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COMPARATIVE ANALYSIS OF MODELS FOR COMPUTER CALCULATIONS OF CROWD BEHAVIOR

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The question of the crowd behavior description in emergency situations has been considered. The review of studies devoted to the description of real streams of people has been executed. Its main characteristics in normal and emergencies have been defined. The comparative analysis of the models which used for computer calculation of crowd behavior in emergency situations has been made. Advantages and disadvantages of the considered models have been defined. Steps on development of the computer models describing crowd behavior have been offered.

KEYWORDS: computer calculation, crowd, stream of people, models, emergency situation

ПОРІВНЯЛЬНИЙ АНАЛІЗ МОДЕЛЕЙ ДЛЯ КОМП'ЮТЕРНОГО РОЗРАХУНКУ ПОВЕДІНКИ СКУПЧЕНЬ ЛЮДЕЙ

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У роботі розглядається питання опису поведінки скупчень людей у надзвичайних ситуаціях. Виконано огляд досліджень присвячених опису реальних потоків людей. Визначено його основні характеристики в нормальних і аварійних умовах. Виконано порівняльний аналіз моделей, що використовуються для комп'ютерного розрахунку поведінки скупчень людей у надзвичайних ситуаціях. Визначено переваги і недоліки розглянутих моделей. Запропоновано кроки з розвитку комп'ютерних моделей описують поведінку скупчення людей.

КЛЮЧОВІ СЛОВА: комп'ютерний розрахунок, скупчення людей, потік людей, моделі, надзвичайна ситуація

СРАВНИТЕЛЬНЫЙ АНАЛИЗ МОДЕЛЕЙ ДЛЯ КОМПЬЮТЕРНОГО РАСЧЕТА ПОВЕДЕНИЯ СКОПЛЕНИЙ ЛЮДЕЙ

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В работе рассматривается вопрос описания поведения скоплений людей в чрезвычайных ситуациях. Выполнен обзор исследований посвященных описанию реальных потоков людей. Определены его основные характеристики в нормальных и аварийных условиях. Выполнен сравнительный анализ моделей используемых для компьютерного расчета поведения скоплений людей в чрезвычайных ситуациях. Определены достоинства и недостатки рассмотренных моделей. Предложены шаги по развитию компьютерных моделей описывающих поведение скопления людей.

КЛЮЧОВІ СЛОВА: компьютерный расчет, скопление людей, поток людей, модели, чрезвычайная ситуация

Now situations in which the many people perish, destroyed material values, harm of environment, are in most cases associated with manifestation of uncontrolled fear at people.

The importance of studying the crowd behavior over time has not decreased. On the contrary, the reality around us concrete examples proves that modern scientists have a need again and again to refer to the works of G. Le Bon, Freud and others. It will allow understanding more clearly those mechanisms which erase identity of the person who got to crowd and force it to work and make the decisions which aren't correlated with his true desires, requirements and interests.

Systematic studying of the crowd in uncontrolled state of fear began in the second half of the XIX century. At that time interest of European scientists concentrated on properties of crowd, mechanisms of collective aggression, etc. Independently from each other two science schools were formed: German - Psychology of people (M. Lazarus, G. Shteyntal, V. Vundt) and French-Italian - psychology of masses (G. Lebon, G. Tard, V. Pareto, Sh. Siegel) [1].

In Russia the researches devoted to a question of the mass phenomena were begun at the end of XIX – the beginning of the XX century by M.G. Mikhaylovsky (subjective sociology) [2], and then continued by V.M. Behterev (collective reflexology) [3] and A.L. Chizhevskij (heliopsychology) [4].

In literature it is rather often possible to meet very general definitions of crowd in which the main focus is placed on a big people congestion that are brought together in one place [5, 6]. However for more detailed studying of this subject of such definition it isn't enough. In it a large number of details are missed, the most important of which Gustave Lebon [7] listed in the work "Psychology of the people and masses" (1895), namely: anonymity, diffusion of responsibility, impunity; distribution of emotions, opinions via the infection mechanism; suggestibility; impulsiveness, aspiration to immediately realize their desires, variability of behavior; inability to consider, lack of a reasoning and criticism; irritability, exaggerated sensitivity; moral certainty; reasonings of crowd are primitive, incoherent and are most often based only on associations; crowd is capable to perceive only images, and, the image is brighter, the perception is better; most awful things can be called by harmonious words (brotherhood, equality, democracy) which are accepted with reverence; leader is necessary for crowd.

Besides in work [1] it is noted that crowd is a people congestion who aren't united by solidarity of purpose and uniform organizational and role structure, but connected among themselves by the general center of attention and an emotional state.

In extraordinary conditions the circular reaction (in literature the term "emotional whirling" is also used [8]) has the greatest value. This is mutual infection. In other words, it is transmission of an emotional status at the psychophysiological level of contact between organisms [1]. Such emotions as fear, rage, boredom, etc. can be transferred.

At the person captured by emotional circling increased sensitivity to impulses which source is located inside crowd and at the same time reduces sensitivity to impulses from outside. Barriers against any rational argument amplify. Therefore at such moment attempt to influence crowd logical arguments can be untimely and simply dangerous.

Depending on the emotions extended in crowd there can be such destructive phenomena as aggression or fear. Experience shows that the crowd captured by fear is the most dangerous of all kinds of crowd [8].

In this state people cease to understand that it is necessary to do, refuse search of independent decisions and act only under the influence of fear. Moreover, at fear people almost always exaggerate danger from which they seek to leave [5, 6, 9].

Under the influence of a fear stability of the stream movement of people is broken, due to attempts of people to overtake each other. Thereof strongly condensed streams on some path sites, especially in places of its narrowing (for example, in the doorway) can form the "traffic jams" leading of people to death [10]. Therefore at buildings design, especially public constructions, it is always necessary to consider possibility of emergencies.

On statistical data of the USA from 1897 for 1930 at the fires at theaters about 2.5 thousand people were lost [11]. From total number of the victims of 49.8% died in communication by evacuation of people from the building, including because of discrepancy of ways of evacuation to the appointment and emergence of a fear – 46.5%. Though statistical data belong to the past, presently, despite fire extinguishing development of technology, evacuation of people at the fires continues to remain a burning issue. It is promoted by volumes increase of buildings.

Now, outdoor sports arenas are calculated on 100 and more thousands of spectators. Capacity of theaters reaches 2 thousand people and, apparently, will increase further. The closed sports constructions and concerthalls are calculated on 10-50 thousand people [12].

Because crowd captured by fear is extreme danger it is necessary to pay much attention to the organization of the people movement, not only in buildings, but also on the outdoor areas where can pass evacuation path. For this purpose it is necessary to carry out simulation of all possible (or the most probable) scenarios of events.

Now, the crowd simulation is an evolving area of science. The developed models aren't universal. They have their own internal constraints, simulation accuracy and application area. It can lead to difficulties at decision-making in a model choice of people behavior in this or that situation.

Thus, carrying out the comparative analysis of crowd behavior models in emergency situations is rather actual task.

GOAL AND RESEARCH TASKS

The goal of this paper is carrying out comparative analysis of models for computer calculations of crowd behavior. It will allow revealing advantages and disadvantages of existing models and making recommendations for developing new models.

For achieve this goal it is necessary to solve the following tasks:

– perform an analysis of studies devoted to research of the real stream of people;

– execute a review of the main models describing behavior of the crowd and define their advantages and disadvantages;

– carry out the comparative analysis of the main models of crowd behavior;

– formulate recommendations for development of new crowd behavior models.

THE REVIEW OF RESEARCHES OF THE REAL STREAM OF PEOPLE

For the first time researches of the streams movement of people were put on a serious scientific basis by the prof. S.V. Beljaev [10]. In 1937 at Architecture institute of the All-Russian Arts Academy (ARAA) about 200 observations over the streams movement of people in public places were made. Results of these researches formed the basis of standards for evacuation of people from buildings. The main scientific achievement of these researches should be considered establishment of dependence between the speed of a stream and its density.

At the same time researches ARAA contain also serious disadvantages. It is necessary to carry rather small number of natural observations which isn't allowing giving rather full process assessment; unsuccessfully chosen places for observations – sidewalks of streets, tram where movement of people strongly differs from the movement in buildings.

The main disadvantage of the ARAA work should be considered as acceptance of constant speeds values of the streams movement of people that significantly differs from observed parameters actually.

Further researches in this field in the USSR were conducted at the Higher school of the USSR Ministry of Internal Affairs by the prof. M.J. Roytman [9]. Unfortunately, they have not brought anything new in settlement data as were based on S.V. Beljaev 's approach and were therefore not free from the disadvantages stated above.

In 1946 – 1948 at All-Union Scientific Research Institute of Fire Protection Defense by A.I. Milinskij researches were conducted [13]. It differed from the previous researches in the wide scale and more perfect technique. More than 6000 natural observations in various public buildings were made. The physical people sizes, density and speed of the movement streams of people and doorways capacity were studied and considered.

The horizontal projection form of person accepted an ellipse which diameters correspond to width and thickness of the person (Fig. 1). Taking into account a variety of physical data and clothes the accepted assumption insignificantly distorts the actual sizes and horizontal projection form.

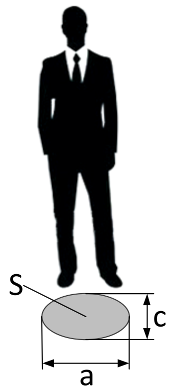


Fig. 1. Horizontal projection of the person in the ellipse form (a – width; c – thickness; S – area of horizontal projections)

Natural observation of streams of people show that in many buildings, for example, in shops and at stations, most of people forming stream (to 80%), have at itself a hand baggage (suitcases, backpacks, bags, briefcases, packages, etc.). Often in a stream there are people with children on hands or leading them by the hand. The most probable combinations of horizontal projections of people and baggage are given in work [9, 14].

In [13] was proposed to express stream density of people by number of people per unit area occupied by stream, people/m^2:

$$D = \frac{N}{wl}, \quad (1)$$

where N – number of people in stream; w – stream width; l – stream length.

Also in [13] was proposed to express stream density of people by areas sum of horizontal projections of people per unit area occupied by stream, m^2/m^2:

$$D = \frac{\sum S}{wl}, \quad (2)$$

where S – area of person horizontal projection in the stream.

Besides in literature, for example in [6], determination of stream density of people through the area per one person also meets, m^2/person:

$$D = \frac{wl}{N}. \quad (3)$$

Expressions for stream density of people (1) and (3) are applicable only at uniform stream, for example, when it consists only of adults in summer clothes or of adults in winter clothes. Expression (2) is applicable at any stream structure.

The maximum stream density established in natural conditions is equal 0.92 m^2/m^2 [13]. This value corresponds to the assumption that the ellipse expressing a horizontal projection of the person isn't exposed to deformations during stream compression. Though in reality as the human body elastic, at considerable compression changes a form and decreases area of its horizontal projection. Therefore, the physical limit of density may exceed 0.92 m^2/m^2.

Researches of streams of people with high density in natural conditions showed that in many cases, mainly in apertures wide up to 1.2 m, density there were more than 0.92 m^2/m^2. However the maximum value didn't exceed 1.0 m^2/m^2.

In paper [15] when carrying out natural experiments it is established that the physical limit of stream density of people which is formed during a crush is equal 1.15 m^2/m^2.

Also A.I. Milinskij [13] determined the most probable stream density of people in normal conditions on the movement paths in various buildings.

In the industrial enterprises and educational institutions stream density of people from 0 to 0.25 m^2/m^2, and in spectacular buildings – from 0 to 0.5 m^2/m^2 are most probable. On ladders, as a rule, low stream density of people from 0 to (0.25 – 0.35) m^2/m^2, and stream density of people close to maximum are observed very rare.

Average speeds of the people movement for various rooms on horizontal paths and ladders in public buildings of different function are established: at theaters and educational institutions movement speed are (15 – 20) m/min, in industrial buildings – (25 – 30) m/min more often, in buildings of transport appointment – (20 - 50) m/min, on ladders – (20 – 25) m/min are most probable.

Further works in this field carried out V.M. Predtechenskij [9], Yu.V. Alekseev, R.M. Duvidzon, V.A. Kalindev, V.V. Holshchevnikov, R.G. Grigoryants and V.S. Gvozdyakov [11]. Researches were conducted with use photo and video equipment that allowed estimating not only quantitative, but also qualitative characteristics of process. Their researches confirmed reliability of the results received by A.I. Milinskij [13].

In the 1950th years foreign authors, such as V. Sholts, K. Ranier and H. Schubert, were also conducted researches on establishment of movement average stream speed of people, its density, etc. [9]. However experiments were made with participation of artificially created streams of people which consisted of people about one age (students). Therefore the speed values of the stream movement of people received under such artificial conditions are significantly overestimated. In this regard these researches can represent only private interest.

Except researches directed at the quantitative indices identification of stream of people in literature also qualitative characteristics of crowd behavior are described.

Observations show that stream of people usually an elongated cigar-shaped form (Fig. 2). Thus the head and closing parts consist of a small number of people moving according to high or small speed, than majority of people in a stream.

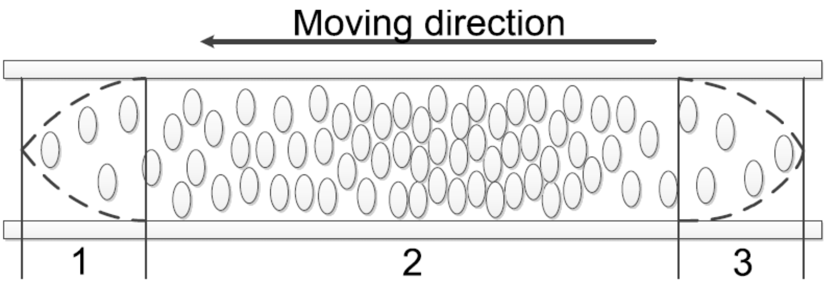


Fig. 2. People stream scheme: 1 – head part; 2 – main part; 3 – closing part

Usually head part of stream leaves with greater speed forward and there is a so-called stream rearrangement by spreading its head part. Therefore it is necessary to consider gradual change of stream density. Than stream density of people is higher, then obstruction of evacuation paths is more probable. Therefore, the probability of accidents increases.

Experience shows movement speed of people at the fire toward external exits isn't identical on length of an evacuation path. It reaches the maximum in the area close to the seat of fire, and, in process of stream of people removal from the center, rates of the movement gradually fade, coming nearer to the normal movement in a place where is already not present, not only dangers, but also noticeable signs of the fire happening in the building. Other picture of the evacuation movement should be expected in buildings at the time of an earthquake: in this case movement speed remains high up to an exit of people from buildings and some removal from them.

Besides, people seek to go the shortest path to the target. They always choose such path even despite need of overcoming of some obstacles and the difficulties associated with it.

In recent years new effects which can be watched in crowd, [14] were found. First of all, it is necessary to mark effect of "herding behavior" which is watched in case of evacuation. This effect is shown that when people aren't sure that it is necessary to do, they aim to imitate behavior of others.

Other interesting effect is formation of movement waves (Stop-and-Go Waves) [16]. It is watched when the crowd with high density is formed. At some moment people throughout some time start moving step by step, then faster, then slower. If the crowd density increases at the further movement, there can be an effect of turbulence of crowd. In more detail about effects which arise in crowd with high density, it is considered in [14, 16].

For people evacuation process modeling it is necessary to use simulation and stochastic model of the stream movement of people which most accurately reflects dynamics of process in various service conditions of buildings [12].

THE ANALYSIS OF THE MAIN MODELS DESCRIBING CROWD BEHAVIOR

Graphic-analytical method and ADPLV model

In the absence of a computer the most accurate and visual teaching mathematical modeling method of crowd behavior is the graphic-analytical method. It was for the first time proposed by A.I. Milinskij [13], and further was improved by V.M. Predtechenskij [20].

The essence of this method can be represented as follows.

Consider the stream movement of people on a horizontal path. In Fig. 3a the floor plan is submitted. It has length L and width W. It is a horizontal portion of the path that ends with the doorway. Moving direction is shown by an arrow. The settlement scheme of the same path is given in the form of straight lines set on a coordinate grid (Fig. 3b): on ordinate axis – path length L, and on abscissa axis – time t.

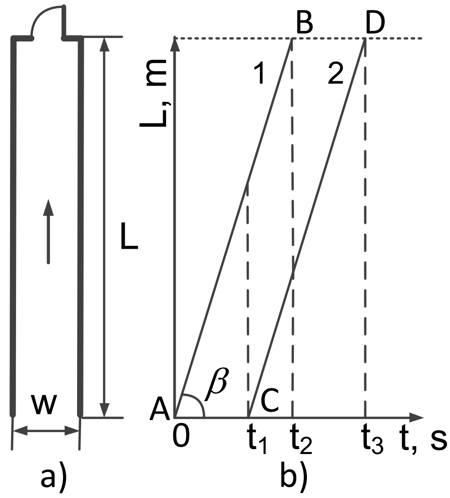


Fig. 3. The analyzed part of the path:
a) floor plan; b) graphic-analytical representation of the stream movement of people

Let in an initial time (t=0) on a path of the movement there is a head part of stream of people. Through time t_1 all people stream enters at path, including also its closing part. Stream of people consists of N people and has density D.

Order of movement diagram creation is following. Knowing stream density of people D the movement speed v, corresponding to this density and traffic conditions (emergency, normal, comfortable) is determined by special tables based on experimental data.

Time necessary for passing of this path is determined by the stream speed of people v and path length: $t_2 = L/v$. Point B with coordinates (t_2, L) connects to the origin of coordinates by straight line AB. The straight line AB shows the movement character of the head part stream of people at the path.

Through time t_1 all stream of people enters at the path. Considering that the stream movement speed of people remains invariable, it is possible without preliminary calculations from the point C corresponding to the beginning of path and time t_1 draw straight line CD, parallel AB, to ordinate, the corresponding length L. The straight line CD expresses the movement of the stream closing part.

In more detail procedure of the stream movement analysis of people on other standard path under various conditions is described in [20].

The advantage of this method is its relative simplicity, visibility and possibility of its application without computer.

At the same time the method isn't devoid of disadvantages, which include: only average speed and average density of stream can be estimated; possibility of its application only for standard path; impossibility to apply a method to research of information distribution and moods in crowd; inapplicability for simulation of people behavior when they have to execute a certain sequence of actions.

Stream simulation of people by a graphic-analytical method, despite the simplicity, is very labor-consuming. Therefore, with the advent of computers were developed approaches to their use for the calculation of stream movement of people in buildings [21, 22].

The ADPLV model was first time presented in V.V. Holshhevnikov's work [21]. It assumes setting of evacuation path in advance and splitting them on "elementary" parts. Then simulation of stream movement of people, based on procedure similar to a graphic-analytical method, is executed. The detailed presentation of the mathematical description of the ADPLV model is presented in work [21].

Simulation model adequacy to real process of the stream movement of people was repeatedly checked by natural observations. This model has high accuracy [21].

The disadvantages of this approach include the fact that it is impossible to define nature of the individual person movement in the crowd. Only probabilistic quantitative characteristics of groups, such as density, number of people, speed are defined. Also this approach isn't suitable for heterogeneous crowds in which different groups of people have various targets and characteristics. Besides, simulation is performed only for a predetermined path.

Consider models at which there is an essence of the individual person.

Models based on cellular automata

Cellular automata were first proposed in Von Neumann's work [23] and had fundamental importance for the whole of science, and also diverse application. For behavior simulate of crowd the most widely used two-dimensional cellular automata to the orthogonal grid which sets space for individuals movement. The size of each cell of grid is equal to the average size of the individual and is defined at entry conditions of simulation and further doesn't change. Besides, each cell can be in one of the following states [24]: it is occupied with stationary object (building, monument, etc.); it is occupied with mobile object (person, car, etc.); movement is forbidden (prohibited zone, obstacle); movement isn't desirable (determined by the rules movement restrictions); it is free. Considered state of cells change synchronously through discrete time intervals according to beforehand set rules, depending on the state of the neighboring cells.

Each individual in the models constructed on the basis of the cellular automata can be characterized by rather extensive list of characteristics, except the linear sizes which at all are identical.

Movement simulation of the person is carried out proceeding from movement probabilities $P_1 – P_8$ in this or that cell which is calculated according to the rules described in the model. The individual moves to that cell the movement probability in which is maximal. In case on the path there are absolute obstacles or considerable congestions of people, the individual is able to look through area round it on some distance and to choose that movement direction which has the smallest number of people and obstacles [14, 25].

The advantages of this approach is the ease of implementation of cellular automata and simple setting of rules, which is quite relevant for describing the crowd behavior, numbering hundreds or even tens of thousands of individuals.

The disadvantages of models based on cellular automata are a discrete behavior of individuals in the crowd, both in time and in space. The horizontal person projection over the width and thickness has averaged the same value that isn’t true. Besides, it is known that the existing models on cellular automata are weak for simulation of the intersecting stream of people [22]. Considering that the cells size need to be set before simulating and that why results of such models strongly depend on user qualification.

Now cellular automata are successfully used when behavior simulation of crowd: in the subway [26], at evacuation from the room [27], building [28], taking into account various obstacles [29].

Models based on the equations of psychophysical interaction

Bases of crowd behavior simulation using the equations of psychophysical interaction were put by D. Helbing in [30]. Then it approved on rather large number of examples [14, 31 – 36]. Under these models each i-th person in the crowd is abstractly represented in the form of the vertical continuous uniform cylinder by radius R_i, and rules of interaction between people are set by potential forces of a psychosocial and physical origin.

Consider a set from N person. Each has weight m_i, where $i = \overline{1, N}$. Let the behavior of each individual person be characterized radius vector $\vec{r}_i$, actual movement speed $\vec{V}_i$ and speed with which the person would like to move $\vec{V}_i^{want}$. Discrepancy of desirable speed and actual speed forms the main force initiating of the movement [30]:

$$\vec{F}_i^t = m_i (\vec{V}_i^{want} - \vec{V}_i)\big/\tau, \qquad (4)$$

where τ – the parameter characterizing time of person involvement in the crowd, s.

Force initiating the movement (4) is arranged in such way that in case of excess of desirable speed over actual speed then person is accelerated. If the person doesn't wish to move anywhere, his movement fades over time.

Besides force initiating the movement (4) the person is affected by other forces associated with person interaction with other people and obstacles. Consider force acting on the person i from the person j [30]:

$$\vec{F}_i^p = (A \exp \frac{D_{ij}}{B} + kH(D_{ij})D_{ij})\vec{n}_{ij} + \eta H(D_{ij})D_{ij} \left\langle \left(\vec{V}_j - \vec{V}_i\right)\vec{\tau}_{ij} \right\rangle \tau_{ij}, \qquad (5)$$

where $D_{ij} = R_i + R_j - \left|\vec{r}_i - \vec{r}_j\right| = R_i + R_j - \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$; $\overrightarrow{n_{ij}}$ and $\overrightarrow{\tau_{ij}}$ – normal and tangential unit vectors; H – Hevisayd's function defining the interaction emergence moment; A, B, k, η – constants which set in the empirical way (in [32] suggested A=430 H, B=0.085 m, k=50000 H/m, $\eta = 55000\,\text{kg}/(\text{m}\times\text{s})$).

First term in (5) represents force, as well as in a formula (4), the psychosocial nature. It describes person unwillingness to come into too close physical contact with other people. The value of this force is greater, than people among themselves are closer and when the distance between persons becomes less than a sum of their radiuses, force of pushing away becomes especially considerable. At long distances the contribution of this term is rather weak. The second and third terms in (5) have traditional view for molecular dynamics. One of them describes elastic pushing away at any collision between persons, and the second – friction force at the tangent movement. Both interactions arise only at positive value therefore these terms are multiplied by Hevisayd's function $H(D_{ij})$.

Similarly (5) force of person interaction with an obstacle, for example with a wall is considered:

$$\vec{F}_{iz}^{w} = (A\exp\frac{D_{iz}^{w}}{B} + kH(D_{iz}^{w})D_{iz}^{w})\vec{n}_{iz}^{w} + \eta H(D_{iz}^{w})D_{iz}^{w}(\overrightarrow{V_i}\vec{\tau}_{iz}^{w})\vec{\tau}_{iz}^{w}, \quad (6)$$

where $D_{iz}^{w} = R_i - \left|\vec{r}_i - \vec{r}_z\right|$, $\vec{r}_z$ – radius vector from i-th person to the nearest point of a wall z. The values of other variables and parameters as in (5).

Taking into account formulas (4 – 6) equations of the crowd dynamics can be written down in a look, usual for mechanics:

$$m_i \frac{d\overrightarrow{V_i}}{dt} = \vec{F}_i^{t} + \sum_{i \neq j} \vec{F}_{ij}^{p} + \sum_{z} \vec{F}_{iz}^{w},$$
$$\frac{d\vec{r}_i}{dt} = \overrightarrow{V_i}. \quad (7)$$

Finally behavior simulation of the crowd is carried out by means of the decision of 4N differential equations system.

Advantages of these models are opportunity to describe the people movement continuously in time, unlike cellular automata. Forces which affect people and making some psychophysical sense are considered.

However, in the proposed equations describing the crowd dynamics present values aren't measured directly, so researchers have to be assigned to the values of these parameters. Existence of such coefficients, generally speaking, reduces results reliability, so and reduces the predictive force of models.

Also, psychological repulsion force equation, modeling the human reluctance to enter into close contact with other people, is constructed in such a way that makes it impossible to simulate real crush in emergency situations when there is a change or even a reduction in the horizontal projection profile of person [9]. Besides, this profile is described in these models as a circle, but not an ellipse which most precisely corresponds to empirical measurements [9].

In [30, 32], it was noted that by using these models are reproduced typical for crowd phenomena such as avalanche formation of a crush after the people speed exceeds 1.5 m/s (Fig. 4). The desirable speed is higher, the more slowly the crowd leaves the room. In crowd as a result of a crush there are affected crowds obstructing the traffic.

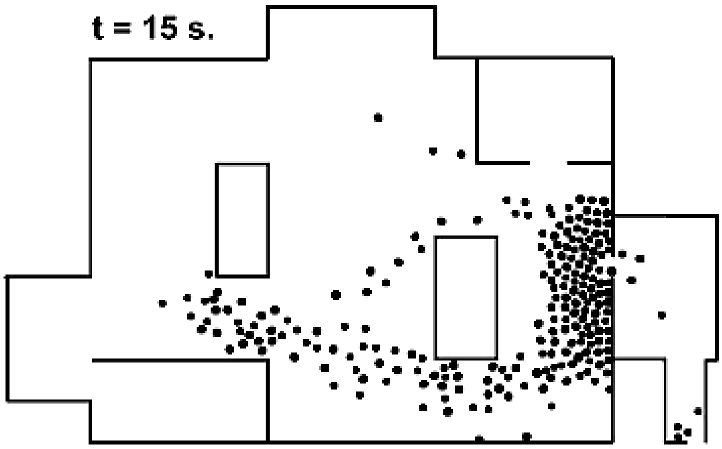


Fig. 4. Simulation of crush formation arising in the crowd [32]

Models based on the hybrid multiagent architecture

The main attention of the models based on cellular automata and the equations of psychophysical interaction concentrates on interaction simulation of people among themselves and obstacles. These approaches initially weren't aimed at the description of information transfer mechanisms between people, ensuring the movement to any target or following to some action plan. Any addition in them of these possibilities leads to subjectivism in the model which doesn't have physical.

Multiagent approach, in which each person is considered as the intellectual agent, allowed raising crowd simulation on new level [14, 36 – 38]. Thus, decentralized system isn't functioning within global rules and laws, but on the contrary, these global rules and laws are result of agent's individual activity.

Agent is essence which simulates of person behavior in processes of data collection and processing and independently moves in information space in the direction of the target [37 – 40].

Now there are three main creation concepts of agent architecture [22, 37, 39]: deliberative (reasoning, logical deduction); reactive (rules of action, reaction to events); hybrid (combining of the two previous).

Bases of strict formalization of knowledge and actions of the deliberative agent are put by K. Konolige in work [41]. He proposed hierarchical meta-language for the description and a logical conclusion on knowledge and actions of agents.

Each deliberative agent has library of plans (scenarios) defining options of possible actions which can be undertaken by the agent for achievement of its intentions. Plans, thus, realize procedural agent knowledge. Each plan contains some components. The trigger or conditions of a call define circumstances under which the plan has to be considered as possible for application. The plan has the context or preconditions defining circumstances under which implementation of the plan can begin. The plan also has a body which may contain the targets and primitive actions.

The advantage of deliberative architecture is application possibility of the strict formal methods and well fulfilled technologies of traditional artificial intelligence allowing it is rather easy to represent knowledge in a symbolical form. To do this we can be used languages of manipulation with formal representations, such as Prolog, JESS, etc. [22, 37, 42].

Solutions search of the problems arising when using classical methods of artificial intelligence in the agent-based systems led to appearance of a new agent's class based on reactive architecture. Considered to be the founder of this direction is R. Brooks who so formulated key ideas of a behaviorist view of intelligence [39]: intellectual behavior can be created without obvious symbolical representation of knowledge; intellectual behavior can be created without obvious abstract logical conclusion; intelligence is suddenly arising property of some difficult systems.

In the real world the intelligence isn't expert system or inference engine, and the intellectual behavior arises as interaction result of the agent with environment. Instead of world simulation and planning the reactive agents should have a collection of simple behavioral schemes which react to changes in the environment in the "stimulus – response" form.

Reactive agents, at least in several experiments, proved ability to solve limited number of simple problems in the real world [22, 33]. However they face problems when performing tasks demanding knowledge of the world which are obtained by logical deduction or from memory. Moreover, the reactive agents are often made "rigidly" and have no abilities to training.

Recently, some researchers admit [39, 40] that the agent should have a high-level and low-level output reactive abilities. It led to creation of hybrid agents.

When simulate of the crowd then the level describing reactive agent behavior is carried out, based on cellular automata [22], or on the equations of psychophysical interaction [33, 34, 43, 44]. Deliberative level is usually described by a predicate logic [34, 43, 45, 46].

Advantages of hybrid architecture are: agent's possibility to simulate the difficult behavior based on long-term planning of the actions and solutions of short-term tasks of movement in surrounding area; ability of communication simulation between people in crowd; formations of groups; possibility of training. Unfortunately, the agent's hybrid architecture isn't deprived of the disadvantages peculiar to deliberative and reactive architecture; however the basic is management complexity of interaction between different levels.

Models based on the potential scalar field

Currently the fastest models of large-scale crowd simulation (thousands people) are models based on potential scalar fields [47]. It allows describing difficult crowd behavior in the closed room, on city streets, on the open areas with a difficult terrain. They are plausibly to movement simulation of people in a single stream, and in crossing streams. You can find more information about research of the crossed streams in [16].

Basic provisions of this approach were put in works [48, 49], and its rather complete description is presented in A. Treuille's work [47]. Subsequently approach gained further development in R.V. Grebennikov's works [50, 51].

In models based on the potential scalar fields each agent submits to a number of assumptions [47]:

Hypothesis 1: Each person tries to achieve the geographical target g.

The target field is set by the potential decreasing in proportion to a distance between the current position of the agent in a point x and the next target with coordinates x_g [48]: $g(x) = 1/\left|x - x_g\right|$.

Hypothesis 2: The person moves with the greatest possible speed.

Environment influences the movement speed of the person by its reduction on rises and increases on descents. Also maximum movement speed of the person depends on crowd density surrounding agent. Generally, such dependence is expressed as follows:

$$\frac{dx}{dt} = f(x, q)\vec{n}_q ,$$

where f – potential field of the maximum speed for the agent in a point x, moving in the target q,

$\vec{n}_q = [\cos(q), \sin(q)]^T$ – unit vector in target q.

The field of the maximum agent speed is described by the following expressions [48]:

$$f(x,q) = f_T(x,q) + \left(\frac{p(x + rn_q) - p_{min}}{p_{max} - p_{min}}\right)\left(f_U(x,q) - f_T(x,q)\right),$$

$$f_T(x,q) = f_{max} + \left(\frac{\nabla h(x) n_q - s_{min}}{s_{max} - s_{min}}\right)(f_{min} - f_{max}),$$

$$f_U(x,q) = u(x + rn_q)n_q,$$

where p – crowd density; p_{min} and p_{max} – threshold value of crowd density; f_T – topographical speed; f_{min} and f_{max} – minimal and maximal agent speed; s_{min} and s_{max} – minimal and maximal terrain slope; $\nabla h(x)n_q$ – terrain slope in the point q; f_U – stream speed; u – average stream speed taken in the current position of the agent with shift r aside q.

Hypothesis 3: In the process of moving a person tries to avoid collisions with other people, walls and other obstacles.

For a description of this statement in [47] it was proposed to introduce an additional field of discomfort g(x). According to it if all other things being equal then the crowd participant will prefer to be in a point x, than in a point x', if $g(x') > g(x)$. This idea can be realized by setting the linear combination of three following parameters: path length; time spent for the movement; discomfort degree.

Hypothesis 4. Let Π – set of all possible path from a point x to the target g. The agent chooses such path $P \in \Pi$ which corresponds to the minimum expression:

$$\alpha\int_P 1ds + \beta\int_P 1dt + \gamma\int_P gdt \qquad (8)$$

where $\int ds$ – integral on path length; $\int dt$ – integral on time; α, β, γ – weight coefficients. Considering that $\int ds = \int vdt$, where v – agent speed, write formula (8) in the form:

$$\int_p Cds,$$

$$C = \frac{\alpha v + \beta + \gamma g}{v}, \qquad (9)$$

$$\alpha\int_P 1ds + \beta\int_P \frac{1}{v}ds + \gamma\int_P \frac{g}{v}dt.$$

Proposed criterion (9) is used to calculate the optimal path. Let there is a function $\varphi \in R$, on all the range of definition equal to optimality criterion value. It is obvious that for simultaneous achievement of the target and minimization of optimality criterion, the agent has to move aside, opposite to value of function gradient φ. This potential function in a target point of path is zero, and on all other interval satisfies to Eykonal's equation [52]:

$$\|\nabla\varphi(x)\| = C,$$

where $\nabla\varphi(x)$ – function gradient $\varphi(x)$.

Thus, all people move to the opposite side from a gradient, which normalized on speed in this point:

$$\frac{dx}{dt} = -f(x,q)\frac{\nabla\varphi(x)}{\|\nabla\varphi(x)\|},$$

In Fig. 5 visual display of the fields used when simulate crowd behavior is presented.

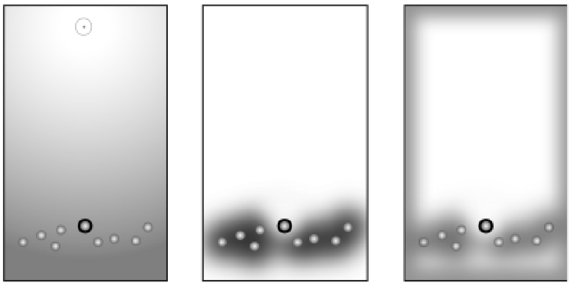
In paper [50] it was proposed to introduce an additional hypothesis according to which each agent has a limited visibility distance out of which all fields are equal to zero. Besides, for search of an optimum path a number of methods were analyzed and was chosen particle swarm optimization method [53]. It provides the acceptable results with the minimum computing expenses (there is no need directly to calculate gradient value $\varphi(x)$).

Thus, the proposed hypothesis leads to the basic movement equations of agents in a crowd stream for a two-dimensional case [54]:

$$-\frac{\partial p}{\partial t}+\frac{\partial}{\partial x}\left(pg(p)f^2(p)\frac{\partial \varphi}{\partial x}\right)+\frac{\partial}{\partial y}\left(pg(p)f^2(p)\frac{\partial \varphi}{\partial y}\right)=0$$

$$g(p)f(p)=\frac{1}{\sqrt{\left(\frac{\partial \varphi}{\partial x}\right)^2+\left(\frac{\partial \varphi}{\partial y}\right)^2}},$$

where φ – time before the end of the movement; p – crowd density; $f(p)$ – agents speed as function from density; $g(p)$ – discomfort as function from density; (x, y, t) – agent's coordinates and current time.



a) b) c)

Fig. 5. Visual display of the field used when simulate crowd behavior [51]: a) global target; b) density; c) discomfort

The advantages of these models include the ability to describe the crowd as a dynamic system, and the ability to perform a large crowd simulation in real time (Fig. 6).

The disadvantage of these models is its inapplicability to the crowd simulation with a high density, when there is considerable friction between people [50].

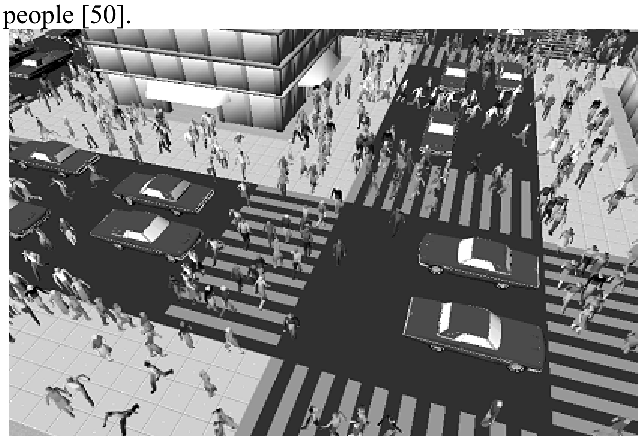


Fig. 6. Behavior simulation of pedestrians and motor transport on city streets [47]

COMPARATIVE ANALYSIS OF MODELS FOR COMPUTER CALCULATIONS OF CROWD BEHAVIOR

The review of the main crowd models showed that now there are many models, the main ones are:

– ADLPV model based on a graphic-analytical method and assuming splitting all evacuation path into standard path and then stream simulation of people, using tabular movement speed values of people in these or those conditions [21];

– models based on cellular automata [24 – 28]. In similar models the environment on which agents moves, is presented as a set of cells forming a periodic grid with given rules of transition. These rules define the cell state in the next time through state of the neighboring cells located on it at a certain distance at the current time;

– models based on the equations of psychophysical interaction [30 – 35]. In these models all social forces acting on the agent in crowd are expressed as physical forces, and the agent movement is described on the basis of classical mechanics laws;

– models based on multi-agent approach [37 – 46], in which the simulation of agents' behavior is described by a rules set of movement and interaction of individual agents;

– models based on potential scalar fields [47, 50 – 54]. In such models for each agent it is set: a set of the potential fields describing target location to which aspire the agents; movement speed; discomfort which is felt by the agent at movement.

The review of the papers devoted to real crowd research allowed defined their main characteristics, concerning which it is necessary to carry out the comparative analysis of crowd models: profile of person horizontal projection;

simulation possibility of the crowd with a high density (more than 0.92 m^2/m^2); simulation possibility of heterogeneous crowd; crowd behavior based on stream simulation of people in general or on individual behavior of each person; evacuation path is known before simulation; implementation possibility of the actions scenario; floor plane in which simulation is carried out.

Also it is necessary to consider of additional factors, such as: modeling accuracy; user qualification; computational complexity.

The results of the comparison of crowd models on the above parameters are presented in Table.

Table.

Comparative analysis of models for computer calculations of crowd behavior

Parameter	Model				
	1	2	3	4	5
Profile of person horizontal projection	–	Square	Circle	Circle	Circle
Crowd simulation with high density	Yes	No	No	Yes	No
Heterogeneous crowd	No	No	Yes	Yes	Yes
Stream/person	Stream	Person	Person	Person	Person
Evacuation path is known in advance	Yes	No	Yes	No	No
Running of the action scenario	No	No	No	Yes	No
Floor plane	Typically	Discrete	Any	Any	Any
Accuracy	High	Low	Middle	High	High
User qualification	Low	High	High	High	Low
Computational complexity	Low	Low	High	High	Low

1 – Model ADLPV, based on the graphic-analytical method; 2 – Models based on cellular automata; 3 – Models based on the equations of psychophysical interaction; 4 – Models based on the hybrid multiagent architecture; 5 – Models based on the potential scalar fields

The comparative analysis revealed general disadvantage of the modern models – the profile of a horizontal projection isn't an ellipse as it is noted in the researches devoted to real crowds. It in turn can lead to distortion of simulation results. Especially when trying to simulation of high-density crowd in which the crush is watched. From the models considered, based on the description of the crowd behavior through the individual person behavior, only multi-agent models allows to simulate plausible crush by setting specific rules of conduct.

Thus, it is necessary to propose the creation of new models describing the crowd's behavior, the following recommendations.

For the description difficult social and psychological behavior of people to use hybrid architecture of agents that will allow not only simulate person response to environment, but also different actions scenarios, communication between people, training, etc.

In model to consider opportunity to simulate a heterogeneous crowd in which crowd participants differ not only the speed, physiological restrictions, targets, but also overall dimensions.

Base model on the horizontal projection profile of person in the ellipse form, which would correspond to the real observations. Also to provide possibility setting of other profile configurations.

For plausibility of movement simulation of people in indoor and outdoor areas to use elements of the approach based on potential scalar fields.

The developed model should be verified by the field observations results. For this purpose the ADLPV model which has high accuracy in case of the stream description of crowd can be also used.

CONCLUSION

The conducted researches suggest the following conclusions:

1. Analysis of studies devoted to research of the real stream of people has been executed. It allowed defining the main qualitative and quantitative crowd characteristics (horizontal projection profile of person, maximum density of crowd, person speed in standard locations, etc.) in normal and emergencies.

2. The review of the main models describing the crowd's behavior has been executed: model ADLPV, based on the graphic-analytical method; models based on cellular automata; models based on the equations of psychophysical interaction; models based on the hybrid multiagent architecture; models based on the potential scalar fields.

Advantages and disadvantages of the models have been defined.

3. The comparative analysis of the main models of crowd behavior has been executed.

4. Recommendations which need to be considered in case of new models creation have been formulated:

– use multi-agent hybrid architecture of agents that will allow describing difficult social and psychological agent's behavior in crowd besides reaction to environment;

– base model on horizontal projection profile of person in the ellipse form that will correspond to real observations. Also to provide possibility to setting of other profile configurations;

– use for the description of agent interaction with environment elements of the approach based on potential scalar fields. It will allow is probable acting agent as indoors and on outdoor areas, and also to set influence areas of dangerous factors;

– verify developed model using results of field observations. For this purpose the ADLPV model based on a graphic-analytical method which has high accuracy in case of the stream description of crowd can be also used.

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