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# EXPLORING BIANCHI TYPE-VI$_o$ VISCOUS HOLOGRAPHIC RICCI DARK ENERGY COSMOLOGICAL MODEL IN BRANS-DICKE THEORY OF GRAVITATION

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- A viscous holographic Ricci dark energy (VHRDE) model is investigated within the framework of Brans–Dicke (BD) gravity.
- Bulk viscosity generates effective negative pressure, while the BD scalar field dynamically regulates the gravitational coupling.
- A natural transition from an early matter-dominated decelerated phase to a late-time dark-energy-dominated accelerated expansion is obtained.
- The model extends beyond ΛCDM, exhibiting a dynamically evolving equation of state that can approach or cross the phantom divide $\omega < -1$
- Anisotropy and shear decay with cosmic time, leading to late-time isotropization consistent with observational (CMB) constraints.

We investigate a viscous holographic Ricci dark energy (VHRDE) model in Bianchi type (BT) - VI$_o$ spacetime within Brans–Dicke (BD) gravity, incorporating a pressure-less matter component. Assuming a relationship between metric potentials and a parameterised bulk viscosity coefficient, exact solutions of the BD field equations are found. The combined effects of anisotropy, Ricci holographic cutoff, and bulk viscosity yield a dynamical framework that can describe multiple phases of cosmic evolution. We examine the progression of significant measurements, including the scalar field, energy densities, bulk viscosity, viscous pressure, equation of state (EoS), expansion variables, and the deceleration parameter, revealing a transition from a decelerated, matter-dominated epoch to an accelerated, dark-energy-dominated phase. Bulk viscosity generates effective negative pressure driving acceleration, while the BD scalar field modulates gravitational coupling. Comparison with previous studies confirms that this model provides a viable mechanism for cosmic acceleration without requiring a cosmological constant.

**Keywords:** *Bianchi type- VI$_o$ metric; Holographic Ricci dark energy (HRDE); Bulk Viscosity; Brans-Dicke theory*

## 1. INTRODUCTION

The accelerated expansion of the Universe, revealed through observations of SN Ia, CMB anisotropies, and large-scale structure surveys, stands as one of the most profound and unresolved challenges in modern cosmology. This phenomenon is generally attributed to an unknown component characterised by a negative pressure, known as dark energy (DE), which currently predominates the cosmic energy amount. Our current understanding suggests that dark energy constitutes approximately 73% of the universe, with dark matter comprising 23% and baryonic matter making up 4%, while radiation contributes negligibly at the present epoch [1–5].Two essential methodologies have been established to better understand this late-time acceleration. The first introduces new dynamical energy components, such as quintessence, phantom fields, k-essence, tachyon models, and Chaplygin gas. The second modification extends Einstein's general relativity, leading to alternative gravitational frameworks, such as scalar–tensor (ST) theories and $f(R)$ or $f(R,T)$ gravities [6–12]. Among the many DE candidates, the holographic dark energy (HDE) model [13] is particularly compelling, as it emerges naturally from the holographic principle, which relates the degrees of freedom of a physical system to the area of its boundary surface [14, 15]. The holographic framework was initially developed by Cohen *et al.*, who related the vacuum energy density (ED) to the Hubble scale ($L \simeq H^{-1}$), while Li proposed the HDE density as $\rho_d = 3C^2 M_{\rm pl}^2 L^{-2}$. Gao *et al.*, modified this formulation by adopting the Ricci scalar cutoff ($L \simeq R^{-\frac{1}{2}}$), leading to the RDE model, while Granda and Oliveros [14–19] introduced a generalized form involving both $H^2$ and $\dot{H}$. Chen and Jing [20] further extended it to the modified HRDE model, expressed as

$$\rho_d = \frac{3}{8\pi G}\left(\varepsilon_0 H^2 + \eta\,\dot{H} + \zeta\,\frac{\ddot{H}}{H}\right) \tag{1}$$

These models, and their extensions, have been widely studied and shown to describe key features of cosmic acceleration [21–24]. Parallel to these developments, bulk viscosity has been proposed as a natural mechanism for generating effective negative pressure. In the early Universe, neutrino interactions led matter to behave like a viscous fluid, decreasing viscosity as expansion progressed. Bulk viscosity arises when a system departs from local thermodynamic equilibrium and has been shown to drive both early inflation and late-time acceleration without a cosmological constant [25–28]. Recent works have applied viscosity to holographic models in anisotropic spacetimes, highlighting its role as a viable candidate for DE [29].

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Scalar–tensor theories, particularly BD gravity, provide a natural framework for extending general relativity. In BD theory, the gravitational constant is replaced by a dynamical scalar field $\phi$, which couples to geometry through the dimensionless parameter $\omega$. The BD field equations can be written as

$$G_{\mu\vartheta} = -\frac{8\pi}{\phi} T_{\mu\vartheta} - \frac{\omega}{\phi^2}\left(\phi_{,\mu}\phi_{,\vartheta} - \frac{1}{2} g_{\mu\vartheta}\phi_{,k}\phi^{,k}\right) - \frac{1}{\phi}\left(\phi_{;\mu\vartheta} - g_{\mu\vartheta}\Box\phi\right) \tag{2}$$

and

$$\Box\phi = \frac{8\pi}{3+2\omega} T \tag{3}$$

together with the conservation law

$$T^{\mu\vartheta}_{;\vartheta} = 0 \tag{4}$$

This framework has been successfully applied to both isotropic and anisotropic cosmologies, offering flexibility in explaining dynamical features of cosmic evolution [30–35]. Motivated by these considerations, the present work investigates a VHRDE model in a BT–VI$_o$ anisotropic spacetime within BD gravity, incorporating a pressureless matter component. The combined effects of anisotropy, Ricci holographic cutoff, and bulk viscosity generate a rich dynamical system capable of reproducing different cosmic phases. Exact solutions of the BD field equations are obtained by assuming a relation among metric potentials and a parameterized bulk viscosity coefficient. The time evolution of key cosmological parameters is analyzed to examine the roles of viscous effects and anisotropies in shaping cosmic dynamics. The structure of the paper is as follows: Section 2 formulates the governing field equations, Section 3 develops the exact analytical solutions, Section 4 provides a detailed analysis of the physical characteristics of the model, and Section 5 synthesizes the principal findings and implications.

## 2. METRIC AND FIELD EQUATIONS

We consider a spatially homogeneous BT - VI$_o$ metric form.

$$ds^2 = dt^2 - A_1^2\, dx^2 - A_2^2 e^{-2x}\, dy^2 - A_3^2 e^{2x}\, dz^2 \tag{5}$$

Where $A_1, A_2$, and $A_3$ are functions of 't' only. The HRDE energy-momentum tensor with bulk viscosity is

$$T_{\mu\vartheta} = T^m_{\mu\vartheta} + T^d_{\mu\vartheta}$$

Where $T^m_{\mu\vartheta}$ and $T^d_{\mu\vartheta}$ are the energy-momentum tensor for matter and the HRDE, respectively. They are

$$T^d_{\mu\vartheta} = (\rho_d + \bar{p}_d)\, u_\mu u_\vartheta - \bar{p}_d\, g_{\mu\vartheta} \tag{6}$$

and

$$T^m_{\mu\vartheta} = \rho_m\, u_\mu u_\vartheta \tag{7}$$

Where $\rho_m$ and $\rho_d$ are the matter and the holographic Ricci dark energy densities, respectively, and $\bar{p}$ means the viscous HDE pressure.
In the co-moving frame of reference, we will get

$$T_1^1 = T_2^2 = T_3^3 = -\bar{p}_d, \qquad T_4^4 = \rho_m + \rho_d \tag{8}$$

Where $\rho_m, \rho_d$ and $\bar{p}_d$ are all functions of time 't' only.
The pressure of VHRDE is governed by the corresponding relation. $\bar{p}_d = p_d - 3\epsilon H$. In the late-time evolution of the Universe, the inclusion of bulk viscosity in the DE sector facilitates a rapid phantom-like expansion. Throughout this analysis, dark matter is treated as a pressureless component, while the effective pressure of VHRDE is consistently modelled as the combined contribution of the HRDE pressure and the bulk viscous term.
The BD field equations (2), using metric (5) and supported by equations (6)-(8) are given by:

$$\frac{\ddot{A}_2}{A_2} + \frac{\ddot{A}_3}{A_3} + \frac{\dot{A}_2\dot{A}_3}{A_2A_3} + \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{A}_2\dot{\phi}}{A_2\phi} + \frac{\dot{A}_3\dot{\phi}}{A_3\phi} + \frac{1}{A_1^2} = -8\pi\phi^{-1}\bar{p}_d \tag{9}$$

$$\frac{\ddot{A}_1}{A_1} + \frac{\ddot{A}_3}{A_3} + \frac{\dot{A}_1\dot{A}_3}{A_1A_3} + \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{A}_1\dot{\phi}}{A_1\phi} + \frac{\dot{A}_3\dot{\phi}}{A_3\phi} - \frac{1}{A_1^2} = -8\pi\phi^{-1}\bar{p}_d \tag{10}$$

$$\frac{\ddot{A}_1}{A_1} + \frac{\ddot{A}_2}{A_2} + \frac{\dot{A}_1\dot{A}_2}{A_1A_2} + \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{A}_1\dot{\phi}}{A_1\phi} + \frac{\dot{A}_2\dot{\phi}}{A_2\phi} - \frac{1}{A_1^2} = -8\pi\phi^{-1}\bar{p}_d \tag{11}$$

$$\frac{\dot{A}_1\dot{A}_2}{A_1A_2}+\frac{\dot{A}_2\dot{A}_3}{A_2A_3}+\frac{\dot{A}_1\dot{A}_3}{A_1A_3}-\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2}+\frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}_1}{A_1}+\frac{\dot{A}_2}{A_2}+\frac{\dot{A}_3}{A_3}\right)-\frac{1}{A_1^2}=-8\pi\phi^{-1}(\rho_m+\rho_d) \tag{12}$$

$$\frac{1}{A_1^2}\left(\frac{\dot{A}_2}{A_2}-\frac{\dot{A}_3}{A_3}\right)=0 \tag{13}$$

$$\ddot{\phi}+\dot{\phi}\left(\frac{\dot{A}_1}{A_1}+\frac{\dot{A}_2}{A_2}+\frac{\dot{A}_3}{A_3}\right)=\frac{8\pi}{3+2\omega}\left(\rho_m+\rho_d-3\bar{p}_d\right) \tag{14}$$

$$\frac{\ddot{A}_1}{A_1}-\frac{\ddot{A}_2}{A_2}+\frac{\dot{A}_2}{A_2}\left(\frac{\dot{A}_1}{A_1}-\frac{\dot{A}_2}{A_2}\right)+\frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}_1}{A_1}-\frac{\dot{A}_2}{A_2}\right)-\frac{2}{A_1^2}=0 \tag{15}$$

## 3. VHRDE COSMOLOGICAL MODELS

From equation (13), we get $A_2 = c_1A_3$,by taking $c_1 = 1$ without loss of generality, we have

$$A_2 = A_3 \tag{16}$$

The field equations (9)–(15) constitute a set of independent, nonlinear relations involving seven unknown functions of cosmic time t: $A_1, A_2, A_3, \rho_m, \rho_d, \bar{p}_d,$ & $\phi$.The following physically motivated conditions are imposed to ensure a well-determined solution.

1. The relation between the scalar field and the average scale factor a(t) is established as given by Johri and Kalyani [36]

$$\phi = a^m = (A_1A_2A_3)^{\left(\frac{m}{3}\right)}, \quad \text{where } m \text{ is a constant} \tag{17}$$

2. Since $\sigma \propto \theta$ leads to the metric potentials relation (Collins et al. [37])

$$A_1 = A_2^n, \quad \text{where } n \text{ is an arbitrary constant.} \tag{18}$$

We adopt the parameterized form of bulk viscosity as proposed by Ren and Meng [38]

$$\epsilon = \epsilon_1 + \epsilon_2\frac{\dot{a}}{a} + \epsilon_3\frac{\ddot{a}}{\dot{a}} \tag{19}$$

Where $\epsilon_1, \epsilon_2, \&\epsilon_3$ are constants. From equations (9)-(11), (17), and (18), we get

$$\frac{\ddot{A}_2}{A_2}+(n+1)\frac{\dot{A}_2^2}{A_2^2}+\frac{\dot{A}_2\dot{\phi}}{A_2\phi}-\frac{2}{(n-1)\,A_2^{2n}}=0 \tag{20}$$

Using equations (17) & (18), from equation (20), we get

$$A_2 = A_3 = \left(\frac{k}{l}\right)^{-\frac{1}{n}}\left[\tanh^2(nkt)-1\right]^{-\frac{1}{2n}} \tag{21}$$

$$A_1 = \left(\frac{l}{k}\right)\left[\tanh^2(nkt)-1\right]^{-\frac{1}{2}} \tag{22}$$

From equations (17), (21), and (22), we get

$$\phi = \left(\frac{k}{l}\right)^{\frac{n+2}{n}}\left[\tanh^2(nkt)-1\right]^{\frac{n+2}{2n}} \tag{23}$$

Where $l^2 = \frac{2}{n(1-n)}$ & $k$ is an integrating constant.

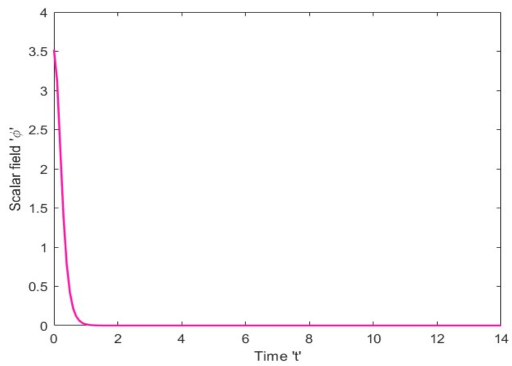


**Figure 1.** Variation of the BD scalar field $\phi(t)$ vs. time 't' for k=1.5 & n=2.45

For the specified parameters, Figure 1 shows how the BD scalar field $\phi(t)$ changes over time in the VHRDE model. Initially, $\phi(t)$ starts at a high value ($\phi \approx 3.6$) and decreases rapidly due to the combined influence of viscous effects and holographic corrections, reflecting a strong dynamical contribution in the early universe. Around $t \gtrsim 2$, the decay slows, and the field stabilizes near zero, remaining effectively constant at late times. This behaviour indicates a transition from an early scalar-dominated regime to a dark energy dominated phase, consistent with the dissipative nature of VHRDE and supporting a stable, slowly evolving dark energy component in the present epoch. Unlike Lambda CDM, where the gravitational coupling is constant and the scalar field plays no dynamical role, the VHRDE framework allows $\phi(t)$ to evolve, reflecting richer dynamics influenced by viscosity and holographic effects.

The Hubble parameter H is

$$H = \frac{k(n+2)}{3} \tanh(nkt) \tag{24}$$

Now, metric (5) can be written as

$$ds^2 = dt^2 - \frac{1}{k^2}\left[\tanh^2(nkt) - 1\right]^{-1} dx^2 - \left(\frac{k}{l}\right)^{-\frac{2}{n}} \left[\tanh^2(nkt) - 1\right]^{-\frac{1}{n}} \left(e^{-2x}dy^2 + e^{2x}dz^2\right) \tag{25}$$

The HRDE energy density can be evaluated from equations (1) and (24) as follows

$$\rho_d = \frac{3\phi k^2}{8\pi}\left[\epsilon_0\left(\frac{n+2}{3}\tanh(nkt)\right)^2 + \eta\frac{n(n+2)}{3}\operatorname{sech}^2(nkt) - 2\zeta n^2 \operatorname{sech}^2(nkt)\right] \tag{26}$$

Where $\phi = \left(\frac{k}{l}\right)^{\frac{n+2}{n}} \left[\tanh^2(nkt) - 1\right]^{\frac{n+2}{2n}}$

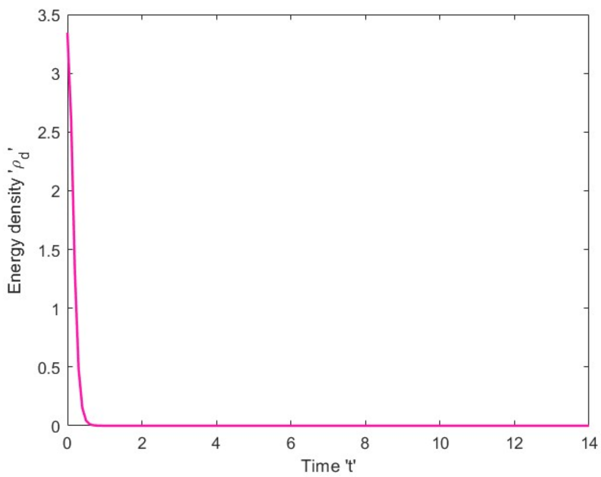


**Figure 2.** Variation of HRDE density $\rho_d$ with time $t$ for $k = 1.5$, $n = 2.45$, $\epsilon_0 = 0.025$, $\eta = 1.155$, $\zeta = 0.055$.

In the VHRDE model, Figure 2 shows how the ED $\rho_d$ has evolved across cosmic time. Initially, $\rho_d$ starts at a relatively high value ($\rho_d \approx 3.4$), reflecting a dominant DE component influenced by strong holographic and viscous effects. As time progresses, the ED undergoes a steep decline, approaching near-zero values around $t \approx 4$, after which it asymptotically flattens and remains nearly constant. This behaviour indicates rapid energy dissipation due to viscous interactions during the early universe, followed by a quasi-static regime corresponding to a stabilized DE-dominated era. The late-time plateau suggests a de Sitter–like accelerated expansion, which is consistent with observations and the expected behaviour of HRDE models with bulk viscosity. At late times, it shows that the VHRDE effectively mimics a cosmological constant. Additionally, $\rho_d$ gradually decreases over time, ensuring the physical condition $\rho_d \geqslant 0$. Unlike Lambda CDM, where the DE density remains constant, the VHRDE framework exhibits dynamic decay before stabilizing, reflecting richer dissipative physics. A minimal interaction is assumed between pressure-less matter and the viscous HRDE component, allowing them to evolve independently, in line with the approach of Sarkar [39]. As a result, the conservation equation yields:

$$\dot{\rho}_m + \left(\frac{\dot{A}_1}{A_1} + \frac{\dot{A}_2}{A_2} + \frac{\dot{A}_3}{A_3}\right)\rho_m = 0 \tag{27}$$

$$\dot{\rho}_d + \left(\frac{\dot{A}_1}{A_1} + \frac{\dot{A}_2}{A_2} + \frac{\dot{A}_3}{A_3}\right)(\rho_d + \bar{p}_d) = 0 \tag{28}$$

From eqs. (21), (22), and (27), we get

$$\rho_m = k_1 \left(\frac{k}{l}\right)^{\frac{n+2}{n}} \left[\tanh^2(nkt) - 1\right]^{\frac{n+2}{2n}} \tag{29}$$

Where $k_1$ is an integrating constant.

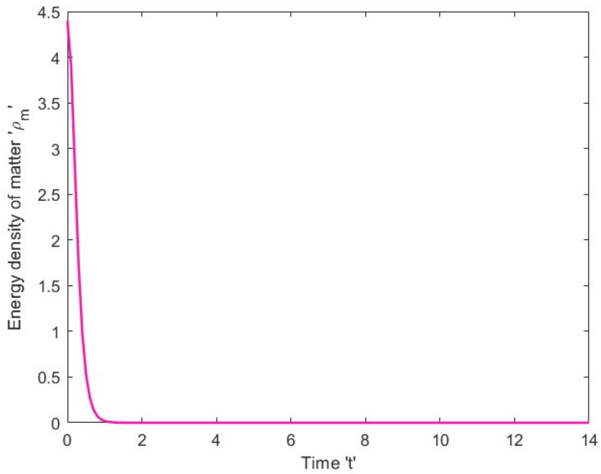


**Figure 3.** Variation of the ED of matter $\rho_m$ vs. time 't' for $k = 1.5, n = 2.45$ & $k_1 = 1.25$

In the VHRDE cosmological framework, Figure 3 displays the variation of the matter ED $\rho_m$ across time. Initially, $\rho_m$ is high ($\rho_m \approx 4.3$), reflecting a matter-dominated epoch in the early universe. The curve declines sharply as time progresses, indicating rapid dilution of matter density due to cosmic expansion. Around $t \approx 3$, $\rho_m$ approaches zero and stabilizes at an almost negligible value, marking the transition from matter domination to DE domination. The asymptotic behaviour demonstrates that matter becomes dynamically irrelevant in the late-time universe, consistent with the current DE-dominated phase. The steep decay further highlights the combined influence of bulk viscosity and holographic constraints in accelerating the dissipation of matter's contribution in VHRDE cosmologies. In Lambda CDM, matter dilutes as $a^{-3}$ without viscous corrections, while in VHRDE, the decay is sharper, reflecting more substantial dissipative effects.
The bulk viscosity coefficient is expressed by

$$\epsilon = \epsilon_1 + \epsilon_2 \left[\frac{k(n+2)}{3}\tanh(nkt)\right] + \epsilon_3(nk)\left[\frac{\operatorname{sech}^2(nkt)}{\tanh(nkt)} + 2\tanh(nkt) \quad + 2\frac{7n+2}{6n}\frac{\tanh(nkt)}{\operatorname{sech}^2(nkt)}\right] \tag{30}$$

Figure 4 illustrates the evolution of the coefficient $\epsilon(t)$ of bulk viscosity in the VHRDE model. During the early and intermediate epochs($t \lesssim 12$), bulk viscosity remains extremely small and nearly constant, indicating negligible viscous effects in the initial stages of cosmic evolution. However, as the universe approaches the far-future regime, $\epsilon(t)$ exhibits a dramatic rise, diverging sharply and reaching values of the order $10^{45}$ near $t \approx 14$. This rapid growth suggests that dissipative processes become increasingly dominant at late times. The divergence of $\epsilon$ may signal the onset of a non-equilibrium state or a possible future phase transition driven by strong viscous effects. Physically, such behaviour could correspond to a highly accelerated expansion or a phantom-like scenario, where bulk viscosity mimics or amplifies dark energy. These results emphasize the crucial role of viscosity in shaping the future dynamics of the universe, particularly within holographic and scalar–tensor cosmologies. In contrast, Lambda CDM assumes no viscous terms, making bulk viscosity identically zero. The sharp divergence in VHRDE highlights a key departure from the Lambda CDM paradigm.

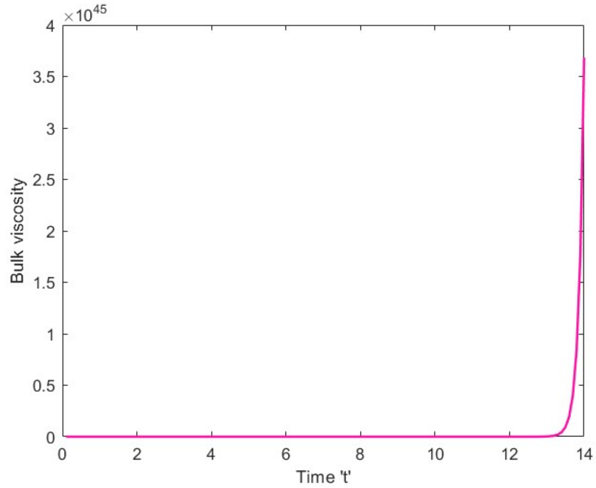


**Figure 4.** Variation of the bulk viscosity $\epsilon$ vs. time 't' for $k = 1.5, n = 2.45, \epsilon_1 = 2.035, \epsilon_2 = 1.025 \& \epsilon_3 = 3.15$

By solving equations (21) through (23), we may determine the HRDE's viscous pressure as

$$\bar{p}_d = \frac{\phi k^2}{16\pi}\left\{(n+1)\left[(n-1)\ \mathrm{sech}^2(nkt)+1\right]+\left[(3n+1)-(\omega+2)(n+2)^2\right]\tanh^2(nkt)\right\} \tag{31}$$

where $\phi = \left(\frac{k}{l}\right)^{\frac{n+2}{n}}\left[\tanh^2(nkt)-1\right]^{\frac{n+2}{2n}}$

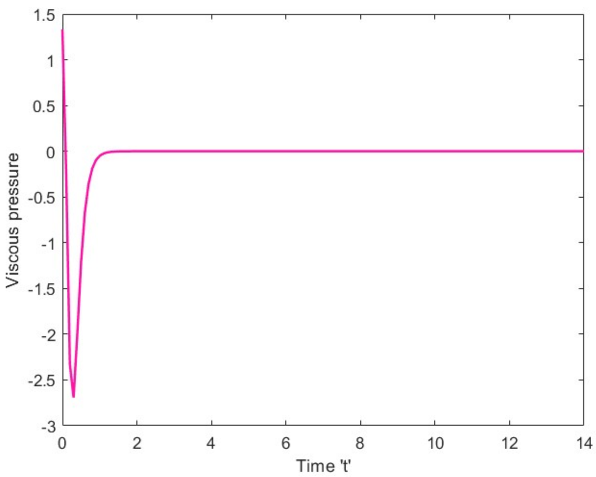


**Figure 5.** Variation of the viscous pressure $\bar{p}_d$ vs. time 't' for $k = 1.5, n = 2.45, \epsilon_0 = 0.025, \eta = 1.155, \omega = 2.25 \& \zeta = 0.055$

In the VHRDE model, Figure 5 depicts how the viscous pressure $\bar{p}_d$ alters across cosmic time. Initially, $\bar{p}_d$ is strongly positive, exceeding unity, but it rapidly falls into the negative region, reaching a minimum of about $\bar{p}_d \approx -2.8$ at early times ($t \lesssim 1$). This sharp decline indicates that viscous effects acted as an effective negative pressure in the early universe, potentially driving an acceleration of the inflation-like phase. Thereafter, $\bar{p}_d$ gradually increases and approaches zero from below, stabilizing for $t > 4$. This asymptotic flattening implies that viscous contributions become negligible late, consistent with a quasi-equilibrium, DE-dominated phase. The overall trend highlights the dissipative nature of the cosmic fluid and its significant role in shaping both early and intermediate expansion dynamics. In Lambda CDM, the effective pressure is constant, whereas VHRDE introduces time-dependent viscous contributions that enrich the early and mid-time dynamics.

The HRDE's pressure as

$$p_d = \frac{\phi k^2}{16\pi}\left\{(n+1)\left[(n-1)\,\mathrm{sech}^2(nkt)+1\right]+\left[(3n+1)-(\omega+2)(n+2)^2\right]\tanh^2(nkt)\right\}$$
$$+\left[k(n+2)\tanh(nkt)\right]\left\{\varepsilon_1+\varepsilon_2\left[\frac{k(n+2)}{3}\tanh(nkt)\right]\right.$$
$$\left.+\varepsilon_3(nk)\left[\frac{\mathrm{sech}^2(nkt)}{\tanh(nkt)}+2\tanh(nkt)+\frac{2(7n+2)}{6n}\,\frac{\tanh(nkt)}{\mathrm{sech}^2(nkt)}\right]\right\} \tag{32}$$

where $\phi = \left(\frac{k}{l}\right)^{\frac{n+2}{n}}\left[(\tanh(nkt))^2-1\right]^{\frac{n+2}{2n}}$

From equations (26) and (32), we obtain The HRDE's EoS parameter as

$$\omega_d = \frac{p_d}{\rho_d} \tag{33}$$

Where

$$p_d = \frac{\phi k^2}{16\pi}\left\{(n+1)\left[(n-1)\,\mathrm{sech}^2(nkt)+1\right]+\left[(3n+1)-(\omega+2)(n+2)^2\right]\tanh^2(nkt)\right\}$$
$$+\left[k(n+2)\tanh(nkt)\right]\left\{\varepsilon_1+\varepsilon_2\left[\frac{k(n+2)}{3}\tanh(nkt)\right]\right.$$
$$\left.+\varepsilon_3(nk)\left[\frac{\mathrm{sech}^2(nkt)}{\tanh(nkt)}+2\tanh(nkt)+\frac{2(7n+2)}{6n}\,\frac{\tanh(nkt)}{\mathrm{sech}^2(nkt)}\right]\right\}$$

and $\rho_d = \frac{3\phi k^2}{8\pi}\left[\epsilon_0\left(\frac{n+2}{3}\tanh(nkt)\right)^2+\eta\frac{n(n+2)}{3}\,\mathrm{sech}^2(nkt)-2\zeta n^2\,\mathrm{sech}^2(nkt)\right]$

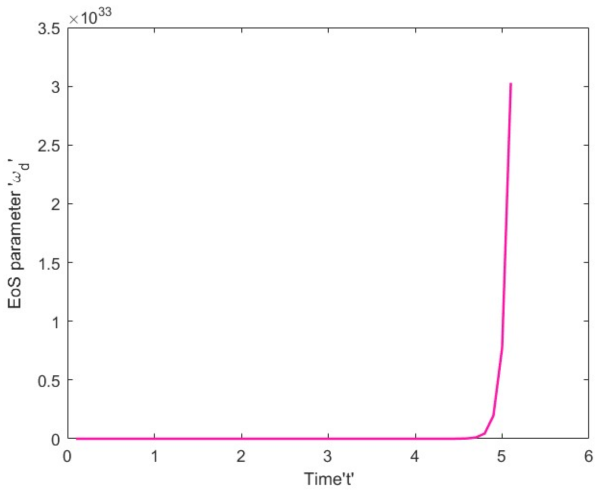


**Figure 6.** Variation of the EoS parameter $\omega_d$ vs. time 't' for$k = 1.5, n = 2.45, \epsilon_0 = 0.025, \eta = 1.155, \omega = 2.25$ & $\zeta = 0.055$

The EoS parameter is defined as $\omega_d = \frac{p}{\rho}$. Different values of $\omega_d$ correspond to distinct evolutionary phases of the Universe. During the decelerated expansion, $\omega_d = 1$ denotes stiff fluid, $\omega_d = \frac{1}{3}$ represents radiation, and $\omega_d = 0$ corresponds to a dust-dominated epoch. In contrast, the accelerated expansion phase includes quintessence $-1 < \omega_d < \frac{1}{3}$, the cosmological constant ($\omega_d = -1$), and phantom energy ($-1 < \omega_d$). Figure 6 shows how the VHRDE model's EoS parameter has changed over time. At early times, $\omega > -1$, placing the universe in a quintessence-like regime with relatively weaker negative pressure. As cosmic time increases, viscous and holographic effects dominate, driving $\omega_d$ toward more negative values. Depending on the parameter set, $\omega_d$ either asymptotically approaches the cosmological constant boundary ($\omega_d = -1$) or crosses into the phantom regime ($\omega_d < -1$). In the latter case, the model indicates a possible big-rip–like future driven by rapidly increasing negative pressure. The substantial time dependence of $\omega_d$ reflects the combined influence of bulk viscosity, scalar field dynamics, and the Ricci cutoff. In contrast, the standard Lambda CDM model assumes a strictly constant $\omega_d = -1$, corresponding to a true cosmological constant. The deviation of $\omega_d$ , the fixed value in the VHRDE framework highlights its dynamical character, offering a richer phenomenology that can capture transitions between quintessence, Λ-like, and phantom regimes. Such flexibility may help reconcile theoretical predictions with Planck and other observational constraints that mildly favour evolving DE over a pure cosmological constant.

The coincidence parameter is

$$r = \frac{\rho_d}{\rho_m} = \frac{\frac{3\phi k^2}{8\pi}\left[\ \xi_0\left(\frac{n+2}{3}\tanh(nkt)\right)^2 + \eta\,\frac{n(n+2)}{3}\,\mathrm{sech}^2(nkt) - 2\zeta n^2\,\mathrm{sech}^2(nkt)\ \right]}{k_1\left(\frac{k}{l}\right)^{\frac{n+2}{n}}\left[\tanh^2(nkt) - 1\right]^{\frac{n+2}{2n}}} \tag{34}$$

## 4. SOME ESSENTIAL PROPERTIES OF THE MODEL

The spatial volume is

$$V = \left(\frac{k}{l}\right)^{-\frac{n+2}{n}}\left(\tanh^2(nkt) - 1\right)^{-\frac{n+2}{2n}} \tag{35}$$

The average scale factor is

$$a(t) = V(t)^{1/3} = \left(\frac{k}{l}\right)^{-\frac{n+2}{3n}}\left(\tanh^2(nkt) - 1\right)^{-\frac{n+2}{6n}} \tag{36}$$

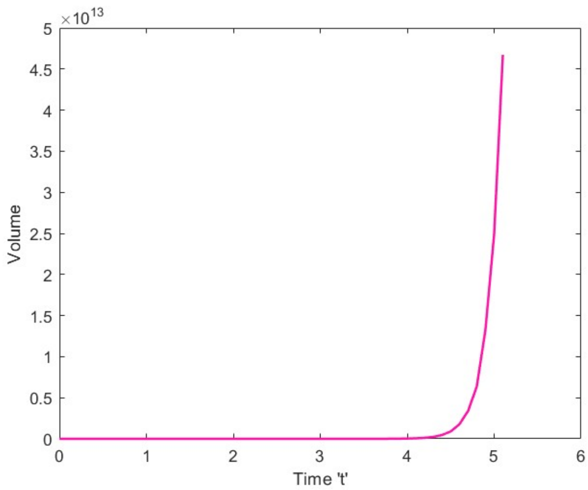


**Figure 7.** Variation of the Volume V vs. time 't' for k=1.5 & n=2.45

Figure 7 presents the time progression of the volume scale factor $V(t)$ in the VHRDE model. During the early epoch ($t \lesssim 4$), the volume remains extremely small and nearly constant, reflecting a slowly expanding or quasi-static universe. A sharp transition occurs around $t \approx 4.5$, where $V(t)$ grows rapidly by several orders of magnitude. This exponential-like rise signals late-time accelerated expansion, driven by the combined influence of HDE and bulk viscosity. The asymptotic behaviour of $V(t)$ at later times indicates a dark energy–dominated regime that may evolve toward a de Sitter–like or phantom scenario. In contrast, the standard Lambda CDM model predicts smooth exponential expansion with a constant DE density and no viscosity effects. The VHRDE framework, by incorporating dissipative processes, provides a richer dynamical picture that can capture both the observed acceleration and possible deviations from Lambda CDM in the universe's long-term evolution.

For the flow vector $u^{\mu}$, the corresponding expressions for the expansion and shear scalars are obtained as follows.

$$\theta = 3H = k(n+2)\tanh(nkt) \tag{37}$$

$$\sigma^2 = \frac{7}{18}\theta^2 = \frac{7}{18}\left[k(n+2)\tanh(nkt)\right]^2 \tag{38}$$

Figure 8 shows the evolution of the Hubble parameter $H(t)$ (solid line) and the expansion scalar $\theta(t)$ (dashed line) in the

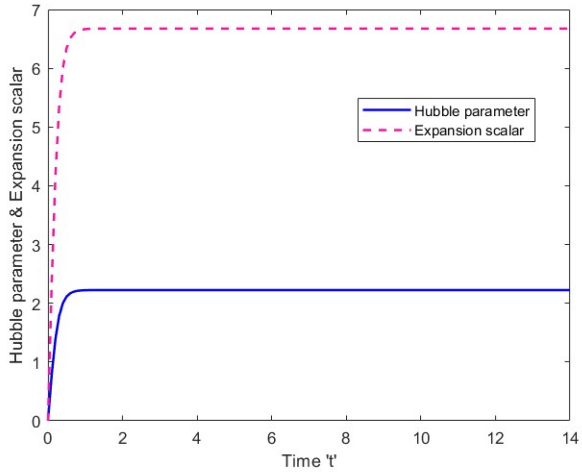


**Figure 8.** Variation of the Hubble parameter & Expansion scalar vs. time't' for k=1.5 & n=2.45

VHRDE model. Initially, both quantities rise rapidly, reflecting an early phase of significant expansion. As time increases, the Hubble parameter stabilizes near a constant value $H \approx 2.3$, while $\theta$, proportional to $3H$, asymptotically approaches $\theta \approx 6.9$, consistent with theoretical expectations. This convergence signals the transition from a dynamic early epoch to a quasi–de Sitter regime in the late universe. The constancy of $H$ and $\theta$ at late times indicates a steady accelerated expansion, driven by the interplay of the Ricci holographic cutoff and bulk viscous effects, which together guide the universe toward an asymptotic inflation-like state without a future singularity. In contrast, due to a true cosmological constant, the Lambda CDM model predicts a constant $H$ only at asymptotic times. The VHRDE scenario generalizes this behaviour by allowing viscosity-driven dynamics, yielding a richer description of cosmic acceleration.
The average anisotropy parameter is defined by

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{2\,(n-1)^2}{(2n+1)^2} \tag{39}$$

The deceleration parameter given by

$$q = -\frac{3\dot{\theta}}{\theta^2} - 1 = -\frac{3n}{(n+2)}\,\operatorname{csch}^2(nkt) - 1 \tag{40}$$

Figure 9 illustrates the evolution of the deceleration parameter $q(t)$ in the VHRDE model. Initially, $q(t)$ takes a very large negative value ($q \approx -13$), indicating a phase of rapid super-acceleration driven by strong viscous and holographic effects in the early universe. As time progresses, $q(t)$ increases toward an asymptotic value around $q \approx -1$, signalling a transition to steady accelerated expansion resembling a de Sitter phase. The model exhibits super-acceleration ($q < -1$) at early times and normal accelerated expansion ($-1 \leq q < 0$) at late times. At $t_0 = 13.8$ Gyr, $q$ is consistent with current observational estimates of a nearly de Sitter–like universe. In contrast, the Lambda CDM model predicts $q \approx -0.55$ at the present epoch and approaches $q = -1$ only asymptotically due to a constant cosmological constant. The VHRDE framework, with viscosity and holographic corrections, captures stronger early acceleration and a richer time-dependent behaviour, distinguishing it from standard Lambda CDM predictions.

## 5. COMPARISON OF OUR RESULTS

Recent VHRDE investigations have been carried out in both isotropic and anisotropic cosmologies under different gravity theories. In spatially flat FRW models, linear bulk viscosity has been shown to generate viable late-time acceleration and to regulate the transition from decelerated to accelerated expansion [40,41]. Extensions to anisotropic settings, such as axially symmetric, Ruban, and BT–III spacetimes, indicate that bulk viscosity coupled with scalar–tensor dynamics plays

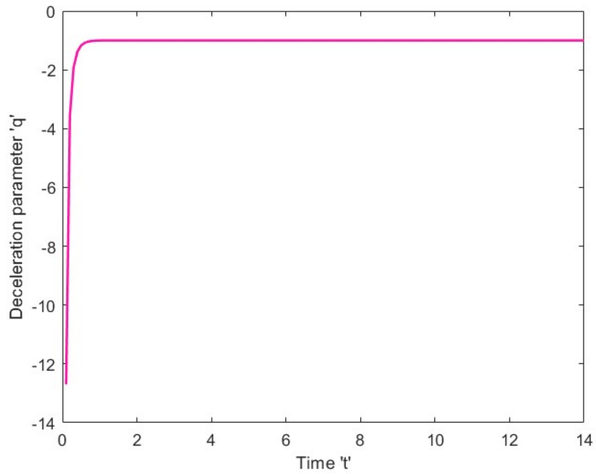


**Figure 9.** Variation of the deceleration parameter q vs.time ‘t’ for k=1.5 & n=2.45

a crucial role in suppressing anisotropy and initiating accelerated expansion [29, 42, 43]. Furthermore, studies in modified gravity frameworks, including $f(R,T)$ theory, demonstrate that viscosity ensures thermodynamic consistency, supports the generalized second law, and accounts for transitions between cosmic phases [44]. Our BT–VI$_o$ model under BD gravity is consistent with these earlier results yet extends them in several important respects. First, the BT - VI$_o$ geometry introduces a richer anisotropic structure, allowing us to track the detailed decay of shear and anisotropy parameters. Second, the BD scalar field provides a dynamical gravitational coupling that modulates the influence of viscosity, offering new insight into how anisotropy and scalar–tensor effects jointly shape cosmic evolution. Third, we obtain explicit solutions showing that (i) anisotropy and shear decay monotonically, (ii) the deceleration parameter undergoes a sign flip, indicating a transition from decelerated to accelerated expansion, and (iii) A late-time de Sitter phase is indicated by the fact that the EoS value is getting close to $-1$. These behaviours mirror the general trends observed in FRW and anisotropic VHRDE models, but are derived in a more general anisotropic background with explicit scalar–tensor coupling. Unlike Lambda CDM, where dark energy is constant and acceleration emerges gradually, the VHRDE–BD model exhibits a dynamical evolution in which viscosity and scalar–tensor effects drive early super-acceleration, while the EoS parameter approaches $-1$ at late times, yielding a de Sitter–like phase that aligns with our findings. Table 1 compares the key cosmological parameters of the Lambda CDM and VHRDE models in BD gravity, showing the effect of viscosity and HRDE on cosmic evolution.

**Table 1.** Comparison of Key Cosmological Parameters between Lambda CDM and the Present VHRDE Model

| **Parameter** | **Lambda CDM Model** | **VHRDE Model** |
|---|---|---|
| Gravitational coupling / scalar field | Constant gravitational coupling (no scalar field; $G$ = const.). | Dynamical Brans–Dicke scalar field $\phi(t)$; positive, initially large ($\approx 3.6$) and decreases to a nearly constant value at late times. |
| Dark energy density $\rho_d$ | Constant (cosmological constant). | Time-dependent; begins large ($\approx 3.4$), decays rapidly, and stabilizes at a small constant value at late times. |
| Matter density $\rho_m$ | Varies as $a^{-3}$; dominates early epoch. | Decays faster than $a^{-3}$; initially $\approx 4.3$ and becomes negligible around $t \approx 3$ due to viscous dissipation. |
| Bulk viscosity coefficient $\varepsilon(t)$ | Zero (no viscosity term). | Positive and time-dependent; nearly constant early, rises sharply and diverges ($\sim 10^{45}$) at late times. |
| Viscous pressure $\bar{p}_d(t)$ | Absent. | Initially positive, then becomes negative ($\approx -2.8$ early), approaching zero from below. |
| Equation of state $w_d = p_d/\rho_d$ | Constant, $w_d = -1$. | Dynamic; evolves from quintessence ($-1 < w_d < -1/3$) toward $w_d \to -1$ or crosses the phantom divide ($w_d < -1$). |
| Deceleration parameter $q(t)$ | Present value $q_0 \approx -0.55$; tends to $-1$ at late times. | Strongly time-dependent; early super-acceleration ($q \approx -13$), converging to $q \approx -1$ at late times. |
| Hubble parameter $H(t)$ | Decreases and approaches constant value at late times. | Evolves dynamically; stabilizes near $H \approx 2.3$ (model units). |
| Anisotropy / shear | Zero (isotropic FRW). | Non-zero initially (Bianchi VI$_o$) but decays monotonically. |
| Spatial volume / scale factor | Smooth exponential-like expansion at late times. | Slow initial growth followed by sharp exponential increase near $t \approx 4$. |
| Coincidence parameter $r = \rho_d/\rho_m$ | Decreases slowly; $\approx 1$ at present epoch. | Falls rapidly; matter becomes dynamically negligible earlier than in $\Lambda$CDM. |
| Thermodynamic consistency | Trivial (no viscous contribution). | Maintained by $\varepsilon(t) > 0$. |
| Observational concordance | Consistent with SNe Ia, CMB, and BAO data. | Qualitatively consistent at late times ($w \to -1$, $q \to -1$); quantitative fitting required. |

Table 1 clearly demonstrates that the VHRDE model provides an extended framework beyond Lambda CDM by incorporating bulk viscosity, anisotropy, and scalar–tensor dynamics. The model reproduces late-time acceleration without invoking a cosmological constant and aligns well with observed cosmological behaviour.

## 6. CONCLUSIONS

This paper investigates a VHRDE model in a BT–VI$_o$ anisotropic spacetime under BD gravity with a pressure-less matter component. Precise solutions of the BD field equations are obtained by assuming a relation among the metric potentials and a parameterized bulk viscosity coefficient. The combined role of anisotropy, Ricci infrared cutoff, and viscosity is analysed to explain different phases of cosmic evolution. The Brans–Dicke scalar field $\phi$ remains positive for all cosmic times (Fig. 1). It decreases monotonically with the expansion, which indicates a gradual strengthening of the effective gravitational coupling in the late epoch. The scalar field's evolution significantly affects the HRDE and expansion dynamics. The VHRDE density $\rho_d$ is positive throughout the evolution (Fig. 2). It starts with large values in the early anisotropic phase and decreases steadily to a small constant value at late times, supporting accelerated expansion. The matter ED $\rho_m$ is also positive and decreases monotonically (Fig. 3). This behaviour corresponds to the expected dilution of pressure-less matter due to cosmic expansion. The bulk viscosity coefficient $\epsilon$ remains positive, ensuring thermodynamic consistency. It increases with time, which shows that viscous effects become more important in the DE–dominated era. This enhancement contributes to the negative pressure required for acceleration. The effective viscous pressure is negative for the entire evolution (Fig. 5). Its magnitude is large in the early epoch, providing a strong acceleration-driving mechanism. It decreases gradually with time, leading to a more stable expansion phase. The EoS parameter $\omega_d$ starts with positive values, decreases as time evolves, and approaches zero in the late epoch. This indicates a smooth transition from a matter-like regime to a dust-like regime driven by the combined effects of HRDE and viscosity. The expanding nature of the Universe is confirmed by the spatial volume increasing monotonically from zero at $t = 0$. Both the mean Hubble parameter $H(t)$ and the expansion scalar $\theta(t)$ are always positive. In the early Universe, both were large and in later eras, they tend to be constant, corresponding to a steady accelerated expansion similar to de Sitter behaviour. The deceleration parameter $q(t)$ remains negative throughout the evolution, indicating continuous accelerated expansion of the universe. At early times, $q(t) < -1$ corresponds to a super-accelerated phase, while at late times it approaches $q = -1$, indicating a de Sitter-like expansion. Finally, the VHRDE model in BD theory demonstrates how anisotropy, bulk viscosity, and HRDE can drive cosmic evolution shifted from matter domination to DE acceleration. The scalar field regulates the effective gravitational coupling, while viscosity provides the mechanism for sustained acceleration. The results show that a cosmological constant is unnecessary to explain late-time acceleration in anisotropic backgrounds.

**Author contributions**
**K.P.S. Suryanarayana:**conceptualisation, theoretical formulation, and supervision of the research work.
**K.V.S. Sireesha(Corresponding Author):** Methodology, validation, manuscript review and editing, project administration, and correspondence with the journal.
**R. Sathibabu:** Analytical calculations,data analysis, preparation of figures, and drafting of initial sections of the manuscript.
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## ДОСЛІДЖЕННЯ КОСМОЛОГІЧНОЇ МОДЕЛІ В'ЯЗКОЇ ГОЛОГРАФІЧНОЇ КОСМОЛОГІЧНОЇ МОДЕЛІ ТЕМНОЇ ЕНЕРГІЇ РІЧЧІ ТИПУ БІАНКІ В ТЕОРІЇ ГРАВІТАЦІЇ БРАНСА-ДІКЕ

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- Досліджено в'язку голографічну модель темної енергії Річчі (VHRDE) в рамках гравітації Бранса-Діке (BD).
- Об'ємна в'язкість генерує ефективний негативний тиск, тоді як скалярне поле BD динамічно регулює гравітаційний зв'язок.
- Отримано природний перехід від ранньої уповільненої фази з домінуванням матерії до пізньої прискореної фази розширення з домінуванням темної енергії.
- Модель виходить за межі ΛCDM, демонструючи динамічно еволюціонуюче рівняння стану, яке може наближатися або перетинати фантомну межу $\omega < -1$
- Анізотропія та зсувний спад з космічним часом, що призводить до ізотропізації наприкінці часу, що узгоджується з обмеженнями спостережень (CMB).

Ми досліджуємо в'язку голографічну модель темної енергії Річчі (VHRDE) у просторі-часі типу Б'янкі (BT) - $VI_o$ в межах гравітації Бранса-Дікке (BD), включаючи компонент матерії без тиску. Припускаючи зв'язок між метричними потенціалами та параметризованим коефіцієнтом об'ємної в'язкості, знайдено точні розв'язки рівнянь поля BD. Комбінований вплив анізотропії, голографічного обрізання Річчі та об'ємної в'язкості дають динамічний каркас, який може описувати кілька фаз космічної еволюції. Ми досліджуємо прогресію важливих вимірювань, включаючи скалярне поле, густину енергії, об'ємну в'язкість, в'язкий тиск, рівняння стану (EoS), змінні розширення та параметр уповільнення, виявляючи перехід від уповільненої епохи, де домінує матерія, до прискореної фази, де домінує темна енергія. Об'ємна в'язкість генерує ефективний негативний тиск, що зумовлює прискорення, тоді як скалярне поле BD модулює гравітаційний зв'язок. Порівняння з попередніми дослідженнями підтверджує, що ця модель забезпечує життєздатний механізм космічного прискорення без необхідності космологічної константи.

**Keywords:** *метрика типу Б'янкі - $VI_o$; голографічна темна енергія Річчі (HRDE); об'ємна в'язкість; теорія Бранса-Дікке*