East European Journal of Physics. 3. 138–145 (2026)

DOI: 10.26565/2312-4334-2026-3-12 ISSN 2312-4334

# FINITE-SIZE SCALING OF THERMAL SUSCEPTIBILITY AND SPECIFIC HEAT DENSITY NEAR THE QCD DECONFINEMENT PHASE TRANSITION

B. Moussaoui[1,2], A. Ait El Djoudi[1*], H. Mouloudj[1,3], M. A. Lakehal[1,4]

[1]*Particle and Statistical Physics Laboratory, Ecole Normale Supérieure-Kouba, Vieux-Kouba, B.P.92, 16050 Algiers, Algeria*

[2]*Theoretical Physics Laboratory, Faculty of Physics, USTHB Bab-Ezzouar, El Alia, BP 32, 16111 Algiers, Algeria*

[3]*Hassiba Benbouali University, Chlef, Algeria*

[4]*M'hamed Bougara University, Boumerdes, Algeria*

[*]*Corresponding Author e-mail: amal.aiteldjoudi@g.ens-kouba.dz*

Received June 25, 2026; accepted August 17, 2026

The properties of the thermally driven deconfinement phase transition (DPT) from a hadronic gas (HG) to a color-singlet quark-gluon plasma (QGP) containing gluons and massless up and down quarks, at a nonzero quark chemical potential $\mu$, are investigated by considering the volumetric coexistence of the hadronic gas and the QGP in a finite-size system. Finite-size effects are analyzed for the thermally driven DPT at different $\mu$ values. The critical exponents are determined using a numerical finite-size scaling (FSS) analysis, by fitting the results as a function of the system size. The effective transition temperature ($T_c$) exhibits a shift toward higher values as the system size decreases, indicating a critical behavior in the region of the thermally driven DPT from a HG to the QGP. Crucially, all three scaling critical exponents are found to be independent of the quark chemical potential $\mu$, establishing the universality of the finite-size scaling structure with respect to $\mu$.

**Keywords:** *Thermally driven deconfinement phase transition (DPT); Color-singlet; Quark chemical potential; Finite-size scaling (FSS) analysis; Transition temperature*

## 1. INTRODUCTION

Understanding the phase transition between hadronic gas (HG) phase and the quark-gluon plasma (QGP) phase represents one of the most fundamental phenomena in contemporary high-energy and nuclear physics, and continues to be the subject of intense theoretical and experimental investigation [1, 2]. Quarks and gluons, the fundamental components of hadrons, interact via the strong interaction governed by Quantum Chromodynamics (QCD), the theory of the strong force. Their behavior is studied in heavy-ion collisions at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) [1]. The Deconfinement phase transition (DPT) describes the transition from the HG phase to the QGP near effective transition temperature of 150-175MeV for three quark flavors [3], a result that finds consistent support from recent high-precision lattice QCD simulations conducted at both vanishing and finite chemical potential [4, 5].

While most QCD studies assume infinite systems, the QGP formed in experiments is finite, with volumes ranging from 50 $\mathrm{fm}^3$ to 250 $\mathrm{fm}^3$ [6, 7]. This fundamental discrepancy between infinite volume theoretical frameworks and the genuinely finite nature of the physical system motivates a rigorous and systematic investigation of finite-size effects on the DPT. Recent model-based studies have confirmed that a reduction in system volume leads to significant modifications of key thermodynamic response functions, including the pressure, energy density, specific heat, speed of sound, and conserved-charge susceptibilities, and that the QCD crossover transition is generally observed to become progressively smoother as the system size decreases [8]. Finite-size effects strongly influence the position of the effective transition point and other thermodynamic quantities in the phase coexistence model (PCM) [9–11]. Recent lattice QCD analyses have further demonstrated that the volume and quark chemical potential dependence of the deconfinement crossover differs completely from that of the chiral transition [12], underscoring the necessity of a careful and dedicated finite-size analysis for the deconfinement sector specifically. Furthermore, the recent application of finite-size scaling (FSS) methodology to experimental net-proton cumulant data from RHIC has yielded the first experimental evidence for a QCD critical point within a range consistent with theoretical predictions [13], imparting further importance to comprehensive FSS investigations.

The present paper extends the FSS analysis of the thermally driven DPT in finite volume ($V$), first conducted at vanishing chemical potential in Ref. [10], to the case of nonzero quark chemical potential within the color-singlet partition function framework. While Ref. [10] established the FSS exponents at $\mu = 0$, the explicit dependence of the FSS structure on $\mu$, in particular whether the critical exponents remain universal as $\mu$ is varied, has not previously been demonstrated for the thermally driven transition. This constitutes the principal new contribution of the present work. We systematically demonstrate, for the first time, that the FSS critical exponents governing the transition width, and the temperature shift are all independent of $\mu$, thereby establishing the universality of the finite-size scaling laws with respect to the quark chemical potential.

***Cite as:*** B. Moussaoui, A. Ait El Djoudi , H. Mouloudj, M. A. Lakehal, East Eur. J. Phys. 3, 138 (2026), https://doi.org/10.26565/2312-4334-2026-3-12

This paper is organized as follows. In Sec. 2, we describe the mixed HG-QGP in finite volume system within the MIT bag model, using the PCM, and we derive the exact total color-singlet partition function (CS-PF) at nonzero quark chemical potential $\mu$ using the group theoretical projection method [14, 15]. In Sec. 3, we introduce the key thermodynamic response functions: the order parameter, the thermal susceptibility, and the specific heat density. We also present a detailed discussion of the numerical results characterizing the finite-size behavior of these quantities at $\mu = 0$ and $\mu = 100$ MeV. Section 4 is devoted to the numerical FSS analysis and the determination of the critical exponents. Section 5 summarizes the principal conclusions of our study.

## 2. THE MIXED HG-QGP EQUATION OF STATE AT NON-VANISHING CHEMICAL POTENTIAL

To describe the QCD-DPT in a finite system, we use the PCM [11]. This model assumes a mixed system consisting of HG and QGP phases coexisting within a total volume $V$. The HG phase occupies a fractional volume $V_{HG} = hV$ and the QGP phase occupies the complementary volume $V_{QGP} = (1-h)V$, where the fraction $h \in [0,1]$ parametrizes the partitioning of the volume between the two phases. The PCM is employed here within the MIT bag model, using massless up and down quarks and treating the two phases as non-interacting. There is evidence suggesting that the effects of quark masses on the transition point in the $T-\mu$ plane at lower densities are indeed small [16], and the model is well-established for qualitative finite-size analyses and captures the essential thermodynamics of the DPT. Using the PCM, the mean value of a physical quantity $<A(T,\mu,V)>$ of the mixed system can be written as [11]:

$$\langle A(T,\mu,V)\rangle = \frac{\int_0^1 A(h,T,\mu,V)\,Z(h,T,\mu,V)\,dh}{\int_0^1 Z(h,T,\mu,V)\,dh} \tag{1}$$

where $A(h,T,\mu,V)$ is the total thermodynamic quantity in the state $h$, and $Z(h,T,\mu,V)$ is the total partition function of the system with the requirement of color-singletness accounted for using the group theoretical projection method [14, 15]. Under the assumption of a non-zero quark chemical potential and considering non-interacting phases, $Z(h,T,\mu,V)$, calculated in our previous work [16] for a HG of massless pions and a QGP containing massless up and down quarks, can be written as:

$$Z(h,T,\mu,V) = \frac{4}{9\pi^2}\exp\left(\frac{\pi^2}{30}VT^3\right)\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} d\phi\, d\psi\; M(\phi,\psi)\exp\left[(1-h)R(T,\mu,V;\phi,\psi)\right], \tag{2}$$

where $M(\phi,\psi)$ represents the Haar measure for the $SU(3)$ group integration:

$$M(\phi,\psi) = \left[\sin\left(\frac{1}{2}\left(\psi+\frac{\phi}{2}\right)\right)\sin\left(\frac{\phi}{2}\right)\sin\left(\frac{1}{2}\left(\psi-\frac{\phi}{2}\right)\right)\right]^2, \tag{3}$$

and

$$R(T,\mu,V;\phi,\psi) = VT^3\left(g(\phi,\psi,\mu/T) - \frac{\pi^2}{30} - \frac{B}{T^4}\right), \tag{4}$$

where $g(\phi,\psi,\mu/T) = g_0(\phi,\psi) + g_1(\phi,\psi,\mu/T)$, with $g_0(\phi,\psi)$ given by:

$$\begin{aligned} g_0(\phi,\psi) = {} & \frac{\pi^2}{12}\left(\frac{21}{30}d_Q + \frac{16}{15}d_G\right) + \frac{\pi^2}{12}\frac{d_Q}{2}\sum_{q=r,b,g}\left\{\left[\left(\frac{\theta_q}{\pi}\right)^2 - 1\right]^2 - 1\right\} \\ & - \frac{\pi^2}{12}\frac{d_G}{2}\sum_{G=1}^{4}\left[\left(\frac{\theta_G-\pi}{\pi}\right)^2 - 1\right]^2, \end{aligned} \tag{5}$$

where $d_Q$ and $d_G$ are the degeneracy factors of quarks and gluons respectively. The angles $\theta_q$ (for $q = r, g, b$) are given by:

$$\theta_r = \frac{\phi}{2} + \frac{\psi}{3}, \qquad \theta_g = -\frac{\phi}{2} + \frac{\psi}{3}, \qquad \theta_b = \frac{-2\psi}{3}, \tag{6}$$

and $\theta_G$ (for $G = 1, \ldots, 4$) are expressed as follows:

$$\theta_1 = \theta_r - \theta_g, \qquad \theta_2 = \theta_g - \theta_b, \qquad \theta_3 = \theta_b - \theta_r, \qquad \theta_4 = 0, \tag{7}$$

and the function $g_1(\phi,\psi,\mu/T)$ is given by:

$$g_1(\phi,\psi,\mu/T) = \left(1 - \frac{\phi^2}{2\pi^2} - \frac{2\psi^2}{3\pi^2}\right)\left(\frac{\mu}{T}\right)^2 + \frac{1}{2\pi^2}\left(\frac{\mu}{T}\right)^4. \tag{8}$$

## 3. FINITE-SIZE EFFECTS ON THE THERMALLY DRIVEN DPT: THERMAL SUSCEPTIBILITY AND SPECIFIC HEAT DENSITY

The main quantities of interest in this study are the thermal susceptibility and the specific heat density. First, we derive the order parameter $<h(T,\mu,V)>$, represented by the mean value of the HG volume fraction, which may be written as:

$$\langle h(T,\mu,V)\rangle = \frac{\int_0^1 h(T,\mu,V)\, Z(h,T,\mu,V)\, dh}{\int_0^1 Z(h,T,\mu,V)\, dh} \tag{9}$$

The thermal susceptibility is defined as the first derivative of the order parameter $<h(T,\mu,V)>$, with respect to temperature:

$$\chi_T(T,\mu,V) = \left.\frac{\partial\langle h(T,\mu,V)\rangle}{\partial T}\right|_{\mu,V} \tag{10}$$

The specific heat density $c_T(T,\mu,V)$ is calculated from the derivative of the energy density $<\epsilon(T,\mu,V)>$ with respect to temperature as:

$$c_T(T,\mu,V) = \left.\frac{\partial\langle \varepsilon(T,\mu,V)\rangle}{\partial T}\right|_{\mu,V} \tag{11}$$

$$\langle\varepsilon(T,\mu,V)\rangle = \frac{T^2}{V}\left\langle\left(\frac{\partial \ln Z(h,T,\mu,V)}{\partial T}\right)_{\mu,V}\right\rangle + \frac{\mu T}{V}\left\langle\left(\frac{\partial \ln Z(h,T,\mu,V)}{\partial \mu}\right)_{T,V}\right\rangle \tag{12}$$

Figures 1 and 2 show the behavior of the order parameter $<h>$ (left panel) and the thermal susceptibility $\chi_T$ (right panel) as functions of temperature, calculated at $\mu = 0$ and $\mu = 100MeV$, respectively for several system volumes with $B^{1/4} = 200MeV$. In both cases, the DPT exhibits characteristic finite-size rounding: at small volumes, thermal fluctuations strongly smear the transition, resulting in smooth, rounded changes in ⟨h⟩, while for large volumes the order parameter develops a nearly sharp step-like discontinuity at the transition temperature characteristic of a first-order transition, known to occur at the thermodynamic limit [17].

The magnitude of the thermal susceptibility peaks, $|\chi_T^{max}(V)|$, increases systematically with system size, and the position of each peak defines the volume-dependent effective transition temperature $T_c^{\text{Fixed}\,\mu}(V)$. As the system size decreases, the thermal width $\delta T_c(V)$ of the transition region broadens and shifts to higher temperatures, reflecting the reduction of QGP degrees of freedom imposed by color-singletness [11, 14, 15]. The shifting of $T_c^{\text{Fixed}\,\mu}(V)$ to higher values at smaller volumes is a direct consequence of the fact that in finite systems the QGP color-singlet state requires additional thermal energy to be formed compared to the infinite-volume case.

In the thermodynamic limit, the true transition temperatures are found to be $T_c^{(\mu=0)}(\infty) = 143.928MeV$ and $T_c^{(\mu=100MeV)}(\infty) = 139.229MeV$, consistent with theoretical estimates obtained from recent lattice QCD simulations [18, 19]. A nonzero chemical potential shifts the entire transition region to lower temperatures without altering the qualitative finite-size structure.

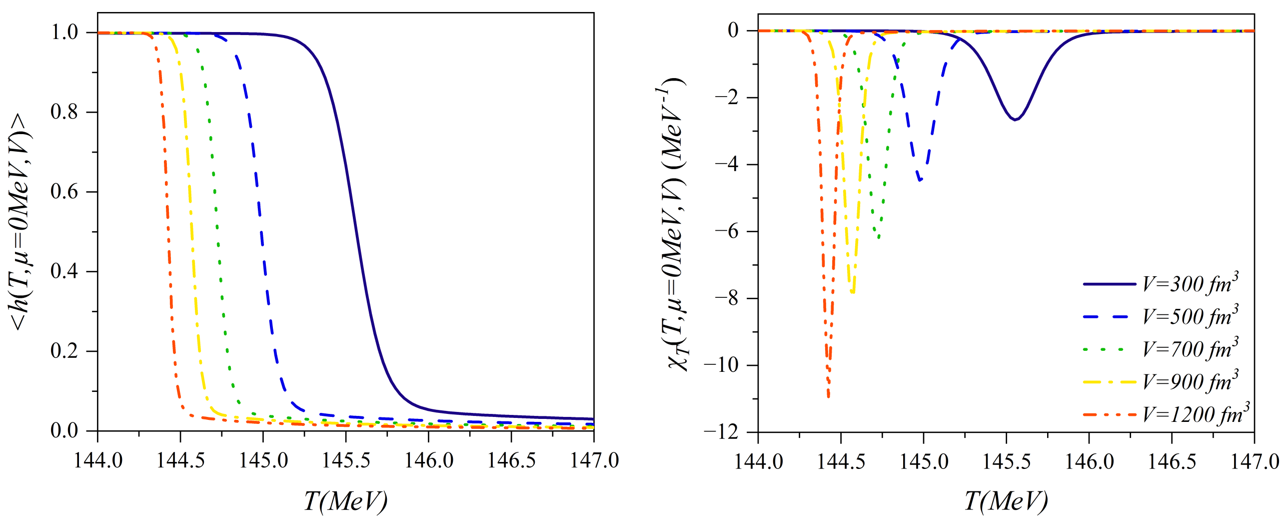


**Figure 1.** Variations of the order parameter $\langle h\rangle$ (left panel) and the thermal susceptibility $\chi_T$ (right panel) as a function of the temperature at $\mu = 0MeV$, for various volume selections with $B^{1/4} = 200MeV$.

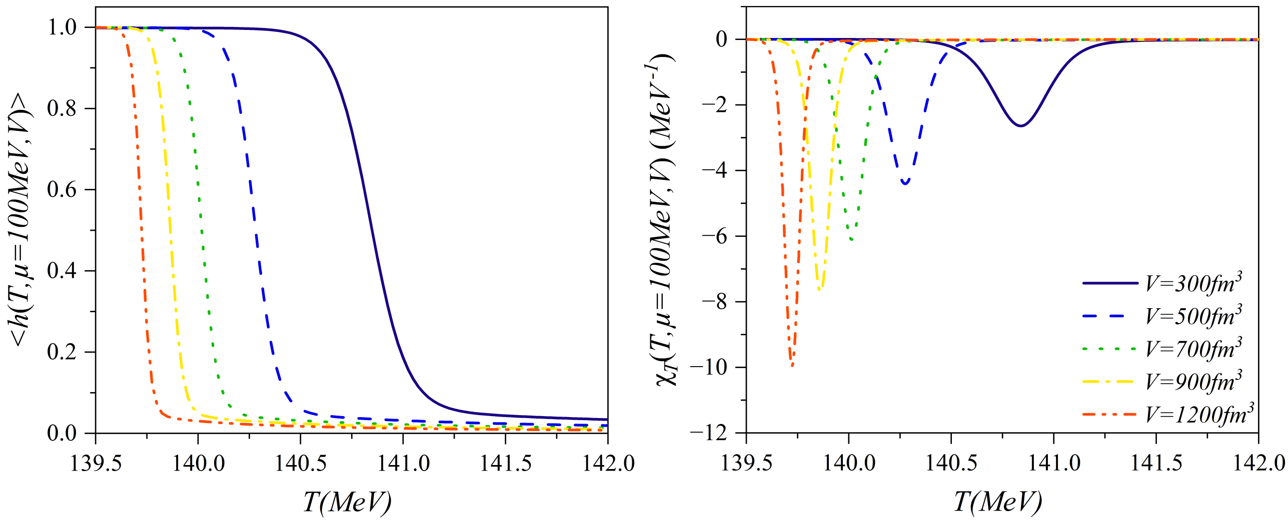


**Figure 2.** Variations of the order parameter $\langle h \rangle$ (left panel) and the thermal susceptibility $\chi_T$ (right panel) as a function of the temperature at $\mu = 100MeV$, for various volume selections with $B^{1/4} = 200MeV$.

Figure 3 shows the specific heat density $c_T$ as a function of temperature at $\mu = 0$ (left panel) and $\mu = 100$ MeV (right panel), for several volume selections. The specific heat density exhibits well-defined peaks whose maxima $c_T^{\max}$ increase sharply with increasing volume, while their widths $\delta T_c(V)$ decrease, in agreement with the expectation that the transition sharpens in larger systems [20]. The localization of the maximum $c_T^{\max}$ at temperature $T_c(V)$ provides an independent determination of the effective transition temperature, consistent with the location of the thermal susceptibility peak. A similar volume dependence of the specific heat density has been reported in Refs. [8,21]. Importantly, the quark chemical potential $\mu$ does not alter the qualitative behavior of either $\chi_T$ or $c_T$ as functions of volume: it merely shifts the entire temperature transition $\delta T_c(V)$ to a lower temperature, while leaving the finite-size scaling structure unchanged. This observation motivates the systematic FSS analysis presented in the next section.

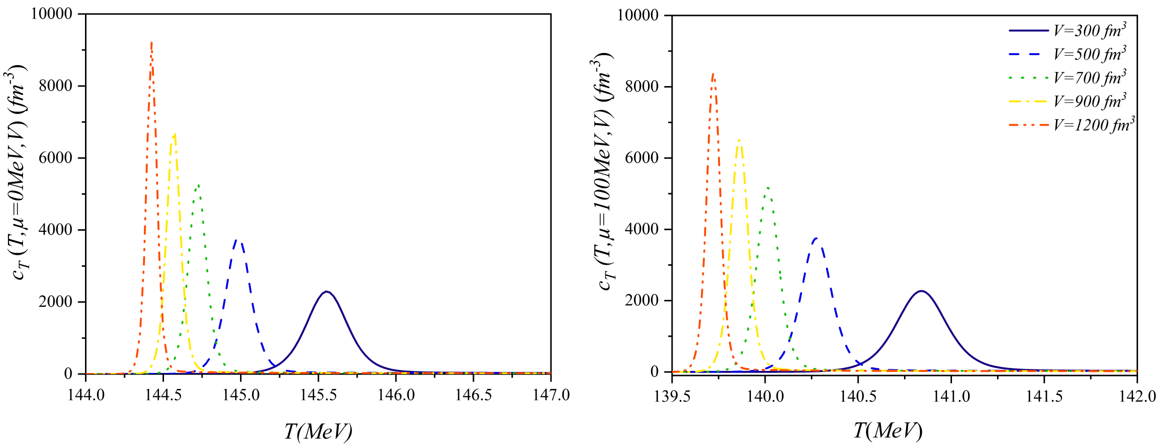


**Figure 3.** Plots of the specific heat density $c_T$ as a function of temperature, (left panel) at $\mu = 0$, and (right panel) $\mu = 100MeV$, for several volume selections with $B^{1/4} = 200MeV$.

## 4. FINITE-SIZE SCALING ANALYSIS: CRITICAL EXPONENTS FOR THE THERMAL DPT

In the thermodynamic limit, several thermodynamic quantities associated with the thermally driven DPT diverge or become discontinuous at the critical temperature. In a finite system of volume $V$, these singularities are rounded over a finite temperature transition region $\delta T_c(V)$, around an effective transition temperature $T_c(V)$, which is shifted relative to the true transition temperature $T_c(\infty)$. The resulting shift is: $\tau_T(V) = T_c(V) - T_c(\infty)$. The FSS theory for a first-order phase transition [22–25] predicts characteristic power-law dependences on the system volume. The critical exponents characterizing their power-law evolution with the volume are [10]:

$$\begin{cases} c_T^{\max}(\text{Fixed } \mu, V) \sim V^{\alpha_T}, \\ \delta T_c(\text{Fixed } \mu, V) = T_2(V) - T_1(V) \sim V^{-\theta_T}, \\ \tau_T(\text{Fixed } \mu, V) = T_c(V) - T_c(\infty) \sim V^{-\lambda_T}. \end{cases} \quad (13)$$

For a first-order phase transition at fixed quark chemical potential, the scaling critical exponents are expected to be equal to unity: $\alpha_T = \theta_T = \lambda_T = 1$.

Figure 4 shows the peak maxima $c_T^{max}$ of the specific heat density as functions of the volume, at $\mu = 0$ (left panel) and $\mu = 100MeV$ (right panel), with $B^{1/4} = 200MeV$. A power-law fit of the form $c_T^{\max}(\text{Fixed } \mu, V) \sim V^{\alpha_T}$ yields: $\alpha_T = 1.012 \pm 0.004$ ($\mu = 0$), $\alpha_T = 1.017 \pm 0.006$ ($\mu = 100MeV$).

The scaling of the specific heat density peak maximum with volume (exponent $\alpha_T \approx 1$) is the hallmark signature of a first-order phase transition [10]. The slight deviation from unity is within the systematic numerical uncertainty of the fitting procedure. It is noteworthy that the exponent values at $\mu = 0$ and $\mu = 100MeV$ are statistically indistinguishable within the quoted uncertainties, providing the first evidence that the FSS of the specific heat density maximum is insensitive to the quark chemical potential.

Figure 5 displays the width of the transition temperature region $\delta T_c(\text{Fixed } \mu, V)$, as a function of the inverse volume $1/V$, at $\mu = 0$ (left panel) and $\mu = 100MeV$ (right panel). This allows the determination of the value of the critical exponent $\theta_T$ defined in the scaling: $\delta T_c(\text{Fixed } \mu, V) \sim V^{-\theta_T}$, and we find the numerical result: $\theta_T = 1.010 \pm 0.001$ ($\mu = 0$), $\theta_T = 1.007 \pm 0.001$ ($\mu = 100MeV$). These values confirm, with high numerical precision, the expected linear scaling of the transition width with $1/V$ for a first-order phase transition. Again, the critical exponents at $\mu = 0$ and $\mu = 100MeV$ are statistically consistent, reinforcing the independence of the FSS structure from the quark chemical potential.

Figure 6 shows the shift of the effective transition temperature $\tau_T(\text{Fixed } \mu, V)$, extracted from the position of the thermal susceptibility peak, as a function of $1/V$. A power-law fit to the form $\tau_T(\text{Fixed } \mu, V) = V^{-\lambda_T}$, gives the numerical result of the shift scaling exponent: $\lambda_T = 0.859 \pm 0.003$ ($\mu = 0$), $\lambda_T = 0.860 \pm 0.003$ ($\mu = 100MeV$). According to [10], such a value $\lambda_T \neq 1$ suggests the contribution of additional terms with higher powers of $V^{-1}$, such that the parametrization should be with the laws: $A_1V^{-\lambda} + A_2V^{-2}$ or $A_1V^{-\lambda} + A_2V^{-2} + A_3V^{-3}$, where $A_1, A_2, A_3$, expected to be independent of the quark chemical potential.

Taken together, the results of the FSS analysis show that the scaling exponents $\alpha_T$ and $\theta_T$ are equal to unity within numerical precision, while $\lambda_T$ deviates slightly from unity due to subleading corrections. All three exponents are independent of the quark chemical potential $\mu$, and are collectively consistent with a first-order phase transition. We conclude that the thermally driven DPT, in the presence of a nonzero quark chemical potential, obeys the same FSS scaling laws as those obtained at $\mu = 0$ [10], and that the critical exponents are universal with respect to $\mu$. This conclusion parallels the analogous finding for the density driven DPT reported in Ref. [16], where the scaling exponents were found to be independent of the fixed temperature at which the density-driven transition is studied.

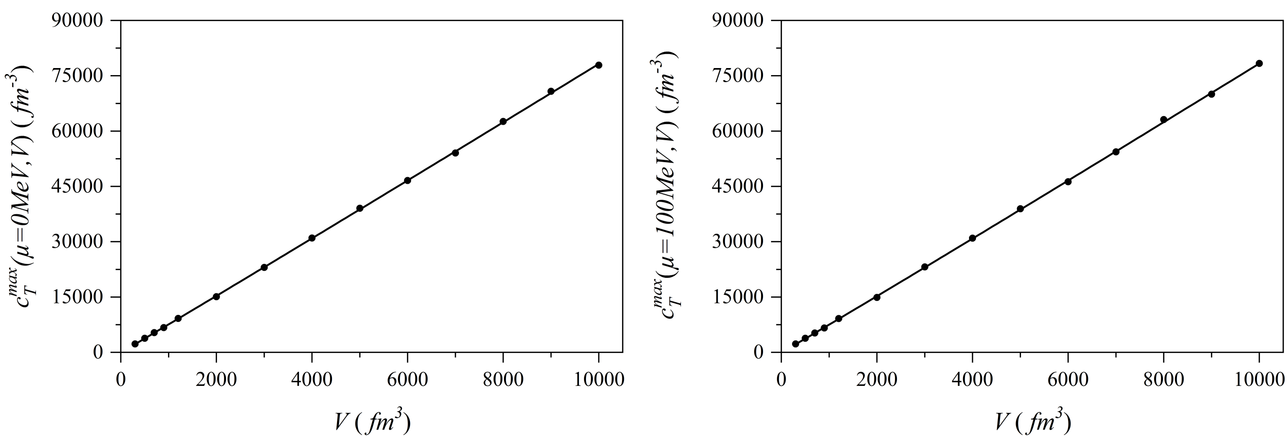


**Figure 4.** Plot of the peak maxima of the specific heat density $c_T^{\max}$ with volume, as a function of volume, at $\mu = 0$ (left panel) and $\mu = 100MeV$ (right panel), with $B^{1/4} = 200MeV$.

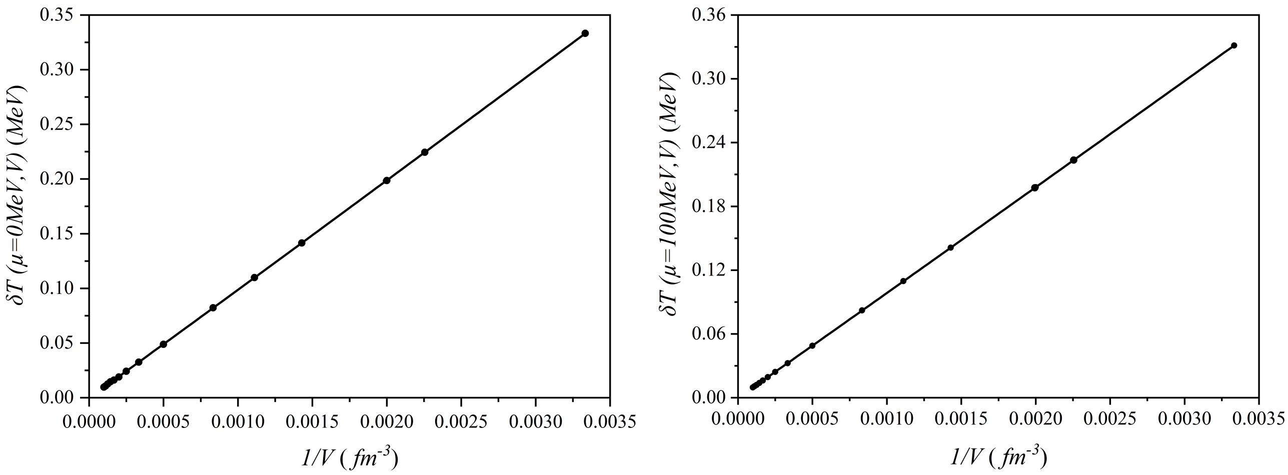


**Figure 5.** Plot of the width of the temperature transition region $\delta T(V)$, as a function of inverse volume, at $\mu = 0$ (left panel) and $\mu = 100MeV$ (right panel), with $B^{1/4} = 200MeV$.

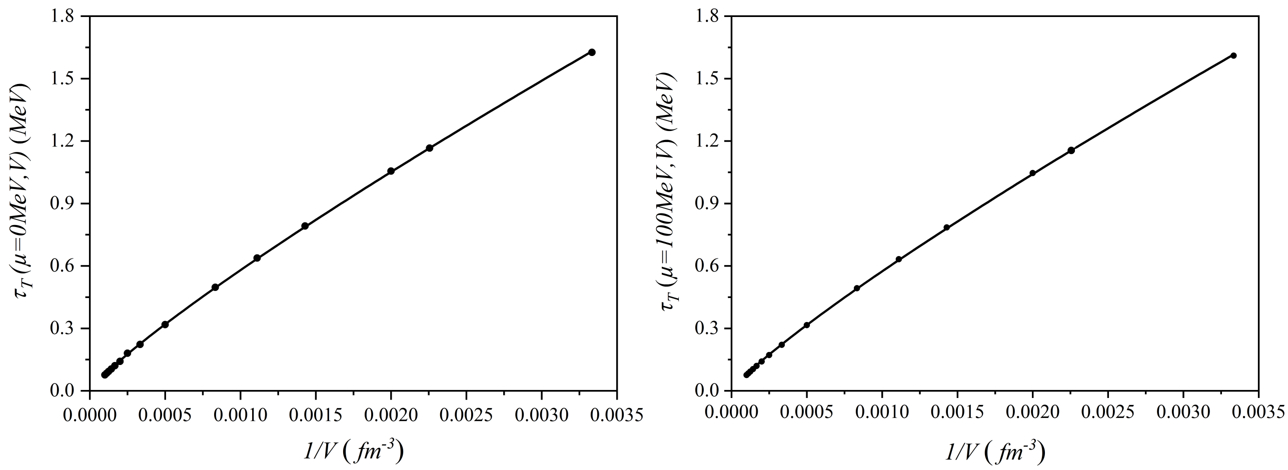


**Figure 6.** Plot of the shift of the effective transition temperature $\tau_T(V)$ from the thermal susceptibility, as a function of inverse volume, at $\mu = 0$ (left panel) and $\mu = 100MeV$ (right panel), with $B^{1/4} = 200MeV$.

## 5. CONCLUSIONS

In the present work, we have investigated the finite-size effects on the thermally driven deconfinement phase transition (DPT) from a hadronic gas (HG) consisting of massless pions to a color-singlet quark-gluon plasma (QGP) containing gluons and massless up and down quarks. Using the exact color-singlet partition function (CS-PF) within the phase coexistence model (PCM), we have computed the key thermodynamic response functions such as the order parameter $\langle \mathrm{h} \rangle$, the susceptibility $\chi_T$, and the specific heat density $c_T$ as functions of temperature, at fixed quark chemical potential $\mu$, and varying volume $V$.

In small volumes, a smooth crossover is observed due to strong thermodynamic fluctuations, while in the thermodynamic limit a sharp discontinuity in the order parameter $\langle h \rangle$ appears at the true transition temperature $T_c(\text{Fixed}\,\mu, \infty)$, signaling a first-order DPT. A systematic variation of the effective transition temperature $T_c(V)$ with system size is observed, which diminishes for larger volumes and converges to $T_c(\text{Fixed}\,\mu, \infty)$ in the thermodynamic limit. The quark chemical potential does not affect the width $\delta T_c(\text{Fixed}\,\mu, V)$ of the transition region; it only shifts the true transition temperature $T_c(\text{Fixed}\,\mu, \infty)$ to lower values. A similar volume dependence has been reported in Refs. [8, 21].

A numerical FSS analysis of the behavior of the maxima of the rounded peaks of the specific heat density $c_T^{\max}$, the width of the temperature transition region $\delta T_c(V)$, and the shift of the effective transition temperature $\tau_T(V)$, yields the scaling critical exponents $\alpha_T$, $\theta_T$, $\lambda_T$. All three exponents converge to unity when subleading corrections are properly accounted for, confirming that the thermally driven DPT is of first order.

The principal new finding of this work is that all three FSS critical exponents are independent of the quark chemical potential $\mu$. Based on our results and those of Refs. [10, 16], we conclude that the thermally driven DPT obeys a universal finite-size scaling law governed by the system volume, with critical exponents characteristic of a first-order transition and independent of $\mu$, identical to those found for the density-driven DPT [16]. This universality of the FSS structure with respect to the thermodynamic control parameters ($\mu$ and $T$) represents a robust property of the color singlet QCD deconfinement transition in the PCM framework.

**Acknowledgments**

This research work is supported by the Algerian Ministry of Higher Education and Scientific Research.

**Funding**

No funding was received for conducting this study.

**Conflict of interest**

The authors declare that there are no conflicts of interest.

**ORCID**

**B. Moussaoui**, https://orcid.org/0000-0001-8580-9412; **A. Ait El Djoudi**, https://orcid.org/0009-0005-7773-0401; **H. Mouloudj**, https://orcid.org/0009-0006-0376-9642; **M. A. Lakehal**, https://orcid.org/0009-0007-8930-0454

**REFERENCES**

[1] K. Adcox, et al., "Formation of dense partonic matter in relativistic nucleus-nucleus collisions at RHIC: Experimental evaluation by the PHENIX Collaboration," Nucl. Phys. A, **757**, 184 (2005). https://doi.org/10.1016/j.nuclphysa.2005.03.086

[2] X. An, et al., "The BEST framework for the search for the QCD critical point and the chiral magnetic effect," Nucl. Phys. A, **1017**, 122343 (2022). https://doi.org/10.1016/j.nuclphysa.2021.122343

[3] F. Karsch, et al., "Quark mass and flavour dependence of the QCD phase transition," Nucl. Phys. B, **605**, 579 (2001). https://doi.org/10.1016/S0550-3213(01)00200-0

[4] S. Borsanyi, R. Kara, Z. Fodor, D. A. Godzieba, P. Parotto, and D. Sexty, "Precision study of the continuum SU(3) Yang-Mills theory: How to use parallel tempering to improve on supercritical slowing down for first-order phase transitions,)" Phys. Rev. D, **105**(7), 074513 (2022). https://doi.org/10.1103/PhysRevD.105.074513

[5] S. Borsanyi, Z.Fodor, J.N. Guenther, P. Parotto, A. Pasztor, L. Pirelli, K.K. Szabó, and C.H. Wong, "QCD deconfinement transition line up to $\mu_B$ = 400 MeV from finite volume lattice simulations," Phys. Rev. D, **110**(11), 114507 (2024). https://doi.org/10.1103/PhysRevD.110.114507

[6] L.F. Palhares, E.S. Fraga, and T. Kodama, "Chiral transition in a finite system and possible use of finite size scaling in relativistic heavy-ion collisions," J. Phys. G, **38**(8), 085101 (2011). https://doi.org/10.1088/0954-3899/38/8/085101

[7] S.A. Basset, et al., "Reaction dynamics in Pb + Pb at the CERN/SPS: From partonic degrees of freedom to freeze-out," Prog. Part. Nucl. Phys. **42**, 313 (1999). https://doi.org/10.1016/S0146-6410(99)00086-1

[8] S. Pal, A. Motornenko, V. Vovchenko, A. Bhattacharyya, J. Steinheimer, and H. Stoecker, "Effect of finite volume on thermodynamics of quark-hadron matter," Phys. Rev. D, **109**(1), 014009 (2024). https://doi.org/10.1103/PhysRevD.109.014009

[9] M.E. Fisher, and M.N. Barber, "Scaling theory for finite-size effects in the critical region," Phys. Rev. Lett. **28**(23), 1516 (1972). https://doi.org/10.1103/PhysRevLett.28.1516

[10] M. Ladrem, and A. Ait-El-Djoudi, "Finite-size effects and scaling for the thermal QCD deconfinement phase transition within the exact color-singlet partition function," Eur. Phys. J. C, **44**(2), 257-265 (2005). https://doi.org/10.1140/epjc/s2005-02357-y

[11] C. Spieles, H. Stöcker, and C. Greiner, "Phase transition of a finite quark-gluon plasma," Phys. Rev. C, **57**(2), 908 (1998). https://doi.org/10.1103/PhysRevC.57.908

[12] S. Borsanyi, Z. Fodor, J.N. Guenther, R. Kara, P. Parotto, A. Pasztor, L. Pirelli, and C.H. Wong, "Chiral versus deconfinement properties of the QCD crossover: Differences in the volume and chemical potential dependence from the lattice," Phys. Rev. D, **111**(1), 014506 (2025). https://doi.org/10.1103/PhysRevD.111.014506

[13] A. Sorensen, and P. Sorensen, "Locating the critical point for the hadron to quark-gluon plasma phase transition from finite-size scaling of proton cumulants in heavyion collisions," arXiv:2405.10278 [nucl-th] (2024). https://doi.org/10.48550/arXiv.2405.10278

[14] H.-T. Elze, U. Heinz, and T. Kodama, "On the color-singlet quark-glue plasma," Phys. Lett. B, **124**(6), 515 (1983). https://doi.org/10.1016/0370-2693(83)91564-2

[15] H.-T. Elze, U. Heinz, and T. Kodama, "Finite size effects for quark-gluon plasma droplets," Phys. Lett. B, **179**(4), 385 (1986). https://doi.org/10.1016/0370-2693(86)90498-3

[16] B. Moussaoui, A. Ait El Djoudi, and A. Tawfik, "Finite-Size Effects Near QCD Critical Point: Quark Number Susceptibility," Int. J. Theor. Phys. **61**(6), N181 (2022). https://doi.org/10.1007/s10773-022-05159-x

[17] T.D. Lee and C.N. Yang, "Statistical theory of equations of state and phase transitions. II. Lattice gas and Ising model," Phys. Rev. **87**, 410 (1952). https://doi.org/10.1103/PhysRev.87.410

[18] E.S. Fraga, “QCD under extreme conditions: An informal discussion,” (2013). https://doi.org/10.48550/arXiv.1310.6656

[19] O. Philipsen, “The QCD equation of state from the lattice,” Prog. Part. Nucl. Phys. **70**, 55 (2013). https://doi.org/10.1016/j.ppnp.2012.09.003

[20] S. Basu, S. Chatterjee, R. Chatterjee, T.K. Nayak, and B.K. Nandi, “Specific heat of matter formed in relativistic nuclear collisions,” Phys. Rev. C, **94** (4), 044901 (2016). https://doi.org/10.1103/PhysRevC.94.044901

[21] N. Magdy, M. Csanad, and R.A. Lacey, “Influence of finite volume and magnetic field effects on the QCD phase diagram,” J. Phys. G, **44** (2), 025101 (2017). https://doi.org/10.1088/1361-6471/44/2/025101

[22] M.E. Fisher and A.N. Berker, “Scaling for first-order phase transitions in thermodynamic and finite systems,” Phys. Rev. B, **26** (5), 2507 (1982). https://doi.org/10.1103/PhysRevB.26.2507

[23] V. Privman and M.E. Fisher, “Finite-size effects at first-order transitions,” J. Stat. Phys. **33** (2), 385 (1983). https://doi.org/10.1007/BF01009803

[24] K. Binder, and D.W. Heermann, “Monte Carlo Simulations in Statistical Physics,” 2nd ed. (Springer-Verlag, New York, 2002).

[25] H. Meyer-Ortmanns, “Phase transitions in quantum chromodynamics,” Rev. Mod. Phys. **68**(2), 473 (1996). https://doi.org/10.1103/RevModPhys.68.473

## СКІНЧЕННО-РОЗМІРНЕ МАСШТАБУВАННЯ ТЕПЛОВОЇ СПРИЙНЯТЛИВОСТІ ТА ПИТОМОЇ ТЕПЛОЄМНОСТІ ПОБЛИЗУ КХД ФАЗОВОГО ПЕРЕХОДУ З ДЕКОНФАЙНМЕНТОМ

**Б. Мусауї[1,2], А. Айт Ель Джуді[1], Н. Мулудж[1,3], М. А. Лейкхал[1,4]**

[1]*Лабораторія фізики елементарних частинок та статистичної фізики, Вища нормальна школа-Куба, В'є-Куба, B.P.92, 16050 Алжир, Алжир*

[2]*Лабораторія теоретичної фізики, Фізичний факультет, USTHB Bab-Ezzouar, Ель-Алія, BP 32, 16111 Алжир, Алжир*

[3]*Університет Хассіби Бенбуалі, Шлеф, Алжир*

[4]*Університет Мхамеда Бугари, Бумердес, Алжир*

Властивості термічно керованого фазового переходу деконфайнменту (DPT) від адронного газу (HG) до кольорової синглетної кварк-глюонної плазми (QGP), що містить глюони та безмасові верхні та нижні кварки, при ненульовому хімічному потенціалі кварка $\mu$, досліджуються з урахуванням об'ємного співіснування адронного газу та QGP у системі скінченних розмірів. Ефекти скінченних розмірів аналізуються для термічно керованого DPT при різних значеннях $\mu$. Критичні показники визначаються за допомогою числового аналізу масштабування скінченних розмірів (FSS) шляхом апроксимації результатів як функції розміру системи. Ефективна температура переходу ($T_c$) демонструє зсув до вищих значень зі зменшенням розміру системи, що вказує на критичну поведінку в області термічно керованого DPT від високорозмірної форми до QGP. Важливо, що всі три критичні показники масштабування виявляються незалежними від хімічного потенціалу кварків $\mu$, що встановлює універсальність структури масштабування скінченних розмірів відносно $\mu$.

**Ключові слова:** *термічно керований фазовий перехід деконфайнменту (DPT); кольоровий синглет; хімічний потенціал кварків; аналіз масштабування скінченних розмірів (FSS); температура переходу*