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COSMOLOGICAL EVOLUTION OF AN ANISOTROPIC BIANCHI TYPE-VI$_0$ UNIVERSE WITH RÉNYI HOLOGRAPHIC DARK ENERGY

Chandra Rekha Mahanta, **Rajashree Mahanta***
Department of Mathematics, Gauhati University, Guwahati - 781014 (INDIA)
**Corresponding Author e-mail: rajashree.mahanta@gauhati.ac.in*

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In this work, we investigate a spatially homogeneous and anisotropic Bianchi type-VI$_0$ cosmological model within the framework of General Relativity by considering a non-interacting mixture of pressureless dark matter and Rényi holographic dark energy (RHDE), with the Hubble horizon serving as the infrared cutoff. Exact solutions of the Einstein field equations are obtained by adopting the hyperbolic scale factor $a(t) = (\sinh(\beta t))^{1/n}$. The model parameters are constrained through a Markov Chain Monte Carlo (MCMC) analysis using 77 Observational Hubble Data (OHD) measurements of the Hubble parameter, yielding the best-fit values $H_0 = 67.9\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ and $n = 1.3$. A comparison with the standard ΛCDM model reveals a slightly lower minimum χ^2 value together with negative values of ΔAIC and ΔBIC, indicating a modest statistical preference for the proposed model. The reconstructed cosmic evolution exhibits a smooth transition from a matter-dominated decelerating phase to the present accelerated epoch, with the transition redshift lying within the observationally favored range $0.5 \lesssim z_t \lesssim 0.7$. Furthermore, the evolution of the matter and RHDE energy densities, dark energy pressure, equation-of-state parameter, anisotropy parameter and skewness parameter demonstrates that the model successfully reproduces the observed late-time accelerated expansion while undergoing a gradual isotropization. Overall, the proposed RHDE model provides a physically consistent and observationally viable framework for describing the late-time evolution of the Universe.

Keywords: *Cold dark matter; Bianchi type-VI_0 spacetime; Cosmic acceleration; Rényi holographic dark energy; Deceleration parameter; Equation of state parameter*

1. INTRODUCTION

Hubble's 1929 work [1] represented a major milestone in cosmology, providing strong evidence for one of the twentieth century's most significant discoveries - the expansion of the universe. In the late twentieth century, observations from two independent teams - the High-redshift Supernova Search Team led by A. G. Riess [2] and the Supernova Cosmology Project led by S. Perlmutter [3] demonstrated that the universe is presently undergoing an accelerated expansion. Since that period, multiple astrophysical and cosmological observations, such as CMB temperature anisotropies [4–6], large-scale structures including galaxy clustering [7–9], and Baryon Acoustic Oscillations (BAO) [10] have consistently supported the evidence for the universe's accelerated expansion. But the origin of the observed late-time cosmic acceleration remains unclear and continues to be a significant open problem in modern cosmology, even after more than two decades since its discovery. Dark matter is another exotic component of the universe that does not interact with electromagnetic radiation, and hence cannot be directly observed, but its existence is inferred from its gravitational effects on visible matter [11]. Observational evidence indicates that the dark components together make up about 95% of the total energy density of the universe, with dark matter (DM), a non-relativistic component that interacts very weakly with baryonic matter, contributing nearly 27%, and dark energy (DE) accounting for approximately 68%, while only about 5% consists of ordinary baryonic matter. To understand the nature of dark energy and the current accelerated expansion of the universe, several dark energy candidates have been proposed in the literature. The cosmological constant Λ, introduced by Einstein in his field equations, is the simplest and most natural candidate for dark energy. It can be interpreted as vacuum energy density with an equation of state parameter $\omega_{de} = \frac{p_{de}}{\rho_{de}} = -1$. Although it is in good agreement with observational data, it suffers from certain cosmological issues, including the fine-tuning and cosmic coincidence problems [12–14]. Therefore, a variety of dark energy models have been widely studied, including scalar field models such as quintessence [15, 16], k-essence [17], phantom [18], tachyon [19, 20], quintom [21], as well as exotic fluid models like the Chaplygin gas [22] etc.

In recent years, the holographic dark energy (HDE) model has gained significant interest as an alternative approach to describe the evolution of the universe. It is based on the holographic principle from quantum gravity, connecting dark energy with fundamental aspects of quantum physics. The development of this model is strongly influenced by Bekenstein's work on black hole thermodynamics [23]. Again, Hawking [24] demonstrated that the surface area of a black hole cannot decrease over time. According to this result, in a system of multiple black holes, the area of each black hole remains non-decreasing, and when two black holes combine, the final area is at least equal to the total area of the original ones [25]. This property closely resembles the behavior of entropy in thermodynamics. Bekenstein proposed that the entropy of a black hole is proportional to the area of its event horizon and can be expressed as $S_{BH} = \frac{A}{4\pi}$, where A denotes the area of the horizon [23]. Later, G.t'Hooft [26] introduced the holographic principle within the context of black hole physics, based on the relationship between entropy and the surface area of the black hole. Fischler and Susskind [27] subsequently

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generalized this idea to a cosmological context. The principle suggests that the entropy of a system is governed by its surface area rather than its volume. In this framework, the ultraviolet (UV) cutoff at short distances is linked to the infrared (IR) cutoff at large distances through limits set by black hole formation. Additionally, if the total energy within a region exceeds the mass of a black hole of the same size, gravitational collapse may occur. Extending this idea to the entire universe, one can interpret it as a system whose vacuum energy has a holographic origin, thereby giving rise to a dynamical model of holographic dark energy. Based on the holographic principle, Cohen *et al.* [28] established a connection between the system entropy S, the infrared (IR) cutoff L, and the ultraviolet (UV) cutoff Λ, given by $L^3\Lambda^4 \leq LM_p^2$, where $M_p = (8\pi G)^{-1/2}$ is the reduced Planck mass. By considering the largest possible IR cutoff L and using the relation $\rho_{\rm de} \approx \Lambda^4$, the holographic dark energy density can be written as $\rho_{\rm de} = 3c^2 M_p^2 L^{-2}$, where c is a dimensionless constant [29]. Several other suitable choices for the IR cutoff have also been proposed in the literature, including the Hubble horizon H^{-1}, event horizon, particle horizon, the universe's conformal age, the Ricci scalar radius, and the Granda-Oliveros cutoff. However, each of these choices can lead to new cosmological challenges. Li [29] proposed a holographic dark energy model in 2004, observing that cosmic acceleration could not be explained using either the particle horizon or the Hubble horizon. The future event horizon, however, can provide a viable explanation but a significant drawback of this model is its dependence on the universe's future evolution, making it challenging to precisely determine the universe's age [30]. To address this limitation, alternative infrared (IR) cutoffs have been proposed to describe the universe's current and future evolution. Granda and Oliveros [31, 32] introduced a new IR cutoff that depends on both the square of the Hubble parameter and its time derivative. This approach leads to the New Holographic Dark Energy (NHDE) model, with the energy density given by $\rho_{\rm de} = 3c^2 M_p^2 \left(\alpha H^2 + \beta \dot{H}\right)$,where α and β are constants. Gao *et al.* [33] developed an alternative holographic dark energy model, known as the Holographic Ricci Dark Energy (HRDE) model, in which the future event horizon is replaced by the inverse of the Ricci scalar, $L \approx R^{-1/2}$. Consequently, the energy density becomes proportional to the Ricci scalar R.

In recent years, different entropy frameworks have been proposed and applied to study various cosmological scenarios. In 2018, the Tsallis holographic dark energy (THDE) model was introduced [34], employing the Tsallis generalized entropy $S_\delta = \gamma A^\delta$ where δ is a non-additive parameter, A denotes the area of the event horizon, and γ is a constant [35]. Barrow [36] proposed a new approach to black hole entropy that incorporates quantum gravity effects, and Saridakis [37] constructed the Barrow Holographic Dark Energy (BHDE) model by using Barrow entropy in place of the standard Bekenstein-Hawking entropy. Recent studies indicate that the Bekenstein entropy $S = \frac{A}{4}$ can be interpreted as a special case of Tsallis entropy [38–44], which leads to a convenient expression for Rényi entropy [45] given by $S = \frac{1}{\delta} \ln\left(\frac{\delta}{4}A + 1\right)$. By applying this relation together with the assumption $\rho_{\rm de} dV \propto T dS$ [46], the energy density of Rényi holographic dark energy (RHDE) is obtained as $\rho_{\rm de} = \frac{3c^2H^2}{8\pi\left(1+\frac{\pi\delta}{H^2}\right)}$, where c^2 is a numerical constant. Jahromi *et al.* [47] proposed a new holographic dark energy model, known as Sharma–Mittal HDE, by incorporating Sharma–Mittal entropy [48] within the framework of the holographic principle, where the Hubble horizon is taken as the IR cutoff. Their analysis shows that the model exhibits behavior in good agreement with observational results. Kaniadakis introduced another form of entropy, which represents a one-parameter generalization of the classical entropy, expressed as $S_K = \frac{1}{K}\sinh(KS_{BH})$, where K is a dimensionless parameter [49]. The usual Bekenstein-Hawking entropy is recovered in the limit $K \to 0$. Using this entropy framework along with the holographic dark energy concept, a new model called Kaniadakis Holographic Dark Energy (KHDE) has been proposed in the literature [50, 51].

The RHDE model has been investigated using different choices of infrared (IR) cutoffs [52, 53]. Prasanthi and Aditya [53] constructed anisotropic and spatially homogeneous Bianchi type-VI$_0$ Rényi holographic dark energy (RHDE) models by taking both the Hubble horizon and the Granda-Oliveros horizon as infrared (IR) cutoffs. Their analysis shows that the squared speed of sound indicates stability for the RHDE model with the Hubble horizon, whereas it exhibits instability when the Granda-Oliveros horizon is used. Younas *et al.* [54] examined the cosmological implications, such as stability and the equation of state (EoS) parameter, of Rényi entropy-based holographic dark energy models within the framework of Chern–Simons gravity. Maity and Debnath [55] studied Rényi holographic dark energy in a D-dimensional fractal universe by exploring its cosmological implications. T. Chinnappalanaidu *et al.* [56] studied the dynamical behavior of a homogeneous and anisotropic Kantowski-Sachs universe filled with Rényi holographic dark energy, using a generalized scalar-tensor theory as the gravitational framework. Iqbal and Jawad [57] studied the cosmological parameters of the RHDE model by considering the Hubble horizon as the infrared (IR) cutoff within the DGP brane-world framework, and found that the model exhibits unstable behavior. Jawad *et al.* [58] examined the RHDE model in the framework of loop quantum cosmology by considering the Hubble horizon as the infrared (IR) cutoff. The RHDE model has been examined by several researchers in different frameworks to study the universe's present expansion and its associated characteristics.

Almost all these studies in the literature focus on anisotropic and homogeneous universe models because although the present universe appears to be homogeneous and isotropic on large scales and is well described by the Friedmann-Lemaître-Robertson-Walker framework, there is no definitive observational evidence that completely excludes the possibility of anisotropy. It is possible that anisotropies were present in the early universe, as suggested by the observations of anisotropies in the Cosmic Microwave Background (CMB). Spatially homogeneous and anisotropic Bianchi models are widely studied to improve our understanding of the dynamics of the evolving universe. These models provide some of the simplest descriptions of anisotropic backgrounds and play an important role in explaining the large-scale behavior of the universe. The FLRW cosmological model, which is homogeneous and isotropic, can be considered as a particular case of the Bianchi type I universe. As a result, spatially homogeneous and anisotropic Bianchi cosmological models have been widely studied in various contexts. Motivated by the above discussion, our aim is to examine a Rényi holographic dark energy model in a Bianchi type VI_0 universe within the framework of General Relativity.

The structure of the paper is as follows: In Section 2, we consider a spatially homogeneous and anisotropic Bianchi type VI_0 universe consisting of pressureless cold dark matter and non-interacting Rényi holographic dark energy. In this section, the Hubble horizon $L = H^{-1}$ is chosen as the infrared (IR) cutoff, and the corresponding Einstein field equations are derived. Section 3 presents exact solutions of the field equations obtained by considering a hyperbolic form of the scale factor. Section 4 focuses on the physical and kinematical aspects of the model through the study of the evolution of key cosmological parameters. Section 5 summarizes the main conclusions of this work.

2. METRIC AND GOVERNING EQUATIONS

We consider the Bianchi type-VI_0 spacetime in the following form:

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{2\alpha x}dy^2 + C^2e^{-2\alpha x}dz^2 \tag{1}$$

The scale factors $A(t)$, $B(t)$ and $C(t)$ vary only with cosmic time t and $\alpha \neq 0$ is taken as a constant. The metric reduces to the Bianchi type-I case when $\alpha = 0$,

It is assumed that the universe contains cold dark matter along with a proposed anisotropic fluid, which serves as the components of holographic dark energy.

The energy-momentum tensor T^i_j for cold dark matter is

$$T^i_j = diag[-1, 0, 0, 0]\rho_m \tag{2}$$

where ρ_m represents the energy density of cold dark matter.

The energy-momentum tensor for Rényi holographic dark energy $\bar{T}^i_j$, takes the following form

$$\bar{T}^i_j = diag[-\rho_{de}, p^{de}_x, p^{de}_y, p^{de}_z] \tag{3}$$

where, ρ_{de} denotes the energy density of Rényi holographic dark energy [46], while p^{de}_x, p^{de}_y, and p^{de}_z signify the pressures of Rényi holographic dark energy in the x, y and z directions, respectively.

Density of Rényi holographic dark energy ρ_{de}, is defined by

$$\rho_{de} = \frac{3D^2H^2}{8\pi\left(1 + \frac{\delta\pi}{H^2}\right)} \tag{4}$$

where D^2 is treated as a numerical constant and H is the Hubble parameter.

By permitting anisotropy in the pressure of Rényi holographic dark energy, p_{de} and in its equation of state parameter, $\omega_{de} = \frac{p_{de}}{\rho_{de}}$, we obtain.

$$\begin{aligned}\bar{T}^i_j &= diag[-1, \omega^{de}_x, \omega^{de}_y, \omega^{de}_z]\rho_{de} \\ &= diag[-1, \omega_{de}, (\omega_{de} + \eta), (\omega_{de} + \eta)]\rho_{de}\end{aligned} \tag{5}$$

where $\omega^{de}_x = \omega_{de}$, $\omega^{de}_y = (\omega_{de} + \eta)$, and $\omega^{de}_z = (\omega_{de} + \eta)$ are the directional equation of state parameters for the Rényi holographic dark energy along the x, y and z axes, respectively. The skewness parameter η indicates the deviations from ω_{de} along the directions of y and z. The 4-velocity vector $u^i = (1, 0, 0, 0)$ is taken to satisfy the condition $u^iu_i = -1$.

In Relativitistic units, Einstein's field equation is defined as

$$R_{ij} - \frac{1}{2}g_{ij}R = T_{ij} + \bar{T}_{ij} \tag{6}$$

In this equation, R_{ij} denotes the Ricci tensor, g_{ij} refers to the metric tensor, and $R = g^{ij}R_{ij}$ represents the Ricci scalar curvature.

In a comoving coordinate system, Einstein's field equation (6), along with equations (2) and (5) for the metric (1), results in the following set of field equations.

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{C}}{C}\frac{\dot{A}}{A} - \left(\frac{\alpha}{A}\right)^2 = \rho_m + \rho_{de} \tag{7}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \left(\frac{\alpha}{A}\right)^2 = -\omega_{de}\rho_{de} \tag{8}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{B}}{B}-\left(\frac{\alpha}{A}\right)^2=-\left(\omega_{de}+\eta\right)\rho_{de} \tag{9}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}}{A}\frac{\dot{C}}{C}-\left(\frac{\alpha}{A}\right)^2=-\left(\omega_{de}+\eta\right)\rho_{de} \tag{10}$$

$$\frac{\dot{C}}{C}-\frac{\dot{B}}{B}=0 \tag{11}$$

where the overdot symbol indicates the derivative with respect to the cosmic time t.

From equations (7)–(11), we derive the continuity equation as:

$$\dot{\rho}_m+3H\rho_m+\dot{\rho}_{de}+3H\left(1+\omega_{de}\right)\rho_{de}+\eta\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\rho_{de}=0 \tag{12}$$

We assume that the two components of the universe evolve independently without interaction. Hence, continuity equation (12) leads to separate conservation equations for matter and dark energy as

$$\dot{\rho}_m+3H\rho_m=0 \tag{13}$$

$$\dot{\rho}_{de}+3H\left(1+\omega_{de}\right)\rho_{de}+\eta\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\rho_{de}=0 \tag{14}$$

3. THE SOLUTIONS OF THE FIELD EQUATIONS

The Hubble parameters corresponding to the directions of x, y, and z for the Bianchi type-VI$_0$ metric are

$$H_x=\frac{\dot{A}}{A},\qquad H_y=\frac{\dot{B}}{B},\qquad H_z=\frac{\dot{C}}{C} \tag{15}$$

The average scale factor a and the volume V are defined as follows

$$a=(ABC)^{\frac{1}{3}}\qquad V=a^3=ABC \tag{16}$$

The average value of Hubble parameter H is defined as

$$H=\frac{1}{3}\frac{\dot{V}}{V}=\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=\frac{1}{3}\left(H_x+H_y+H_z\right) \tag{17}$$

The mathematical form of the average anisotropy parameter A_m is

$$A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{18}$$

Here, $\Delta H=H_i-H$, where H_i (for $i=x,y,z$) denotes the directional Hubble parameters along the x, y and z directions. The anisotropy parameter measures how much the expansion deviates from isotropy. Evaluating this parameter is essential for determining whether the cosmological model tends toward isotropy over time.

From Eq. (11), we obtain

$$B=mC \tag{19}$$

where $m>0$ is an integration constant.

Using equations (15) , (17) and (19), the expression for the anisotropy parameter in (18) can be written as:

$$A_m=\frac{2}{9H^2}\left(H_x-H_z\right)^2 \tag{20}$$

Substituting Eq. (19) into the field equations (7)- (11), we obtain

$$\left(\frac{\dot{C}}{C}\right)^2+2\frac{\dot{C}}{C}\frac{\dot{A}}{A}-\left(\frac{\alpha}{A}\right)^2=\rho_m+\rho_{de} \tag{21}$$

$$\left(\frac{\dot{C}}{C}\right)^2 + 2\frac{\ddot{C}}{C} + \left(\frac{\alpha}{A}\right)^2 = -\omega_{de}\rho_{de} \tag{22}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \left(\frac{\alpha}{A}\right)^2 = -\left(\omega_{de} + \eta\right)\rho_{de} \tag{23}$$

By subtracting equation (22) from equation (23), we obtain:

$$\frac{\dot{A}}{A} - \frac{\dot{C}}{C} = H_x - H_z = \frac{\lambda}{V} + \frac{1}{V}\int\left(\frac{2\alpha^2}{A^2} - \eta\rho_{de}\right)V dt \tag{24}$$

Here, λ is an integration constant, and the term η appears due to the possible intrinsic anisotropy of the fluid,

Finally, substituting equation (24) into equation (20), we obtain the anisotropy parameter of the expansion as follows:

$$A_m = \frac{2}{9}\frac{1}{H^2}\left[\lambda + \int\left(\frac{2\alpha^2}{A^2} - \eta\rho_{de}\right)V dt\right]^2 V^{-2} \tag{25}$$

Following [59], we take

$$\eta = \frac{2\alpha^2}{\rho_{de}A^2} \tag{26}$$

Substituting equation (26) into equation (25), we obtain the reduced anisotropy parameter of the expansion as

$$A_m = \frac{2}{9}\frac{\lambda^2}{H^2}V^{-2} \tag{27}$$

Equations (21)-(23) of the Einstein field equations, along with (26), lead to:

$$\left(\frac{\dot{C}}{C}\right)^2 + 2\frac{\dot{C}}{C}\frac{\dot{A}}{A} = \rho_m + \left(1 + \frac{\eta}{2}\right)\rho_{de} \tag{28}$$

$$\left(\frac{\dot{C}}{C}\right)^2 + 2\frac{\ddot{C}}{C} = -\left(\omega_{de} + \frac{\eta}{2}\right)\rho_{de} \tag{29}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \left(\frac{\alpha}{A}\right)^2 = -\left(\omega_{de} + \frac{\eta}{2}\right)\rho_{de} \tag{30}$$

The relative rate of expansion (or contraction) of the universe is described by the expansion scalar θ, while the shear scalar σ measures the deformation caused by density fluctuations. The scalar expansion θ and shear scalar σ are defined as

$$\theta = 3H = H_x + 2H_z \tag{31}$$

$$\sigma^2 = \frac{1}{2}[H_x^2 + 2H_z^2] - \frac{\theta^2}{6} = \frac{1}{3}\left(H_x - H_z\right)^2 \tag{32}$$

The three equations, (28)–(30) involve six unknowns, namely $A, C, \rho_m, \rho_{de}, \omega_{de}$ and η, together with (4) indicate that the system remains unsolved without additional constraints .

Thus, in order to obtain a determinate solution, two additional conditions are required. We consider the following physically relevant constraints:

(i) We assume that the shear scalar is proportional to the scalar expansion which gives

$$A = C^k \tag{33}$$

where, k is a nonzero integration constant.

(ii) Next, we assume the average scale factor $a(t)$ as

$$a(t) = \left(\sinh\left(\beta t\right)\right)^{\frac{1}{n}} \tag{34}$$

where, β is an arbitrary constant and n represents a positive constant.

Equation (34), is a generalization of the scale factor obtained by Amirhashchi *et al.* [60] and Pradhan *et al.* [61] for dark energy models in FRW and Bianchi type-VI_0 space-times, respectively.

Substituting the average scale factor from Equation (34) into Equations (16), (19) and (33), we derive the directional scale factors.

$$A = \left(\frac{\sinh^{\frac{3}{n}}(\beta t)}{m}\right)^{\frac{k}{(k+2)}} \tag{35}$$

$$B = m\left(\frac{\sinh^{\frac{3}{n}}(\beta t)}{m}\right)^{\frac{1}{(k+2)}} \tag{36}$$

$$C = \left(\frac{\sinh^{\frac{3}{n}}(\beta t)}{m}\right)^{\frac{1}{(k+2)}} \tag{37}$$

Thus, the metric (1) takes the form:

$$ds^2 = -dt^2 + \left(\frac{\sinh^{\frac{3}{n}}(\beta t)}{m}\right)^{\frac{2k}{(k+2)}} dx^2 + m\left(\frac{\sinh^{\frac{3}{n}}(\beta t)}{m}\right)^{\frac{2}{(k+2)}} e^{2\alpha x} dy^2 + \left(\frac{\sinh^{\frac{3}{n}}(\beta t)}{m}\right)^{\frac{2}{(k+2)}} e^{-2\alpha x} dz^2 \tag{38}$$

4. RELEVANT PHYSICAL PROPERTIES

The average Hubble parameter, H, is a fundamental cosmological quantity that characterizes the expansion rate of the universe. By employing the scale factor $a(t) = [\sinh(\beta t)]^{1/n}$, from which the Hubble parameter can be expressed in terms of the cosmic time t as:

$$H = \frac{\dot{a}}{a} = \frac{\beta}{n}\coth(\beta t) \tag{39}$$

To describe the cosmic expansion in terms of the observable redshift, z, we employ the standard relation between the scale factor and redshift $a(t) = \frac{1}{1+z}$, where the present value of the scale factor is normalized to unity, i.e., $a_0 = 1$. Therefore, the Hubble parameter in terms of the redshift is obtained as

$$H(z) = \frac{\beta}{n}\sqrt{1 + (1+z)^{2n}} \tag{40}$$

Imposing the normalization condition at the present epoch, $H_0 \equiv H(z = 0)$, yields $H_0 = \frac{\beta}{n}\sqrt{2}$ from which it follows that $\frac{\beta}{n} = \frac{H_0}{\sqrt{2}}$. Consequently, the normalized Hubble parameter assumes the form

$$H(z) = H_0\sqrt{\frac{1 + (1+z)^{2n}}{2}} \tag{41}$$

which will be employed throughout the subsequent analysis.

The deceleration parameter is evaluated using the kinematic relation

$$q(z) = -1 + \frac{(1+z)}{H(z)}\frac{dH(z)}{dz}$$

Substituting the normalized Hubble parameter (41) and performing the differentiation with respect to the redshift yields

$$q(z) = -1 + \frac{n(1+z)^{2n}}{1 + (1+z)^{2n}} \tag{42}$$

which provides the evolution of the cosmic expansion directly in terms of the observable redshift. For the proposed cosmological model to be physically viable, the model parameter n must satisfy the following conditions:

1. In the high-redshift limit ($z \gg 1$), the deceleration parameter approaches $q(z) \to n - 1$.. To ensure a decelerating early Universe, which is essential for the formation of large-scale structures, the condition $q(z) > 0$ must be satisfied, implying $n > 1$.
2. At the present epoch ($z = 0$), the deceleration parameter is given by $q_0 \equiv q(0) = \frac{n-2}{2}$. The observed late-time accelerated expansion requires $q_0 < 0$, which leads to the constraint $n < 2$.

Accordingly, the physically admissible range of the model parameter is $1 < n < 2$. Within this interval, the transition from decelerated to accelerated expansion occurs at the transition redshift z_t, defined by the condition $q(z_t) = 0$. Solving this condition yields

$$z_t = \left(\frac{1}{n-1}\right)^{\frac{1}{2n}} - 1 \tag{43}$$

For all $n \in (1, 2)$, the transition redshift is real, positive, and uniquely determined, thereby ensuring a physically consistent cosmic evolution from an early decelerating phase to the present accelerated epoch.
To investigate the evolution of the cosmological quantities, namely the dark energy density $\rho_{de}(z)$, matter density $\rho_m(z)$, dark energy pressure $p_{de}(z)$, equation-of-state parameter $\omega_{de}(z)$, and deceleration parameter $q(z)$, the model parameters H_0 and n are first constrained using the observational Hubble data (OHD) set. The resulting best-fit values are subsequently employed in the analysis of the model.

4.1. Observational constraints

To constrain the free parameters, $\{H_0, n\}$, of the proposed HDE model, a statistical analysis is performed using the normalized Hubble parameter given in Eq. (41). The resulting best-fit values are subsequently employed in the cosmological analysis. A total of 77 observational Hubble parameter $[H(z)]$ data points are employed to constrain the model parameters, H_0 and n. The dataset spans the redshift range $0 \leq z \leq 2.36$ and is adopted from Ref. [62]. The corresponding chi-square (χ^2) statistic is defined as

$$\chi^2_{OHD} = \sum_{i=1}^{77}\left[\frac{H_{\rm obs}(z_i) - H_{\rm th}(z_i; H_0, n)}{\sigma_{H,i}}\right]^2 \tag{44}$$

where $H_{\rm obs}(z_i)$ and $H_{\rm th}(z_i; H_0, n)$ denote the observed and theoretical values of the Hubble parameter at redshift z_i, respectively, while $\sigma_{H,i}$ represents the corresponding observational uncertainty. The model parameters are constrained using the Markov Chain Monte Carlo (MCMC) technique through the minimization of the chi-square statistic, χ^2. Under the assumption of Gaussian-distributed observational uncertainties, the likelihood function is defined as $\mathcal{L}(\boldsymbol{\theta}) \propto \exp\left(-\frac{\chi^2(\boldsymbol{\theta})}{2}\right)$, where $\boldsymbol{\theta} = \{H_0, n\}$ denotes the set of free model parameters. The corresponding marginalized 1σ and 2σ confidence regions are displayed through the likelihood contour plots. The parameter constraints obtained from the OHD dataset are presented in Fig. 1. To assess the viability of the proposed HDE model, its performance is compared with that of the spatially flat ΛCDM model, whose Hubble parameter is given by $H(z) = H_0\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}$, where Ω_m and Ω_Λ denote the present-day matter density parameter and cosmological constant density parameter, respectively. The relative performance of the competing models is further evaluated using the Akaike Information Criterion (AIC) [63] and the Bayesian Information Criterion (BIC) [64], defined as $\mathrm{AIC} = \chi^2_{\rm min} + 2\kappa$, and $\mathrm{BIC} = \chi^2_{\rm min} + \kappa \ln N$, respectively. Here, κ represents the number of free model parameters, while N denotes the total number of observational data points. In general, models with lower AIC and BIC values are statistically preferred, as they provide a better balance between the goodness of fit and model complexity. Table 1 presents the best-fit values of the model parameters together with their corresponding 1σ confidence intervals for both the proposed HDE model and the standard ΛCDM cosmology. For the OHD dataset, the proposed model yields a marginally lower minimum chi-square value, $\chi^2_{\rm min}$, than the ΛCDM model, indicating a slightly better agreement with the observational data. To further assess the relative performance of the two models, the differences in the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) are evaluated. For the OHD dataset, these are obtained as $\Delta\mathrm{AIC} = \mathrm{AIC}_{\rm HDE} - \mathrm{AIC}_{\Lambda\rm CDM} = -2.484$, and $\Delta\mathrm{BIC} = \mathrm{BIC}_{\rm HDE} - \mathrm{BIC}_{\Lambda\rm CDM} = -2.484$. The negative values of both ΔAIC and ΔBIC indicate that the proposed HDE model is statistically preferred over the standard ΛCDM model. Consequently, the analysis based on the OHD dataset provides statistical support for the proposed model. The best-fit value of the Hubble constant obtained in this work, $H_0 = 67.90 \pm 0.88$ km s^{-1} Mpc^{-1}, is consistent with the Planck 2018 CMB measurement, $H_0 = 67.4 \pm 0.5$ km s^{-1} Mpc^{-1} [65], and the recent DESI Data Release 1 BAO determination, $H_0 = 67.97 \pm 0.38$ km s^{-1} Mpc^{-1} [66].

Table 1. Comparison of the best-fit parameter values obtained from OHD datasets for both our model and the ΛCDM model.

Model	H_0	Ω_m	n	$\chi^2_{\rm min}$	AIC	BIC
Our model	$67.90^{+0.88}_{-0.88}$	–	$1.30^{+0.019}_{-0.019}$	70.566	74.566	79.254
ΛCDM	$67.41^{+1.14}_{-1.16}$	$0.299^{+0.016}_{-0.015}$	–	73.050	77.050	81.738

Now, employing the relation between the scale factor and the redshift, $a(t) = (1+z)^{-1}$, and substituting the obtained expression $a(t) = [\sinh(\beta t)]^{1/n}$, yields $\sinh(\beta t) = (1+z)^{-n}$. Consequently, using the normalization relation $\beta = \frac{nH_0}{\sqrt{2}}$, the cosmic time as a function of the redshift is obtained as

$$t(z) = \frac{\sqrt{2}}{nH_0} \sinh^{-1}[(1+z)^{-n}] \tag{45}$$

4.2. Evolution of the deceleration parameter q(z) and phase transition

Figure 2 depicts the redshift evolution of the deceleration parameter obtained using the best-fit values of the model parameters. As expected, the model exhibits a transition from an early decelerating Universe to the current phase of accelerated expansion. The inferred transition redshift,$z_t = 0.589 \pm 0.052$, is fully consistent with the 1σ range derived from model-independent CC observations [67] and the combined DESI BAO and Pantheon+ Type Ia Supernova (SNe Ia) analyses, which constrain the transition redshift to the range $z_t \in [0.55, 0.70]$ [68]. Furthermore, the present value of the deceleration parameter is obtained as $q_0 = -0.350 \pm 0.0095$. Having constrained the model parameters through observational data and the subsequent analysis focuses on the evolution of the remaining cosmological parameters.

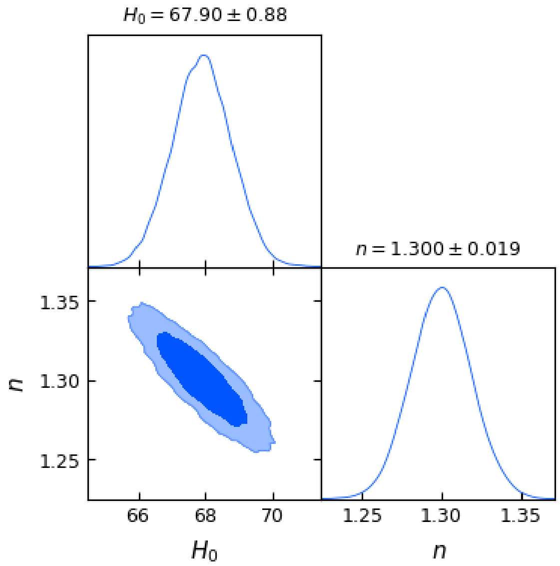

Figure 1. One-dimensional marginalized posterior distributions together with the corresponding two-dimensional marginalized confidence contours at the 1σ and 2σ confidence levels for the proposed cosmological model, constrained using the OHD dataset.

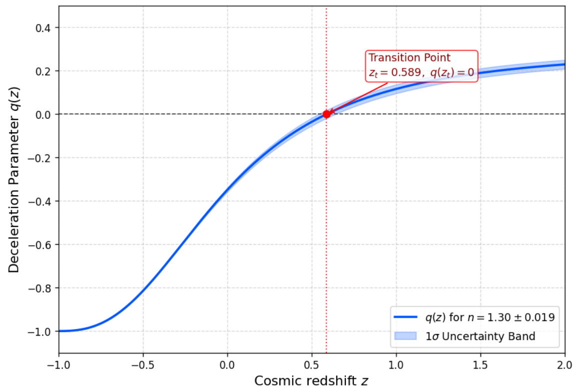

Figure 2. Behaviour of the deceleration parameter

4.3. Evolution of the matter and dark energy density

Using Eq.(13), together with the scale factor $a(t) = [\sinh(\beta t)]^{1/n}$, and the standard scale factor-redshift relation, $a = (1+z)^{-1}$ the matter energy density is obtained as

$$\rho_m(z) = \rho_{m0}(1+z)^3 \tag{46}$$

where ρ_{m0} denotes the present-day matter energy density. The present matter energy density can be expressed in terms of the model parameters H_0 and n by relating it to the present critical density, $\rho_{c0} = 3H_0^2$ through the matter density parameter, $\Omega_{m0} = \frac{\rho_{m0}}{\rho_{c0}}$. At the present epoch ($z = 0$), the field equations for the Bianchi VI$_0$ spacetime satisfy the closure relation $\Omega_{m0} + \Omega_{\text{de}0} = 1$, where the present-day matter and Rényi holographic dark energy density parameters are given by $\Omega_{m0} = \frac{2-n}{2}$ and $\Omega_{\text{de}0} = \frac{n}{2}$ Hence, the present matter energy density is determined as $\rho_{m0} = \Omega_{m0}\rho_{c0} = \left(\frac{2-n}{2}\right)(3H_0^2) = \frac{3}{2}H_0^2(2-n)$. Substituting this expression into the above relation gives the matter energy density as a function of redshift,

$$\rho_m(z) = \frac{3}{2}H_0^2(2-n)(1+z)^3 \tag{47}$$

Substituting the normalized Hubble parameter given by Eq.(41) into Eq.(4), the Rényi holographic dark energy density is obtained as

$$\rho_{de}(z) = \frac{3D^2H_0^4\left[1+(1+z)^{2n}\right]^2}{2H_0^2\left[1+(1+z)^{2n}\right]+4\delta\pi} \tag{48}$$

At the present epoch ($z = 0$), the above expression reduces to $\rho_{de}(0) = \frac{3D^2H_0^4}{H_0^2+\delta\pi}$. Expressing the present-day dark energy density in terms of the critical density, $\rho_{c0} = 3H_0^2$, and using the relation $\Omega_{de}(0) = \frac{\rho_{de}(0)}{\rho_{c0}} = \frac{n}{2}$, one obtains the following algebraic constraint on D:

$$D^2 = \frac{n}{2}\left(1 + \frac{\delta\pi}{H_0^2}\right) \tag{49}$$

Consequently, D^2 can be expressed uniquely in terms of the primary model parameters.

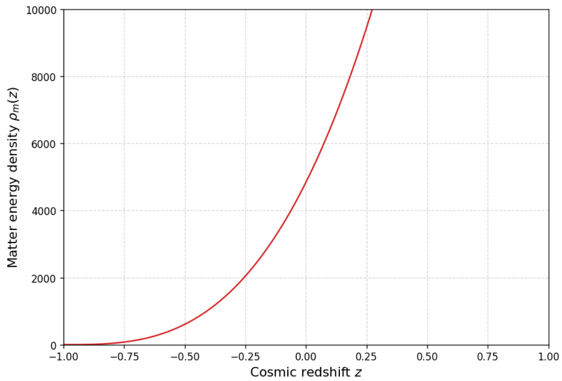

Figure 3. Behaviour of the matter energy density

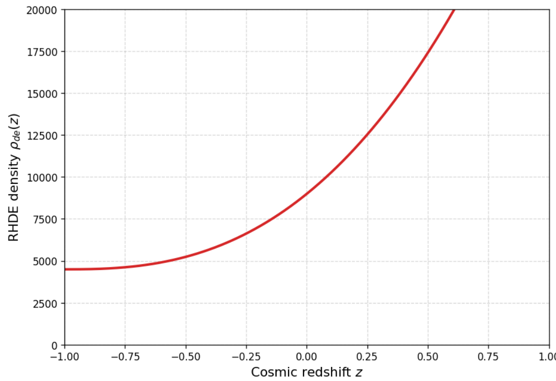

Figure 4. Behaviour of the RHDE density for $\delta = 1.4$

Figures 3 and 4 present the evolution of the matter energy density, $\rho_m(z)$, and the Rényi holographic dark energy (RHDE) density, $\rho_{de}(z)$, as functions of the redshift for the best-fit parameter values $n = 1.3$, $H_0 = 67.9$ km s^{-1} Mpc^{-1}, and $\delta = 1.4$. Figure 3 demonstrates that the matter energy density exhibits the standard cosmological evolution, $\rho_m(z) \propto (1+z)^3$ decreasing continuously with cosmic expansion. It attains large values during the early Universe and progressively diminishes toward zero as the redshift approaches $z = -1$, reflecting the expected dilution of non-relativistic matter. The evolution of the RHDE density is displayed in Fig.4. It is evident that $\rho_{de}(z)$ remains positive throughout the entire redshift range considered. In the high-redshift regime, the matter component dominates over the RHDE density, thereby providing favorable conditions for the formation of cosmic structures. As the Universe evolves toward lower redshifts, the RHDE density gradually overtakes the matter component and eventually approaches a finite positive value in the far future ($z \to -1$). This asymptotic behavior is consistent with a sustained phase of accelerated cosmic expansion. Furthermore, the present-day relation,$\rho_{de}(0) > \rho_m(0)$, indicates that the proposed cosmological model has entered a dark-energy-dominated epoch following the transition from an earlier matter-dominated decelerating phase. The convergence of $\rho_{de}(z)$ to a finite positive constant at late times further suggests that the RHDE component effectively mimics a cosmological constant, ensuring a stable future evolution and excluding the occurrence of Big Rip.

4.4. Evolution of the anisotropy parameter and skewness parameter

Using Eqs.(27) and (40), the anisotropy parameter can be expressed explicitly as a function of the redshift in the form

$$A_m(z) = \frac{4\lambda^2}{9H_0^2}\frac{(1+z)^6}{1+(1+z)^{2n}} \tag{50}$$

By combining Eqs. (40), (48), and (26), the redshift-dependent skewness parameter is obtained as

$$\eta(z) = \frac{2\alpha^2 m^{\frac{2k}{k+2}}(1+z)^{\frac{6k}{k+2}}\left[2H_0^2\left(1+(1+z)^{2n}\right)+4\delta\pi\right]}{3D^2H_0^4\left[1+(1+z)^{2n}\right]^2} \tag{51}$$

where D^2 is determined by $D^2 = \frac{n}{2}\left(1+\frac{\delta\pi}{H_0^2}\right)$

The evolution of the anisotropy parameter is presented in Fig.5. It is evident that the anisotropy parameter decreases monotonically from relatively large values in the early Universe ($z > 0$) to $A_m(0) \approx 0.044$ at the present epoch, and ultimately approaches zero in the asymptotic future ($z \to -1$). This behavior indicates that the spatial anisotropy is progressively suppressed during the cosmic evolution, leading the proposed cosmological model toward an isotropic limit at late times. Such an evolution is consistent with the high degree of isotropy inferred from observations of the Cosmic Microwave Background (CMB). Figure 6 illustrates the redshift evolution of the skewness parameter, $\eta(z)$, which quantifies

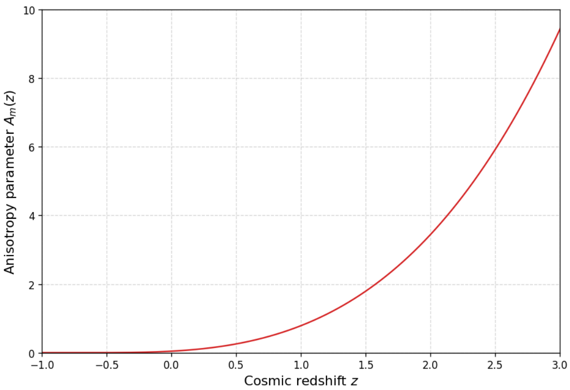

Figure 5. Behaviour of the anisotropy parameter for $\lambda = 30$

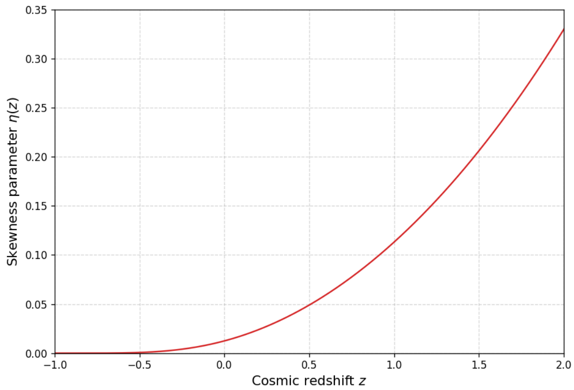

Figure 6. Behaviour of the skewness parameter for $\alpha = 3$, $m = 3$, $k = 10$ and $\delta = 1.4$

the directional anisotropy in the pressure distribution of the cosmic fluid. The parameter decreases continuously from $\eta \approx 0.33$ at $z = 2$ to $\eta(0) \approx 0.013$ at the present epoch and asymptotically vanishes as $z \to -1$. The decay of $\eta(z)$ signifies the gradual disappearance of directional pressure anisotropies, indicating that the dark energy fluid evolves toward an effectively isotropic configuration capable of sustaining the observed late-time accelerated expansion. The simultaneous suppression of both the anisotropy parameter, $A_m(z)$, and the skewness parameter, $\eta(z)$, demonstrates that the proposed cosmological model undergoes a natural isotropization throughout its evolution. Their asymptotic convergence to zero at late times implies that the model approaches an effectively isotropic, in agreement with the stringent isotropy constraints provided by contemporary CMB observations.

4.5. Evolution of the equation of state parameter and the pressure for Rényi holographic dark energy

We now discuss the cosmic role of the equation of state parameter (EoS), which is defined as

$$\omega_{de} = \frac{p_{de}}{\rho_{de}} \tag{52}$$

Here, ρ_{de} and p_{de} represent the energy density and pressure of Rényi holographic dark energy, respectively. The equation of state (EoS) parameter ω_{de} is a key factor in determining the universe's dynamical evolution. Depending on its value, the universe can be classified into different cosmological phases: when $\omega_{de} = 0$, the universe behaves like non-relativistic matter; for $-1 < \omega_{de} < -\frac{1}{3}$, it corresponds to a quintessence phase; a value of $\omega_{de} = -1$ represents a cosmological constant-dominated universe; and when $\omega_{de} < -1$, the universe is in a phantom phase.
From Eqs. (14), (48) and (51), we obtain the equation-of-state parameter as

$$\omega_{\rm de}(z) = -1 - \frac{1}{3H(z)}\left[\frac{\sqrt{2}nH_0H(z)^2(1+z)^{4n}}{H(z)^2+\delta\pi} - \frac{2\sqrt{2}nH_0(1+z)^{2n}}{\sqrt{1+(1+z)^{2n}}} + \frac{4\alpha^2m^{\frac{2k}{k+2}}(1+z)^{\frac{6k}{k+2}}\left(H(z)^2+\delta\pi\right)}{D^2(k+2)H(z)^3}\right] \tag{53}$$

From Eqs. (48), (52) and (53), the pressure equation for R enyi holographic dark energy is obtained as

$$p_{\rm de}(z) = -\frac{3D^2H(z)^4}{H(z)^2+\delta\pi} - \frac{3D^2H(z)^4}{H(z)^2+\delta\pi}\left[\frac{nH_0^2(1+z)^{2n}}{3\left(H(z)^2+\delta\pi\right)} - \frac{4n(1+z)^{2n}}{3\left[1+(1+z)^{2n}\right]}\right] - \frac{4\alpha\, m^{\frac{2k}{k+2}}(1+z)^{\frac{6k}{k+2}}}{k+2} \tag{54}$$

where $H(z) = \frac{H_0}{\sqrt{2}}\sqrt{1+(1+z)^{2n}}$ and $D^2 = \frac{n}{2}\left(1+\frac{\delta\pi}{H_0^2}\right)$

Figures 7 and 8 present the redshift evolution of the dark energy pressure, $p_{de}(z)$, and the dark energy equation-of-state (EoS) parameter, $\omega_{de}(z)$, respectively. As illustrated in Fig.7, the dark energy pressure remains negative throughout the entire cosmic evolution. Its magnitude evolves smoothly from relatively large negative values at high redshifts to a finite negative constant in the asymptotic future ($z \to -1$). The persistence of negative pressure throughout the cosmic history provides the repulsive effect required to account for the observed late-time accelerated expansion of the Universe. Figure 8 depicts the evolution of the EoS parameter, $\omega_{de}(z)$. The results indicate that the EoS remains in the phantom regime ($\omega_{de} < -1$) over a finite period of cosmic evolution before gradually approaching the cosmological-constant limit, $\omega_{de} = -1$, as $z \to -1$. Such behavior is characteristic of a transient phantom phase, in which the phantom nature of dark energy is only temporary and the cosmological dynamics eventually converge toward a de Sitter phase. At the present epoch, the model yields $\omega_{\rm de}(0) \approx -0.73$, implying that the current accelerated expansion is driven by a quintessence-like dark energy component. In contrast to the standard ΛCDM model, where the dark energy equation-of-state remains

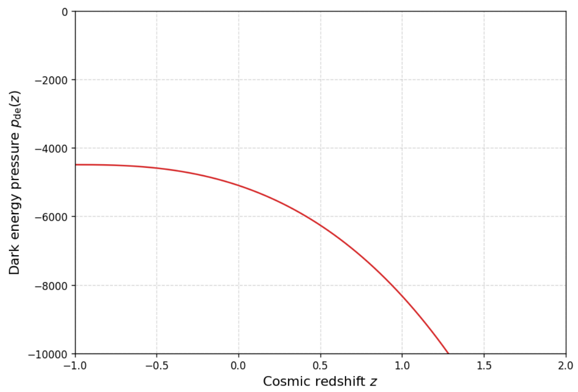

Figure 7. Behaviour of the dark energy pressure for $\alpha = 3$, $m = 3$, $k = 10$ and $\delta = 1.4$

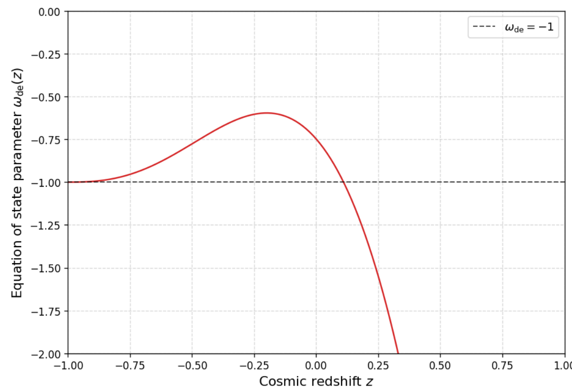

Figure 8. Behaviour of the EoS parameter for $\alpha = 3$, $m = 3$, $k = 10$ and $\delta = 1.4$

fixed at $\omega_{de} = -1$, the Rényi holographic dark energy model predicts a dynamical evolution of $\omega_{de}(z)$ from a transient phantom phase to a present quintessence phase, before asymptotically approaching the de Sitter limit in the far future. This late-time convergence to the cosmological-constant boundary suggests a stable future evolution and is consistent with dynamical dark energy scenarios that avoid future singularities, including the Big Rip, through suitable cosmological evolution mechanisms [69, 70].

5. DISCUSSION AND CONCLUSION

In the present work, we have investigated a spatially homogeneous and anisotropic Bianchi type-VI$_0$ cosmological model within the framework of General Relativity by considering Rényi holographic dark energy (RHDE), with the Hubble horizon serving as the infrared cutoff. The cosmic matter content is assumed to comprise pressureless dark matter and RHDE, with no interaction between the two components. Exact solutions of the Einstein field equations have been obtained by adopting the average scale factor $a(t) = (\sinh(\beta t))^{1/n}$. The free model parameters have been constrained through a Markov Chain Monte Carlo (MCMC) analysis using a compilation of 77 Observational Hubble Data (OHD) measurements of the Hubble parameter, $H(z)$. The analysis yields the best-fit values $H_0 = 67.9\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ and $n = 1.3$. Based on these observationally constrained parameters, the physical and kinematical properties of the model have been investigated through the evolution of the relevant cosmological observables. The main findings of the present analysis are summarized as follows:

- The matter energy density evolves according to the standard relation $\rho_m(z) \propto (1+z)^3$, decreasing continuously as the Universe expands and tending to zero in the asymptotic future ($z \to -1$). By contrast, the Rényi holographic dark energy density remains finite and positive throughout the cosmic evolution, asymptotically approaching a constant value at late times. This behavior naturally supports a sustained phase of cosmic acceleration.

- The anisotropy parameter, $A_m(z)$, together with the skewness parameter, $\eta(z)$, decreases continuously during the cosmic evolution and approaches zero in the asymptotic future ($z \to -1$). The vanishing of these quantities signifies the gradual disappearance of both spatial and fluid anisotropies, indicating that the proposed cosmological model evolves toward an isotropic state at late times. This behavior is in agreement with the high degree of isotropy revealed by contemporary Cosmic Microwave Background (CMB) observations.

- The dark energy pressure, $p_{de}(z)$, remains negative throughout the cosmic evolution, while the equation-of-state (EoS) parameter, $\omega_{de}(z)$, undergoes a transient phantom phase ($\omega_{de} < -1$), passes through a quintessence phase at the present epoch, before asymptotically approaching the cosmological constant boundary, $\omega_{de} \to -1$, as $z \to -1$. The convergence of the EoS parameter to the de Sitter limit ensures a stable late-time evolution and prevents the occurrence of future singularities, including the Big Rip.

ORCID

Chandra Rekha Mahanta, https://orcid.org/0000-0002-8019-8824; **Rajashree Mahanta**, https://orcid.org/0009-0009-2656-115X

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КОСМОЛОГІЧНА ЕВОЛЮЦІЯ АНІЗОТРОПНОГО ВСЕСВІТУ ТИПУ Б'ЯНКІ VI$_0$ З ГОЛОГРАФІЧНОЮ ТЕМНОЮ ЕНЕРГІЄЮ РЕНЬЇ

Чандра Рекха Маханта, **Раджашрі Маханта**

Кафедра математики, Університет Гаухаті, Гувахаті - 781014 (ІНДІЯ)

У цій роботі ми досліджуємо просторово однорідну та анізотропну космологічну модель типу Б'янкі VI$_0$ в рамках загальної теорії відносності, розглядаючи невзаємодіючу суміш темної матерії без тиску та голографічної темної енергії Рен'ї (ГТЕ), де горизонт Хаббла служить інфрачервоним обрізом. Точні розв'язки рівнянь поля Ейнштейна отримані шляхом прийняття гіперболічного масштабного коефіцієнта $a(t) = (\sinh(\beta t))^{1/n}$. Параметри моделі обмежені за допомогою аналізу методом Монте-Карло з використанням ланцюгів Маркова (MCMC) з використанням 77 вимірювань параметра Хаббла, отриманих за допомогою спостережних даних Хаббла (OHD), що дає значення найкращого наближення $H_0 = 67.9\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ та $n = 1.3$. Порівняння зі стандартною моделлю ΛCDM показує дещо нижче мінімальне значення χ^2 разом із негативними значеннями ΔAIC та ΔBIC, що вказує на помірну статистичну перевагу запропонованої моделі. Реконструйована космічна еволюція демонструє плавний перехід від фази уповільнення з домінуванням матерії до сучасної прискореної епохи, при цьому червоне зміщення переходу знаходиться в межах діапазону $0.5 \lesssim z_t \lesssim 0.7$, що є сприятливим для спостережень. Крім того, еволюція матерії та густини енергії, тиску темної енергії, параметра рівняння стану, параметра анізотропії та параметра асиметрії демонструють, що модель успішно відтворює спостережуване прискорене розширення наприкінці часу, одночасно зазнаючи поступової ізотропізації. Загалом, запропонована модель РДДЕ забезпечує фізично узгоджену та спостережливо життєздатну основу для опису еволюції Всесвіту наприкінці часу.

Ключові слова: *холодна темна матерія; простір-час типу Б'янкі VI_0; космічне прискорення; голографічна темна енергія Реньї; параметр уповільнення; параметр рівняння стану*