(\frac{R}{L} + \frac{G}{C} \right) \frac{\partial U}{\partial t} + \frac{R}{L} \cdot \frac{G}{C} \cdot U = \frac{1}{LC} \cdot \frac{\partial^2 U}{\partial x^2}, \tag{10}$$

$$\frac{\partial^2 I}{\partial t^2} + \left(\frac{R}{L} + \frac{G}{C} \right) \frac{\partial I}{\partial t} + \frac{R}{L} \cdot \frac{G}{C} \cdot I = \frac{1}{LC} \cdot \frac{\partial^2 I}{\partial x^2}. \tag{11}$$

We see that the Esq. (10), (11) have the same form. But these equations have got the second degree, i.e. the degree of the equation for each unknown functions increases in comparison with the degree of equations in the system (9). To derive the set of equations for each $2J+1$ unknown functions (the analog of the set (10), (11)) the action of the - $\left(i\hat{\partial} + M \right)$ operator on the system (8) has been considered in Ref. [18]:

$$(-\Box)^l \left(\Box + M^2 \right) U(x)^l_\mu = -\left(i\hat{\partial} + M \right) j(x)^l_\mu . \tag{12}$$

It is the set of the differential equations for the HSF interactions and it is similar to the non-homogeneous Klein-Gordon equation for the $\frac{1}{2}$-spin particles. The set (12) includes the first derivatives of $j(x)^l_\mu$. The set of the differential equations for $2J+1$ independent components of $U(x)_{\mu}^{l}$ (the analog of set (10), (11)) cannot be derived as components of $j(p)_{\mu}^{l}$ obey the conditions (2)-(4). In consequence of these conditions (and the similar conditions for $U(x)_{\mu}^{l}$) the physical independent functions are implicit functions for $J>1$. For the components corresponding to $|\lambda| \neq J$ we have more than one equation. (e. g. at $J=1, \lambda=0$ we have different equations for $U_0(x)$, $U_3(x)$). These differential equations for the state with $J=1, \lambda=0$ can be reduced to one equation if we shall differentiate these equations with respect to different variables (for $J=1, \lambda=0$ the equation for $U_0(x)$ we must differentiate with respect to x_3 and equation for $U_3(x)$ with respect to x_0). Thus, we must differentiate to derive the $2J+1$ independent functions (the states with definite J, λ). In consequence of this differentiation, the largest degree of the continuous derivatives for $J(x)_{\mu}^{l}$ has to equal two [18]. Therefore, in Ref.[5] it is derived $m(j)=2$.

Now we take into account the difference between the requirements for the functions of one and two variables which can be expanded in the Fourier series or the Fourier transformation. It is known that the functions of one variable with the finite number of the ordinary discontinuities on any finite interval (e.g. if the function obey the Dirichlet conditions) can be expanded in the Fourier series. Note that the function for the Fourier integral must be absolutely integrated in addition. For such function the Fourier series and integral converges to the magnitudes of this function in the points of its continuity. But the double Fourier series converges to the magnitude of the function of two variables in the point if this function is continuous and have continuous derivatives of the first degree and the mixed derivative of the second degree [26]. We assume that such absolutely integrable functions can be expanded in the Fourier integral. The magnitude of $m(j)$ must be increased to 4.

The currents (5) include bispinors with the $\sqrt{p_{10}+m_N}$ - or $\sqrt{p_{20}+m_N}$ - factor. These factors behave as $\sqrt{p_o/2}$ at $p_0 \to \infty$. Therefore, to take into account these factors we put $m(j)=5$.

For the common currents $\eta(p)_{\mu}^{l}$ we can write the Fourier integral similarly to Eq.(6). In the same way we can write the condition for the common spin-tensor $\eta(p)_{\mu}^{l}$ (similar to Eq. (7)). The integrals

$$\int_{-\infty}^{\infty} dp_0 \int_{-\infty}^{\infty} dp_3 \left|\eta(p)_{\mu}^{l}\right| |p_0|^{m_0} |p_3|^{m_3}, \tag{13}$$

where $m(\eta)=m(j)+2l=m_0+m_3$, must converge.

Thus, we conclude that the numbers $m(j)$ and $m(\eta)$ depend on the degree of the continuous partial derivatives for the HSF currents and fields.

As result we formulate the theorem on continuity of current derivatives (which can be named as theorem on current asymptotics):

If the $j(x)_{\mu}^{l}$ physical current has the continuous partial derivatives of the $m(j)$ degree then their Fourier – components $j(p)_{\mu}^{l}$ must decrease at $|p_{\nu}| \to \infty$ to provide the convergence of the integrals (7) in all the kinematic regions.

In consequence of the theorem on continuity of current derivatives the Fourier– components of the $j(p)_{\mu}^{l}$ physical currents and the $\eta(p)_{\mu}^{l}$ usual currents which are the rational fractions must decrease as $\left|j(p)_{\mu}^{l}\right| \le |p_{\nu}|^{-w(j)-3}$ and $\left|\eta(p)_{\mu}^{l}\right| \le |p_{\nu}|^{-w(j)-2l-3}$ at $|p_{\nu}| \to \infty$, respectively.

Note that the theorem on continuity of current derivatives is the consequence of the relation between the convergence of the integrals depending on the parameters (x_0 and x_3) and the continuity of this integrals as well as their derivatives with respect to these parameters. In Ref. [21] the example of such relation is considered for the $\varphi(x)=\int_{-\infty}^{\infty} cos\, px\, dp/\left(p^4+a^4\right)$ function.

The important sense of the theorem on continuity of current derivatives is related to the convergence of the integrals (7), (13) at infinite p_0, p_3.(improper integrals of the first kind) as well as at finite p_0, p_3 (improper integrals of the second kind).

FORM FACTORS FOR INTERACTION OF HSF WITH 0-AND 1/2-SPIN PARTICLES

Let us consider the physical current of $J(p)\to\frac{1}{2}(p_2)+o(q_2)$ - transition of Ref. [14]

$$j(p,q)_{\mu_1...\mu_l}=g_lF_l(p,q)\left(p^2\right)^l\cdot \tag{14}$$

$$\varphi^+(q_2)\bar{u}(p_2)\begin{Bmatrix}1\\ i\gamma_5\end{Bmatrix}\Pi(p)_{\mu_1...\mu_l,\nu_1...\nu_l}\,q_{\nu_1}...q_{\nu_l},$$

where $q=q_2-p_2$. The usual current corresponding to Eq. (14) may be written as

$$\eta(p,q)_{\mu_1...\mu_l}=g_lF_l(p,q)\varphi^+(q_2)\bar{u}(p_2)\begin{Bmatrix}1\\ i\gamma_5\end{Bmatrix}q_{\nu_1}...q_{\nu_l}. \tag{15}$$

In Refs [21, 22] the form factor for the interaction of the higher – spin boson with two spinless particles has been derived

$$f(p,q)=\left[(pq)^{2n_1}+a^{4n_1}\right]^{-1}\left[\left(2\frac{\left(pq'\right)^2}{q'^2}-p^2\right)^{2n_2}+b^{4n_2}\right]^{-1}, \tag{16}$$

where a and b are some positive constants, n_1 and n_2 are some natural numbers (integer positive).

The function (16) permits one to satisfy the theorem on continuity of current derivatives for the interaction of the higher spin boson with two spinless particles. However, the application of this function to the $N^*(J)\leftrightarrow N\pi$ - transitions leads to the contradiction with the results of the partial wave analyses. This contradiction is due to vanishing of $f(p,q)$ (16) at $q^2=0$, which corresponds to $W_0=1341MeV$. $W_0^2=2\left(m_N^2+m_\pi^2\right)$, where m_N and m_π is the nucleon and the pion mass, respectively. Therefore, the real and the imaginary parts of the partial amplitudes corresponding to the $N^*(J)$ contributions with the vertex functions including $f(p,q)$ (16) must vanish. But it is well known that the partial resonance amplitudes, in particular for $\Delta(1232)$ - excitation, in πN - scattering, do not equal zero at $W=W_0$. It is possible that this contradiction can be eliminated by means of some modification of the function (16). We can consider for $F(p,q)$ in Eqs. (14), (15) the product $f(p,q)$ (16) and the $\left(q^2\right)^{-2n_2}\cdot\left[cq^{4n_3}+c^{4n_3}\right]$ factor, where c is the positive constant, n_3 is the natural number. But this $f(p,q)$ form factor gives fairly complicated s - dependence of the resonance excitation amplitudes in the πN - scattering. Therefore, it is of interest to derive another form factor with more simple p^2 - dependence.

Let us consider the non-negative continuous function $f_l(p,q)$ for which the integrals

$$J_{m_0m_3}=\int_{-\infty}^{\infty}dp_0\int_{-\infty}^{\infty}dp_3f_l(p,q)\left|p_0\right|^{m_0}\left|p_3\right|^{m_3} \tag{17}$$

converge $\left(m_0+m_3=0,1,2,...,2l+5\right)$. Then the physical currents (14) with any form factor $F_l(p,q)$ for which

$$\lim_{p_0^2+p_3^2\to\infty}\frac{\left|F_l(p,q)\right|}{f_l(p,q)}\le C,\quad 0<C<\infty, \tag{18}$$

obey the theorem on currents and fields as well as the theorem on continuity of current derivatives. Therefore, in further we shall consider the $f_l(p,q)$ function.

To study the convergence of the integrals (17) we use the method of Refs. [21, 22]. The scalar function $f_l(p,q)$ depends on the invariant variables $p^2,(p,q),q^2$. We put the $q=q_2-p_2$ fixed. Then $f_l(p,q)$ depends on two invariant variables: p^2 and (p,q). We shall use next statements.

Statement 1. If for the non-negative continuous function $g(p,q)$ and the $f_l(p,q)$ function

$$\lim_{p_0^2+p_3^2\to\infty}\frac{g(p,q)}{f_l(p,q)}\ge 1, \tag{19}$$

and the integrals

$$J_{m_0,m_3} = \int_{-\infty}^{\infty} dp_0 \int_{-\infty}^{\infty} dp_3 g(p,q) |p_0|^{m_0} |p_3|^{m_3} \tag{20}$$

converge, then the integrals $J_{m_0 m_3}$ (17) converge too.

Statement 2. If the integral J_{m_0+1,m_3} or J_{m_0,m_3+1} converges then the integral $J_{m_0 m_3}$ converges too.

As the integrand in (17) includes the modulus of p_0 and p_3, $J_{m_0 m_3}$ is not relativistic covariant at arbitrary numbers m_0 and m_3 [21]. However, these integrals can be expressed in terms of relativistic covariants at even m_0, m_3 and they depend on q^2. We denote the minimal even number, which equal or is bigger than 5, as m_1 (i.e., $m_1 = 2l+6$).

Let us consider the function

$$f_l(p,q) = (pq)^{2n_1} \left[\left(p^2\right)^{2n_2} + a^{4n_2} \right]^{-1} \left[(pq)^{2n_3} + b^{4n_3} \right]^{-1}, \tag{21}$$

where a and b are positive constants, n_1, n_2, and n_3 the natural numbers.

At $q^2 < 0$ we can choose the system with $q_0 = 0$ and $q^2 = -\vec{q}_\perp^{\,2} - q_3^2$. Then we have for even m_0 and m_3

$$J_{m_0 m_3}(q_0 = 0) = q_3^{2n_1} \int_{-\infty}^{\infty} \frac{|p_3|^{2n_1+m_3} dp_3}{(p_3 q_3)^{2n_3} + b^{4n_3}} \cdot 2 I_{m_0}(p_3), \tag{22}$$

where

$$I_{m_0}(p_3) = \int_0^{\infty} \frac{p_0^{m_0} dp_0}{\left(p_0^2 - p_3^2\right)^{2n_2} + a^{4n_2}}. \tag{23}$$

Similarly to the Statement 2 we conclude that integral $I_{m_0}(p_3)$ converges if the integral $I_{m_0+1}(p_3)$ converges. We have

$$I_{m_0+1}(p_3) = \frac{1}{2} \int_0^{\infty} \frac{p_0^{\frac{m_0}{2}} dp_0^2}{\left(p_0^2 - p_3^2\right)^{2n_2} + a^{4n_2}} = \frac{1}{2} \int_{-p_3^2}^{\infty} \frac{\left(x + p_3^2\right)^{\frac{m_0}{2}} dx}{x^{2n_2} + a^{4n_2}} < \frac{1}{2} \int_{-\infty}^{\infty} \frac{\left(x + p_3^2\right)^{\frac{m_0}{2}} dx}{x^{2n_2} + a^{4n_2}}, \tag{24}$$

where $x = p_0^2 - p_3^2 = p^2$. For last integral we derive

$$\begin{aligned} \frac{1}{2} \int_{-\infty}^{\infty} \frac{\left(x + p_3^2\right)^{\frac{m_0}{2}}}{x^{2n_2} + a^{4n_2}} dx &= \sum_{k=0}^{\frac{m_0}{2}} \left[1 + (-1)^k\right] \begin{pmatrix} \frac{m_0}{2} \\ k \end{pmatrix} \cdot \left(p_3^2\right)^{\frac{m_0}{2} - k} \int_0^{\infty} \frac{x^k dx}{x^{2n} + a^{4n}} = \\ &= \frac{\pi}{4n_2} \sum_{k=0}^{\frac{m_0}{2}} \begin{pmatrix} \frac{m_0}{2} \\ k \end{pmatrix} \left[1 + (-1)^k\right] \frac{(p_3)^{m_0 - 2k}}{\left(a^2\right)^{2n_2 - k - 1}} \cdot \frac{1}{\sin \frac{\pi(k+1)}{2n_3}} \end{aligned}, \tag{25}$$

$$\text{as} \int_0^{\infty} \frac{z^m dz}{z^{2n} + a^{2n}} = \frac{\pi}{2n a^{2n_2 - m_1 - 1}} \cdot \frac{1}{\sin \frac{\pi(m+1)}{2n}}, \tag{26}$$

where m and n are natural numbers $(m \le 2n - 2)$. The integrals (25) converge for $k_{max} = \frac{m_1}{2}, 2n_2 \ge k_{max} + 2,\ n_2 \ge \frac{m_1}{4} + 1$. Thus, we have

$$\begin{aligned} n_2 &\ge \frac{l+5}{2} \ \ for\ odd\ l \\ n_2 &\ge \frac{l}{2} + 3 \ \ for\ even\ l \end{aligned} \tag{27}$$

After substitution of the integral (25) in double integral (22) we derive

$$J_{m_0+1,m_3}\left(q_0=0\right)<\frac{\pi}{4n_2}\sum_{k=0}^{\frac{m_0}{2}}\begin{pmatrix}\frac{m_0}{2}\\k\end{pmatrix}\left[1+\left(-1\right)^k\right]\cdot\frac{\left(a^2\right)^{-2n_2+k+1}}{sin\frac{\pi}{2n_2}\left(k+1\right)}\cdot q_3^{2n_1}\int_{-\infty}^{\infty}\frac{\left|p_3\right|^{2n_1+m_3+\frac{m_0}{2}-k}dp_3}{\left(p_3q_3\right)^{2n_3}+b^{4n_3}}. \tag{28}$$

In last integral with respect to p_3 the most increasing integrand at $p_3\to\infty$ corresponds to $k=0$ and the maximal value of the natural number $m_3+m_0=2l+6$. Therefore, if the integrals (28) converge for $k=0$ and $m_3=m_1$ then all the integrals $J_{m_0m_3}\left(q_0=0\right)$ (22) converge at $q_3\neq 0$ and at $0\le m_0,\ m_3\le m_1$. To consider the case of $q_0=q_3=0$ (i.e., for $q=\left(0,q_1,q_2,0\right)$, $q^2<0$) we introduce new variable of integration in (28) $y=p\cdot q=p_3q_3$. Then the integrals in (28) for $k=0$ and $m_3=m_1$ may be written as

$$q_3^{2n_1}\int_{-\infty}^{\infty}\frac{\left|p_3\right|^{2n_1+m_1}}{\left(p_3q_3\right)^{2n_3}+b^{4n_3}}dp_3=2q_3^{-1-m_1}\int_{0}^{\infty}\frac{y^{2n_1+m_1}}{y^{2n_3}+b^{4n_3}}dy\ . \tag{29}$$

This integrals do not exist for $q_3=0$. But for natural n_1 the function $f_l\left(p,q\right)$ (22) equals to zero. Thus, we can put $J_{m_0m_3}\left(q_0=q_3=0\right)=0$ at $n_1\ge 1$. In the case $n_1=1$ we have the restriction for n_3 from the convergence of the integrals in (28)

$$n_3\ge l+5 \tag{30}$$

Using the statement 2 we conclude that the integrals $J_{m_0m_3}$ converge for any allowed m_0 and m_3 $\left(0\le m_0,\ m_3\le m_1\right)$ at $q^2<0$.

Now we consider the integrals $J_{m_0m_3}$ at $q^2=q_0^2-q_1^2-q_3^2>0$. We can choice the frame with $q_3=0$ at even numbers m_0 and m_3. Then the integrals (17) are given by

$$J_{m_0m_3}\left(q_3=0,q^2>0\right)=q_0^{2n_1}\int_{-\infty}^{\infty}\frac{\left|p_0\right|^{2n_1+m_0}}{\left(p_0q_0\right)^{2n_3}+b^{4n_3}}\cdot\int_{-\infty}^{\infty}\frac{\left|p_3\right|^{m_3}dp_3}{\left(p_3^2-p_0^2\right)^{2n_2}+a^{4n_2}}\ . \tag{31}$$

If we change $p_0\rightleftarrows p_3, m_0\rightleftarrows m_3$ then the integrals (31) become equal to the integrals (22), (23). For $q^2=0$ we can consider the limit of (31) at $q_0^2\to q_1^2+q_2^2$. Thus, the (14) with $F_l\left(p,q\right)=f_l\left(p,q\right)$ (21) obey the theorem on continuity of current derivatives at $n_1=1$ and the requirements (27), (30) for natural numbers n_2,n_3.

CONSEQUENCES OF THEOREM ON CONTINUITY OF CURRENT DERIVANIVES FOR $N^*\left(J\right)$ CONTRIBUTION TO πN - SCATTERING AT HIGH ENERGY

Let us at first study the energy dependence of $f_l\left(p,q\right)$ for $N^*\left(J\right)\rightleftarrows N\pi$ - transitions when the nucleon and the pion are on its mass shells. This form factor can be used in the $s-$ and $u-$ channel amplitudes of the πN - scattering. For the $s-$channel $p=p_1+q_1=p_2+q_2,\ p^2=s=W^2,\quad q=q_2-p_2,\ p\cdot q=m_\pi^2-m_N^2$, where $p_1\left(p_2\right)$ and $q_1\left(q_2\right)$ are the 4- momentum of the initial (final) nucleon and the pion, respectively. For the form factor $f_l\left(p,q\right)$ (21) we can write in this case

$$f_l\left(p,q\right)<A/\left(s^{2n_2}+d^{4n_2}\right). \tag{32}$$

The A constant is equal to $\left(n_1=1\right)$

$$A=\frac{\left(m_N^2-m_\pi^2\right)^2}{\left(m_N^2-m_\pi^2\right)^{2n_3}+b^{4n_3}}\ . \tag{33}$$

For $u-$ channel we have $p=p_1-q_2=p_2-q_1,\ p^2=u,\ q=p_2+q_1,\ \ q.p=m_N^2-m_\pi^2$ and

$$f_l(p,q) \le A / \left(u^{2n_2} + a^{4n_2}\right), \tag{34}$$

where A is given by Eq. (33).

In particular for the $\Delta(1232) \leftrightarrow N\pi -$ transition we have from (32)

$$f_l(p,q) \le A / \left(p^{12} + a^{12}\right). \tag{35}$$

In the c.m.s. for πN - scattering is $s-$ channel the HSF momentum $p = (W,0,0,0)$ and $p \cdot q = m_\pi^2 - m_N^2, q^2 = 2\left(m_\pi^2 + m_N^2\right) - s$. Then the integral (17) is proportional to

$$J_{m_0 m_3} \sim \int_{-\infty}^{\infty} dp \int_{-\infty}^{\infty} dp_3 f_l(p,q) |p_0|^{m_0} |p_3|^{m_3} \delta(p_3). \tag{36}$$

The integral (36) vanishes at $m_3 \ne 0$. Therefore, we put $m_3 = 0$, $m_0 = 0,1,...,2l + m(j)$. As consequence of the Statement 2 the integrals (36) converge at $m_0 < m_1$ if the integrals (36) converge at $m_0 = m_1 \le 2l + 5$. Thus, the convergence $J_{m_1 0}$ in (36) is related to the convergence of the integrals:

$$J_{m_1 0} \sim A \int_0^{\infty} \frac{W^{3l+6} dW}{W^{4n_2} + a^{4n_2}}, \tag{37}$$

as $\left|\vec{q}\right| \to W/2$ at $W \to \infty$. This integral converges at $4n_2 \ge 3l + 8,$ i.e.:

$$n_2 \ge \frac{3}{4} l + 2. \tag{38}$$

Let compare the restrictions (27) and (38) for some l. From (27) next minimal magnitudes of n_2 can be derived $n_2 = 3,4,5,5,5$ for $l = 1,2,3,4,5,$ respectively. Similar magnitudes derived from (38) are; $n_2 = 3,4,5,5,6$ for $l = 1,2,3,4,5,$ respectively. We see that the restrictions (27) and (38) give the same minimal magnitudes for $l = 1,2,3,4$, i.e., for the $J = \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}$, respectively. For $l = 5$ the restriction (38) gives bigger magnitude. Thus, the restriction (38) can provide the convergence of the integrals (17) in the kinematics with fixed q as well as the kinematics of HSF-resonance at the rest. Therefore, the restriction (38) ought to be used in the form factor (21).

Now we consider the contributions of the HSF –resonances $N^*(J)$ to the $s-$ channel amplitudes of the πN - scattering in our approach. According to Eqs. (23), (24), (31) of Ref. [14] the product of the $N^*(J)$ propagator and two physical currents may be written as

$$\begin{gathered} T\left(\pi N \to N^*(J) \to \pi N\right) = g_2^2 \left[f_l(p,q)\right]^2 \left(p^2\right)^{2l} \cdot \\ \varphi^+(q_2) \varphi(q_1) \bar{u}(p_2) \frac{\widehat{p} \pm M}{p^2 - M^2 + iM\Gamma} \Pi\left(p,q',q\right) u(p_1), \end{gathered} \tag{39}$$

where g_l is the $\pi NN^*(J)$ coupling constant, $\Pi\left(p,q',q\right)$ is the contracted projection operator. The product $\bar{u}(p_2) \frac{\widehat{p} \pm M}{p^2 - M^2 + iM\Gamma} u(p_1) \varphi^+(q_2) \varphi(q_1)$ approaches to some constant at $s \to \infty$. Accordingly to Eq. (15) of Ref. [14] the contracted projection operator in the rest frame of $N^*(J)$ is given by

$$\Pi\left(p,q',q\right) = \frac{l!4^l}{(2l+1)!!} \left|\bar{q}_1\right|^{2l} \left[P'_{l+1}(z) - \frac{\overline{\sigma q_2} \cdot \overline{\sigma q_1}}{\left|\bar{q}_1\right|^2} P'_l(z)\right], \tag{40}$$

where $z = cos\,\theta$, and θ is the scattering angle, $\left|\bar{q}\right| = \left|\bar{q}\right|' = 2\left|\bar{q}_1\right| = 2\left|\bar{q}_2\right|;$ $P_l(z)$ is the Legendre polynomial. The contracted projection operator behaves as $\Pi\left(p,q',q\right)\sim\left|\bar{q}_1\right|^{2l}=s^l/4^l$ at $s\to\infty$. Therefore, we obtain $T\left(\pi N\to N^*(J)\to\pi N\right)\sim\left[f_l(p,q)\right]^2 s^{2l}$. Using the energy dependences (32) of the function $f_l(p,q)$ we derive

$$T\left(\pi N\to N^*(J)\to\pi N\right)\sim\frac{s^{3l}}{s^{4n_2}}, \tag{41}$$

where integer number n_2 obey the restriction (38). Then the energy dependence of the $N^*(J)$ - contributions to the πN - scattering amplitudes is given by

$$T\left(\pi N\to N^*(J)\to\pi N\right)\le\frac{C_1}{s^8}, \tag{42}$$

where C_1 is constant. Thus, in consequence of the theorem on continuity of current derivatives the contributions of $N^*(J)$ to the πN - scattering amplitudes in s - channel for arbitrary $J>\frac{1}{2}$ decrease at least as $1/s^8$ at $s\to\infty$. Compare the restrictions (42) with the result of the quark counting rule [29, 30]. According to this rule the amplitude of the elastic pion-nucleon scattering has to behave as $1/s^8$. Two distinctions are between the restrictions (42) and the result of the quark counting rule. The quark counting rules correspond to a lot of different contributions and are valid for hard processes (i.e., for great s and $|t|, t=(p_1-p_2)^2$). The restrictions (42) correspond to the contribution of one $N^*(J)$-resonance and valid for arbitrary scattering angle. Thus the restrictions (42) do not contradict to the results of quark counting rules.

CONCLUSION

From Ref.[14] we see that the HSF interaction models which contradict to the theorem on currents and fields are not consistent. It is due to the inconsistency of the linear algebraic equation system for the Fourier components of the field spin-tensors. Note that the theorem on currents and fields allows simultaneously to derive the scale dimension of the HSF propagator equal to -1 for any half-integer spin value HJF and to eliminate the ambiguities in the vertex functions of HSF interactions. In addition to this condition of the consistency the continuity of the current components and their partial derivatives in the space-time up to some degree may be considered as the condition of the consistency. Indeed if the current components or their partial derivatives have discontinuities (points or lines) we must indicate a fashion or a direction of an approaching to this discontinuity in the space-time. The model with discontinuities of the current components is inconsistent without such information on the fashion of approaching to these discontinuities.

It is known that amplitudes and cross-sections of reactions are expressed in terms of the interaction current components. As the current components in the space-time are the Fourier-transformations (i.e., the integrals depending on parameters) their discontinuities are related to weak decrease of the current components in the momentum representation at $\left|p_\nu\right|\to\infty$ (i.e., Fourier-components of currents can be diverging intergrals). But from the experimental data we can see that the cross-sections of the reactions involving the higher spin particles are approximately equal to (or less than) the cross-sections of the reactions involving the particles of the lower spin (0 and 1/2) only at high energies. Thus, we may conclude that the current components and their partial derivatives must be continuous in the space-time, i.e., the theorem on continuity of current derivatives is valid.

The calculations of the contributions of the higher spin nucleon resonances $N^*(J)$ to the amplitudes of the elastic πN-scattering in the framework of the model for the $\pi NN^*(J)$ vertex (which obeys the theorem on currents and fields as well as the theorem on continuity of current derivatives) show that these contributions must decrease at least as s^{-8} at $s\to\infty$. We see that the theorems on continuity of current derivatives guarantee the small contributions of $N^*(J)$ to the amplitudes of the elastic πN -scattering at high energy, but formally these contributions are non-zero. Such behavior of the $N^*(J)$-contributions to amplitudes agrees with the experimental data.

There are two different predictions for the contributions of $N^*(J)$-resonances to the partial amplitudes of the πN-scattering at high energies: 1) usual isobar models give the power energy growth; 2) The approach of present paper gives the energy decrease. According to the partial wave analyses of the πN-scattering [27, 28] the partial amplitudes behave approximately as some constants at $W\approx 2$ GeV. These amplitudes correspond to sums of different contributions, in particular to some $N^*(J)$-resonances. Therefore it can be concluded that the contributions of the resonances $P_{33}(1232)$ ($\Delta(1232)$), the $D_{13}(1520)$, the $F_{15}(1680)$, and the $D_{33}(1700)$ to the corresponding partial amplitudes decrease at high energies in reality. It can be considered as confirmation of the predictions of present model.

The interactions like to $N^*(J) \to N\pi\left(J \to \frac{1}{2}+0\right)$ are simplest, as they are determined by one partial amplitude. But the transitions with several partial amplitudes exist also (such as $N^*(J) \to N\rho, N^*(J) \to N\omega\left(J \to \frac{1}{2}+1\right), N^*(J) \to Nf^0\left(J \to \frac{1}{2}+2\right)$, $N^*(J) \to \Delta\pi\left(J \to \frac{3}{2}+0\right)$ with two higher-spin particles and $N^*(J) \to \Delta\rho, N^*(J) \to \Delta\omega\left(J \to \frac{3}{2}+1\right)$ with three higher-spin particles). In these transitions the theorem on currents and fields as well as the theorem on continuity of current derivatives must be valid for each partial amplitude and for each higher spin particle. The validity of the theorem on continuity of current derivatives can be provided by the products of the form factors for each higher spin particles. As example, we can consider the form factors like to (16) for the higher spin bosons and the form factors like to (21) for HSF with own parameters (such as a,b,n_1,n_2,n_3). We can expect that the high-energy decrease of the amplitudes for the higher spin particle interactions enlarges with the number of the higher spin particles involved in the transition and the reaction. Possibly this rule explains the experimental fact formulated as: the higher spin resonances are formed but are not produced in the πN - interaction.

In relations with this rule and the restrictions (42) it is of interest to compare them with the dual models [12]. In dual models it is stated that the reaction amplitude at arbitrary energy can be presented as the sums of infinity quantity of resonances or Regge poles. In dual models the presentation of the amplitude by the sum of resonances in intermediate energies and Regge poles at higher energies is considered as wrong. The restrictions similar to (42) show that contributions of the HSF-resonances at higher energies are small but they do not equal zero. From quark models with constituent quarks and Regge pole model the increase of a resonance quantity with energy can be expected. Therefore it can be assumed that in reality the total contribution of all the resonances to amplitude has got non-zero value at higher energies.

Acknowledgments. I thank the Y.V. Kulish and I.A. Anders, O.A. Osmaev, V.I. Khrabustovskij for interesting discussions.

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